

Performance of DF Incremental Relaying with Energy Harvesting Relays in Underlay CRNs

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Abstract—In this paper, we analyze the throughput performance of incremental relaying using energy harvesting (EH) decode-and-forward (DF) relays in underlay cognitive radio networks (CRNs). The destination combines the direct and relayed signals when the direct link is in outage. From the derived closed-form expressions, we present an expression for the power-splitting parameter of the EH relay that optimizes the throughput performance. We demonstrate that relaying using EH DF relays results in better performance than direct signalling without a relay *only* when the destination combines the direct signal from the source with the relayed signal. Computer simulations demonstrate accuracy of the derived expressions.

I. INTRODUCTION

Cognitive radio networks (CRNs) have shown great promise in alleviating the acute shortage of spectrum. In such networks, secondary (unlicensed) users are allowed to share the spectrum of the primary (licensed) users. Underlay CRNs in particular, have been shown to result in great improvement in spectral utilization efficiency. In these networks, the secondary transmitters transmit simultaneously with the primary transmitters in the same frequency band, but with powers carefully constrained to limit interference to the primary receiver below a specified interference temperature limit.

In the recent years, the use of energy harvesting (EH) is being studied to prolong battery life of nodes. Although energy can be harvested from natural sources, it is wireless EH that shows the greatest promise. In particular, the possibility of simultaneous wireless information and power transfer has spurred research interest in this area. Since practical circuits cannot simultaneously harvest energy and perform information processing, time-switching and power-splitting relaying protocols have been proposed [1]. In the former, the relay is first charged by the source for a fraction of the signalling interval prior to two-hop relaying. In the latter, the received signal is split into two parts, with a fraction (referred to as the power-splitting parameter ρ) being used for energy harvesting, while the rest is used for information processing. It is well known that optimization of this parameter is crucial to maximize throughput.

While the use of EH relays in CRNs is well motivated, analysis of performance of such networks has attracted attention only over the past few years [2]–[9]. Two types of EH methodologies have been proposed in the context of CRNs

In the first type, energy is harvested from the primary signal [2]–[4], [6]–[8]. Note that [4], [6] use the interweave cognitive radio principles, [2], [3], [9] use the underlay signalling mode, while [7], [8] use overlay principles. In the second type, which is of primary interest in this paper, energy is harvested from the secondary source [5], [9] (this requires secondary EH nodes to be in close proximity to the secondary transmitter). Under these circumstances, even in the non-cognitive context, the importance of considering the direct channel from source to destination in addition to the signal relayed by the EH relay has been recognized by only a few authors [10], [11]. In *all existing literature* on two-hop EH relaying in underlay CRNs [3], [5], [9], the direct channel from source to destination has been ignored. In underlay cognitive radio, powers used at secondary transmitters is a random quantity. For this reason, the secondary nodes need to be in close proximity to each other to ensure any reasonable quality of service in the secondary links. In underlay signalling with EH relays, the nodes need to be even closer since energy harvested is typically small. Ignoring the direct channel from source to destination is therefore not reasonable in such situations. In this paper, we demonstrate this fact.

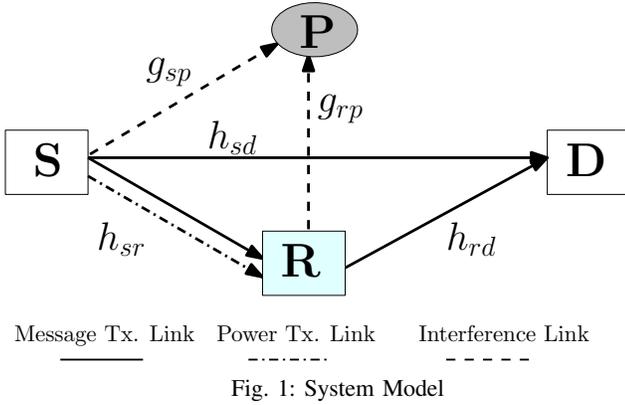
The contributions of this paper are as follows:

- 1) We derive closed-form expressions for the throughput of an incremental relaying scheme in which the destination combines the signal relayed by the EH relay with the direct signal from the source.
- 2) Using approximated throughput expressions, we derive an expression for the power-splitting parameter that maximizes throughput.
- 3) We demonstrate that relaying with an EH relay results in larger throughput than direct signalling only when the destination combines the direct and relayed signals.

Notations: $\mathbb{E}_{\mathcal{C}}(\cdot)$ denotes the expectation over the condition/conditions \mathcal{C} . $\exp(\lambda)$ represents the exponential distribution with parameter λ , and $\mathcal{CN}(0, a)$ denotes the circular normal distribution with mean 0 and variance a . $E_1(\cdot)$ and $Ei(\cdot)$ represent the exponential integrals defined in [12, 5.1.1] and [12, 5.1.2] respectively.

II. SYSTEM MODEL

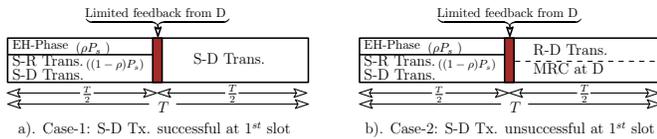
We consider a two-hop decode-and-forward (DF) relay network as depicted in Fig. 1. The primary network consists



of the primary transmitter (not depicted in the figure) and the primary receiver P. The secondary network (SN) consists of three nodes - a source (S), a relay (R) and a destination (D). Each node equipped with one antenna. Denote the channel between S and R by $h_{sr} \sim \mathcal{CN}(0, \lambda_{sr}^{-1})$, and that between the R and D by $h_{rd} \sim \mathcal{CN}(0, \lambda_{rd}^{-1})$. Similarly, denote the channel between S and P by $g_{sp} \sim \mathcal{CN}(0, \lambda_{sp}^{-1})$, and that between R and P by $g_{rp} \sim \mathcal{CN}(0, \lambda_{rp}^{-1})$. We assume that R is equipped with a super-capacitor, and acts as an EH node with EH factor η . The energy harvested in the first phase is used by R to relay the signal to D. As in most literature on underlay CRN, we neglect the primary signal at R and D. This is reasonable because of the large distance between the primary transmitter and the secondary nodes [13] (this assumption has been justified on information theoretic grounds [14]). All the channels are reversible and quasi-static.

III. TRANSMISSION PROTOCOL

We assume fixed-rate transmission at rate R_s by all nodes. Signalling based on the incremental protocol is completed in two time-slots as depicted in Fig.2. Message transmission is based on the incremental relaying protocol [15], and EH is based on the power-splitting protocol [1].



In the first transmission time-slot, S transmits information symbols to D and R as depicted in Fig.2. R harvests energy from this signal using power splitting, and attempts to decode the information symbols. Meanwhile, D attempts to decode the symbols. If successful, it sends feedback to the source and the relay¹. The relay discards the decoded symbols, and the source re-transmits a new block of symbols to D as depicted in

¹We assume that feedback time is extremely small and can be neglected in the analysis without loss of generality.

Fig.2a². Else, R relays the symbols using the energy harvested, and D combines the signals in the first and the second time-slots as depicted in Fig.2b.

In the first time-slot, S transmits a message signal x with power P_s in underlay mode to R and D. In PS-EH case, a component y_{d_1} of the received signal with ρ fraction of the power is utilized for EH, while the remaining signal with fraction $1 - \rho$ of the power is used to decode the symbols. Clearly, y_r at R and y_{d_1} at D in the first phase are given by:

$$y_r = \sqrt{(1 - \rho)P_s} x |h_{sr}|^2 + n_r \quad \text{and} \quad (1)$$

$$y_{d_1} = \sqrt{P_s} x |h_{sd}|^2 + n_{d_1} \quad (2)$$

respectively, where $n_r, n_{d_1} \sim \mathcal{CN}(0, N_o)$ are noise samples at R and D. Clearly, the SNR Γ_{d_1} at D in the first time-slot is $\frac{P_s |h_{sd}|^2}{N_o}$. Energy harvested at R is $\eta \rho P_s |h_{sr}|^2 / 2$ (ignoring noise) so that the power available for relaying is given by:

$$h_{P_r} = \rho \eta P_s |h_{sr}|^2 = \beta P_s |h_{sr}|^2, \quad (3)$$

where $\beta = \eta \rho$. Let I denote the interference temperature limit. In order to ensure that the interference caused to primary receiver P is limited to I , the power P_s at S is chosen to be:

$$P_s = I / |g_{sp}|^2. \quad (4)$$

We consider only the peak interference constraint at S, and ignore the peak power constraint for the following reason:

- 1) It is well known that performance of CRNs exhibits an outage floor, and does not improve with increase in peak power (it is in this low outage and high throughput region that CRNs are typically operated) [16], [17] (and references therein). In this paper, we discuss optimization of PS EH parameter, which is of interest in this high throughput regime.
- 2) Since performance of CRNs is typically limited by interference, and sufficient peak power is typically available, this assumption is quite reasonable.

In the second time-slot, R is used to forward the decoded symbol \hat{x} to D. In order to ensure that the interference at P is constrained to I , the total transmit power P_r at R is chosen to be:

$$P_r = \min \left(h_{P_r}, \frac{I}{|g_{rp}|^2} \right). \quad (5)$$

Received signal y_{d_2} at D can be expressed as

$$y_{d_2} = \sqrt{P_r} \hat{x} |h_{rd}| + n_{d_2}, \quad (6)$$

where $n_{d_2} \sim \mathcal{CN}(0, N_o)$ is the additive white Gaussian noise sample. We assume that D uses MRC to combine the signals obtained in the first phase (2) and second phase (6). Signal-to-noise ratios (SNRs) at R and D are expressed using (1), (2)

²The relay does not harvest energy in this phase. Energy stored in the super-capacitor is assumed to be lost since it lacks the ability to store charge over long intervals.

and (6) as:

$$\Gamma_r = \frac{(1-\rho)P_s|h_{sr}|^2}{N_o} \quad \text{and} \quad (7)$$

$$\Gamma_d = \underbrace{\frac{P_s|h_{sd}|^2}{N_o}}_{\Gamma_{d_1}} + \underbrace{\frac{P_r|h_{rd}|^2}{N_o}}_{\Gamma_{d_2}}. \quad (8)$$

IV. PERFORMANCE ANALYSIS

In this section, we analyze throughput performance of the described incremental relaying protocol. When $\Gamma_{d_1} \geq \gamma_{th}$, the direct link is successful, so that rate achieved is R_s . When $\Gamma_{d_1} < \gamma_{th}$, and the relay can decode successfully ($\Gamma_r \geq \gamma_{th}$), and the SNR at the destination after combining is sufficient ($\Gamma_d = \Gamma_{d_1} + \Gamma_{d_2} \geq \gamma_{th}$), signalling is completed in two hops (rate $R_s/2$). Clearly, throughput τ is given by:

$$\tau = 0.5 R_s \underbrace{\Pr(\Gamma_{d_1} < \gamma_{th}, \Gamma_r \geq \gamma_{th}, \Gamma_{d_1} + \Gamma_{d_2} \geq \gamma_{th})}_{q_1} + \underbrace{R_s \Pr(\Gamma_{d_1} \geq \gamma_{th})}_{q_2}, \quad (9)$$

where $\gamma_{th} = 2^{R_s} - 1$. We note that q_1 , the probability that the relayed link is successful while the direct link is not successful, can alternatively be represented as:

$$q_1 = 1 - \left(\underbrace{\Pr(\Gamma_r < \gamma_{th})}_{p_1} + \underbrace{\Pr(\Gamma_{d_1} \geq \gamma_{th}, \Gamma_r \geq \gamma_{th})}_{p_2} + \underbrace{\Pr(\Gamma_d < \gamma_{th}, \Gamma_r \geq \gamma_{th})}_{p_3} \right). \quad (10)$$

We evaluate each of the terms in what follows. From (7), p_1 can be evaluated as:

$$p_1 = \Pr(\Gamma_r < \gamma_{th}) = \Pr\left(\frac{(1-\rho)P_s|h_{sr}|^2}{N_o} < \gamma_{th}\right). \quad (11)$$

Using $P_s = I/|g_{sp}|^2$, it can be shown that:

$$p_1 = 1 - \frac{1}{1 + \lambda_{sr}\psi/(\lambda_{sp}(1-\rho))}, \quad (12)$$

where $\psi = \frac{\gamma_{th}}{I/N_o}$. Similarly, q_2 is given by:

$$q_2 = \Pr(\Gamma_{d_1} \geq \gamma_{th}) = \frac{1}{1 + \lambda_{sd}\psi/\lambda_{sp}}. \quad (13)$$

We note that Γ_r and Γ_{d_1} are not independent due to their dependence on the random variable $P_s = I/|g_{sp}|^2$. By first conditioning on $|g_{sp}|^2$, exploiting the independence of $\Gamma_r|_{|g_{sp}|^2}$ and $\Gamma_d|_{|g_{sp}|^2}$, and then averaging over $|g_{sp}|^2$, we can show that p_2 can be written as:

$$p_2 = \int_0^\infty \lambda_{sp} e^{\lambda_{sp}|g_{sp}|^2} \int_{\psi|g_{sp}|^2}^\infty \lambda_{sd} e^{-\lambda_{sd}|h_{sd}|^2} \int \frac{\psi|g_{sp}|^2}{(1-\rho)} \lambda_{sr} e^{-\lambda_{sr}|h_{sr}|^2} d|h_{sr}|^2 d|h_{sd}|^2 d|g_{sp}|^2.$$

After integrating over $|h_{sr}|^2$ and $|h_{sd}|^2$, p_2 can be expressed in terms of $|g_{sp}|^2$ as:

$$p_2 = \int_0^\infty \lambda_{sp} e^{\lambda_{sp}|g_{sp}|^2} e^{-\lambda_{sd}\psi|g_{sp}|^2 - \frac{\lambda_{sr}\psi|g_{sp}|^2}{(1-\rho)}} d|g_{sp}|^2.$$

Now after averaging over $|g_{sp}|^2$, the resultant expression can be found out to be:

$$p_2 = \frac{1}{1 + \frac{\psi}{\lambda_{sp}} \left(\lambda_{sd} + \frac{\lambda_{sr}}{(1-\rho)} \right)}. \quad (14)$$

An approximate closed-form expression for p_3 is derived in Appendix-A. The final expression is presented in (15).

Resultant expression of τ can be found out by substituting for p_1 , p_2 , p_3 and q_2 into (9).

A. Value of EH-factor for optimal throughput

Throughput τ is small for $\rho = 0$ (no energy harvested at the relay) and $\rho = 1$ (no decoding is possible at the relay). In both these cases, the relayed signal is not available. When $0 < \rho < 1$, the throughput is larger since the relayed signal is not always in outage. It is clear that the following optimal value of EH parameter (ρ^*) that maximizes throughput is of interest:

$$\rho^* = \arg \max_{\rho} \tau. \quad (17)$$

It is difficult to find an exact solution for ρ^* since the lengthy expression for τ contains several nonlinear functions.

To obtain an expression for τ in a simplified form (say τ_{sim}), we use the high-SNR approximation ($I\lambda_{sp}/N_o \gg \gamma_{th}$) and also neglect the R-P link³. We show through simulations in Fig. 3 of Section V that throughput of a practical system that imposes the peak interference constraint at the relay is indistinguishable from one that neglects it. In other words, SN performance does not depend (or at best very loosely depends) on the statistical parameter (λ_{rp}) of the R-P channel for most practical range of parameters. Intuitively, this is because of the fact that the harvested energy is very small (less than $I/|g_{rp}|^2$) with very high probability.

From the above discussion, throughput can be represented in most simplified form as given in (18) (please refer the Appendix-B for derivation). We omit proof of concavity due to space constraints. Solving $\frac{d\tau_{sim}}{d\rho} = 0$, results in a quadratic equation which has two roots, out of which the one between 0 to 1 is given by⁴:

$$\rho^* \approx \frac{1 - \sqrt{\left(1 + \frac{\lambda_{sp}}{\lambda_{sd}\psi}\right) \frac{\psi\lambda_{sr}}{\lambda_{sp}}}}{1 + \sqrt{\left(1 + \frac{\lambda_{sp}}{\lambda_{sd}\psi}\right) \frac{\eta}{\lambda_{rd}}}}. \quad (19)$$

³We note that the R-P link is ignored only for the simplified analysis to obtain insights into the optimum power-splitting parameter. The relay needs to apply power control as in any other underlay system. We note that all computer simulations are performed with the interference channel from relay to primary receiver.

⁴Since $\lambda_{sr} \ll \lambda_{sd}$, it can be shown that $\rho^* \in [0, 1]$.

$$\begin{aligned}
p_3 \approx & t \left(\left(\frac{a}{c + \lambda_{sp}} \right)^2 \frac{\lambda_{sp} \lambda_{sr}}{(a + b + d \lambda_{sp})} e^{\frac{a \lambda_{sr}}{c + \lambda_{sp}}} \left(E_1 \left(\frac{a \lambda_{sr}}{c + \lambda_{sp}} \right) - E_1 \left(\frac{a \lambda_{sr}}{c + \lambda_{sp}} + a s \right) \right) + \frac{d^2 \lambda_{sp} \lambda_{sr}}{a + b + d \lambda_{sp}} e^{s(-a+b) - d(\lambda_{sp} s + \lambda_{sr})} \right. \\
& \times \left(\text{Ei}(d(\lambda_{sr} + \lambda_{sp} s)) - \text{Ei}(d \lambda_{sr} + b s + d \lambda_{sp} s) \right) - \frac{\lambda_{sr}(a + b)^2 e^{\frac{\lambda_{sr}(a+b)}{\lambda_{sp}}} \left(E_1 \left(\frac{(a + b) \lambda_{sr}}{\lambda_{sp}} \right) - E_1 \left(\frac{(a + b) \lambda_{sr}}{\lambda_{sp}} + (a + b) s \right) \right) \right. \\
& \left. - \frac{\lambda_{sr} e^{s(-a+b)}}{\lambda_{sp} s + \lambda_{sr}} + \frac{\lambda_{sp} \lambda_{sr} e^{-a s}}{(c + \lambda_{sp})(c s + \lambda_{sp} s + \lambda_{sr})} + \frac{c}{c + \lambda_{sp}} \right) + \frac{b^2 \lambda_{sr}}{\lambda_{sp}(b + d \lambda_{sp})} e^{\frac{b \lambda_{sr}}{\lambda_{sp}}} \left(E_1 \left(\frac{b \lambda_{sr}}{\lambda_{sp}} \right) - E_1 \left(\frac{b(\lambda_{sr} + \lambda_{sp} s)}{\lambda_{sp}} \right) \right) \\
& - \frac{d^2 \lambda_{sp} \lambda_{sr}}{b + d \lambda_{sp}} e^{-b s - d(\lambda_{sp} s + \lambda_{sr})} (\text{Ei}(d(\lambda_{sr} + \lambda_{sp} s)) - \text{Ei}(d \lambda_{sr} + b s + d \lambda_{sp} s)) + \frac{\lambda_{sr} e^{-b s}}{\lambda_{sp} s + \lambda_{sr}} + \frac{c \lambda_{sp} s^2}{(\lambda_{sp} s + \lambda_{sr})(c s + \lambda_{sp} s + \lambda_{sr})} \\
& - \frac{\lambda_{sp} \lambda_{sr}}{(c + \lambda_{sp})(c s + \lambda_{sp} s + \lambda_{sr})} - \frac{c}{c + \lambda_{sp}}, \tag{15}
\end{aligned}$$

$$\text{where } a = \frac{\lambda_{rp}}{\beta}, \quad b = \frac{\psi \lambda_{rd}}{\beta}, \quad c = \psi \lambda_{sd}, \quad d = \frac{\lambda_{rd}}{\beta \lambda_{sd}}, \quad \text{and } s = \frac{(1 - \rho)}{\psi}. \tag{16}$$

$$\tau_{sim} \approx 0.5 R_s \left(1 - \left(\frac{1}{\left(1 + \frac{\lambda_{sp}}{\lambda_{sd} \psi} \right) \left(1 + \frac{\eta \lambda_{sp} \rho}{\psi \lambda_{rd} \lambda_{sr}} \right)} + \frac{1}{\frac{\lambda_{rd} \lambda_{sp} (1 - \rho)}{\eta \rho \lambda_{sr} \psi}} + \frac{1}{\frac{\psi \lambda_{sd}}{\lambda_{sp}} + 1} \right) \right) + R_s q_2 \tag{18}$$

As a special case, value of ρ which maximizes the throughput if the S-D link ignored⁵ (i.e. $\lambda_{sd} \rightarrow \infty$) becomes:

$$\rho_{nd}^* \approx \lim_{d_{sd} \rightarrow \infty} \rho^* = \frac{1 - \frac{\psi \lambda_{sr}}{\lambda_{sp}}}{1 + \frac{\eta}{\lambda_{rd}}}, \tag{20}$$

where subscript *nd* is used to emphasize the fact that no direct path is present. In this case, the throughput is derived from (9) by using $\lambda_{sd} \rightarrow \infty$, and given by:

$$\tau_{nd} = \lim_{\lambda_{sd} \rightarrow \infty} \tau. \tag{21}$$

V. SIMULATION RESULTS

In this section, we validate the derived expressions by computer simulations. The normalized S-R, R-D, and S-D distances are assumed to be 1.2, 1.8 and 3 respectively. The normalized S-P and R-P distances are assumed to be 3. Path loss exponent ϵ is assumed to be 4. We assume $\eta = 0.7$ and $I/N_o = 6$ dB unless stated otherwise.

The importance of optimizing ρ for maximizing throughput is clearly brought out in Fig.3 (concavity is clear from the plots). The graph depicts a plot of τ versus ρ for $R_s = 3$ bpcu for different d_{sr} .

The value of ρ^* indicated by (19) for d_{sr} of 1.2, and 1.7 are 0.87 and 0.62 respectively, which are in close agreement with simulations. Similarly, in the absence of the direct link, ρ_{nd}^* of 0.89 and 0.68 are indicated by (20), which are in close agreement with simulations. We note that $\rho_{nd}^* > \rho^*$, as can be intuitively expected. It is clear that a) incremental relaying results in higher throughput than relay-less signalling from S to D, b) two-hop relaying that ignores the S-D link results in throughput that is quite poor as compared to direct S-D signalling without the relay, and c) a smaller d_{sr} results in

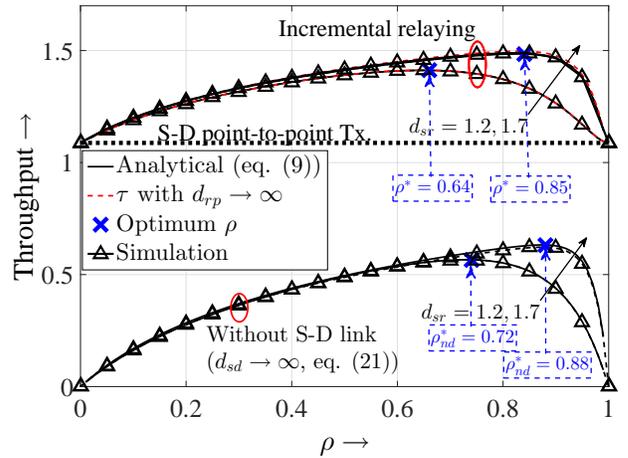


Fig. 3: Throughput vs. ρ for different d_{sr}

larger throughput, especially for large ρ (note that $d_{sr} + d_{rd} = d_{sd}$ so that a smaller d_{sr} implies a larger d_{rd}).

In Fig.4, throughput is plotted versus R_s for two different values of I . For each point, the optimum value of ρ^* is computed and used. The superiority of incremental relaying over direct point-to-point transmission is apparent. Moreover, the gap between the two is higher for larger value of I . This happens because relayed signalling has a higher chance of non-outage for larger value of I . It is observed that an optimum value of R_s exists which maximizes throughput. It is apparent from (9), that throughput is limited by R_s when it is small, and by outage when R_s is large. The optimum value needs to be obtained by numerical search.

VI. CONCLUSION

In this paper, we derived a closed-form expression for the throughput performance of an underlay two-hop network with

⁵In this case, there is no involvement of direct path and a case of two-hop transmission between nodes S to D via R.

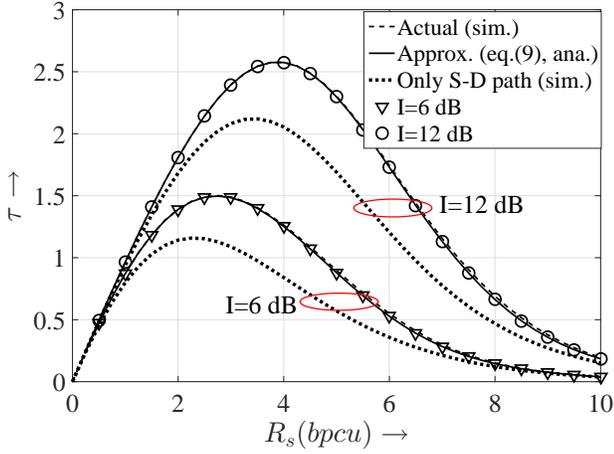


Fig. 4: Throughput vs. R_s for different I

a power-splitting based energy harvesting relay. We present a closed-form expression for the throughput maximizing power-splitting parameter.

APPENDIX

A. Derivation of p_3

In this Appendix, we derive an expression for p_3 . To this end, we first define X as:

$$X \triangleq \min \left(hP, \frac{I}{|g_{rp}|^2} \right) |h_{rd}|^2 \quad (22)$$

Its Cumulative distribution function (CDF) conditioned on hP can be derived as:

$$F_{X|h_P}(x) = 1 - e^{-\frac{\lambda_{rd}x}{\beta P_s |h_{sr}|^2}} \left(1 - e^{-\frac{\lambda_{rp}I}{\beta P_s |h_{sr}|^2}} \left(\frac{\frac{\lambda_{rd}x}{I\lambda_{rp}}}{1 + \frac{\lambda_{rd}x}{I\lambda_{rp}}} \right) \right) \quad (23)$$

From (10), p_3 can be expressed as:

$$p_3 = \Pr \left(\frac{P_s |h_{sd}|^2}{N_o} + \frac{X |h_{sr}|^2}{N_o} \leq \gamma_{th}, \frac{(1-\rho)P_s |h_{sr}|^2}{N_o} > \gamma_{th} \right).$$

We derive p_3 by successive averaging over each random variable and keeping other r.v.s. in terms of condition. We first average w.r.t. X by using (23) to get:

$$\begin{aligned} p_3 &= \mathbb{E}_{\mathbb{C}_1} \left[F_{X|h_P} \left(\gamma_{th} - \frac{P_s |h_{sd}|^2}{N_o} \right) \right] \\ &= \mathbb{E}_{\mathbb{C}_1} \left[1 - e^{-\frac{\lambda_{rd}}{\beta P_s |h_{sr}|^2} \left(\gamma_{th} - \frac{P_s |h_{sd}|^2}{N_o} \right)} \left(1 - e^{-\frac{\lambda_{rp}I}{\beta P_s |h_{sr}|^2}} \right. \right. \\ &\quad \left. \left. \times \left(1 - \frac{1}{1 + \frac{\lambda_{rd}}{I\lambda_{rp}} \left(\gamma_{th} - \frac{P_s |h_{sd}|^2}{N_o} \right)} \right) \right) \right], \end{aligned}$$

where the condition $\mathbb{C}_1 = \left\{ \frac{(1-\rho)P_s |h_{sr}|^2}{N_o} > \gamma_{th}, \frac{P_s |h_{sd}|^2}{N_o} \leq \gamma_{th} \right\}$. Unfortunately, averaging the above with respect to random variables $|h_{sr}|^2$ and $|g_{sp}|^2$ results in intractable expressions. We need to make use of the following fact: $1 \gg \frac{\lambda_{rd}}{I\lambda_{rp}} (\gamma_{th} N_o - P_s |h_{sd}|^2)$ (since $P_s |h_{sd}|^2 / N_o < \gamma_{th}$ and $I\lambda_{rp} \gg \lambda_{rd}$). Hence for tractability, $\frac{\lambda_{rd}}{I\lambda_{rp}} (\gamma_{th} N_o - P_s |h_{sd}|^2)$ can be replaced by its mean i.e. $\frac{\lambda_{rd}}{I\lambda_{rp}} (\gamma_{th} N_o - \mathbb{E}_{P_s |h_{sd}|^2 \leq \gamma_{th}} (P_s |h_{sd}|^2))$.

$\mathbb{E}_{P_s |h_{sd}|^2 \leq \gamma_{th}} (P_s |h_{sd}|^2)$ can be derived as:

$$\mathbb{E}_{\mathbb{C}_2} (P_s |h_{sd}|^2) = \frac{I\lambda_{sp}}{\lambda_{sd}} \left(\log \left(\frac{\gamma_{th} \lambda_{sd} N_o}{I\lambda_{sp}} + 1 \right) + \frac{\frac{\gamma_{th} \lambda_{sd} N_o}{I\lambda_{sp}}}{\frac{\gamma_{th} \lambda_{sd} N_o}{I\lambda_{sp}} + 1} \right), \quad (24)$$

where $\mathbb{C}_2 = \{P_s |h_{sd}|^2 \leq N_o \gamma_{th}\}$. Now, p_3 can further be represented in approximated form as:

$$p_3 \approx \mathbb{E}_{\mathbb{C}_1} \left[1 + te^{-\frac{\lambda_{rd}(\gamma_{th} N_o - |h_{sd}|^2 P_s)}{\beta |h_{sr}|^2 P_s} - \frac{I\lambda_{rp}}{\beta |h_{sr}|^2 P_s}} - e^{-\frac{\lambda_{rd}(\gamma_{th} N_o - |h_{sd}|^2 P_s)}{\beta |h_{sr}|^2 P_s}} \right], \quad (25)$$

where $t = 1 - 1/(1 + \frac{\lambda_{rd}}{I\lambda_{rp}} (\gamma_{th} N_o - \mathbb{E}_{\mathbb{C}_2} [P_s |h_{sd}|^2]))$. The above approximation is tight for $\lambda_{rd} \ll I\lambda_{rp}$. This is true for general system settings in underlay-CRN.

Now averaging the above expression over $|h_{sr}|^2$ and using the expression for P_s in (4), results in the following expression for p_3 :

$$p_3 \approx \mathbb{E}_{\mathbb{C}_3} \left[\frac{\lambda_{rd} \left(1 - e^{-\frac{\gamma_{th} \lambda_{sd} N_o}{P_s}} \right)}{\lambda_{rd} - \beta |h_{sr}|^2 \lambda_{sd}} - \frac{(\beta |h_{sr}|^2 \lambda_{sd})}{\lambda_{rd} - \beta |h_{sr}|^2 \lambda_{sd}} \left(1 + te^{-\frac{\gamma_{th} \lambda_{rd} N_o + I\lambda_{rp}}{\beta |h_{sr}|^2 P_s}} - te^{-\frac{\beta \gamma_{th} |h_{sr}|^2 \lambda_{sd} N_o + I\lambda_{rp}}{\beta |h_{sr}|^2 P_s}} - e^{-\frac{\gamma_{th} \lambda_{rd} N_o}{\beta |h_{sr}|^2 P_s}} \right) \right],$$

where condition $\mathbb{C}_3 = \left\{ \frac{(1-\rho)|h_{sr}|^2}{\psi} > |g_{sp}|^2 \right\}$. Now let $\psi = \frac{\gamma_{th}}{I/N_o}$, p_3 becomes:

$$p_3 \approx \mathbb{E}_{\mathbb{C}_3} \left[\frac{\lambda_{rd} \left(1 - e^{-|g_{sp}|^2 \gamma_{th} \lambda_{sd} \psi} \right)}{\lambda_{rd} - \beta |h_{sr}|^2 \lambda_{sd}} - \frac{(\beta |h_{sr}|^2 \lambda_{sd})}{\lambda_{rd} - \beta |h_{sr}|^2 \lambda_{sd}} \left(1 + te^{-\frac{|g_{sp}|^2}{\beta |h_{sr}|^2} (\lambda_{rd} \psi + \lambda_{rp})} - te^{-|g_{sp}|^2 (\psi \lambda_{sd} + \frac{\lambda_{rp}}{\beta |h_{sr}|^2})} - e^{-\frac{|g_{sp}|^2 \lambda_{rd} \psi}{\beta |h_{sr}|^2}} \right) \right]$$

Now averaging the above equation over $|g_{sp}|^2$ and then using some straightforward manipulations, the resultant expression is presented in (26). From (26), p_3 is now expressed as:

$$p_3 = \int_0^\infty p_3 |h_{sr}|^2 \lambda_{sr} e^{-\lambda_{sr} |h_{sr}|^2} d|h_{sr}|^2 \quad (27)$$

The above integral can be simplified using the integral presented in [12, 5.1.1]:

$$\int_r^s \frac{e^{-px}}{qx+1} dx = \frac{e^{\frac{r}{q}}}{q} \left(E_1 \left(\frac{(qr+1)p}{q} \right) - E_1 \left(\frac{(qs+1)p}{q} \right) \right).$$

We omit the manipulations due to space limitations and present the approximated p_3 as in (15) (top of page-4).

B. Derivation of τ_{sim}

In the expression for p_3 in (15), $t = 1 - 1/(1 + \frac{\lambda_{rd}}{I\lambda_{rp}} (\gamma_{th} N_o - \mathbb{E}_{\mathbb{C}_2} [P_s |h_{sd}|^2]))$ as defined in (25), with $\mathbb{E}_{\mathbb{C}_2} [P_s |h_{sd}|^2]$ given by (24). Clearly, $t = 0$ when $\lambda_{rp} \rightarrow \infty$, which enables us to simplify p_3 . Resultant expression is given in (28) (second equation on the top of the next page).

Further simplification is possible when $\frac{I\lambda_{sp}}{N_o} \gg \gamma_{th}$, which is the commonly encountered situation. In this case, the

$$p_3 \Big|_{|h_{sr}|^2} = \frac{\lambda_{sp}}{\lambda_{rd} - \beta|h_{sr}|^2\lambda_{sd}} \left(\frac{\lambda_{rd}\lambda_{sd}\psi}{\lambda_{sd}\lambda_{sp}\psi + \lambda_{sp}^2} - \beta|h_{sr}|^2\lambda_{sd} \left(\frac{1}{\lambda_{sp}} - \beta|h_{sr}|^2 \left(t \left(\frac{1}{\beta|h_{sr}|^2(\lambda_{sd}\psi + \lambda_{sp}) + \lambda_{rp}} \right) \right) \right) \right) + \frac{1}{\beta|h_{sr}|^2\lambda_{sp} + \lambda_{rd}\psi} \Big) \quad (26)$$

$$p_3^{d_{rp} \rightarrow \infty} = \frac{(\lambda_{sr}\psi/\beta)^2}{\lambda_{sp} \left(\frac{\lambda_{rd}\lambda_{sp}}{\beta\lambda_{sd}} + \frac{\lambda_{rd}\psi}{\beta} \right)} \left(\lambda_{sr} e^{\frac{\lambda_{rd}\lambda_{sp}\psi}{\beta\lambda_{sp}}} \left(E_1 \left(\frac{\lambda_{rd}\lambda_{sr}\psi}{\beta\lambda_{sp}} \right) - E_1 \left(\frac{\lambda_{rd}\psi}{\beta\lambda_{sp}} \left(\lambda_{sr} + \frac{\lambda_{sp}(1-\rho)}{\psi} \right) \right) \right) \right) \quad (28)$$

$$- \frac{\lambda_{sr}\psi \left(1 - e^{-\frac{\lambda_{rd}(1-\rho)}{\beta}} \right)}{\lambda_{sp}(1-\rho) + \lambda_{sr}\psi} - \frac{\lambda_{sp}\lambda_{sr}}{\frac{\lambda_{rd}\lambda_{sp}}{\beta\lambda_{sd}} + \frac{\lambda_{rd}\psi}{\beta}} \left(\frac{\lambda_{rd}}{\beta\lambda_{sd}} \right)^2 e^{-\frac{\lambda_{rd}}{\beta\lambda_{sd}} \left(\frac{\lambda_{sp}(1-\rho)}{\psi} + \lambda_{sr} \right) - \frac{\lambda_{rd}(1-\rho)}{\beta}}$$

$$\times \left(\text{Ei} \left(\frac{\lambda_{rd}}{\beta\lambda_{sd}} \left(\lambda_{sr} + \frac{\lambda_{sp}(1-\rho)}{\psi} \right) \right) - \text{Ei} \left(\frac{\lambda_{rd}}{\beta\lambda_{sd}} \left(\lambda_{sr} + \frac{\lambda_{sp}(1-\rho)}{\psi} \right) + \frac{\lambda_{rd}(1-\rho)}{\beta} \right) \right)$$

$$\tau_{sim} = 0.5R_s \left(1 - \left(\frac{1}{\left(1 + \frac{\lambda_{sp}}{\lambda_{sd}\psi} \right) \left(1 + \frac{\eta\lambda_{sp}\rho}{\psi\lambda_{rd}\lambda_{sr}} \right)} + \frac{1}{\left(\frac{\lambda_{rd}}{\eta\rho} + 1 \right) \left(\frac{\lambda_{sp}(1-\rho)}{\psi\lambda_{sr}} + 1 \right)} + \frac{1}{\frac{\psi}{\lambda_{sp}} \left(\lambda_{sd} + \frac{\lambda_{sr}}{1-\rho} \right) + 1} \right) \right) \quad (30)$$

arguments of $e^{-x}\text{Ei}(x)$ and $e^x E_1(x)$ terms in p_3 increase and decrease respectively. Using the fact that $e^{-x}\text{Ei}(x) \rightarrow 0$ for $x \rightarrow \infty$ (here $x \propto \frac{\lambda_{sp}}{\psi}$), the term associated with $\text{Ei}(x)$ vanishes from the expression for p_3 . We further use the fact that $E_1(x) \gg E_1((x+A))$, for $A > 0$ when A is very large (as it is in this case since $(1-\rho)I\lambda_{sp}/(\gamma_{th}N_o) > 0$). The term with $e^x E_1((x+A))$ can be neglected in the high SNR region. From (28), the resultant approximated expression can be written as:

$$p_3^{d_{rp} \rightarrow \infty} \stackrel{A \gg 0}{\approx} \left(\frac{\psi\lambda_{rd}}{\beta\lambda_{sp}} \right)^2 \frac{(\lambda_{sp}\lambda_{sr})}{\frac{\psi\lambda_{rd}}{\beta} + \frac{\lambda_{rd}\lambda_{sp}}{\beta\lambda_{sd}}} e^{\frac{\psi\lambda_{rd}\lambda_{sr}}{\beta\lambda_{sp}}} E_1 \left(\frac{\psi\lambda_{rd}\lambda_{sr}}{\beta\lambda_{sp}} \right) \quad (29)$$

$$+ \frac{\lambda_{sr}}{\frac{\lambda_{sp}(1-\rho)}{\psi} + \lambda_{sr}} \left(e^{-\frac{\lambda_{rd}(1-\rho)}{\beta}} - 1 \right).$$

By using the following very tight lower and upper bounds $e^x E_1(x) \geq (1+x)^{-1}$ and $e^{-\lambda_{rd}(1-\rho)/\rho} \leq (1 + \lambda_{rd}(1-\rho)/\rho)^{-1}$ respectively into (29), and from (12), (14) and (13), τ can be approximated as τ_{sim} in (30).

It is worth noting that the above approximation is valid for $\frac{I\lambda_{sp}}{N_o} \gg \gamma_{th}$. However, it continues to follow the throughput τ in other cases. To get a closed form expression, we further approximate the above by utilizing the fact $\frac{\lambda_{rd}}{\eta\rho} \gg 1 - \frac{\lambda_{rd}}{\eta}$ (since $\eta, \rho \leq 1$) and $\frac{I\lambda_{sp}(1-\rho)}{\gamma_{th}N_o\lambda_{sr}} \gg 1$ except when $\rho \approx 1$ and represented in (18) (top of the page-4). Please note that $\rho \approx 1$ is unlikely, since it results in outage of the relayed link.

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