Improving Resource Efficiency with Partial Resource Muting for Future Wireless Networks

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Abstract—We propose novel resource allocation algorithms that have the objective of finding a good tradeoff between resource reuse and interference avoidance in wireless networks. To this end, we first study properties of functions that relate the resource budget available to network elements to the optimal utility and to the optimal resource efficiency obtained by solving max-min utility optimization problems. From the asymptotic behavior of these functions, we obtain a transition point that indicates whether a network is operating in an efficient noiselimited regime or in an inefficient interference-limited regime for a given resource budget. For networks operating in the inefficient regime, we propose a novel partial resource muting scheme to improve the efficiency of the resource utilization. The framework is very general. It can be applied not only to the downlink of 4G networks, but also to 5G networks equipped with flexible duplex mechanisms. Numerical results show significant performance gains of the proposed scheme compared to the solution to the max-min utility optimization problem with full frequency reuse.

I. INTRODUCTION

Future network architectures are envisioned to provide seamless connections anywhere and anytime with asymmetric traffic. In particular, in the media access control (MAC)/link layer, one of the key challenges to bring this vision to reality is to devise service-centric resource allocation mechanisms able to find a good balance between interference avoidance and resource (spectrum) reuse. Promising approaches to address this challenge are the muting schemes, which include the almost blank subframe (ABS) approach in time domain [1] and the mutually exclusive resource block allocation approach in frequency domain [2]. However, in these approaches two questions remain largely unanswered:

- At which point full resource reuse becomes inefficient? In other words, when is it appropriate to activate resource muting?
- How to develop efficient resource muting schemes that deal with complex interference patterns caused by disruptive architectures such as flexible duplex in future networks?

In this paper, we address these questions by studying properties of solutions to a large class of max-min utility fairness problems. In more detail, building upon the seminal work of Yates [3], which has introduced the concept of *standard interference functions (SIFs)* in wireless networks (see Definition 1), many researchers have devised efficient algorithms able to solve a large class of max-min fairness problems with the signal-to-interference-plus-noise ratio (SINR) [4]–[7] as the utility function. More recently, fairness problems with rate-related utilities characterized by nonlinear load coupling models have also been increasingly gaining attention [6], [8], [9]. Most studies devoted to these problems focus heavily on the development of numerical solvers, and, by doing so, issues such as properties of the utility and the network efficiency as a function of the budget available to network elements have remained largely unexplored. This gap has been addressed in the recent study in [10], which has used the concept of asymptotic functions in nonlinear analysis [11] to derive tight bounds for the network performance obtained by solving maxmin utility problems. That study has also derived a transition point that, for a given resource budget, indicates whether a network operates in a resource efficient region.

In this study we build upon the findings in [10] to develop novel efficient resource allocation algorithms for current and future network technologies. Our main contributions can be summarized as follows:

- We develop partial resource muting schemes based on the characterization of resource-efficient regions of solutions to a general class of resource allocation problems.
- The above schemes, which are based on the solutions to a series of subproblems using fixed point iterations, efficiently detect a set of bottleneck users that should be assigned to the muting region, and the schemes optimize the service-centric resource allocation.
- The framework proposed here can be applied both to the downlink (DL) of 4G networks and to 5G networks equipped with flexible duplex mechanisms.

This study is organized as follows. In Section II we introduce notation and mathematical background on i) max-min fairness problems and *constrained eigenvalue problems (CEVPs)*, and ii) properties of the solutions to CEVPs. In Section III we propose a resource muting scheme for DL resource allocation. This scheme is enhanced in Section IV to cater for complex interference models in 5G networks. Conclusions are summarized in Section V.

II. NOTATIONS, DEFINITIONS, AND PRELIMINARIES

The following definitions are used in this paper. The nonnegative and positive orthant in K dimensions are denoted by \mathbb{R}^{K}_{+} and \mathbb{R}^{K}_{++} , respectively. Let $\mathbf{x} \leq \mathbf{y}$ denote the componentwise inequality between two vectors. A norm $\|\cdot\|$ on \mathbb{R}^{K}_{+} is said to be monotone if $(\forall \mathbf{x} \in \mathbb{R}_{+}^{K})(\forall \mathbf{y} \in \mathbb{R}_{+}^{K}) \mathbf{0} \leq \mathbf{x} \leq \mathbf{y} \Rightarrow \|\mathbf{x}\| \leq \|\mathbf{y}\|$. By diag(**x**) we denote a diagonal matrix with the elements of **x** on the main diagonal. The cardinality of set \mathcal{A} is denoted by $|\mathcal{A}|$. The positive part of a real function is defined by $[f(x)]^{+} := \max\{0, f(x)\}$. Standard interference functions (SIFs) are defined as follows:

Definition 1. [3] A function $f : \mathbb{R}^K \to \mathbb{R}_{++} \cup \{\infty\}$ is an SIF if the following axioms hold: 1) (Monotonicity) $(\forall \mathbf{x} \in \mathbb{R}_+^K) (\forall \mathbf{y} \in \mathbb{R}_+^K) \mathbf{x} \leq \mathbf{y} \Rightarrow f(\mathbf{x}) \leq f(\mathbf{y}); 2)$ (Scalability) $(\forall \mathbf{x} \in \mathbb{R}_+^K) (\forall \alpha > 1) \alpha f(\mathbf{x}) > f(\alpha \mathbf{x}); and 3)$ (Nonnegative effective domain) dom $f := \{\mathbf{x} \in \mathbb{R}^K \mid f(\mathbf{x}) < \infty\} = \mathbb{R}_+^K$. A vector function $\mathbf{f} : \mathbb{R}_+^K \to \mathbb{R}_{++}^K : \mathbf{x} \mapsto [f_1(\mathbf{x}), \dots, f_N(\mathbf{x})]$ is called an SIF if each of the component functions is an SIF.

We now turn our attention to the general description of the problems we address in this study. In more detail, a large array of utility maximization problems in wireless networks can be seen as particular instances of the following optimization problem [4]–[6], [8]:

$$\underset{\mathbf{x} \in \mathbb{R}_{+}^{K}}{\operatorname{maximize}} \min_{k \in \mathcal{K}} u_{k}(\mathbf{x})$$
(1a)

subject to
$$\|\mathbf{x}\| \le \theta$$
 (1b)

where $\mathcal{K} := \{1, \ldots, K\}$ is the set of network elements, $u_k : \mathbb{R}_+^K \to \mathbb{R}_+$ is the utility function of network element $k \in \mathcal{K}$, and $\|\cdot\|$ is a monotone norm used to constrain the resource utilization $\mathbf{x} = [x_1, \ldots, x_K]$ to a given budget $\theta > 0$. In the above problem formulation, the *k*th component x_k of the optimization variable \mathbf{x} is the resource utilization of network element $k \in \mathcal{K}$. Now, consider the following conditional eigenvalue problem (CEVP):

(The conditional eigenvalue problem:) Given a monotone norm $\|\cdot\|$, a budget $\theta \in \mathbb{R}_{++}$, and a mapping $\mathbf{T} : \mathbb{R}_{+}^{K} \to \mathbb{R}_{++}^{K} : \mathbf{x} \mapsto [T^{(1)}(\mathbf{x}), \dots, T^{(K)}(\mathbf{x})]$, where $T^{(k)} : \mathbb{R}_{+}^{K} \to \mathbb{R}_{++}$ is an SIF for each $k \in \mathcal{K}$, the CEVP is stated as follows:

Find
$$(\mathbf{x}, c) \in \mathbb{R}_{+}^{K} \times \mathbb{R}_{++}$$

such that $\mathbf{T}(\mathbf{x}) = \frac{1}{c}\mathbf{x}$ and $\|\mathbf{x}\| = \theta$. (2)

As an implication of the results in [7], if the utility functions in (1) and the SIF **T** in (2) are related by $(\forall k \in \mathcal{K})(\forall \mathbf{x} \in \mathbb{R}^{K})$ $u_{k}(\mathbf{x}) = x_{k}/T^{(k)}(\mathbf{x})$, then \mathbf{x}^{*} solves (1) if (\mathbf{x}^{*}, c^{*}) solves (2). Furthermore, we have $(\forall k \in \mathcal{K}) u_{k}(\mathbf{x}^{*}) = c^{*}$. Therefore, to solve (1), we only need to devise efficient algorithmic solutions to (2). To this end, [7] has proved that (2) has a unique solution $(\mathbf{x}^{*}, c^{*}) \in \mathbb{R}_{++}^{K} \times \mathbb{R}_{++}$ and that $\mathbf{x}^{*} \in \mathbb{R}_{++}^{K}$ is the limit of the sequence $(\mathbf{x}^{(n)})_{n \in \mathbb{N}}$ generated by

$$\mathbf{x}^{(n+1)} = \frac{\theta}{\left\|\mathbf{T}\left(\mathbf{x}^{(n)}\right)\right\|} \mathbf{T}\left(\mathbf{x}^{(n)}\right), \text{ with } \mathbf{x}^{(0)} \in \mathbb{R}_{+}^{K}.$$
 (3)

With knowledge of \mathbf{x}^* , we recover c^* from the equality $c^* = \theta / \|\mathbf{T}(\mathbf{x}^*)\|$.

Note that later studies have also established the connection between Problem (1) and Problem (2) by using arguments with different levels of generality [6]. Recently, [10] has studied the influence of the budget θ on the solution to these problems, and we summarize some of the results of that study because they are crucial to the contributions that follow.

One of the key tools used in the analysis in [10] is the notion of asymptotic functions associated with SIFs:

Proposition 1. [10, Prop. 1] In \mathbb{R}_+^K , the asymptotic function $T_{\infty} : \mathbb{R}^K \to \mathbb{R} \cup \{\infty\}$ associated with an SIF $T : \mathbb{R}^K \to \mathbb{R}_{++} \cup \{\infty\}$ is given by

$$\left(\forall \mathbf{x} \in \mathbb{R}_{+}^{K}\right) T_{\infty}(\mathbf{x}) = \lim_{h \to \infty} T(h\mathbf{x})/h \in \mathbb{R}_{+}.$$
 (4)

For convenience, before we show relations between properties of asymptotic functions and properties of the solution to Problem (2), let us the recall the following definitions:

Definition 2 (Utility and resource efficiency). [10, Def. 4] Let $(\mathbf{x}_{\theta}, c_{\theta}) \in \mathbb{R}_{++}^{K} \times \mathbb{R}_{++}$ denote the solution to Problem (2) for a given budget $\theta \in \mathbb{R}_{++}$. The utility and the resource efficiency function are defined by, respectively, $U : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++} : \theta \mapsto c_{\theta}$ and $E : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++} : \theta \mapsto U(\theta)/||\mathbf{x}_{\theta}||.$

We can now state selected properties of the solution to Problem (2) (and hence to Problem (1)):

Proposition 2. [10, Prop. 3] Assume that the following CEVP: Find $(\mathbf{x}_{\infty}, \lambda_{\infty}) \in \mathbb{R}^{K}_{+} \times \mathbb{R}_{+}$ such that

$$\Gamma_{\infty}(\mathbf{x}_{\infty}) = \lambda_{\infty} \mathbf{x}_{\infty}, \ \|\mathbf{x}_{\infty}\| = 1$$
(5)

has a unique positive solution $(\mathbf{x}_{\infty}, \lambda_{\infty}) \in \mathbb{R}_{++}^{K} \times \mathbb{R}_{++}$, where each coordinate $T_{\infty}^{(k)}$ of $\mathbf{T}_{\infty} : \mathbb{R}_{+}^{K} \to \mathbb{R}_{+}^{K}$ is an asymptotic function associated with the SIF $T^{(k)}$ in (2). Then the solution to (2) has the following properties:

(i) Asymptotic utility and resource efficiency:

$$\sup_{\theta \to 0} U(\theta) = \lim_{\theta \to \infty} U(\theta) = 1/\lambda_{\infty},$$
(6)

$$\sup_{\theta > 0} E(\theta) = \lim_{\theta \to 0^+} E(\theta) = 1/\|\mathbf{T}(\mathbf{0})\|.$$
(7)

(ii) Upper bound for the utility: $(\forall \theta \in \mathbb{R}_{++})$,

$$U(\theta) \leq \begin{cases} \theta / \|\mathbf{T}(\mathbf{0})\|, & \text{if } \theta \leq \theta^{(trans)} \\ 1/\lambda_{\infty}, & \text{otherwise}, \end{cases}$$
(8)

where $\theta^{(trans)}$ is the transition point defined by

$$\theta^{(trans)} := \|\mathbf{T}(\mathbf{0})\| / \lambda_{\infty}. \tag{9}$$

(iii) Upper bound for the resource efficiency: $(\forall \theta \in \mathbb{R}_{++})$,

$$E(\theta) \le \min\left\{1/\|\mathbf{T}(\mathbf{0})\|, 1/(\lambda_{\infty}\theta)\right\}.$$
 (10)

The transition point $\theta^{(\text{trans})}$ in (9) serves as a coarse indicator of whether we can obtain substantial gains in utility by increasing the budget θ . More precisely, if the given budget θ is greater than $\theta^{(\text{trans})}$, then the network is likely operating in a regime where the performance is limited by interference, so increasing θ even by orders of magnitude typically brings only marginal gains in utility. In contrast, if $\theta < \theta^{(\text{trans})}$, then noticeable gains in utility can be obtained by increasing the budget θ . Fig. 1 illustrates this observation, which is heavily exploited by the algorithms proposed in the next sections.





III. PROPOSED ALGORITHMS FOR DL RESOURCE ALLOCATION

In this section, we build upon the results in [7] and Proposition 2 to devise novel algorithms for DL resource allocation. Then, in Section IV, we study problems with more complex interference models incorporating disruptive architectures envisioned for 5G networks.

A. System Model and Problem Formulation

We consider an orthogonal frequency division multiplexing (OFDM)-based wireless network system, consisting of a set of base stations (BSs) $\mathcal{N} := \{1, \ldots, N\}$ and a set of communication links $\mathcal{K} := \{1, \ldots, K\}$ in DL. Hereafter, we use the terms "communication links", "services", and "users" interchangeably (without loss of generality, we can assume that each user sets up one communication link for one service at an unit of time). Let \overline{W} denote the total bandwidth in Hz, and let $\mathbf{w} \in [0, 1]^K$ be a vector collecting the fraction of bandwidth allocated to the services. We assume that the transmit power spectral density (in Watts/Hz) of all services are given and collected in a vector $\mathbf{p} \in \mathbb{R}_{++}^K$. Let the data rate demand of all services (in bit/s) be collected in $\bar{\mathbf{r}} \in \mathbb{R}_{++}^K$. The matrix $\mathbf{A} \in \{0, 1\}^{N \times K}$ denotes the BS-to-service assignment matrix, where $a_{n,k} = 1$ if k is served by BS n, and 0 otherwise.

By $v_{k,l}$ we denote the channel gain between the transmitter of service l and the receiver of service k. For $k \neq l$, $v_{k,l}$ is the interference channel gain, which is positive if service lcauses interference to service k, otherwise $v_{k,l} = 0$. The scalar $v_{k,k} > 0$ is the channel gain of link k. By σ_k^2 we denote the noise spectral density (in Watt/Hz) in the receiver of service k. By interpreting w as the probability of generating interference from the transmitter of a link to the receiver of another link (on any resource block) [8], [12], the SINR that link k experiences can be approximated by

$$\operatorname{SINR}_{k}(\mathbf{w}) \approx \frac{p_{k} v_{k,k}}{\sum\limits_{l \neq k} v_{k,l} p_{l} w_{l} + \sigma_{k}^{2}} = \frac{p_{k}}{\left[\tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p}) \mathbf{w} + \tilde{\boldsymbol{\sigma}}\right]_{k}},$$
(11)

where $\tilde{\mathbf{V}} \in \mathbb{R}_{+}^{K \times K}$ denotes the *interference coupling matrix* with the (k, l)-th entry defined as $v_{k,l}/v_{k,k}$, and $\tilde{\boldsymbol{\sigma}} := [\sigma_1^2/v_{1,1}, \ldots, \sigma_K^2/v_{k,k}] \in \mathbb{R}_{++}^K$.

The achievable rate (in bit/s) of service k is computed by $r_k(\mathbf{w}) = w_k \overline{W} \log_2(1 + \text{SINR}_k(\mathbf{w})).$ (12)

The objective of the first algorithm proposed here is to maximize the worst-case service-specific *quality of service* (QoS) satisfaction level, defined as the ratio of the achievable rate $r_k(\mathbf{w})$ to the rate demand \bar{r}_k , subject to a per-BS load constraint. Formally, the problem is stated as follows:

$$\underset{\mathbf{w} \in \mathbb{R}_{+}^{K}}{\operatorname{maximize}} \min_{k \in \mathcal{K}} r_{k}(\mathbf{w}) / \bar{r}_{k}$$
(13a)

subject to
$$\|\mathbf{A}\mathbf{w}\|_{\infty} \le \theta$$
, (13b)

where $\|\cdot\|_{\infty}$ denotes the L^{∞} -norm. Note that (13b) implies a resource reuse factor of 1, and with full load constraint we have $\theta = 1$; i.e., $(\forall n \in \mathbb{N}) \sum_{k \in \mathcal{K}} a_{n,k} w_k \leq 1$.

In this work, we are interested not only in the solution to the above problem, but also in answering the following question: *Is the load limit* $\theta = 1$ *with resource reuse factor* 1 *an efficient operation point*?

B. Optimal Resource Allocation and Performance Limits

To study the resource efficiency of the solution to Problem (13) as a function of the budget θ , we start with next technical result. The proof is omitted because it is similar to that in [13, Ex. 2].

Lemma 1. Suppose that the SINR is modeled as in (11), then the mapping $\mathbf{T} := [T^{(1)}(\mathbf{w}), \ldots, T^{(K)}(\mathbf{w})]$, where $(\forall k \in \mathcal{K}) T^{(k)}(\mathbf{w}) : \mathbb{R}_{+}^{K} \to \mathbb{R}_{++} : \mathbf{w} \mapsto \bar{r}_{k}/(\overline{W}\log_{2}(1 + \mathrm{SINR}_{k}(\mathbf{w})))$, is an SIF.

By using Lemma 1 and [14, Prop. 1] we verify that the optimal solution \mathbf{w}^* to (13) satisfies $(\forall k \in \mathcal{K}) \ u_k(\mathbf{w}^*) = c^*$ [14, Prop. 1] for some $c^* \in \mathbb{R}_{++}$. This result can be alternatively obtained as follows. Assuming that each BS serves at least one user and that every user is served by a BS, which guarantees that each BS serves a nonempty and unique set of users, we have that all rows of the assignment matrix A are linearly independent. Therefore, A is a nonnegative full (row) rank matrix, so the function $g: \mathbb{R}^K \to \mathbb{R}_+ : \mathbf{w} \mapsto \|\mathbf{A}\|_{\mathbf{w}}\|_{\infty}$ (which in particular implies $q(\mathbf{x}) = \|\mathbf{A}\mathbf{w}\|$ for $\mathbf{w} \in \mathbb{R}_+^K$), where $|\cdot|$ denotes the coordinate-wise absolute value of a vector, is a monotone norm. Thus, the problem in (13) is an instance of that in (1), and the solution to (13) can be easily obtained with the iterations in (3) as explained in Section II. Furthermore, by using Proposition 1, we can compute the asymptotic mapping T_{∞} associated with T. By doing so, as shown below, we are able to compute $(\mathbf{w}_{\infty}, \lambda_{\infty})$ defined in Proposition 3, and the performance limits in Proposition 2 become readily available.

Proposition 3. Let $\mathbf{T} : \mathbb{R}_{+}^{K} \to \mathbb{R}_{++}^{K} : \mathbf{w} \mapsto [T^{(1)}(\mathbf{w}), \dots, T^{(K)}(\mathbf{w})]$ be as defined in Lemma 1. Suppose that $\mathbf{p} \in \mathbb{R}_{++}^{K}$ and $\tilde{\mathbf{V}}$ is irreducible, implying that each service is interfered by at least another of the services. The asymptotic mapping $\mathbf{T}_{\infty} : \mathbb{R}_{+}^{K} \to \mathbb{R}_{+}^{K} : \mathbf{w} \mapsto [T_{\infty}^{(1)}(\mathbf{w}), \dots, T_{\infty}^{(K)}(\mathbf{w})]$ associated with \mathbf{T} is given by:

$$\mathbf{T}_{\infty}(\mathbf{w}) = \operatorname{diag}(\mathbf{p})^{-1} \mathbf{\Phi} \tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p}) \mathbf{w}$$
(14)
where $\mathbf{\Phi} := \frac{\ln(2)}{\overline{W}} \operatorname{diag}(\overline{r}_1, \dots, \overline{r}_K).$

Furthermore, there exists a unique positive solution $(\mathbf{w}_{\infty}, \lambda_{\infty}) \in \mathbb{R}_{++}^{K} \times \mathbb{R}_{++}$ to the CEVP

$$\mathbf{T}_{\infty}(\mathbf{w}_{\infty}) = \lambda_{\infty} \mathbf{w}_{\infty}, \ \|\mathbf{A}\mathbf{w}_{\infty}\|_{\infty} = 1, \tag{15}$$

which is given by

$$\lambda_{\infty} = \lambda_*, \ \mathbf{w}_{\infty} = \mathbf{w}_* / \|\mathbf{A}\mathbf{w}_*\|_{\infty}, \tag{16}$$

where λ_* and \mathbf{w}_* are, respectively, the unique largest real eigenvalue and a corresponding real eigenvector of the matrix $\mathbf{G} := \operatorname{diag}(\mathbf{p})^{-1} \mathbf{\Phi} \tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p}).$

Proof. Applying Proposition 1, we have

$$(\forall k \in \mathcal{K}) (\forall \mathbf{w} \in \mathbb{R}_{+}^{K}) \ T_{\infty}^{(k)} = \lim_{h \to \infty} T^{(k)}(h\mathbf{w})/h$$

= $\lim_{h \to \infty} \bar{r}_{k} / \left(h \overline{W} \log_{2} \left(1 + p_{k} / \left[h \tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p}) \mathbf{w} + \tilde{\boldsymbol{\sigma}} \right]_{k} \right) \right).$

Defining $g(h) := \log_2 \left(1 + p_k / \left[h \tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p}) \mathbf{w} + \tilde{\boldsymbol{\sigma}} \right]_k \right)$ and $f(h) := \bar{r}_k/(\overline{W}h)$, we have that $\lim_{h\to\infty} g(h) = 0$, $\lim_{h\to\infty} f(h) = 0, g'(h) \neq 0$ for $h \in \mathbb{R}_+$, and that $\lim_{h\to\infty} f'(h)/g'(h)$ exists. By using L'Hôpital's rule, we verify that $T_{\infty}^{(k)} = \ln(2)\bar{r}_k \left[\tilde{\mathbf{V}}\operatorname{diag}(p)\mathbf{w}\right]_k/(\overline{W}p_k)$. We obtain (14) by writing $\mathbf{T}_{\infty} := \left[T_{\infty}^{(1)}, \ldots, T_{\infty}^{(K)}\right]$ in matrix form. We can also verify that G is nonnegative and irreducible. As a result, by Perron-Frobenius theory [15, p. 673], **G** has a simple positive eigenvalue $\lambda_* \in \mathbb{R}_{++}$ associated with a positive right eigenvector \mathbf{w}_* . Furthermore, any other real eigenvalue λ satisfies $|\lambda| \leq \lambda_*$, and if $T_{\infty}(\mathbf{w}) = \mathbf{G}\mathbf{w} = \lambda \mathbf{w}$ for some $\mathbf{w} \in \mathbb{R}^{K}_{+} \setminus \{\mathbf{0}\}$, then we have $\lambda = \lambda_*$ and $\mathbf{w} = c\mathbf{w}_*$ for some $c \in \mathbb{R}_{++}$. In particular, with $c = 1/||\mathbf{A}\mathbf{w}_*||_{\infty}$, $\mathbf{w}_{\infty} = c\mathbf{w}_*$, and $\lambda_{\infty} = \lambda_*$, we deduce $T_{\infty}(\mathbf{w}_{\infty}) = \mathbf{G}\mathbf{w}_{\infty} = \lambda_{\infty}\mathbf{w}_{\infty}$ and $\|\mathbf{A}\mathbf{w}_{\infty}\|_{\infty} =$ $\|\mathbf{A}(\mathbf{w}_*/\|\mathbf{A}\mathbf{w}_*\|_{\infty})\|_{\infty} = \|\mathbf{A}\mathbf{w}_*\|_{\infty}/\|\mathbf{A}\mathbf{w}_*\|_{\infty} = 1.$ The above implies that (16) is the unique solution to the CEVP in (15), and the proof is complete.

Remark 1. Proposition 3 shows an efficient method to compute $(\mathbf{w}_{\infty}, \lambda_{\infty})$ as the solution to the CEVP (15). Note that, in this specific case, the asymptotic mapping associated with a nonlinear mapping (defined in Lemma 1) becomes linear, as shown in (15). Moreover, to compute the eigenvalues of $\mathbf{G} = \operatorname{diag}(\mathbf{p})^{-1} \Phi \tilde{\mathbf{V}} \operatorname{diag}(\mathbf{p})$, we can equivalently compute the eigenvalues of the matrix $\Phi \tilde{\mathbf{V}}$, which is independent of \mathbf{p}^{1} .

C. Partial Resource Muting

Having $(\mathbf{w}_{\infty}, \lambda_{\infty})$ in hand, we are now able to compute $\theta^{(\text{trans})}$ with (9) and answer the following question related to the resource efficiency raised in Section III-A:

If the transition point yields $\theta^{(trans)} < 1$, full resource reuse (i.e., $\theta = 1$) may not be an efficient operation point because we are likely operating in an interference limited region where the resource availability θ can be decreased without noticeable changes in utility – see Fig 1.



Fig. 2: Resource muting region in downlink.

The new challenge arises:

How to improve the resource efficiency if the network is operating in an interference-limited inefficient region?

Since the bottleneck users usually consume most of the resources and impair the performance, we consider muting partial resources in the neighboring cells to mitigate the interference received in (and generated by) the bottleneck users. Based on $(\lambda_{\infty}, \mathbf{w}_{\infty})$ obtained in (16) and the derived transition point $\theta^{(\text{trans})}$, we propose a resource muting scheme consisting of the following steps.

1) Triggering the Resource Muting Scheme: $\theta^{(\text{trans})} < 1$ implies inefficient usage of resources in the region $[\theta^{(\text{trans})}, 1]$ due to heavy interference. Instead of allocating all resources in at least one BS to achieve only a slight increase in utility, the network may better benefit from muting partial resources to reduce the interference of bottleneck users. Therefore, the resource muting scheme can be triggered if $\theta^{(\text{trans})} < 1$.

2) Modifying the Interference Pattern and Load Constraints: Suppose that a set of bottleneck users, denoted by $\mathcal{K}^{(b)}$, is selected (details about the selection of bottleneck users are given in the next subsection). The motivation is to allocate $\mathcal{K}^{(b)}$ to a muting region, such that each user $k \in \mathcal{K}^{(b)}$ neither receives interference from the neighboring cells nor generates interference to those cells, as shown in Fig. 2. Since the received/generated interference of the bottleneck users from/to the neighboring cells is canceled, the interference coupling matrix is updated as follows

$$(\forall k \in \mathcal{K}^{(b)})(\forall m \in \mathcal{N}_k)(\forall l \in \mathcal{K}_m)v_{k,l} = v_{l,k} = 0,$$
 (17)
where \mathcal{N}_k denotes the set of neighboring cells of cell k.

On the other hand, to incorporate the muting region in the constraint, we modify the load constraint in (13b) as shown below:

$$g(\mathbf{w}) \le 1, \tag{18a}$$
$$g: \mathbb{R}^{K} \to \mathbb{R}_{+}: \mathbf{w} \mapsto \max_{n \in \mathcal{N}} \left(\sum_{k \in \mathcal{K}_{n}} |w_{k}| + \sum_{m \in \mathcal{N}_{n}} \sum_{l \in \mathcal{K}_{m}^{(b)}} |w_{l}| \right), \tag{18b}$$

where $\mathcal{K}_m^{(b)}$ denotes the set of bottleneck users in cell m. Constraint (18) implies that, for each cell n, the sum of resources allocated to its serving users \mathcal{K}_n (including both the bottleneck and non bottleneck users) and the resources allocated to all the bottleneck users served by its neighboring cells $m \in \mathcal{N}_n$ is limited to the total amount of resources. Note

¹Suppose that $\mathbf{D} \in \mathbb{R}^{K \times K}$ is an invertible matrix and $\mathbf{X} \in \mathbb{R}^{K \times K}$, the eigenvalues of the matrices \mathbf{X} and $\mathbf{D}\mathbf{X}\mathbf{D}^{-1}$ are the same [16, Rem. 1].

that the modified function g is a monotone norm. Thus, the problem with the modified constraint (18) is still an instance of that in (1).

3) Detecting Bottleneck Users: The asymptotic behavior $(\lambda_{\infty}, \mathbf{w}_{\infty})$ provides a good guide to detect efficiently the bottleneck services, because it indicates the existing limits of the utility and the fraction of allocated resources as $\theta \to \infty$. More precisely, let the *k*th entry of \mathbf{w}_{∞} be denoted by $w_{\infty}^{(k)}$. The larger the value of $w_{\infty}^{(k)}$, the higher the chance that the corresponding user impairs the system performance owing to the large amount of occupied resources, and, consequently, the higher the possibility that this user causes heavy interference.

To show how \mathbf{w}_{∞} can be useful in the development of efficient heuristics for selecting the set of bottleneck users, we compare the following two approaches: *exhaustive search* and *successive selection*. As we discuss below, the latter approach is suboptimal, but its computational complexity is subtantially smaller than that of the former approach.

- *Exhaustive search.* Given a set of candidate users, denoted by $\mathcal{K}^{(c)}$, we find an optimal subset $\mathcal{K}^{(b)} \subseteq \mathcal{K}^{(c)}$ that provides the maximum utility. This problem poses a serious computational challenge: the number of subsets is exponential in the size of the domain. It might require an exhaustive search over $\sum_{i=1,\ldots,|\mathcal{K}^{(c)}|} \binom{|\mathcal{K}^{(c)}|}{i}$ possible subsets.
- Successive selection: By sequentially selecting users with the highest values $w_{\infty}^{(k)}$, the resulting optimized utility has a general trend of first increasing and then decreasing, as shown in Fig. 3. Therefore, instead of exhaustively searching for the best set of bottleneck users, we propose an efficient heuristic to approximate the solution. We successively add the users with the next highest value of $w_{\infty}^{(k)}$ until the utility does not increase.

Fig. 3 illustrates the motivation for successive selection. It shows the optimized utility depending on the number of bottleneck users for four simulation instances, each of them with a different distribution for the user locations and traffic demands. Suppose that the selection of bottleneck users is based on the rank order the components of \mathbf{w}_{∞} . Let us sort the entries in \mathbf{w}_{∞} in descending order, with the order of $w_{\infty}^{(k)}$ denoted by o_k . For example, if the number of bottleneck users is 3, then, the highest ranked users, i.e., $\{k \in \mathcal{K} : o_k \in \{1, 2, 3\}\}$ are selected. The curves generated by different instances show a common trend of utility increase when the users with the highest values of $w_{\infty}^{(k)}$ are allocated in the muting region. Then, if too many bottleneck users are selected, the utility decreases. Such pattern reflects the tradeoff between resource utilization and interference mitigation. Therefore, although the curves are not always piecewise monotonic, numerical results show that adding users successively until the utility starts to decrease provides a simple means of identifying bottleneck users.

4) Fixed Point Iteration: Once the set of bottleneck users is updated, the load constraint is modified as shown in (18) to incorporate the muting scheme, and the fixed point iteration (3) (with $\theta = 1$ and $\|\cdot\| = g(\cdot)$ as defined in (18)) can



Fig. 3: Example of successive selection of bottleneck users.

Algorithm 1: Partial Resource Muting
input : $n \leftarrow 0, \mathcal{K}^{(b)}(0) = \emptyset$
output: $\mathcal{K}^{(b)}_{*}, \mathbf{w}^{*}$
1 Compute $\mathbf{w}^*(0), c^*(0)$ using (3) with $\theta = 1$;
2 Compute $\mathbf{w}_{\infty}, \lambda_{\infty}$ and transition point $\theta^{(\text{trans})}$ using
Proposition 2 and 3;
3 if $\theta^{(trans)} < 1$ then
4 Sort $w_{\infty}^{(k)}$ in descending order;
5 while $n = 0$ or $c^*(n) - c^*(n-1) \ge 0$ do
6 $\mathcal{K}^{(b)}_* \leftarrow \mathcal{K}^{(b)}_*(n), \mathbf{w}^* \leftarrow \mathbf{w}^*(n);$
7 $n \leftarrow n+1;$
8 $ \mathcal{K}^{(b)}(n) \leftarrow \mathcal{K}^{(b)}(n-1) \cup \{k o_k = n\}; $
9 Update interference pattern and load constraints
with (17) and (18);
10 Compute $c^*(n), \mathbf{w}^*(n)$ corresponding to $\mathcal{K}^{(b)}(n)$
with fixed point iteration (3);
11 else if $\theta^{(trans)} \geq 1$ then
12 $\qquad \mathcal{K}^{(b)}_* \leftarrow \mathcal{K}^{(b)}_*(0), \mathbf{w}^* \leftarrow \mathbf{w}^*(0)$

be applied to solve Problem (1). Algorithm 1 summarizes the proposed resource muting mechanism.

D. Numerical Results

We consider a real-world scenario with 15 three-sector macro BSs and 10 pico BSs in the city center of Berlin, Germany. The locations of the macro BSs are given by the real data set [17], while the pico BSs are placed uniformly at random in the playground. The macro BSs are equipped with directional antennae with transmit power of 43dBm, while the pico BSs have omnidirectional antennae with transmit power of 30dBm. The total bandwidth is 10 MHz. The macrocell pathloss is obtained from the real data set [17], and the picocell pathloss uses the 3GPP LTE model in [18]. Uncorrelated fast fading characterized by Rayleigh distribution is implemented on top of the pathloss. A fixed number of users are randomly and uniformly distributed on the playground. The traffic demand per user is uniformly distributed between [0, 10] MBit/s with an average value of 5 MBit/s.

1) Resource Efficiency with or without Resource Muting: Fig. 4 illustrates how the transition point of the utility and the resource efficiency change when the resource muting scheme is activated. Fig. 4a shows that the resource efficient region increases (or, equivalently, the interference-limited region decreases) when applying resource muting. The achieved utility $U(\theta)$ and resource efficiency $E(\theta)$ with the muting scheme are significantly higher at $\theta = 1$ (with the load constraint (18)).

2) Efficient Selection of Bottleneck Users: To show that \mathbf{w}_{∞} is useful to identify bottleneck users, we use Monte Carlo simulations to generate random locations and demands of users, and we compare the number of bottleneck users $|\mathcal{K}^{(b)}|$ selected by *exhaustive search* and by *successive selection* as described in Section III-C3. In the upper subfigure of Fig. 5, 500 points of $\left(\left|\mathcal{K}^{(b)}_{(exh)}\right|, \left|\mathcal{K}^{(b)}_{(suc)}\right|\right)$ are plotted (note that there are overlapping points), with each corresponding to a distribution of user locations and traffic demands, where $\mathcal{K}^{(b)}_{(exh)}$ and $\mathcal{K}^{(b)}_{(suc)}$ denote the set of bottleneck users selected by *exhaustive search* and by *successive selection*, respectively. The lower subfigure shows the empirical probability density of the number of bottleneck users. Although Fig. 5 shows that *successive selection* generally selects less members than *exhaustive search*, it is shown in Fig. 6 that *successive selection*.

3) Performance Improvement: In Fig. 6 we compare the performance of four protocols for K = 200 users and K = 400 users. The performance is characterized by the cumulative distribution function (CDF) of the achievable utility derived from 1000 Monte Carlo simulations with random distribution of user locations and traffic demands. The four protocols are described below:

- (I) *Non-muting*: We solve Problem (13) with $\theta = 1$ by applying the fixed point iteration in (3).
- (II) Muting based on \mathbf{w}_{∞} , successive selection: We solve Problem (13) with Algorithm 1. The interference coupling and the load constraints are modified according to (17) and (18), respectively.
- (III) Muting based on \mathbf{w}_{∞} , exhaustive search: Similar to Algorithm 1, but the bottleneck users are selected by exhaustive search.
- (IV) Muting based on I, successive search: Similar to Algorithm 1, but the bottleneck users are selected successively based on a different indicator $I_k :=$ $\sum_{l \neq k} (p_l v_{k,l} w_l^* + p_k v_{l,k} w_k^*)$, which reflects the sum of the received and generated interference by user k.

Comparing the performance of Protocol I with the other muting schemes in Fig. 6, we observe that the muting schemes significantly improve the desired utility. Comparing Protocol II and III, we verify that successive selection performs only slightly worse than exhaustive search, but the computational effort is significantly reduced. Furthermore, comparing Protocol III and IV, we verify that \mathbf{w}_{∞} is a more appropriate indicator for bottleneck selection than the indicator based on interference measurements.

IV. EXTENSION OF THE FRAMEWORK TO 5G NETWORKS

The framework can be extended to optimize 5G networks equipped with flexible duplex mechanisms. Flexible duplex





(b) Log-log plot of resource efficiency depending on θ .

Fig. 4: Comparison between the performance limits of nonmuting scheme and muting scheme, K = 200.



Fig. 5: Number of bottleneck users selected by exhaustive searching and by successive selection.

is one of the key technologies in 5G to adapt to asymmetric uplink (UL) and DL traffic with flexible resource allocation in joint time-frequency domain. The main challenge is the newly introduced inter-mode interference (IMI) between UL and DL as shown in Fig.7, which makes Problem (13) far more complex. In previous work [14] we have shown how to incorporate IMI into the interference model, and we have developed a novel algorithm called successive approximation of fixed point (SAFP) to approximate the solution to (13). In this study we briefly describe the interference model and the SAFP algorithm, and we put more focus on a new resource



Fig. 6: Performance comparison between max-min utility fairness with and without muting for K = 200 and 400.

muting scheme based on the spectral properties of asymptotic mappings.

A. Joint UL/DL System Model

We now generalize the DL system model defined in Section III-A by considering a set of services \mathcal{K} , including both UL and DL services. As in previous sections, let the BS-to-service assignment be denoted by $\mathbf{A} \in \{0,1\}^{N \times K}$. Without resource muting, the utility max-min fairness problem can also be written in the general form (13). However, the SINR model needs to be modified to take IMI into account. As shown in Fig. 7, inter-cell interference appears in the overlapping resource region. We introduce the overlapping factors $(c_{k,l})$, collected in $\mathbf{C}(\mathbf{w}) \in \mathbb{R}_{++}^{K \times K}$, to incorporate the probability that intra- and inter-mode interference appear for a given resource allocation \mathbf{w} . Let $\boldsymbol{\nu}^{(x)} := \left[\nu_1^{(x)}, \ldots, \nu_N^{(x)}\right]$, $x \in \{\mathbf{u}, \mathbf{d}\}$ denote the UL or DL load (i.e., the fraction of occupied resources) of all cells that can be computed with \mathbf{w} . For example, $\nu_n^{(\mathbf{u})} \subseteq \mathcal{K}_n$ is the set of UL services in BS n. The SINR of service k incorporating IMI is approximated by

$$\operatorname{SINR}_{k}(\mathbf{w}) \approx \frac{p_{k}}{\left[\left(\mathbf{C}(\mathbf{w}) \circ \tilde{\mathbf{V}}\right) \operatorname{diag}(\mathbf{p})\mathbf{w} + \tilde{\boldsymbol{\sigma}}\right]_{k}}$$
(19)

with $\mathbf{C}(\mathbf{w}) := (c_{k,l}) \in \mathbb{R}^{K \times K}_+$,

$$c_{k,l} := \begin{cases} \left[\left(\nu_{n_l}^{(x_l)} + \nu_{n_k}^{(x_k)} - 1 \right) / \nu_{n_k}^{(x_k)} \right]^+ & \text{if } x_l \neq x_k \\ \min\left\{ 1, \nu_{n_l}^{(x_l)} / \nu_{n_k}^{(x_k)} \right\} & \text{if } x_l = x_k \end{cases}$$

where \circ denotes the Hadamard product, $x_k \in \{u, d\}$ denotes the duplex mode of service k, and n_k denotes the serving BS of k. The approximation is based on two probabilities:

• The overlapping factor $c_{k,l}$ is the ratio of the overlapping area between load $\nu_{n_l}^{(x_l)}$ and $\nu_{n_k}^{(x_k)}$ to the load $\nu_{n_k}^{(x_k)}$, which roughly indicates the probability that an arbitrary resource unit allocated to mode x_l in BS n_l causes interference to BS n_k in mode x_k . Note that the first case $x_l \neq x_k$ corresponds to the IMI between UL and DL, while $x_l = x_k$ corresponds to the intra-mode interference. • w_l collected in w serves as the probability that an arbitrary resource unit in n_l is allocated to service l.

The multiplication of $c_{k,l}$ by w_l loosely approximates the probability that a resource unit allocated to l in duplex mode x_l in BS n_l causes interference to service k in duplex mode x_k in BS n_k .

B. Successive Approximation of Fixed Point

Introducing C into (20) removes the properties of T given in Lemma 1, which further leads to possibly more than one fixed point of the resulting CEVP (2). In [14] we developed a novel algorithm SAFP to approximate the near-optimal fixed point of the CEVP. The novel proposed algorithm assisted with *random initialization* and *successive approximation* is summarized below:

- The algorithm runs for $Z^{(\max)}$ times, where at the *i*-th time different random initializations of $\mathbf{w}^{(0)} := \hat{\mathbf{w}}_i$ and the corresponding $\hat{\mathbf{C}} := \mathbf{C}(\hat{\mathbf{w}}_i)$ are used.
- For each initialization, we iteratively perform the following two steps: 1) we use the fixed point iteration (3) with respect to the approximated $\hat{\mathbf{C}}$ and derive $\mathbf{w}^{(t)}$, and 2) update $\hat{\mathbf{C}} = \mathbf{C}(\mathbf{w}^{(t)})$ and increment *t*. The iteration stops if $\|\mathbf{w}^{(t)} - \mathbf{w}^{(t-1)}\| \le \epsilon$, where ϵ is a distance threshold.
- Each random initialization converges to a fixed point (not necessarily different from those derived from other initializations). We choose the solution with the maximum utility.

As shown in [14], the algorithm converges for each random initialization. With a limited number of random initializations, the algorithm is able to find a solution with low computational complexity.

C. Partial Resource Muting

(20)

We proposed partial resource muting scheme for 4G DL wireless networks in Section III-C to improve the resource efficiency. Along similar lines, the resource muting scheme can be tailored for IMI mitigation in 5G networks enabling flexible duplex. To deal with the complex interference caused by IMI, we have introduced SAFP as the tool to find efficiently the approximated solution to the CEVP with the existence of possibly multiple fixed points. Hence, by replacing the fixed point iteration with SAFP (line 10 in Algorithm 1), the same steps proposed in Algorithm 1 can be applied to optimize the joint UL/DL resource allocation, particularly the set of bottleneck users allocated to the muting region and the resource fraction allocated to them.

It is worth mentioning that the steps used for triggering the muting scheme and for detecting the bottleneck users require the computation of the transition point $\theta^{(\text{trans})}$ and of the tuple $(\lambda_{\infty}, \mathbf{w}_{\infty})$. However, with the modified SINR model (19), $\mathbf{T}(\mathbf{w})$ is not an SIF, and Proposition 3 cannot be applied in a straightforward manner to compute the desired values. Therefore, we propose to approximate $\mathbf{T}(\mathbf{w})$ by replacing $\mathbf{C}(\mathbf{w})$ with the converged approximation of $\hat{\mathbf{C}}$ achieved by SAFP. Let the approximation of $\mathbf{T}(\mathbf{w})$ be denoted by $\mathbf{T}_{\hat{\mathbf{C}}}(\mathbf{w})$. Since $\hat{\mathbf{C}}$ is a matrix with known entries, $\mathbf{T}_{\hat{\mathbf{C}}}(\mathbf{w})$ is an SIF.



Fig. 7: Regions where inter-cell interference appears.

Hence, we can use Proposition 1 to compute $\mathbf{T}_{\hat{\mathbf{C}},\infty}(\mathbf{w})$, and further derive $(\lambda_{\hat{\mathbf{C}},\infty}, \mathbf{w}_{\hat{\mathbf{C}},\infty})$ with respect to $\mathbf{T}_{\hat{\mathbf{C}},\infty}(\mathbf{w})$ along similar lines to Proposition 3. Using the same technique described in Section III-C, we obtain the triggering parameter $\theta^{(trans)}$ and the set of bottleneck users from $(\lambda_{\hat{\mathbf{C}},\infty}, \mathbf{w}_{\hat{\mathbf{C}},\infty})$.

D. Numerical Results

We consider the same network introduced in Section III-D. In addition, flexible duplex is enabled such that resources can be dynamically allocated to UL or DL services. Owing to the space limitation, we only present one exemplary numerical result in Fig. 8. We compare the performance of the following four protocols: (I)"FIX" for fixed ratio between UL and DL resources; (II)"SAFP" for dynamic UL/DL resource configuration with SAFP without muting; (III)"Resource muting based on I" for the resource muting scheme based on interference indicator I as introduced in Section III-D, and (IV)"Resource muting based on \mathbf{w}_∞ " for the resource muting scheme based on the asymptotic behavior \mathbf{w}_{∞} . The performance is obtained by averaging 1000 Monte Carlo simulations with random distribution of user locations and traffic demands conditioned on the given traffic asymmetry for K = 200. We define a measure called inter-cell traffic distance to reflect both the inter-cell traffic asymmetry and the intra-cell UL/DL traffic asymmetry between a pair of cells m, n, computed as $D_{m,n} :=$ $\|\mathbf{d}_n - \mathbf{d}_m\|_2$, where $\|\cdot\|_2$ denotes the L^2 -norm, and the per-cell UL/DL traffic demands $\mathbf{d}_n := \begin{bmatrix} d_n^{(u)}, d_n^{(d)} \end{bmatrix}$ are normalized. Fig. 8 shows that dynamic UL/DL configuration achieves a twofold increase in the average utility compared to fixed UL/DL ratio. Resource muting brings further improvement, varying from 20% to 100%, by adapting to the traffic asymmetry. Note that the performance gains increase with the traffic asymmetry.

V. CONCLUSION

We have characterized an efficient resource utilization region by studying the asymptotic behavior of solutions to maxmin utility optimization problems with interference models based on the load imposed by services. Building upon this result, we have developed a partial resource muting scheme that is suitable for both conventional 4G DL networks and flexible duplex enabled 5G networks. Simulations show significant performance gains in both scenarios, measured by the worstcase QoS satisfaction level, compared to the optimal solution to the max-min optimization problem without resource muting mechanisms. We have also shown that the gain obtained with the proposed muting scheme increases when the traffic becomes highly asymmetric.



Fig. 8: Average utility with different inter-cell traffic distances.

REFERENCES

- 3GPP, "TR 36.133, Requirements for support of radio resource management, Rel-14," http://www.3gpp.org, Oct. 2016.
- [2] G. Hegde, O. D. Ramos-Cantor, Y. Cheng, and M. Pesavento, "Optimal resource block allocation and muting in heterogeneous networks," in *ICASSP*. IEEE, 2016, pp. 3581–3585.
- [3] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on selected areas in communications*, vol. 13, no. 7, pp. 1341–1347, 1995.
- [4] R. D. Yates and C.-Y. Huang, "Integrated power control and base station assignment," *IEEE Trans. on Veh. Technol.*, vol. 44, no. 3, pp. 638–644, 1995.
- [5] N. Vucic and M. Schubert, "Fixed point iteration for max-min SIR balancing with general interference functions," in *ICASSP*. IEEE, 2011, pp. 3456–3459.
- [6] L. Zheng, Y.-W. P. Hong, C. W. Tan, C.-L. Hsieh, and C.-H. Lee, "Wireless max-min utility fairness with general monotonic constraints by Perron-Frobenius theory," *IEEE Trans. on Information Theory*, vol. 62, no. 12, pp. 7283–7298, 2016.
- [7] C. J. Nuzman, "Contraction approach to power control, with nonmonotonic applications," in *GLOBECOM*. IEEE, 2007, pp. 5283–5287.
- [8] C. K. Ho, D. Yuan, L. Lei, and S. Sun, "Power and load coupling in cellular networks for energy optimization," *IEEE Trans. on Wireless Commun.*, vol. 14, no. 1, pp. 509–519, 2015.
- [9] R. L. G. Cavalcante, M. Kasparick, and S. Stańczak, "Max-min utility optimization in load coupled interference networks," *IEEE Trans. on Wireless Commun.*, vol. 16, no. 2, pp. 705–716, 2017.
- [10] R. L. G. Cavalcante and S. Stańczak, "Performance limits of solutions to network utility maximization problems," *arXiv preprint*, 2017. [Online]. Available: http://arxiv.org/abs/1701.06491
- [11] A. Auslender and M. Teboulle, Asymptotic cones and functions in optimization and variational inequalities. Springer Science & Business Media, 2006.
- [12] P. Mogensen *et al.*, "LTE capacity compared to the Shannon bound," in 65th VTC-Spring. IEEE, 2007, pp. 1234–1238.
- [13] R. L. Cavalcante, S. Stanczak, M. Schubert, A. Eisenblätter, and U. Türke, "Toward energy-efficient 5G wireless communications technologies: Tools for decoupling the scaling of networks from the growth of operating power," *IEEE Signal Processing Magazine*, vol. 31, no. 6, pp. 24–34, 2014.
- [14] Q. Liao, "Dynamic uplink/downlink resource management in flexible duplex-enabled wireless networks," in *IEEE ICC Workshop on 5G RAN Design*, 2017.
- [15] C. D. Meyer, Matrix analysis and applied linear algebra. SIAM, 2000.
- [16] R. L. Cavalcante, Y. Shen, and S. Stańczak, "Elementary properties of positive concave mappings with applications to network planning and optimization," *IEEE Trans. on Signal Process.*, vol. 64, no. 7, pp. 1774– 1783, 2016.
- [17] MOMENTUM, "Models and simulations for network planning and control of UMTS," http://momentum.zib.de/, 7 2004.
- [18] 3GPP, "TR 36.814, Further advancements for E-UTRA physical layer aspects, Rel-9," http://www.3gpp.org, Mar. 2010.