Performance and Complexity Tradeoffs of Several Constellations for Non Coherent Massive MIMO

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Abstract—The need to estimate a large number of channels in massive MIMO (mMIMO) has led to the proposal of non coherent (NC) detection, where the channel state information (CSI) is not necessary. In this paper, we discuss the tradeoff between NC designs based on phase-detection (PD) and those based on energy-detection (ED) for mMIMO in terms of performance and complexity. Similarly, we analyze NC schemes with respect to their coherent counterpart. In general, the results show that the PD ones are the best option regarding performance against the ED. Moreover, we analyze the combination of the two detection schemes as an attractive solution at the performance level facing the schemes based merely on energy or phase, in exchange for incrementing the complexity of the receiver. In addition, we propose new constellation schemes for phase based detection multiuser NC-mMIMO systems which allow to multiplex an increased number of users in an easier way.

I. INTRODUCTION

Massive Multiple-Input Multiple-Output (mMIMO) is considered as one of the key technologies for the fifth Generation (5G) and beyond due to its high spectral and energy efficiency [1]. mMIMO uses a large number of antennas at the base station (BS), much higher than the number of served users. The drawback is that the more antennas, the more channel coefficients need to be estimated. In order for this estimation to be feasible, mMIMO systems tend to assume Time Division Duplex (TDD) operation. In this scenario the channel state information (CSI) may be estimated using pilot signals transmitted from each user to the BS, assuming reciprocity in the radio link between uplink and downlink. However, the non-orthogonallity of the pilot sequences compromises the performance of these systems, which is known as pilot contamination [2].

The differential encoding (DE) and non coherent (NC) detection techniques in the transmitter and receiver, respectively, are emerging as a solution to the need of acquiring CSI in the receiver side, now aggravated by the excessive number of antennas in mMIMO. In the field of the NC schemes, the constellation design in the transmitter side is a key aspect of the system in order to carry out an adequate demodulation in the receiver without CSI. In [3], several principles based on space-time codes for designing NC constellations are proposed, although they do not take into account the benefits of mMIMO on signal processing yet. In general, proposals to apply NC schemes to mMIMO in the literature consider the DE and NC detection. In the transmitter, we can employ differential encoding in the amplitude (amplitude differential encoding, ADE) or in the phase (phase differential encoding, PDE) of the signal can be employed. From the point of view of the NC detection, we can use *energy based detection* (ED) or *phase based detection* (PD).

The amplitude shift keying (ASK) scheme is the most commonly used model so far for the NC-ED schemes, although without DE [4]. The main reason is that ASK is a simple scheme, albeit its performance is not so good and the required number of antennas compared to its PD counterparts is bigger as will be shown in this analysis.

Against ED, several PD based schemes were proposed in [5] and [6] with NC detection using differential phase shift keying (DPSK), which overcomes the performance of the ASK in [4]. These designs separate the signals of multiple users merely relying on the knowledge of their received signal powers. In [5], a constellation is proposed with the goal of achieving unequal error performance (UEP) among users. By contrast, in [6] the presented design does not rely on strict power control, providing the same performance for all users.

In order to increase the data rate, some other constellation schemes have been proposed for NC communications, such as amplitude phase shift keying (APSK), recently proposed for mMIMO in [7]. The differential quadrature amplitude modulation (DQAM) encompasses the schemes which perform differential operation in the amplitude of the signal. In [8] a DQAM scheme for mMIMO was proposed. By contrast to APSK, DQAM suffers from an error floor. Differential-APSK (DAPSK) [9] which combines ADE with PDE, enables increasing the order of the constellation and improves the performance as compared to ASK and in certain cases to DPSK.

All these schemes have been proposed separately and there is no framework comparing them. In this contribution, we analyze the different schemes for NC-mMIMO, presenting a tradeoff from the point of view of the performance and complexity. In addition, we propose new constellations based on PDE and PD for a multiuser environment facilitating the multiplexing of more users in the constellation in an easier way, because they can be combined with fewer restrictions. These designs are based on non orthogonal techniques as proposed in the new waveforms for 5G [11]. The remainder of the paper is organized as follows. The system model is introduced in Section II. The proposed constellations are presented in Section III, followed by the analysis of the performance in Section IV and complexity tradeoffs in Section V. Finally, we conclude in Section VI.

II. SYSTEM MODEL

We evaluate the constellation designs for an uplink NCmMIMO system with *J* users and *R* antennas at the BS. Each user maps $\mathbf{b} = b_a + b_p$ bits to $s_n = a_n e^{i\theta_n}$ symbols at the time instant *n*, belonging to a constellation \mathfrak{M} of size $M = M_A M_P$. \mathfrak{M} is composed of M_A concentric M_P -PSK schemes, with $b_a = \log_2(M_A)$ and $b_p = \log_2(M_P)$. The amplitudes of the symbol s_n are $a_n \in A = \{1/M_A, \dots, \beta^{2M_A-2}/M_A\}$, with β being the ratio between amplitudes, typically 2 or 1.4 for Rayleigh fading channels [7]. Then, for the case of $M_A = 2$, the amplitudes are differentially encoded as

$$a_n = \begin{cases} \beta^{-1}, & \text{if } b_n = 1 & \text{and } a_{n-1} = \beta \\ 1, & \text{if } b_n = 0 \\ \beta, & \text{if } b_n = 1 & \text{and } a_{n-1} = 1 \end{cases}$$
(1)

For $M_A > 2$, a look-up table is used to identify the relation among amplitudes a_n according to β , as shown in [7]. In the PD case, $M_A = 1$ and $a_n = 1$, the phase θ_n is differentially encoded $\omega_n = \omega_{n-1}\theta_n$. Finally, we obtain the symbol to be transmitted $x = a_n e^{i\omega_n}$.

The mMIMO wireless channel is modeled using a matrix \boldsymbol{H} of size $R \times J$, whose elements $h_{rj} \sim CN(0, 1)$ represent the propagation between user j and the *r*-th antenna at the BS. The channel matrix accounts for Rayleigh fading with zero mean and variance 1. Each of the antennas at the BS receives the vector \boldsymbol{y}_r , which is obtained as $\boldsymbol{y}[n] = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}[n]$. The AWGN vector \boldsymbol{v} consists of $(R \times 1)$ elements, where $v_j \sim CN(0, \sigma^2)$. The reference SNR is evaluated as $\boldsymbol{\rho} = \frac{\sum_{j=1}^{J} |a_j|^2}{\sigma^2}$.

At the receiver, we assume that $h_{rj}[n] = h_{rj}[n-1] = h_{rj}$, r = 1, ..., R, j = 1, ..., J. In a real scenario there will be a small variation between these two channels, that are considered equal to simplify the analysis. However, it is shown in [12] that PD schemes are very robust to the channel variability that is likely to happen in realistic scenarios. Then, we apply the NC detection as

$$z_n = \frac{1}{R} \sum_{r=1}^{R} y_r[n] y_r^*[n-1].$$
⁽²⁾

The decision variable z_n may be used for detecting the amplitude and the phase of the symbols s_n as

$$\hat{a}_n = \min_{a^k \in A} ||z_n| - a^k|^2$$
 (3)

$$\hat{\theta}_n = \min_{\theta^k \in M_P - PSK} |\angle z_n - 2\pi k/M_P|^2, \tag{4}$$

where $\angle z$ is the phase of z. The detection procedures are explained in detail in [4] for ED and [6] for PD. In the next section, we outline the key points of these procedures involved in the constellation designs.

III. CONSTELLATION SCHEMES

In this section, the constellation schemes used in our analysis are briefly summarized, including new proposals for PD.

A. Constellations for energy detection (ED)

In an ASK receiver the information is only conveyed in the amplitude of the signal and DE is not performed $(M_P = 1)$. The positive real axis is divided into $M = M_A$ non-intersecting intervals *I*, corresponding to each of the transmitted possible power levels \mathfrak{p}_m of the constellation \mathfrak{M} .

$$\mathfrak{M} = \left\{ \mathfrak{p}_{1} = 0, \mathfrak{p}_{2} = \frac{2}{M-1}, \mathfrak{p}_{m} = \frac{2(m-1)}{M-1}, \mathfrak{p}_{M} = 2 \right\},\$$
$$I_{1} = \left[0, \frac{d_{min}}{2}\right], I_{2} = \left[\frac{d_{min}}{2}, \frac{3d_{min}}{2}\right], \cdots, I_{M} = \left[2 - \frac{d_{min}}{2}, \infty\right],\tag{5}$$

where d_{min} is the minimum distance (MD) between two consecutive symbols in \mathfrak{M} . Then, the receiver computes z_n in (2) as the average received power across all the antennas as $\frac{||y||^2}{R}$ and, due to the Law of Large Numbers, can choose p_m related to the interval I_m to which z_n belongs, dispensing with CSI.

B. Constellations for phase detection (PD)

In this case, we have $M_A = 1$, $(M = M_p)$, then the symbols belong to a classical differential M_p -PSK scheme, where the information is only conveyed in the phase of the signal. For the PD schemes proposed in [5] and [6] the received constellation at the BS, so-called "joint constellation", is the combination of all the individual user schemes \mathfrak{M} . This joint constellation is represented by \mathcal{M} of order M^J . More explicitly, received constellation \mathcal{M} is composed of all legitimate combinations of the constellation points of \mathfrak{M}_j resulting in M^J uniquely distinguishable points named joint symbols $\varsigma[n]$,

$$\varsigma[n] = \sum_{j=1}^{J} s_j[n]. \tag{6}$$

As long as this is accomplished, the individual users' symbols $s_j[n]$ can be directly obtained from the detected joint symbols $\hat{\varsigma}[n]$ by demapping in the joint constellation. All designs proposed for PD perform a MD based detection with the symbols in \mathcal{M} as follows

$$\hat{\varsigma}_n = \arg\min_{\varsigma_n \in \mathcal{M}} |z_n - \varsigma_n|.$$
⁽⁷⁾

The UEP design proposed in [5] requires a specific scheme for each user providing different performance among them. This design intercalates the scheme of one user in the previous user's constellation. This design denoted by A is

$$\mathfrak{M}_{j}^{A} = \left\{\frac{2\pi m}{M}\mathsf{L}^{1-j}, m = 0, 1, ..., M-1\right\}, j = 1, ..., J, \mathsf{L} \ge M,$$
(8)

where L is the number of symbols intercalated, we usually use L = M. By contrast, the second design in [5] shown

in (9), denoted by B in this work, uses the same M_P -PSK constellation scheme for all users.

$$\mathfrak{M}^{B} = \left\{\frac{2\pi m}{M}, m = 0, 1, ..., M - 1\right\}, j = 1, ..., J.$$
(9)

Each user is separated in reception using different power profiles denoted by α_j . These parameters represent the power terms associated with the transmission of the different users. This way, the joint symbol formed by combining of *J* individual users is now as follows

$$\varsigma[n] = \sum_{j=1}^{J} \alpha_j s_j[n] \text{ with } \begin{cases} \alpha_1 = 1 \text{ for user } 1\\ \alpha_j > 1 \text{ for user } j, j \neq 1 \end{cases}.$$
(10)

The PD design in (11) from [6] provided the same performance for all users, thanks to achieving the same MD among all users and their symbols. This design C is called equal error protection (EEP).

$$\mathfrak{M}_{j}^{C} = \left\{\frac{2\pi}{JM}[(m+1)J - 1 + j], m = 0, 1, ..., M - 1\right\}, j = 1, ..., J$$
(11)

The main advantage of these schemes is the capability of multiplexing multiple users in the constellation, and separating them in the receiver merely based on the knowledge of the used constellation and their power profiles.

C. Combined constellation schemes

APSK: this scheme comes from a QAM scheme by applying the DPSK principle to the phases, whereas the amplitudes of the symbols are directly transmitted without DE. The APSK detector performs PD and ED. This observes the phase changes between every pair of consecutive received symbols (PD) which determinate the quadrant, and then, demodulates the amplitude by a quantizer (ED).

DAPSK: the amplitude and phase are differentially encoded and are the two terms jointly detected, to improve with respect to APSK. The detector is expressed in [7] as follows

$$(\hat{x}_{n-1}, \hat{x}_n) = \arg\min_{x_{n-1}, x_n \in M} \left| \frac{z_n}{\beta} - x_n x_{n-1}^* \right|.$$
 (12)

D. New PD based constellation

The advantage offered by NC-mMIMO aided DPSK is the non-orthogonal multiplexing of users in the constellation. In order to increase the number of users we propose three alternative schemes. Moreover, theses designs provide different degrees of UEP which add more flexibility to the designs to offer multiple quality of services, to be applied for example to multimedia services. In addition, the degradation on the performance suffered by the intercalated user between only two symbols belonging to another user is relieved. First, we search the optimal MD for the intercalated user in the design (8) by an exhaustive search, being this distance d = 0.56 for the case of J = 2 users and M = 4, as shown in Fig. 1 (a) for each individual user and, in Fig. 1(d) the received constellation. This design is referred to as *design D* in this work.

In the second proposal, with the goal of increasing the distance for the intercalated user, maintaining the unequal performance and the same power gain α_j , we intercalate half of the constellation for the last user between the first half of the previous user while the second one is placed in the opposite quadrants. An example for J = 2 users and M = L = 4 symbols is shown in Fig. 1(b) and (e) for the individual users and received constellation, respectively. The constellation points for this scheme are calculated for the user 1 as

$$\mathfrak{M}_{1}^{E} = \left\{ \frac{2\pi m}{M}, \ m = 1, \cdots, M \right\} \text{ for user } 1, \qquad (13)$$

while for the remaining users they have to be intercalated as

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$$\mathfrak{M}_{j}^{E} = \left\{ \begin{array}{l} \frac{2\pi m}{M} (\mathsf{L}-1)^{1-j}, \ m = 1, \cdots, \mathsf{L}/2\\ \pi + \frac{2\pi m}{M} (\mathsf{L}-1)^{1-j}, \ m = \mathsf{L}/2, \cdots, M \end{array} \right\} \text{ for } j > 1.$$
(14)

For the third proposal the purpose is obtaining the same distance among all symbols in the received constellation from the design (9). We keep the unequal power between individual user signals. In this case, the new design consists in a rotation of the individual constellations in (9). The points for the individual constellations \mathfrak{M}^F are defined as

$$\mathfrak{M}^{F} = \left\{ \frac{\pi(2m+1)}{M}, m = 0, 1, ..., M - 1 \right\}.$$
 (15)

An example of the users' and received constellations is shown in Fig. 1(c) and (f) for J = 2 users and M = 4 symbols. Note that amplitude detection is not needed for PD, merely making MD with the received symbol and the direct mapping between individual constellation.

IV. PERFORMANCE ANALYSIS

For the simulations, a Rayleigh random channel is generated at each iteration (minimum 100,000 iterations) following the model explained in Section II and it is kept constant for 1,000 symbols. The propagation channels of all the different users are uncorrelated. In Fig. 2, firstly, we compare the constellations A to the new proposals E and D for the case $\rho = 0$ dB, J = 2 users and M = 4 symbols. We can see that the performance is unequal between the two users for all schemes, being always better for user 1, since it has higher d_{min} than user 2. This difference between both users for BER= 10^{-3} is measured on the basis of R, obtaining 2000, 150 and 200 extra antennas for user 2 with respect to 1, for the A, E and D schemes, respectively. In the scheme E and D, we increase the MD for user 2 in the individual constellation, therefore the difference between both users is lower than for scheme A. In addition, for the scheme D, we achieve the same MD for all joint symbols, hence, we need a lower R to reach a lower BER than with the schemes A and E. However, since the difference between both users in E and D is approximately the same, the fact of achieving the same MD in design D among joint symbols does not help to obtain an equal performance between users. Also, in Fig. 2 we compare the equal performance



Fig. 1. New constellation schemes for individual users and the received constellations at the BS.

design given by C to unequal designs A and D. In this case, note that the design D for user 1 is close to scheme C, reaching it both users for $R > 10^3$ antennas. This is due to the fact that the user 1 in both cases has the same MD. Regarding the design A, this is worse than C, although for low R the user 1 in A achieves a lower BER.

In Fig. 3 we compare the designs B and F which employ a power gain of $\alpha_2 = 2$ for user 2 with respect to user 1. The BER for $\rho = 0$ dB, J = 2 users and M = 4 is shown. The difference between schemes B and F lies in the received constellation, resulting the same distance for all received symbols in F. However, this feature does not affect the performance, since we have only rotated the individual constellation. These results are consistent with that already discussed for schemes A, D and E in Fig. 2. We conclude that the MD influences exclusively the individual constellation design for each user.

The PD schemes based on DPSK are proposed also for multiuser schemes. The BER for J = 4 users, M = 2 and $\rho = 0$ is shown in Fig. 4 using (8) and (15) schemes. The difference between A and F in terms of R is greater for user 4 than for user 1, since 4 has a lower MD, being this differenced 100, 500, 3000 and 3500 antennas for user 1 up to user 4,



Fig. 2. BER comparison of A, C, D and E constellation designs for $\rho=0$ dB, J=2 users and M=4.

respectively.

We compare an ED scheme using ASK to a PD using DPSK for single user, $\rho = 0$ dB and different sizes of constellation



Fig. 3. BER performance comparison of B and F constellation designs with $\alpha_2 = 2$ for user 2 for $\rho = 0$ dB, J = 2 users and M=4.



Fig. 4. BER performance comparison of design A and F for J = 4 users, M=2 and $\rho = 0$ dB.

 $M \in \{2, 4, 8, 16\}$ symbols. This analysis is shown in Fig. 5. In all cases the required *R* for PD is lower than for ED, approximately 550 antennas less for M = 4; 1,000 for M = 8 and over 10,000 antennas less for M = 16. These results match with the expected ones, since DPSK has always been better than ASK also in the single antenna case [13]. In Fig. 6, a comparison among APSK, DAPSK and DPSK proposals is shown for M = 16, R = 128, 500 and 1000 antennas and single user. We can see that for the APSK scheme increasing over 500 antennas we do not have any improvement on the performance, mMIMO does not help. In addition, we need a higher SNR with respect to DPSK for the same BER. By contrast, the difference between DPSK and DAPSK is not too abrupt. For R = 128 antennas, DAPSK shows a better performance than



Fig. 5. BER comparison between ED (ASK) and PD (design E) schemes for single user.

DPSK, conversely, similar to APSK, DAPSK does not offer any improvement over 500 antennas.



Fig. 6. BER comparison among APSK, DAPSK and DPSK for M= 16.

In addition, we analyze how far our proposal is from its coherent counterpart. For that purpose, we also compare the performance of our EEP design to that achieved by a coherent Maximum Ratio Combining (MRC) receiver which is widely used in the literature for m-MIMO [2]. An MRC receiver normally works worse than ZF and MMSE. However, as power levels are reduced, the cross-talk introduced by the MRC receiver eventually falls below the noise level, and hence, it becomes a viable and advisable option for mMIMO schemes. For this comparison, we assume that the CSI is estimated and, hence, it is subject to a realistic estimation error, which is assumed to be Gaussian. Moreover, for a fair comparison we should take into account the effective throughput reduction due to the insertion of pilots for the channel estimation in the coherent scheme. We will assume a rate-loss of 33% due to pilot overhead as shown in [5]. This implies that for the same rate, we should compare non-coherent differential QPSK to coherent 8-PSK. In Fig. 7 a comparison between a NC-m-MIMO for J = 2 users, a size of constellation of M = 4 and a coherent scheme is shown. We use the UEP designs A, B and C and the EEP design for this comparison. We can see that for the same order of constellation, M = 4 the coherent systems need 30 antennas less than the NC. However, considering a fair comparison the proposed EEP scheme reaches the coherent performance with M = 8 when increasing the SNR by 2 dB. Note that the difference is lower than the classical 3 dB. By contrast, the UEP designs need more antennas or SNR to reach their coherent counterpart.



Fig. 7. BER Performance for both NC and coherent detection.

V. ANALYSIS OF THE COMPLEXITY

The complexity for each scheme is computed in terms of signal processing as the number of multiplications and comparisons that the receiver performs.

Regarding pure ED, the ASK scheme performs $M = M_A M_P$ comparisons as well as DPSK. Although APSK shows worse performance due to the fact that the amplitude is not encoded, it performs $M_A + M_P$ comparisons and one multiplication for each received sample, less than DPSK schemes and the same as DAPSK. Another advantage over conventional QAM without DE for practical implementation is that APSK presents a lower number of possible amplitude levels, resulting in fewer problems with non-linear amplifiers. Conversely, DPSK has only one power level, not showing problems for the amplifier. This scheme performs $M = M_A M_P$ comparisons and no multiplication.

The constellation designs proposed for NC-PD schemes are valid for more users, however the complexity is much higher for a good performance, requiring a number of antennas over 100,000. Hence, for achieving a system with more users we can employ orthogonal techniques such as Time Division Multiplex Access.

VI. CONCLUSIONS

We have shown that PD proposals perform better than the designs based on ED regarding BER. APSK gets worse performance than DAPSK. Whilst DPSK for high R is better than DAPSK, their performance tends to be the same as BER is lower. Moreover, APSK and DAPSK have been proposed in the literature just for single user, whilst DPSK is proposed for multiple users. This allows increasing the capacity of the systems based on DPSK. In summary, DPSK shows a better performance than DPASK and APSK, but DAPSK requires fewer operations in the receiver, showing a lower complexity.

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