Analytic Performance Evaluation of M-QAM Based Decode-and-Forward Relay Networks Over **Enriched Multipath Fading Channels**

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Abstract—The present work is devoted to the analysis of a regenerative multi-node dual-hop cooperative system over enriched multipath fading channels. Novel analytic expressions are derived for the symbol-error-rate for M-ary quadrature modulated signals in decode-and-forward relay systems over both independent and identically distributed as well as independent and nonidentically distributed Nakagami-q (Hoyt) fading channels. The derived expressions are based on the moment-generating-function approach and are given in closed-form in terms of the generalized Lauricella series. The offered results are validated extensively through comparisons with respective results from computer simulations and are useful in the analytic performance evaluation of regenerative cooperative relay communication systems. To this end, it is shown that the performance of the cooperative system is, as expected, affected by the number of employed relays as well as by the value of the fading parameter q, which accounts for pre-Rayleigh fading conditions that are often encountered in mobile cellular radio systems.

I. INTRODUCTION

Spatial diversity is among the most classical and effective methods to overcome multipath fading in wireless channels. Multi-antenna communication systems constitute one method that realises spatial diversity; however, due to the required space and power constraints for efficient operation, this method is often considered relatively inefficient. Another, efficient technique to overcome such kind of resource limitations is cooperative or relay communications [1], [2].

Cooperative transmission schemes exploit the broadcast nature and the inherent spatial diversity of fading channels without the need to install multiple antennas in a single mobile radio terminal, as in conventional MIMO systems. In cooperative systems, wireless terminals share and coordinate their resources for relaying messages to each other and for transmitting information signals over the numerous independent paths in the wireless network. Based on this, receivers exploit received signals often through different combining methods which have been extensively shown to provide efficient and robust operation in signal distortion by fading effects. Cooperative communication systems can be typically realised

by means of either regenerative or non-regenerative methods. The former typically refer to decode-and-forward (DF) relaying protocols whereas the latter refer to amplify-andforward relaying protocols (AF), [3]-[8] and the references therein. It has been shown that the digital processing nature of DF relaying makes it more practical than AF relaying which requires expensive RF transceivers to scale up the analog signal for avoiding to additionally relay the noisy version of the signal [1], [2].

It is widely known that fading phenomena have a significant effect on the performance of conventional and cooperative communications [9]-[12]. As a result, the limits of various communication scenarios have been studied by several researchers over the most basic multipath fading models. In this context, the authors in [3] proposed performance bounds of a DF multi-relay node networks over non-identical Nakagami-m fading channels. Specifically, the authors derived upper and lower bounds for the outage probability (OP) for the case of maximum-ratio-combining (MRC) diversity. Likewise, two alternative lower tight bounds that approximate the corresponding OP were reported in [8] whereas the symbol error-rate (SER) of multi-node DF systems over fading channels was addressed in [13]-[16], and the references therein.

In spite of the numerous investigations on relay communications over fading channels, the majority of the analyses assume that multipath fading effects follow either the Rayleigh or the Nakagami-m distribution. Nevertheless, the last years witnessed numerous advances in wireless channel characterization and modeling and in this context, the Nakagami-q, or Hoyt, distribution has been shown to provide accurate fading characterization of realistic indoor and outdoor multipath fading scenarios, particularly in the context of mobile cellular radio systems [17], [18]. The distinct feature of the Hoyt fading model is its capability to account for enriched multipath fading conditions which are characterized by the absence of dominant components [9], [19]-[25] and the references therein. Nevertheless, in spite of its proved usefulness, this

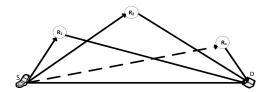


Fig. 1. Multi-node dual hop cooperative relay network

model has not been widely investigated in the context of relay communications. Motivated by this, the present work is devoted to the analytic performance evaluation of decode-and-forward systems over Hoyt distributed enriched multipath fading channels. Specifically, novel analytic expressions are derived for the case M-ary modulated signals over independent and identically distributed (i.i.d) and independent and non-identically distributed (i.n.i.d) Hoyt fading channels. The offered results are expressed in closed-form and their validity is justified through comparisons with results from respective computer simulations. As expected, it is shown that the performance of the considered system is highly dependant on both the number of employed relays and the value of the fading parameter q, which accounts for enriched fading conditions that are in the range of pre-Rayleigh fading severity.

The reminder of this paper is organized as follows: Section II presents the considered DF system and channel model. The exact SER analysis for M-QAM modulated signals over i.i.d and i.n.i.d Hoyt channels is provided in Section III. The corresponding performance analysis is provided in Section IV while closing remarks are given in Section V.

II. SIGNAL AND SYSTEM MODELS

A. Regenerative Relaying

We consider a dual-hop cooperative radio access architecture with intermediate nodes between a source S and a destination D as illustrated in Fig. 1. Each relay node is equipped with a single antenna assuming half-duplex communication in the context of a decode and forward cooperative protocol. Furthermore, the nodes in the system transmit signals through orthogonal channels for avoiding inter-relay interference using for example time division multiple access (TDMA). Based on this, in phase I the source broadcasts the information signal to the destination and to the relay nodes which is expressed as,

$$y_{S,D} = \sqrt{P_S h_{S,D} x} + n_{S,D} \tag{1}$$

and

$$y_{S,R_k} = \sqrt{P_S} h_{S,R_k} x + n_{S,R_k}, \quad k \in \{1, 2, 3, ..., K\}$$
 (2)

respectively, where P_S denotes the transmit source power, x is the transmitted symbol with normalized unit energy, $h_{S,D}$ and h_{S,R_k} are the complex channel gains of the source-destination and source-relay links, and $n_{S,D}$ and n_{S,R_k} are the corresponding additive-white Gaussian noise (AWGN) terms with zero mean and variance N_0 . In the next k+1 available time slot, if a relay k decodes correctly, it forwards the information signal to the destination with power $\bar{P}_R = P_R$.

Otherwise, if the decoding is unsuccessful the relay remains silent i.e., $\bar{P}_R = 0$. Based on this, the received signal at the destination terminal is given by,

$$y_{R_k,D} = \sqrt{\bar{P}_{R_k}} h_{R_k,D} x + n_{R_k,D}, \quad k \in \{1, 2, 3, ..., K\}$$
 (3)

where $h_{R_{k,D}}$ is the complex channel gain from the $k^{\rm th}$ relay to-destination and $n_{R_{k,D}}$ is the corresponding AWGN. Using the maximal-ratio-combining technique, the destination node coherently combines the signals received from the K-relays and the source as follows [26]:

$$y_D = w_1 y_{S,D} + \sum_{k=1}^{K} w_2 y_{R_k,D}$$
 (4)

where $w_1=\sqrt{P_s}h_{S,D}^*/N_0$ and $w_2=\sqrt{\bar{P}_{R_k}}h_{R_k,D}^*/N_0$ are the MRC coefficients for the $y_{S,D}$ and $y_{R_k,D}$ signals, respectively.

B. Nakagami-q (Hoyt) Multipath Fading

As already mentioned, the Hoyt fading model has been shown to represent effectively enriched in non-line-of-sight (NLOS) communication scenarios. The PDF of the instantaneous SNR is given by [9, eq. (2.11)], namely,

$$p_{\gamma}(\gamma) = \frac{1+q^2}{2q\overline{\gamma}} \exp\left(-\frac{(1+q^2)^2\gamma}{4q^2\overline{\gamma}}\right) I_0\left(\frac{(1-q^4)\gamma}{4q^2\overline{\gamma}}\right) \tag{5}$$

where $\overline{\gamma} = E(\gamma) = E(|h|^2)P/N_0 = \Omega P/N_0$ represents the average SNR with $E(\cdot)$ denoting statistical expectation. The corresponding CDF and MGF expressions are given by [21, eq. (9)] and [9, eq. (2.12)], respectively, namely,

$$P_{\gamma}(\gamma) = Q_{1} \left(\sqrt{\frac{(1+q^{4})(1+q)\gamma}{8q(1-q)\overline{\gamma}}}, \sqrt{\frac{(1-q^{4})(1-q)\gamma}{8q(1+q)\overline{\gamma}}} \right) - Q_{1} \left(\sqrt{\frac{(1-q^{4})(1-q)\gamma}{8q(1+q)\overline{\gamma}}}, \sqrt{\frac{(1+q^{4})(1+q)\gamma}{8q(1-q)\overline{\gamma}}} \right)$$
(6)

and

$$M_{\gamma}(s) = \left(1 - 2s\overline{\gamma} + \frac{(2s\overline{\gamma})^2 q^2}{(1+q^2)^2}\right)^{-\frac{1}{2}}$$
 (7)

where $Q_1(a,b)$ denotes the Marcum Q-function [27]–[29]. Notably, the above expressions have a convenient and easily computed mathematical representation. It is also recalled that the fading severity of Hoyt fading channels is inverse proportional to the value of the fading parameter q and it includes Rayleigh distribution as a specific case for q=1, [9].

III. EXACT SYMBOL ERROR RATE ANALYSIS

The average symbol-error-rate for the considered K-relays dual-hop cooperative system is given by [13], [26],

$$P_{SER}^{D} = \sum_{z=0}^{2^{K}-1} P(e|\mathbf{B} = E_z) P(\mathbf{B} = E_z)$$
 (8)

where the vector $\mathbf{B} = [B(1), B(2), B(3), \dots, B(K)]$ accounts for the state of relay nodes with B(k) representing the state

of a each relay and takes values 1 and 0 for the case of successful and unsuccessful decoding, respectively. Furthermore, $\mathbf{E_z} = [E(1), E(2), E(3), \dots, E(K)]$ denotes different possible combinations of decoding results of the relay nodes where z is a particular decoding result combination ranging from 0 to $2^K - 1$. Also, $P(e|\mathbf{B} = E_z)$ denotes the conditional error probability whereas $P(\mathbf{B} = E_z)$ is the corresponding probability of the relay node decoding outcomes. Thus, for the case of statistically independent channels, the joint probability of particular combination of state values can be expressed as $\mathbf{P}(\mathbf{B}) = P(B(1))P(B(2))P(B(3))\dots P(B(K))$.

In the case of MRC in (4), the instantaneous SNR for a given particular E_z at the destination is expressed as [26]

$$\gamma_{MRC}(E_z) = \frac{P_S |h_{S,D}|^2 + \sum_{k=1}^K E_z P_{R_k} |h_{R_k,D}|^2}{N_0}$$
(9)

with the corresponding MGF given by [30],

$$M_{\gamma_{MRC}}(s) = M_{\gamma_{S,D}}(s) \prod_{k=1}^{K} E_z M_{\gamma_{R_k,D}}(s).$$
 (10)

Importantly, the MGF in (7) can be equivalently expressed in terms of the Craig representation, namely,

$$M_{\gamma}\left(-\frac{g}{\sin^2\theta}\right) = \frac{1}{\sqrt{\left(1 + \frac{2\bar{\gamma}g}{(1+q^2)\sin^2\theta}\right)\left(1 + \frac{2\bar{\gamma}q^2g}{(1+q^2)\sin^2\theta}\right)}}.$$
(11)

To this effect, for the case M-QAM mapping one obtains,

$$P(\gamma_{MRC}) = \frac{4C}{\pi} \underbrace{\int_{0}^{\pi/2} M_{\gamma_{MRC}} \left(-\frac{g_{QAM}}{\sin^{2}\theta}\right) d\theta}_{\triangleq \mathcal{I}_{1}} - \frac{4C^{2}}{\pi} \underbrace{\int_{0}^{\pi/4} M_{\gamma_{MRC}} \left(-\frac{g_{QAM}}{\sin^{2}\theta}\right) d\theta}_{\triangleq \mathcal{I}_{2}}$$
(12)

where $C = (1-1/\sqrt{M})$ and $g_{QAM} = 3/2(M-1)$. Evidently, determining the above decoding error probability is subject to analytic evaluation of the \mathcal{I}_1 and \mathcal{I}_2 integrals in (12).

A. The case of i.n.i.d Hoyt fading channels

This subsection is devoted to the derivation of a the closed-form expression for the error probability for the generic case that the value of q and γ is not necessarily equal in each wireless link. To this end, with the aid of (10) and (11) the \mathcal{I}_1 term can be expressed as follows:

$$\mathcal{I}_{1} = \int_{0}^{\pi/2} \prod_{k=1}^{K} \frac{E_{z} d\theta}{\sqrt{\left(1 + \frac{A_{1}}{\sin^{2}\theta}\right) \left(1 + \frac{A_{2}}{\sin^{2}\theta}\right) \left(1 + \frac{B_{1k}}{\sin^{2}\theta}\right) \left(1 + \frac{B_{2k}}{\sin^{2}\theta}\right)}}$$
(13)

where

$$A_1 = \frac{2g_{QAM}\overline{\gamma}_{S,D}}{1 + q_{S,D}^2},\tag{14}$$

$$A_2 = \frac{2q_{S,D}^2 g_{QAM} \overline{\gamma}_{S,D}}{1 + q_{S,D}^2},\tag{15}$$

$$B_{1k} = \frac{2g_{QAM}\overline{\gamma}_{R_k,D}}{1 + q_{R_k,D}^2},\tag{16}$$

and

$$B_{2k} = \frac{2g_{QAM}q_{R_k,D}^2 \overline{\gamma}_{R_k,D}}{1 + q_{R_k,D}^2}.$$
 (17)

By letting $u = \sin^2(\theta)$ in (13) and therefore, $d/du = 2\cos(\theta)\sin(\theta)$ and $\cos(\theta) = \sqrt{1-u}$, it follows that,

$$\mathcal{I}_{1} = \int_{0}^{1} \frac{u^{E_{z}K + \frac{1}{2}}(1 - u)^{-\frac{1}{2}}}{2(u + A_{1})^{\frac{1}{2}}(u + A_{2})^{\frac{1}{2}}} \prod_{k=1}^{K} \frac{E_{z} du}{(u + B_{1k})^{\frac{1}{2}}(u + B_{2k})^{\frac{1}{2}}}.$$
(18)

By factoring out the constant terms, the above integral can be explicitly written as in (19) at the top of the next page. It is recalled here that the channel fades and powers in each wireless link are non-identical i.e. $q_{R_1,D} \neq q_{R_2,D} \neq \cdots \neq q_{R_K,D}$ and $\overline{\gamma}_{R_1,D} \neq \overline{\gamma}_{R_2,D} \neq \cdots \neq \overline{\gamma}_{R_K,D}$. Based on this and after basic algebraic manipulations, a closed-form expression is deduced for \mathcal{I}_1 in (20), at the top of the next page. where,

$$F_D^{(n)}(a;b_1,b_2,b_3,\dots,b_n;c;x_1,x_2,x_3,\dots,x_n) \triangleq \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 \frac{t^{a-1}(1-t)^{c-a-1}}{(1-x_1t)^{b_1}\dots(1-x_nt)^{b_n}} dt$$
(21)

is the generalized Lauricella function of n variables [27], [31]. Importantly, the integrals \mathcal{I}_1 and \mathcal{I}_2 have the same integrand and differ in the upper limit. Therefore, by following exactly the same methodology and additionally letting y=2u leads to (22), which yields straightforwardly the closed-form expression for \mathcal{I}_2 in (23) at the top of the next page.

B. The case of i.i.d Hoyt fading channels

The case of identical links assumes $q_{R_1,D}=q_{R_2,D}=\cdots=q_{R_K,D}=q$ and $\overline{\gamma}_{R_1,D}=\overline{\gamma}_{R_2,D}=\cdots=\overline{\gamma}_{R_K,D}=\overline{\gamma}$, and therefore, $B_{11}=B_{12}=\cdots=B_{1K}=B_1$ and $B_{21}=B_{22}\cdots=B_{2K}=B_2$. Based on this and having derived the general case of non-identically distributed links, the \mathcal{I}_1 and \mathcal{I}_2 for the i.i.d case can be straightforwardly deduced as a special case yielding (24) and (25), respectively, at the top page five.

It is evident that with the aid of the derived closed-form expressions for \mathcal{I}_1 and \mathcal{I}_2 , for both i.n.i.d and i.i.d scenarios, the conditional error probability at the destination terminal after the MRC can be straightforwardly determined by,

$$P(e|\mathbf{B} = E_z) = \frac{4C}{\pi} \mathcal{I}_1 - \frac{4C^2}{\pi} \mathcal{I}_2.$$
 (26)

In order to derive a closed-form expression for the overall SER of the considered system, we additionally need to determine the decoding probability of the relay nodes $P(\mathbf{B}=E_z)$ which is a direct product of the element terms $P(\overline{\gamma}_{S,R_k})$ i.e. decoding error at the relay nodes R_k and $(1-P(\overline{\gamma}_{S,R_k}))$. This is also

$$\mathcal{I}_{1} = \frac{1}{2\sqrt{A_{1}A_{2}} \prod_{k=1}^{K} \sqrt{E_{z}B_{1k}B_{2k}}} \int_{0}^{1} \frac{u^{1/2 + E_{z}K} (1 - u)^{-1/2}}{\left(1 + \frac{u}{A_{1}}\right)^{1/2} \left(1 + \frac{u}{A_{2}}\right)^{1/2}} \prod_{k=1}^{K} \frac{1}{\left(1 + \frac{u}{B_{1k}}\right)^{1/2} \left(1 + \frac{u}{B_{2k}}\right)^{1/2}} du.$$
(19)

$$\sqrt{\pi}\Gamma\left(\frac{3}{2} + E_z K\right) F_D^{(2E_z K + 2)} \left(\frac{\frac{2K + 2}{1}}{\frac{1}{2} + E_z K}; \underbrace{\frac{1}{1}, \frac{1}{2}, \cdots, \frac{1}{2}}_{\frac{1}{2}}; 2 + E_z K; -\frac{1}{A_1}, -\frac{1}{A_2}, \underbrace{-\frac{1}{B_{11}}, \cdots, -\frac{1}{B_{1K}}, -\frac{1}{B_{21}}, \cdots, -\frac{1}{B_{2K}}}_{\frac{1}{2} + E_z K}\right) \right) \\
\mathcal{I}_1 = \frac{2K}{2\sqrt{A_1 A_2} \Gamma\left(2 + E_z K\right) \prod_{k=1}^{K} \sqrt{E_z B_{1k} B_{2k}}} \tag{20}$$

$$\mathcal{I}_{2} = \frac{1}{2^{(EzK+5/2)}\sqrt{A_{1}A_{2}}\prod_{k=1}^{K}\sqrt{E_{z}B_{1k}B_{2k}}} \int_{0}^{1} \frac{y^{1/2+E_{z}K}(1-\frac{y}{2})^{-1/2}}{\left(1+\frac{y}{2A_{1}}\right)^{1/2}\left(1+\frac{y}{2A_{2}}\right)^{1/2}} \prod_{k=1}^{K} \frac{1}{\left(1+\frac{y}{2B_{1k}}\right)^{1/2}\left(1+\frac{y}{2B_{2k}}\right)^{1/2}} dy. \quad (22)$$

$$\Gamma(\frac{3}{2} + E_z K) F_D^{(2E_z K + 3)} \left(\underbrace{\frac{3}{2} + E_z K; \frac{1}{2}, \frac{1}{2}, \cdots, \frac{1}{2}; \frac{5}{2} + E_z K; -\frac{1}{2A_1}, -\frac{1}{2A_2}, -\frac{1}{2B_{11}}, \cdots, -\frac{1}{2B_{1K}}, -\frac{1}{2B_{21}}, \cdots, -\frac{1}{2B_{2K}}, \frac{1}{2}} \right) \\
\mathcal{I}_2 = \frac{2K}{2^{(EzK + 5/2)} \sqrt{A_1 A_2} \Gamma(\frac{5}{2} + E_z K) \prod_{k=1}^{K} \sqrt{E_z B_{1k} B_{2k}}} \tag{23}$$

obtained by applying the aforementioned MGF approach for the source to relay nodes links, namely,

$$P(\gamma_{S,R_k}) = \frac{4C}{\pi} \underbrace{\int_0^{\pi/2} M_{\gamma_{S,R_k}} \left(-\frac{g_{QAM}}{\sin^2 \theta} \right) d\theta}_{\triangleq \mathcal{I}_3} - \frac{4C^2}{\pi} \underbrace{\int_0^{\pi/4} M_{\gamma_{S,R_k}} \left(-\frac{g_{QAM}}{\sin^2 \theta} \right) d\theta}_{\triangleq \mathcal{I}_3}.$$
(27)

In order to evaluate \mathcal{I}_3 and \mathcal{I}_4 in closed-form, we follow the same procedure as in the derivation of the closed-form solutions for \mathcal{I}_1 and \mathcal{I}_2 . This is achieved thanks to the similar algebraic formulation and representation, except from the fact that there is no involvement of $q_{R_k,D}$. Based on this and after long but basic algebraic manipulations, it immediately follows that the \mathcal{I}_3 and \mathcal{I}_4 integrals can be expressed as,

$$\mathcal{I}_3 = \frac{\pi}{4\sqrt{C_1C_2}} F_D^{(2)} \left(\frac{3}{2}; \frac{1}{2}, \frac{1}{2}; 2; -\frac{1}{C_1}, -\frac{1}{C_2}\right)$$
(28)

and

$$\mathcal{I}_4 = \frac{1}{6\sqrt{2C_1C_2}} F_D^{(3)} \left(\frac{3}{2}; \frac{1}{2}, \frac{1}{2}; \frac{1}{2}; \frac{5}{2}; -\frac{1}{2C_1}, -\frac{1}{2C_2}, \frac{1}{2}\right)$$
(29)

respectively, where

$$C_1 = \frac{2g_{QAM}\overline{\gamma}_{S,R_k}}{1 + q_{S,R_k}^2} \tag{30}$$

 $q_{S,R_k}^2 g_{QAM} \overline{\gamma}_{S,R_k}$

$$C_2 = \frac{2q_{S,R_k}^2 g_{QAM} \overline{\gamma}_{S,R_k}}{1 + q_{S,R_k}^2}.$$
 (31)

To this effect, the decoding error probability of the relay nodes can be readily obtained as follows:

$$P(\gamma_{S,R_k}) = P(B=0) = \frac{4C}{\pi} \mathcal{I}_3 - \frac{4C^2}{\pi} \mathcal{I}_4.$$
 (32)

It is recalled here that the SER of the considered system is given in (8) and is obtained with the aid of the derived results on the probability for decoding error at the destination after the MRC, which is computed in terms of \mathcal{I}_1 and \mathcal{I}_2 , and the individual decoding probabilities of the nodes for a given combination $\mathbf{E}_{\mathbf{z}}$. As a result, a novel analytical solution is obtained for the average SER of the cooperative network over Nakagami-q (Hoyt) fading channels. It is also noted that the offered analytic expressions are given in terms of well known functions that are defined by single finite integrals which consist of elementary functions and can be computed using scientific software packages such as MAPLE and MATLAB.

IV. NUMERICAL RESULTS

This section is devoted to the analysis of the offered results on the SER of the considered model for different communication scenarios. The variance of the noise is normalized to unity i.e. $N_0=1$ while square M-QAM constellation is employed assuming equally allocated transmit powers to the

$$\mathcal{I}_{1} = \frac{\sqrt{\pi}\Gamma\left(\frac{3}{2} + E_{z}K\right)F_{D}^{(4)}\left(\frac{3}{2} + E_{z}K; \frac{1}{2}, \frac{1}{2}, \frac{E_{z}K}{2}, \frac{E_{z}K}{2}; 2 + E_{z}K; -\frac{1}{A_{1}}, -\frac{1}{A_{2}}, -\frac{1}{B_{1}}, -\frac{1}{B_{2}}\right)}{2\sqrt{A_{1}A_{2}}\Gamma\left(2 + E_{z}K\right)(B_{1}B_{2})^{E_{z}K/2}}.$$
(24)

$$\mathcal{I}_{2} = \frac{\Gamma(\frac{3}{2} + E_{z}K) F_{D}^{(5)}\left(\frac{3}{2} + E_{z}K; \frac{1}{2}, \frac{1}{2}, \frac{E_{z}K}{2}, \frac{E_{z}K}{2}, \frac{E_{z}K}{2}, \frac{1}{2}; \frac{5}{2} + E_{z}K; -\frac{1}{2A_{1}}, -\frac{1}{2A_{2}} - \frac{1}{2B_{1}}, -\frac{1}{2B_{2}}, \frac{1}{2}\right)}{2^{(EzK+5/2)}\sqrt{A_{1}A_{2}} \Gamma(\frac{5}{2} + E_{z}K)(B_{1}B_{2})^{E_{z}K/2}}.$$
(25)

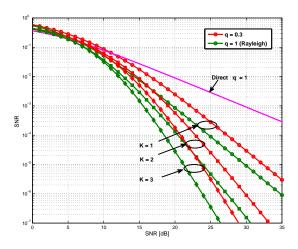


Fig. 2. SER performances over Nakagami-q (*Hoyt*) fading channels for $q=0.3, \ q=1$ with $\Omega_{S,D}=\Omega_{S,R_k}=\Omega_{R_k,D}=1$ for 4-QAM Signals with different number of relays and direct reference.

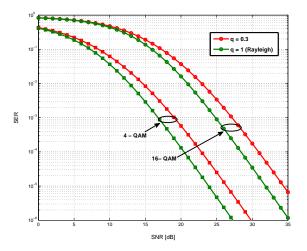


Fig. 3. SER performances over Nakagami-q (*Hoyt*) fading channels q=0.3, q=1 and $\Omega_{S,D}=\Omega_{S,R_k}=\Omega_{R_k,D}=1$ for $4-{\rm QAM}$ and $16-{\rm QAM}$ Signals for the case of two relays.

source and the relay nodes. Furthermore, the average power of the wireless links $\Omega_{i,j}$ is assumed equal to unity.

We firstly plot the corresponding SER as a function of SNR employing one, two and three relay nodes for symmetrical channel scenarios. In Fig. 2, the Hoyt fading parameter is

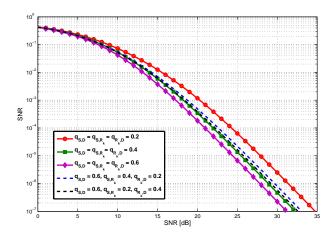


Fig. 4. SER performances over Nakagami-q (*Hoyt*) fading channels with different fading parameters $q_{S,D},q_{S,R_k},q_{R_k,D}$ and $\Omega_{S,D}=\Omega_{S,R_k}=\Omega_{R_k,D}=1$ for 4–QAM Signals employing two relays.

indicatively set to q=0.3 and q=1.0 while all average channel fading powers (channel variances) are assumed to be unity. One can observe the effect of the variation of the severity of enriched fading on the overall system performance. This figure also includes the performance for the case of direct transmission for q=1 (Rayleigh fading) as a reference for demonstrating the effect of enriched fading. In this context, it is also shown that, as expected, the dual hop regenerative communication systems improves significantly the overall system performance regardless of the limited number of relays, considered in this case.

In the same context, Fig. 3 illustrates the SER for different modulation order, namely, M=4 and M=16 in the case of both severe and Rayleigh fading conditions i.e. q=0.3 and q=1.0, respectively. One can notice that the respective SERs differ to each-other for one or even two orders of magnitude in the moderate and high SNR regime. Likewise, Fig. 4 demonstrates the cooperation performance of 4–QAM system of two relay scenario over Hoyt fading channels for different fading parameters of $q_{S,D},q_{S,R_k}$ and $q_{R_k,D}$. By varying the value of $q_{i,j}$, we observe from the plots the effect of q on the symbol error rates over the radio fading channels. Based on this, we can verify that increasing the values of q in the symmetric channel condition improves the performance, i.e. minimizes the SER, of the cooperation system. In particular, as the fading

parameter approached unity, the performance of the system approached gradually the respective Rayleigh performance. In addition, it is shown that the SER degradation is affected more by the value of $q_{R_k,D}$ than by the value of q_{S,R_k} and thus, possessing knowledge of the fading characteristics of the wireless links is rather important in the design and deployment of regenerative relay systems.

V. CONCLUSION

This paper analyzed the performance of multi-node dual-hop regenerative relay wireless system in the presence of enriched multipath fading conditions. Both the case of independent and identically distributed channels and independent and non-identically distributed channels were considered and novel closed-form expressions were derived for the corresponding symbol error rate for the case of M-QAM modulated signals employing maximum ratio combining. The involved analysis was based on the MGF approach and it was shown that the system performance is affected by enriched fading conditions, particularly for small values of the Hoyt fading parameter, regardless of the number of employed relay nodes. In general, the provided analytical results can be used to analyze the performance of general M-QAM based decode-and-forward relay networks over Nakagami-q (Hoyt) fading channels.

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