Optimal Packet Scheduling in a Multiple Access Channel with Energy Harvesting Transmitters

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Abstract: **In this paper, we investigate the optimal packet scheduling problem in a two-user multiple access communication system, where the transmitters are able to harvest energy from the nature. Under a deterministic system setting, we assume that the energy harvesting times and harvested energy amounts are known before the transmission starts. For the packet arrivals, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. Our goal is to minimize the time by which all packets from both users are delivered to the destination through controlling the transmission powers and transmission rates of both users. We first develop a *generalized iterative backward waterfilling* algorithm to characterize the maximum departure region of the transmitters for any given deadline T. Then, based on the sequence of maximum departure regions at energy arrival instants, we decompose the transmission completion time minimization problem into convex optimization problems and solve the overall problem efficiently.**

Index Terms: **Energy-harvesting communications, iterative backward waterfilling, multi-access channel, throughput maximization.**

I. INTRODUCTION

Efficient energy management is crucial for wireless communication systems, as it increases the throughput and improves the delay. Energy efficient scheduling policies have been well investigated in traditional battery powered (unrechargeable) systems [1]–[6]. On the other hand, there exist systems where the transmitters are able to harvest energy from the nature. Such energy harvesting abilities make sustainable and environmentally friendly deployment of communication systems possible. This renewable energy supply feature also necessitates a completely different approach to energy management.

In this work, we consider a multi-user rechargeable wireless communication system, where data packets as well as the harvested energy arrive at the transmitters as random processes in time. As shown in Fig. 1, we consider a two-user multiple access channel, where each transmitter node has two queues. The data queue stores the data arrivals, while the energy queue stores the energy harvested from the environment. Our objective is to adaptively change the transmission rate and power according to the instantaneous data and energy queue sizes, such that the *transmission completion time* is minimized.

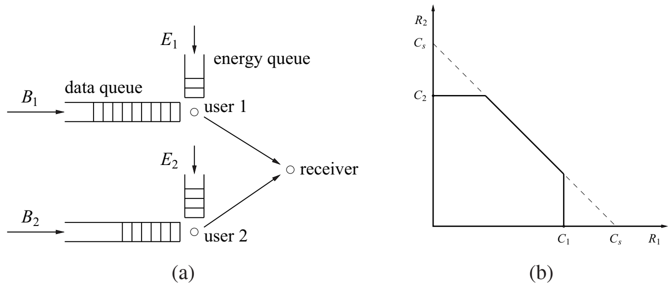

Fig. 1. (a) An energy harvesting multiple access channel model with energy and data queues and (b) the capacity region of the additive white Gaussian noise multiple access channel.

In general, the arrival processes for the data and the harvested energy can be formulated as stochastic processes, and the problem requires an *on-line* solution that adapts transmission power and rate in *real-time*. This seems to be an intractable problem for now. We simplify the problem by assuming that the data packets and energy will arrive in a deterministic fashion, and we aim to develop an *off-line* solution instead. In this paper, we consider the scenario where packets have already arrived before the transmissions start. Specifically, we consider two nodes as shown in Fig. 2. For the traffic load, we assume that there are a total of B_1 bits and B_2 bits available at the first and second transmitter, respectively, at time $t = 0$. We assume that energy arrives (is harvested) at points in time marked with $\circ$. In Fig. 2, E_{1k} denotes the amount of energy harvested for the first user at time s_k. Similarly, E_{2k} denotes the amount of energy harvested for the second user at time s_k. If there is no energy arrival at one of the nodes, we simply let the corresponding amount be zero, which are denoted by the dotted arrows in Fig. 2. Our goal then is to develop methods of transmission to minimize the time, T, by which all of the data packets from both of the nodes are delivered to the destination.

The optimal packet scheduling problem in a single-user energy harvesting communication system is investigated in [8] and [9]. In [8] and [9], we prove that the optimal scheduling policy has a "majorization" structure, in that, the transmit power is kept constant between energy harvests, the sequence of transmit powers increases monotonically, and only changes at some of the energy harvesting instances; when the transmit power changes, the energy constraint is tight, i.e., the total consumed energy equals the total harvested energy. In [8] and [9], we develop an algorithm to obtain the optimal off-line scheduling policy based on these properties. Reference [10] extends [8] and [9] to the case where rechargeable batteries have finite sizes. We extend [8]–[10] in [11] to a fading channel. We solve the transmission

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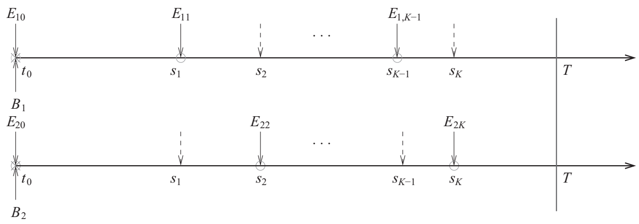

Fig. 2. System model with all packets available at the beginning. Energies arrive at points denoted by $\circ$.

completion time minimization problem in a two-user broadcast channel, independently and concurrently with [12]. Both works assume that the transmitter battery size is unlimited. In [13] we extend these works to the case of a transmitter with a finite capacity rechargeable battery. In the two-user multiple access channel setting studied in this paper, the scheduling problem is significantly more complicated. This is because the two users interfere with each other, and we need to select the transmission powers for both users as well as the rates from the resulting rate region, to solve the problem. In addition, because the traffic load and the harvested energy for both users may not be well-balanced, the final transmission durations for the two users may not be the same, further complicating the problem.

We first investigate a problem which is *dual* to the transmission completion time minimization problem. In this dual problem, we aim to characterize the maximum number of bits both users can transmit for any given time T. These two problems are *dual* to each other in the sense that, if (B_1, B_2) lies on the boundary of the maximum departure region for time T^*, then, T^* must be the solution to the transmission completion time minimization problem with initial number of bits (B_1, B_2). We propose a *generalized iterative backward waterfilling* algorithm to achieve the boundary points of the maximum departure region for any given time T. Then, based on the solution of this dual problem, we go back to the transmission completion time minimization problem, simplify it into standard convex optimization problems, and solve it efficiently. In particular, we first characterize the maximum departure region for every energy arrival instant, and based on the location of the given (B_1, B_2) on the maximum departure region, we narrow down the range of the minimum transmission completion time to be between two consecutive epochs. Based on this information, we propose to solve the problem in two steps. In the first step, we solve for the optimal power policy sequences to achieve the minimum T, so that (B_1, B_2) is on the maximum departure region for this T. This step can be formulated as a convex optimization problem. Then, with the optimal power policy obtained in the first step, we search for the optimal rate policy sequences from the capacity regions defined by the power sequences to finish B_1, B_2 bits. The second step is formulated as a linear programming problem. In addition, we further simplify the problem by exploiting the optimal structural properties for two special scenarios.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system model is as shown in Figs. 1 and 2. As shown in Fig. 1, each user has a data queue and an energy queue. The physical layer is modeled as an additive white Gaussian noise channel, where the received signal is

$$Y = X_1 + X_2 + Z \tag{1}$$

where X_i is the signal of user i, and Z is a Gaussian noise with zero-mean and unit-variance. The capacity region for this two-user multiple access channel is [15]

$$R_1 \leq f(P_1) \tag{2}$$

$$R_2 \leq f(P_2) \tag{3}$$

$$R_1 + R_2 \leq f(P_1 + P_2) \tag{4}$$

where $f(p) = \frac{1}{2}\log(1+p)$. We denote the region defined by these inequalities above as $\mathcal{C}(P_1, P_2)$. This region is shown on the right figure in Fig. 1.

As shown in Fig. 2, user i has B_i bits to transmit which are available at transmitter i at time $t = 0$. Energy is harvested at times s_k with amounts E_{ik} at transmitter i. Our goal is to solve for the transmit power sequence, the rate sequence, and the corresponding duration sequence that minimize the time, T, by which all of the bits are delivered to the destination.

We assume that the transmitters can adapt their transmit powers and rates according to the available energy level and number of bits remaining. The energy consumed must satisfy the causality constraints, i.e., for each user, the total amount of energy consumed up to time t must be less than or equal to the total amount of energy harvested up to time t by that user.

Let us denote the transmit power for the first and second user at time t as $p_1(t)$ and $p_2(t)$, respectively. Then, the transmission rate pair $(r_1(t), r_2(t))$ must be within the capacity region defined by $p_1(t)$ and $p_2(t)$, i.e., $\mathcal{C}(p_1, p_2)(t)$. For user i, $i = 1, 2$, the energy consumed up to time t, denoted as $E_i(t)$, and the total number of bits departed up to time t, denoted as $B_i(t)$, can be written as:

$$E_i(t) = \int_0^t p_i(\tau)d\tau, \quad B_i(t) = \int_0^t r_i(\tau)d\tau, \quad i = 1, 2. \tag{5}$$

Here, r_i and powers p_i are related through the f function as shown in (2)–(4). Then, the transmission completion time minimization problem can be formulated as:

$$\begin{aligned} \min_{p_1,p_2,r_1,r_2} \quad & T \\ \text{s.t.} \quad & E_1(t) \leq \sum_{n:s_n<t} E_{1n}, \quad 0 \leq t \leq T, \\ & E_2(t) \leq \sum_{n:s_n<t} E_{2n}, \quad 0 \leq t \leq T \\ & B_1(T) \geq B_1, \quad B_2(T) \geq B_2 \\ & (r_1, r_2)(t) \in \mathcal{C}(p_1, p_2)(t),\ 0 \leq t \leq T. \end{aligned} \tag{6}$$

We first investigate a problem which is *dual* to this transmission completion time minimization problem. Specifically, we aim to characterize the *maximum departure region*, which is the region of (B_1, B_2) the transmitters can depart within a deadline T. Based on the solution for this dual problem, we will go back and decompose the original transmission completion time minimization problem into convex optimization problems, and solve the overall problem in an efficient way.

III. CHARACTERIZING $\mathcal{D}(T)$: LARGEST (B_1, B_2) REGION FOR A GIVEN DEADLINE T

In this section, our goal is to characterize the *maximum departure region* for a given deadline T. We define the maximum departure region as follows.

Definition 1: For any fixed transmission duration T, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of (B_1, B_2) under any feasible power and rate allocation policy over the duration $[0, T)$.

We call any policy which achieves the boundary of $\mathcal{D}(T)$ to be optimal.

Lemma 1: Under the optimal policy, the transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes at an energy harvesting epoch.

Proof: Without loss of generality, we assume that one of the transmitters changes its transmission power between two energy harvesting instances s_i, s_{i+1}. Denote the instant when the rate changes as s_i', as shown in Fig. 3. Denote the transmit powers for the first and second user over those two consecutive epochs as p_{1n}, $p_{1,n+1}$, and p_{2n}, $p_{2,n+1}$, respectively. Now, consider the duration $[s_i, s_{i+1})$. We equalize the transmit power of both users by letting

$$p_1' = \frac{p_{1n}(s_i' - s_i) + p_{1,n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}$$
$$p_2' = \frac{p_{2n}(s_i' - s_i) + p_{2,n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}$$

If only one user changes its power, then we may have either $p_1' = p_{1n}$ or $p_2' = p_{2n}$. It is easy to check that the energy constraints are satisfied under this new power allocation policy, thus this new policy is feasible. On the other hand, the total number of bits departed over this duration under this new policy is a pentagon bounded by

$$\begin{aligned} f(p_1')(s_{i+1} - s_i) &\geq f(p_{1n})(s_i' - s_i) \\ &\quad + f(p_{1,n+1})(s_{i+1} - s_i') \\ f(p_2')(s_{i+1} - s_i) &\geq f(p_{2n})(s_i' - s_i) \\ &\quad + f(p_{2,n+1})(s_{i+1} - s_i') \\ f(p_1' + p_2')(s_{i+1} - s_i) &> f(p_{1n} + p_{2n})(s_i' - s_i) \\ &\quad + f(p_{1,n+1}, p_{2,n+1})(s_{i+1} - s_i') \end{aligned}$$

where the inequality follows from the fact that $f(p)$ is strictly concave in p. We note that the right hand side of these inequalities characterizes the boundary of the departure region under the original policy over $[s_i, s_{i+1})$. Therefore, the departure region under the original policy is strictly inside the departure region under the new policy, which conflicts with the optimality of the original policy. □

Therefore, in the following, we only consider policies where the rates are constant between any two consecutive energy arrivals. In order to simplify the notation, in this section, for any given T, we assume that there are $N-1$ energy arrival instants (excluding $t = 0$) over $(0, T)$. We denote the last energy arrival instant before T as s_{N-1}, and $s_N = T$. We call the duration between energy arrival instants *epochs*, and denote the lengths of

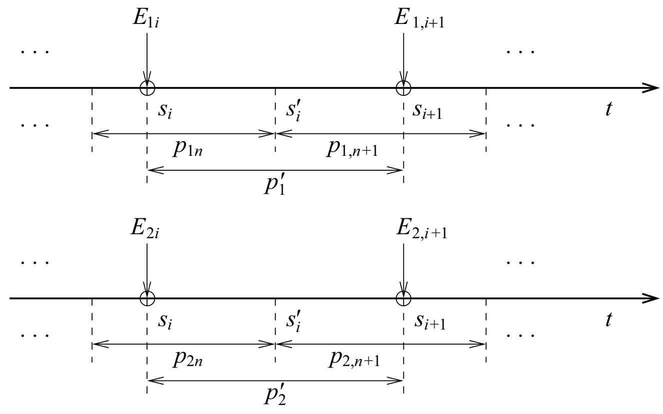

Fig. 3. The power/rate must remain constant between energy harvests.

the epochs with l_n, i.e., $l_n = s_n - s_{n-1}$. Let us define (p_{1n}, p_{2n}) to be the transmit power over $[s_{n-1}, s_n)$.

Lemma 2: For any feasible transmit power sequences $\mathbf{p_1}, \mathbf{p_2}$ over $[0, T)$, the total number of bits departed from both of the users, denoted as B_1 and B_2, is a pentagon defined as

$$B_1 \leq \sum_{n=1}^{N} f(p_{1n}) l_n, \tag{7}$$
$$B_2 \leq \sum_{n=1}^{N} f(p_{2n}) l_n, \tag{8}$$
$$B_1 + B_2 \leq \sum_{n=1}^{N} f(p_{1n} + p_{2n}) l_n. \tag{9}$$

Proof: First we note that the maximum departure region over the first epoch, $\mathcal{D}(l_1)$, is the capacity region $\mathcal{C}(p_{11}, p_{21})$ scaled by the length of the first epoch, l_1. Similarly, the maximum departure region over the second epoch is the capacity region $\mathcal{C}(p_{12}, p_{22})$ scaled by l_{n+1}, denoted as $\mathcal{C}(p_{12}, p_{22})l_2$. Then, we consider the maximum departure region over the first two epochs, i.e., $\mathcal{D}(l_1 + l_2)$. Starting with any point on the boundary of $\mathcal{D}(l_1)$, the feasible departure region is formed by shifting the origin of $\mathcal{C}(p_{12}, p_{22})l_2$ to that boundary point; see Fig. 4. The union of these regions forms a larger pentagon, and the boundary is defined by

$$\begin{aligned} B_1 &\leq f(p_{11})l_1 + f(p_{12})l_2, \\ B_2 &\leq f(p_{21})l_n + f(p_{22})l_2, \\ B_1 + B_2 &\leq f(p_{11} + p_{21})l_1 + f(p_{12} + p_{22})l_2. \end{aligned}$$

The proof of this lemma is completed by applying this argument recursively. □

Lemma 3: $\mathcal{D}(T)$ is a convex region.

Proof: Consider two power policies $(\mathbf{p_1}, \mathbf{p_2})$ and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$ over $[0, T)$. We consider the scenario that the departure region under one power policy is not strictly inside the departure region under the other power policy. Each region is a pentagon as defined in Lemma 2. Without loss of generality, we assume that

$$\sum_{n=1}^{N} f(p_{2n}) l_n > \sum_{n=1}^{N} f(\bar{p}_{2n}) l_n, \tag{10}$$

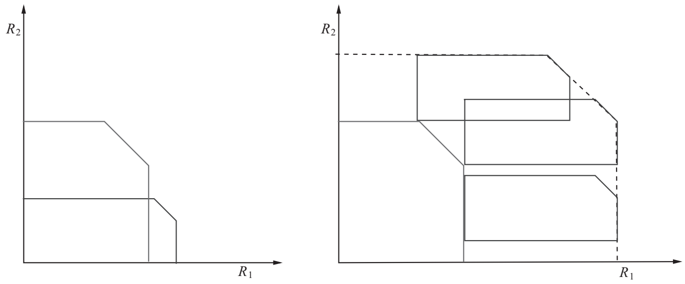

Fig. 4. The maximum departure region over the first two epochs.

$$\sum_{n=1}^{N} f(p_{1n}+p_{2n})l_n \leq \sum_{n=1}^{N} f(\bar{p}_{1n}+\bar{p}_{2n})l_n. \tag{11}$$

Let us construct a new policy as a linear combination of these two policies over $[0,T)$, i.e., $\mathbf{p_i'} = \lambda\mathbf{p_i} + (\mathbf{1}-\lambda)\bar{\mathbf{p}}_\mathbf{i}$, $i=1,2, 0<\lambda<1$. It is straightforward to check that the energy constraints are still satisfied, thus the new policy is feasible. Consider the upper corner points of the departure region under the policies $(\mathbf{p_1}, \mathbf{p_2})$ and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$. Because of the concavity of $f(p)$ in p, we have

$$\sum_{n=1}^{N} f(p'_{2n})l_n > \lambda\sum_{n=1}^{N} f(p_{2n})l_n + (1-\lambda)\sum_{n=1}^{N} f(\bar{p}_{2n})l_n,$$

$$\sum_{n=1}^{N} f(p'_{1n}+p'_{2n})l_n > \lambda\sum_{n=1}^{N} f(p_{1n}+p_{2n})l_n,$$

$$+ (1-\lambda)\sum_{n=1}^{N} f(\bar{p}_{1n}+\bar{p}_{2n})l_n,$$

i.e., the upper corner point of the departure region under the new policy is always above the line connecting these two upper corner points under policies $(\mathbf{p_1}, \mathbf{p_2})$ and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$. Therefore, the union of (B_1, B_2) over all feasible power allocation policies is a convex region. □

Lemma 4: For any $T' > T$, $\mathcal{D}(T)$ is strictly inside $\mathcal{D}(T')$.

Proof: For any policy achieving the boundary point of $\mathcal{D}(T)$, let us fix the power sequence for one user, and change the transmit power of the other user by removing part of its energy consumed before T and spending it over the duration $[T, T')$. Since there is no interference over $[T, T')$, the departures for the user can be potentially improved. Likewise, since some of the interference is removed, the departures for the other user can be potentially improved also. Therefore, $\mathcal{D}(T)$ must be strictly inside $\mathcal{D}(T')$. □

As a first step, we aim to explicitly characterize $\mathcal{D}(T)$ for any given T. Similar to the capacity region of the fading Gaussian multiple access channel [16], where each boundary point is a solution to $\max_{\mathbf{R}\in\mathcal{C}} \boldsymbol{\mu}\mathbf{R}$, here, in our problem, the boundary points also maximize $\boldsymbol{\mu}\mathbf{B}$ for some $\boldsymbol{\mu}$. First, let us examine three different cases separately.

A. $\mu_1 = \mu_2$.

In this subsection, we consider the scenario where $\mu_1 = \mu_2$. Therefore, our problem becomes $\max_{\mathbf{p_1},\mathbf{p_2}} B_1 + B_2$. In [8] and [9], we examined the optimal packet scheduling policy for the single-user scenario. We observe that for any fixed T, the optimal power allocation policy has the "majorization" property. Specifically, we have

$$i_n = \arg\min_{i_{n-1}<i\leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\}, \tag{12}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}}. \tag{13}$$

In this two-user multiple access channel, maximizing the sum of departures is equivalent to maximizing the right hand side of (9), subject to energy causality constraints on both users. We first relax these constraints on each individual user and impose the sum energy constraints on both users instead. Under these constraints, the sum of powers has the same "majorization" property as in the single-user scenario. With the sum power fixed, we can always split the sum power sequence into two individual power sequences, where each individual sequence satisfies its own energy causality constraints. This motivates us to obtain the optimal solution in the following procedure.

First, we merge the energy arrivals from both users, and obtain the sum of energy arrivals as a function of t. We can obtain the optimal sequence of sum of transmit powers, $p_1, p_2, \cdots, p_n$ based on (12) and (13).

The sum of transmit powers and its corresponding duration define $\sum_{n=1}^{N} f(p_n)l_n$. However, we can divide each p_n into p_{1n}, p_{2n} pair in infinitely many ways, such that their sums equal p_n for all n. Each feasible sequence of p_{1n} and p_{2n} gives a feasible region of (B_1, B_2), which is a pentagon. The dominant faces of all of these pentagons are on the same line. Therefore, the union of these pentagons is a larger pentagon. We need to identify the boundary of this larger pentagon, i.e., the end points of its dominant face.

With the sum of powers fixed, we want to find feasible power allocations which maximize B_1 and B_2, individually. As we proved for the single-user case, whenever the sum of powers changes, the total amount of energy consumed up to that instance must be equal to the total amount of energy harvested up to that instance. In other words, both users must deplete their energies completely at that moment. This adds additional energy constraints on both users besides the energy casuality constraints.

In order to maximize B_1, we plot the sum of E_{1n} as a function of t in Fig. 5. Then, we equalize the transmit powers of the first user as much as possible with the casuality constraints on energy and the additional energy consumption constraints. This latter constraint requires us to empty the energy queue at given instances s_{i_1}, s_{i_2}, etc. The former constraint requires us to choose the minimum slope among the lines passing through the origin and any other corner point before the next energy emptying epoch [8], [9]. This gives us the sequence of p_{1n}, as shown in Fig. 5. Based on the concavity of the function $f(p)$, we can prove that this policy maximizes B_1 under the constraint that $B_1 + B_2$ is maximized at the same time.

Once p_{1n} is obtained, p_{2n} can be obtained by subtracting p_{1n} from p_n. Since p_n is always feasible in our allocation, the corresponding p_{2n} must be feasible as well. This power allocation

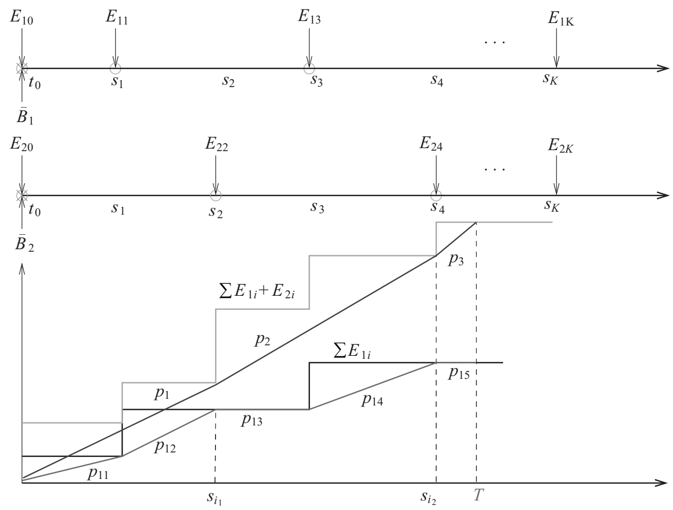

Fig. 5. The total transmit power and the transmit power of the first user.

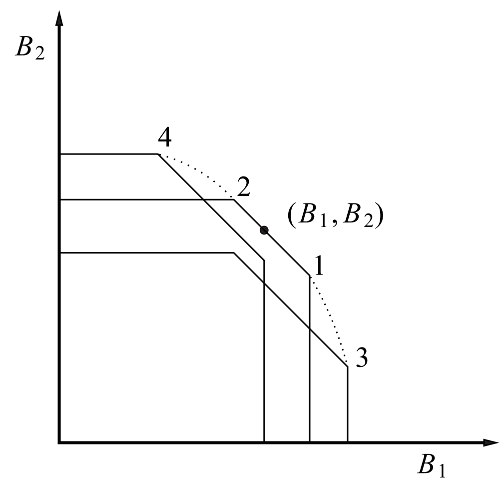

Fig. 6. The departure region $\mathcal{D}(T)$.

defines a pentagon region for (B_1, B_2), where the lower corner point of this pentagon is also the lower point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 1 in Fig. 6. Similarly, we can obtain the upper corner point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 2 in Fig. 6. Since any linear combination of these two policies still achieves the sum rate, any point on the flat part of the dominant face can be achieved. Therefore, the flat part of the dominant face of $\mathcal{D}(T)$ is bounded by these two corner points.

B. $\mu_1 = 0$ or $\mu_2 = 0$.

In this subsection, we aim to maximize the departure from one user only. This procedure is exactly the same as the procedure in the single-user scenario. On top of that, we also want to maximize the departure from the other user. Without loss of generality, we aim to maximize B_1 first. This is a single-user scenario, and the optimal policy can be obtained according to (12) and (13). Given the allocation p^*_{1n}, in order to maximize the departure from the second user, we need to solve the following optimization problem

$$\begin{aligned} \max_{\mathbf{p_2}} \quad & \sum_{n=1}^{N} f(p^*_{1n} + p_{2n}) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{2n} l_n \le \sum_{n=0}^{j-1} E_{2n}, \quad 1 \le j \le N. \end{aligned} \tag{14}$$

Theorem 1: The optimal power allocation for (14) can be found by a *backward waterfilling* process with base water level p^*_{1n} over $[s_{n-1}, s_n)$ for $1 \le n \le N$.

Proof: We note that the constraint in (14) must be satisfied with an equality when $k = N$, otherwise, we can always increase some p_{2n} without conflicting with any other constraint, and the resulting number of departures is thus increased. Based on this observation, (14) can be equivalently expressed as

$$\sum_{n=j}^{N} p_{2n} l_n \ge \sum_{n=j-1}^{N-1} E_{2n}, \quad 1 < j \le N,$$

$$\sum_{n=1}^{N} p_{2n} l_n = \sum_{n=0}^{N-1} E_{2n}.$$

The Lagrangian becomes

$$\begin{aligned} \mathcal{L}(\mathbf{p_2}, \boldsymbol{\lambda}) = & \sum_{n=1}^{N} f(p^*_{1n} + p_{2n}) l_n - \sum_{j=n-1}^{N-1} E_{2j} - \sum_{n=1}^{N} \gamma_n p_{2n} \\ & + \sum_{n=1}^{N} \lambda_n \left(\sum_{j=n}^{N} p_{2j} l_j \right) \end{aligned}$$

where $\lambda_n \ge 0$ when $n > 1$, $\gamma_n \ge 0$, and $\gamma_n p_{2n} = 0$. The optimal solution must satisfy

$$p_{2n} = \left(\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} - p^*_{1n} - 1 \right)^+, n = 1, 2, \cdots, N \tag{15}$$

$1/(\lambda_1 - \sum_{j=1}^{n} \lambda_j)$ can be interpreted as the "water" level over $[s_{n-1}, s_n)$, and $p^*_{1n} + 1$ is the base water level. If $\lambda_n > 0$, no energy flows across the epoch $t = s_{n-1}$, and we have,

$$\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} > \frac{1}{\lambda_1 - \sum_{j=1}^{n-1} \lambda_j}, \tag{16}$$

i.e., the water level over $[s_{n-1}, s_n)$ must be higher than that over $[s_{n-2}, s_{n-1})$.

If $\lambda_n = 0$, energy harvested before flows across the epoch $t = s_{n-1}$, and we have,

$$\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} = \frac{1}{\lambda_1 - \sum_{j=1}^{n-1} \lambda_j}, \tag{17}$$

i.e., the water level over $[s_{n-1}, s_n)$ is equal to that over $[s_{n-2}, s_{n-1})$. Therefore, energy flows across the epoch $t = s_{n-1}$ only when the water level $[s_{n-2}, s_{n-1})$ has the potential to surpass that over $[s_{n-2}, s_n)$, and the energy flow makes the water levels even. A *backward waterfilling* process naturally leads to the optimal power policy. In the backward waterfilling process, we start from $n = N$, fill the energy $E_{2,N-1}$ over

$[s_{N-1}, s_N)$, and get an updated water level as $p_{2N} + p_{1N}^*$; and then, we start to fill energy E_{N-2} over $[s_{N-2}, s_{N-1})$; once the water level exceeds $p_{2N} + p_{1N}^*$, we fill the remaining energy over $[s_{N-2}, s_N)$ until it is depleted. We continue this process until $n = 0$. The difference between the updated water level and the base water level gives us $\mathbf{p_2}$. □

The *backward waterfilling* procedure is shown in Fig. 7. This power allocation defines another pentagon, and its lower corner point maximizes B_1, which is point 3 in Fig. 6. Similarly, we can obtain another pentagon whose upper corner point maximizes B_2, which is point 4 in Fig. 6. In general, points 3 and 4 do not coincide with points 1 and 2, respectively, and consequently, there are curved parts connecting these corner points.

C. General $\mu_1, \mu_2 > 0$.

The curved parts can be characterized through the solution of $\max_{\mathbf{B}\in\mathcal{D}(\mathbf{T})} \boldsymbol{\mu}\mathbf{B}$ for some $\boldsymbol{\mu} > \mathbf{0}$. Since each boundary point corresponds to a corner point on some pentagon, for $\mu_1 > \mu_2$, we need to solve the following problem:

$$\begin{aligned} \max_{\mathbf{p_1},\mathbf{p_2}} \quad & (\mu_1 - \mu_2)\sum_n f(p_{1n})l_n + \mu_2 \sum_n f(p_{1n} + p_{2n})l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \le N \\ & \sum_{n=1}^{j} p_{2n}l_n \le \sum_{n=0}^{j-1} E_{2n}, \quad \forall j : 0 < j \le N. \end{aligned} \tag{18}$$

The problem in (18) is a convex optimization problem with linear constraints, therefore, the unique global solution satisfies the extended KKT conditions as follows:

$$\frac{\mu_1 - \mu_2}{1 + p_{1n}} + \frac{\mu_2}{1 + p_{1n} + p_{2n}} \le \sum_{j=n}^{N} \lambda_j, \quad 1 \le n \le N \tag{19}$$

$$\frac{\mu_2}{1 + p_{1n} + p_{2n}} \le \sum_{j=n}^{N} \beta_j, \quad 1 \le n \le N \tag{20}$$

where the conditions in (19) and (20) are satisfied with equality if $p_{1n}, p_{1n} > 0$. When $\mu_1 \neq \mu_2$, it is difficult to obtain the optimal policy explicitly from the KKT conditions. Therefore, we adopt the idea of *generalized iterative waterfilling* in [14] to find the optimal policy.

Specifically, given the power allocation of the second user, denoted as $\mathbf{p_2^*}$, we optimize the power allocation of the first user, i.e., we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p_1}} \quad & (\mu_1 - \mu_2)\sum_{n=1}^{N} f(p_{1n})l_n + \mu_2 \sum_{n=1}^{N} f(p_{1n} + p_{2n}^*)l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \le N. \end{aligned} \tag{21}$$

Once the power allocation of the first user is obtained, denoted as $\mathbf{p_1^*}$, we do a *backward waterfilling* for the second user to obtain its optimal power allocation. We perform the optimization for both users in an alternating way. Because of the concavity of the objective function and the Cartesian product form of the convex constraint set, it can be shown that the iterative algorithm converges to the global optimal solution [17].

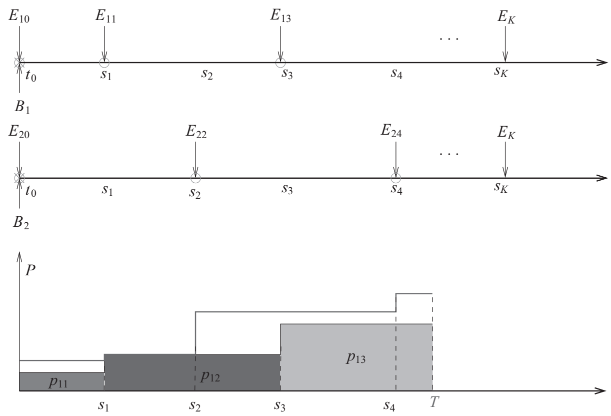

Fig. 7. The optimal transmit power for the second user to maximize its departure.

Because there is more than one term in the objective function of (21), the optimal policy for the first user does not have a backward waterfilling interpretation. However, using the method in [14], we can interpret the procedure for the first user as a *generalized backward waterfilling* operation. In order to see that, given $\mathbf{p_2^*}$, we define a generalized water level $b_n(p_{1n})$ as the inverse of the left hand side of (19), i.e.,

$$b_n(p_{1n}) = \left(\frac{\mu_1 - \mu_2}{1 + p_{1n}} + \frac{\mu_2}{1 + p_{1n} + p_{2n}^*}\right)^{-1} \tag{22}$$

and the base water level as $b_n(0)$, which can be seen as the modified interference plus noise level over the duration $[s_{n-1}, s_n)$. We generalize the form of the water level by taking the priority of users into account. Then, the KKT condition for this single-user problem is

$$\frac{1}{b_n(p_{1n})} \le \sum_{j=n}^{N} \tilde{\lambda}_j, \quad n = 1, 2, \cdots, N. \tag{23}$$

We note that $\tilde{\lambda}_j$ in general is different from the Lagrange multiplier λ_j in (19), since p_{2n}^* need not be the optimal $\mathbf{p_2}$. However, because of the convergence of the iterative algorithm, $\tilde{\lambda}_j$ converges to λ_j eventually as well.

Therefore, under the definition of the generalized water level $b_n(p_{1n})$, we can also interpret the optimal solution for the first user as a *generalized backward waterfilling* process. We first fill $E_{1,N-1}$ over the duration $[s_{N-1}, s_N)$, with the base water level $b_N(0)$. This step gives us an updated water level $b_N(E_{1,N-1}/l_N)$. Then, we move backward to the duration $[s_{N-2}, s_{N-1})$, and fill $E_{1,N-2}$ over that duration until it is depleted, or the water level becomes equal to $b_N(E_{1,N-1}/l_N)$. Once the latter happens, we fill the remaining energy over the durations $[s_{N-2}, s_{N-1})$ and $[s_{N-1}, s_N)$ in a way that the water level always becomes even. We repeat the steps until E_{10} is finished. This allocation gives the optimal $\mathbf{p_1}$ when the power of the second user is fixed. The optimality of this procedure can be proved in the same way as in the proof of Theorem 1.

Therefore, in this section, we determined the largest (B_1, B_2)

region for any given T, i.e., $\mathcal{D}(T)$. We also determined the optimal power/rate allocation policy that achieves the points on the boundary of this (B_1, B_2) region. However, we recall that our goal is to find the minimum time, T, by which we can transmit given fixed number of bits (B_1, B_2). In the next section, we go back to our original problem, and provide a solution for it, using our findings in this section.

IV. MINIMIZING THE TRANSMISSION DURATION: MINIMIZING T FOR A GIVEN (B_1, B_2)

For a given pair (B_1, B_2), in order to minimize the transmission completion time of both users, we need to obtain T such that (B_1, B_2) lies on the boundary of the departure region $\mathcal{D}(T)$, as shown in Fig. 6. However, $\mathcal{D}(T)$ depends on T, which is the objective we want to minimize, and is unknown upfront.

Therefore, in order to solve the problem, we first calculate $\mathcal{D}(\tau)$ for $\tau = s_1, s_2, \cdots, s_K$. Then, we locate (B_1, B_2) on the maximum departure region. If (B_1, B_2) is exactly on the boundary of $\mathcal{D}(\tau)$ for some $\tau = s_i$, then, based on the *duality* of these two problems, we know that this s_i is exactly the minimum transmission completion time the system can achieve, and the corresponding power and rate allocation policy achieving this point is the optimal policy.

If (B_1, B_2) is outside $\mathcal{D}(s_i)$ but inside $\mathcal{D}(s_{i+1})$ for some s_i, then, we conclude that the minimum transmission completion time, T, must lie between these two energy arriving epochs, i.e., $s_i < T < s_{i+1}$. Therefore, $T - s_i$, denoted as t here, is the duration we aim to minimize.

We propose to solve this optimization problem in two steps. In the first step, we aim to find a set of power allocation policies to ensure that (B_1, B_2) is on the boundary of the departure region defined by these power allocation policies. In the second step, with the power allocation policies obtained in the first step, we find a set of rate allocations within the corresponding capacity regions, such that B_1, B_2 are finished by the minimal transmission duration obtained in the first step. The first step guarantees that such a rate allocation exists. Solving the problem through these two steps significantly reduces the complexity for each problem, since the number of unknown variables is about half in each problem. In addition, as we will observe, the first step can be formulated as a standard convex optimization problem, and the second step becomes a linear programming problem. Therefore, both steps can be solved through standard optimization tools in an efficient way.

Let us define the energy spent over $[s_{n-1}, s_n)$ by the first and second transmitter as e_{1n}, e_{2n}, respectively. Then, let $\mathbf{e_1} = [\mathbf{e_{11}}, \mathbf{e_{12}}, \cdots, \mathbf{e_{1,i+1}}]$, and $\mathbf{e_2} = [\mathbf{e_{21}}, \mathbf{e_{22}}, \cdots, \mathbf{e_{2,i+1}}]$, we formulate the optimization problem in the first step as follows

$$
\begin{aligned}
\min_{\mathbf{e_1}, \mathbf{e_2}, \mathbf{t}} \; & t \\
\text{s.t.} \; & \sum_{n=1}^{j} e_{1n} \le \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \le i+1 \\
& \sum_{n=1}^{j} e_{2n} \le \sum_{n=0}^{j-1} E_{2n}, \quad 0 < j \le i+1 \\
& B_1 \le \sum_{n=1}^{i} f\left(\frac{e_{1n}}{l_n}\right) l_n + f\left(\frac{e_{1,i+1}}{t}\right) t \\
& B_2 \le \sum_{n=1}^{i} f\left(\frac{e_{2n}}{l_n}\right) l_n + f\left(\frac{e_{2,i+1}}{t}\right) t \\
& B_1 + B_2 \le \sum_{n=1}^{i} f\left(\frac{e_{1n} + e_{2n}}{l_n}\right) l_n \\
& \qquad + f\left(\frac{e_{1,i+1} + e_{2,i+1}}{t}\right) t
\end{aligned}
\tag{24}
$$

where the last three inequality constraints simply mean that $(B_1, B_2) \in \mathcal{D}(s_i + t)$. We state the problem in this form, so that the constraint set becomes convex, and the problem is transformed into a standard convex optimization problem. The joint concavity of $f(e/t)t$ in (e, t) can be proved through taking second derivatives of the function with respect to e and t, and observing that the Hessian is always negative semidefinite. Therefore, the right hand side of these inequality constraints are all jointly concave, thus the constraint set is convex.

Once we obtain $\mathbf{e_1}, \mathbf{e_2}$ and t, we divide the energy by its corresponding duration, and get the optimal power policy sequences $\mathbf{p_1}$ and $\mathbf{p_2}$. Next, we perform the rate allocation in the second step. Therefore, the problem becomes that of searching for $\mathbf{r_1}$ and $\mathbf{r_2}$ from the sequence of capacity regions defined by the sequences $\mathbf{p_1}$ and $\mathbf{p_2}$ to depart B_1 and B_2. This solution may not be unique. Therefore, we formulate it as a linear programming problem as follows:

$$
\begin{aligned}
\min_{\mathbf{r_1}, \mathbf{r_2}} \quad & r_{1,i+1} \\
\text{s.t.} \quad & \sum_{n=1}^{i} r_{1n} l_n + r_{1,i+1} t = B_1 \\
& \sum_{n=1}^{i} r_{2n} l_n + r_{2,i+1} t = B_2 \\
& (r_{1n}, r_{2n}) \in \mathcal{C}(p_{1n}, p_{2n}), \quad 0 < n \le i+1.
\end{aligned}
\tag{25}
$$

Here, the objective function can be any arbitrary linear function in $\mathbf{r_1}$ and $\mathbf{r_2}$, since our purpose is only to obtain a feasible solution satisfying the constraints. We choose the objective function to be $r_{1,i+1}$ for simplicity. The solution of the optimization problem (24)-(25) gives us optimal power and rate allocation policies, which minimize the transmission completion time for both users.

Obtaining $\mathcal{D}(s_i)$ for every s_i requires a large number of computations, and as we will see, it is not necessary. In order to reduce the computation complexity, we aim to explore two special cases of the problem, and use the algorithm in [8] and [9] to obtain a lower bound for T.

A. (B_1, B_2) *Lies on the Flat Part of the Dominant Face.*

For a given pair of (B_1, B_2), the minimum possible transmission completion time can be achieved if it lies on the flat part of the dominant face of $\mathcal{D}(T)$ for some T. This corresponds to the scenario discussed in subsection III-A. Therefore, we can also treat these two users as a single-user system, and identify the value of T through the method discussed in [8] and [9].

Specifically, we calculate the minimum energy required to finish $B_1 + B_2$ by s_1. This is equal to $2^{2((B_1+B_2)/s_1)} - 1$, denoted as A_1. Then, we compare A_1 with $E_{10} + E_{20}$. If A_1 is smaller than $E_{10} + E_{20}$, then, the minimum possible transmission completion time is the solution to the following equation

$$f\left(\frac{E_{10}+E_{20}}{T}\right) = \frac{B_1+B_2}{T}. \tag{26}$$

In this case, the maximum departure region $\mathcal{D}(T)$ is a pentagon defined by $\mathcal{C}\left(E_{10}/T, E_{20}/T\right)T$. If $B_1 < f\left(E_{10}/T\right)T$ and $B_2 < f\left(E_{20}/T\right)T$, then, we always select a rate from $\mathcal{C}\left(E_{10}/T, E_{20}/T\right)$ to achieve the minimum transmission completion time.

If A_1 is greater than $E_{10} + E_{20}$, then, we continue to calculate the minimum energy required to finish $B_1 + B_2$ by $s_2, s_3, \cdots$, denoted as $A_2, A_3, \cdots$, and compare these with $\sum_{j=0}^{1} E_{1j} + E_{2j}, \sum_{j=0}^{2} E_{1j} + E_{2j}, \cdots$, until the first A_i that becomes smaller than $\sum_{j=0}^{i-1} E_{1j} + E_{2j}$. Then, the minimum possible transmission completion time is the solution of

$$f\left(\frac{\sum_{j=0}^{i-1} E_{1j}+E_{2j}}{T}\right) = \frac{B_1+B_2}{T}. \tag{27}$$

Then, we need to determine whether this constant sum of transmit powers is feasible when the energy arrival times are imposed. We merge the energy arrivals from both users and plot the sum of energies as a function of time. Then, we connect the corner points up to T with the origin, and the smallest slope among the lines gives us the first sum of the transmit powers, p_1, [8], [9]. We repeat this process, to obtain $p_2, p_3, \cdots$, until all of $B_1 + B_2$ bits are transmitted. This gives the shortest possible transmission completion time, T_1, for the system.

Next, we need to determine whether (B_1, B_2) lies on the flat part of the dominant face of $\mathcal{D}(T_1)$. We obtain the region $\mathcal{D}(T_1)$ and find the corner points of the flat part on its dominant face through the method described in subsection III-A, and compare them with (B_1, B_2). If (B_1, B_2) lies within the bound, as shown in Fig. 6, this means that it is feasible to empty both queues by time T_1. The only remaining step is to identify a feasible power and rate allocation sequence to achieve this lower bound.

In order to obtain a feasible power allocation, we simplify the optimization problem in (24) into the following form

$$\begin{aligned} \min_{\mathbf{p_1},\mathbf{p_2}} \quad & p_{11} \\ \text{s.t.} \quad & p_{1n} + p_{2n} = p_n, \quad 0 < n \leq i+1 \\ & B_1 \leq \sum_{n=1}^{i} f(p_{1n}) l_n + f(p_{1,i+1})(T_1 - s_i) \\ & B_2 \leq \sum_{n=1}^{i} f(p_{2n}) l_n + f(p_{2,i+1})(T_1 - s_i). \end{aligned} \tag{28}$$

Again, the objective function can be arbitrary since our purpose is only to obtain a feasible solution satisfying the constraints. We choose p_{11} for simplicity. Once the feasible power allocation is obtained, the optimal rate allocation can be obtained by solving (25).

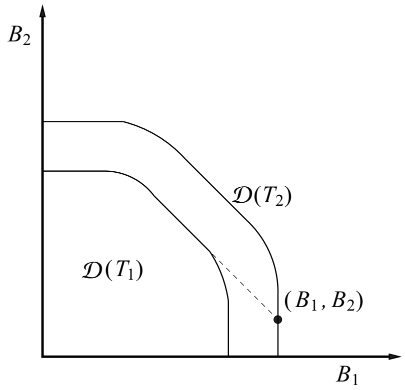

Fig. 8. The minimum transmission duration T to depart (B_1, B_2).

B. (B_1, B_2) Lies on the Vertical or Horizontal Part.

If (B_1, B_2) does not lie on the flat part of the dominant face of $\mathcal{D}(T_1)$, then, it either lies on the vertical or horizontal parts of the boundary of $\mathcal{D}(\mathcal{T})$ for some T, or lies on the curved part of the boundary of $\mathcal{D}(T)$ for some T. Specifically, we assume that (B_1, B_2) is beyond the lower corner point of the flat part of the dominant face of $\mathcal{D}(T_1)$, as shown in Fig. 8. This implies that if we keep transmitting with any policy corresponding to the point on the flat part of the boundary of $\mathcal{D}(T_1)$, by T_1, we have B_2 bits departed from the second user, however, there are still some more bits left in the queue of the first user. This situation motivates us to put more priority on the first user.

Therefore, as the second step, we consider the scenario that (B_1, B_2) lies on the vertical part of the boundary of $\mathcal{D}(T)$, for some duration T. We first ignore the second user, and treat the first user as the only user in the system. This is exactly the same situation as in the single-user scenario. We apply the algorithm in [8], and obtain the transmission duration for the first user, denoted as T_2. T_2 is the shortest possible transmission completion time for given B_1. If we can depart B_2 bits from the second user by T_2, then T_2 is the shortest transmission completion time for both users; otherwise, we cannot finish both data queues by T_2, and the final transmission time should be greater than T_2.

With T_2 fixed, we obtain the optimal energy allocation for the second user through the *backward waterfilling* procedure described in subsection III-B. Once p_{1n} and p_{2n} are determined, we can calculate the maximum number of bits departed from the second user under the assumption that the first user is the primary user. This gives us a number B_2'. If $B_2' \geq B_2$, as shown in Fig. 8, it implies that our assumption is valid, and we can empty both queues by T_2, which is also the shortest possible transmission duration for the system. If $B_2' < B_2$, this implies that we cannot depart B_2 bits from the second queue by T_2, therefore, the final transmission duration could not be T_2 either for the system. This leaves us with the last possibility that (B_1, B_2) must be on the curved part of some other region with some duration T, where $T > T_1, T_2$.

Therefore, up to this point, we obtained a lower bound for the transmission completion time T, which is $\max(T_1, T_2)$. In order to identify an upper bound for T, we only need to calculate the maximum departure region for the energy arriving epochs right after $\max(T_1, T_2)$, until (B_1, B_2) is included for some $\tau = s_i$.

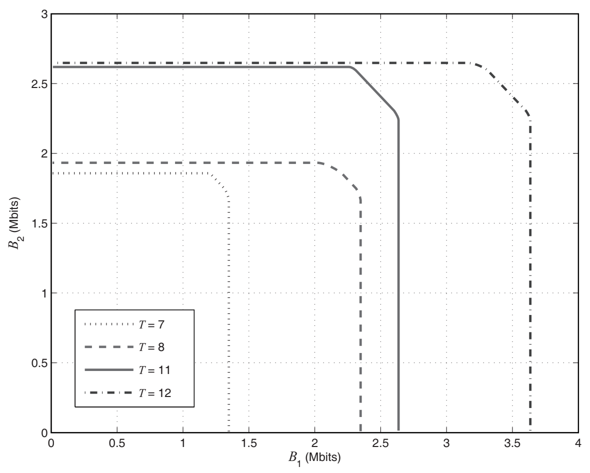

Fig. 9. The maximum departure region of the multiple access channel for various T.

V. SIMULATION RESULTS

We consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1$ MHz and noise power spectral density $N_0 = 10^{-19}$ W/Hz. We assume that the distance between the transmitters and the receiver is 1km, and the path loss is about 110 dB. Then, we have $f(p) = W \log_2(1 + ph/(N_0 W)) = \log_2\left(1 + p/10^{-2}\right)$ Mbps. For the energy harvesting process, we assume that at times $\mathbf{t} = [0, 2, 7, 11]$ s, we have energy harvested with amounts $\mathbf{E} = [5, 5, 10, 10]$ mJ for the first user; at times $\mathbf{t} = [0, 5, 8, 12]$ s, we have energy harvested with amounts $\mathbf{E} = [5, 10, 5, 10]$ mJ for the second user; as shown in Fig. 10. We find the maximum departure region $\mathcal{D}(T)$ for $T = 7, 8, 11, 12$ s, and plot them in Fig. 9. We observe that the maximum departure region is convex for each value of T, each boundary consists of three different parts (flat, vertical/horizontal and curved), and as T increases, the maximum departure region monotonically expands.

We assume that at $t = 0$, we have $B_1 = 2.5$ Mbits from the first user and $B_2 = 2.32$ Mbits from the second user to transmit. We choose the numbers in such a way that the solution is expressable in simple numbers, and can be plotted conveniently. Then, using the proposed algorithm, we obtain the optimal transmission policy, which is shown in Fig. 10. We also determine the transmission rates as $\mathbf{r}_1 = [0.263, 0, 0.585, 0.3]$ Mbps and $\mathbf{r}_2 = [0.1155, 0.585, 0, 0.285]$ Mbps. We note that, for this case, the active transmission is completed by time $T = 10$ s, and the energy harvests at times $t = 11$ s and $t = 12$ s are not used. We also note that (B_1, B_2) lies on the flat part of the dominant face of $\mathcal{D}(10)$, therefore, we finish the transmission of both user simultaneously at $t = 10$ s. Since (B_1, B_2) is not at the corner point, the optimal policy is not unique. We may have different $\mathbf{p_1}$ and $\mathbf{p_2}$ and choose different rates accordingly to have the same departure time. However, the sequence of the sum of transmit powers is unique.

If (B_1, B_2) is not well-balanced, then, it may not be on the dominant face of $\mathcal{D}(10)$, even though the sum $B_1 + B_2$ is the same. For example, if $B_1 = 2.63$ Mbits and $B_2 = 2.19$ Mbits, a

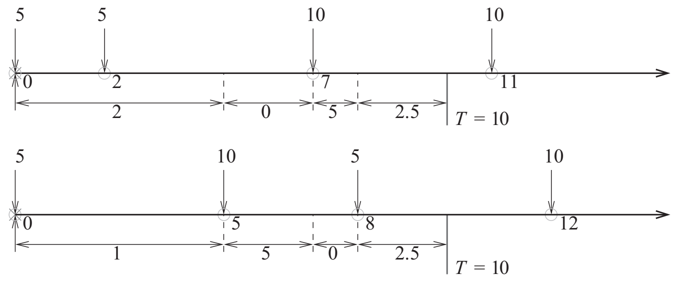

Fig. 10. Optimal transmit powers $\mathbf{p}_1 = [2, 0, 5, 2.5]$ mW, $\mathbf{p}_2 = [1, 5, 0, 2.5]$ mW, with durations $\mathbf{l} = [5, 2, 1, 2]$ s.

simple calculation indicates that (B_1, B_2) lies beyond the range of the dominant face of $\mathcal{D}(10)$, and we cannot finish both queues at $t = 10$ s. Therefore, we take the first user as our primary user, and calculate the minimum possible transmission time for it. The optimal policy for the first user is $p_{11} = 1.43$ mW over $[0, 7)$ s, and $p_{12} = 2.67$ mW over $[7, 10.75)$ s. Based on this allocation, we perform the waterfilling procedure for the second user. The optimal allocation for the second user is shown in Fig. 11, and the maximum number of bits departed from the second user is 2.22 Mbits, which is greater than B_2. This implies that the minimum transmission duration for both users is $T = 10.75$ s, and the data queue of the second user will be emptied earlier than the first user.

The value of (B_1, B_2) may be such that it is neither on the flat part of the dominant face nor on the vertical part of the boundary of any $\mathcal{D}(T)$. For example, let $B_1 = 2.58$ Mbits and $B_2 = 2.24$ Mbits (note that the sum $B_1 + B_2$ is the same as in the previous two examples). From our first example, we know that it is beyond the dominant face of $\mathcal{D}(10)$. Then, we use the method for the second example to find the minimum transmission time for the first user by treating it as the primary user. Calculation indicates that the minimum transmission duration for the first user is $T = 9.7$ s, and the corresponding power allocation is $p_{11} = 1.43$ mW over $[0, 7)$ s, and $p_{12} = 3.7$ mW over $[7, 9.7)$ s. Then, since $T < 10$ s, and 10s is the minimum possible transmission duration for the system, it implies that the total number of bits departed by $T = 9.7$ s is strictly less than $B_1 + B_2$. Therefore, we cannot finish the second queue by $T = 9.7$ s. Based on this analysis, we conclude that (B_1, B_2) must be on the curved part of $\mathcal{D}(T)$ for some T. Then, since it lies within $\mathcal{D}(11)$, together with the lower bound $\max(10, 9.7) = 10$ s, we solve the optimization problem described in (25). The optimal policy is shown in Fig. 12. We observe that the sum of the transmit powers is always increasing, even though they are not monotonically increasing for each individual user. The power changes at $t = 2$ s and $t = 8$ s, where the energy constraints are satisfied with equality for the second user.

These three pairs of (B_1, B_2) are plotted in Fig. 13. Although the sum of B_1, B_2 is the same, they correspond to different scenarios discussed before, and lie on different parts of the boundaries of their corresponding maximum departure regions.

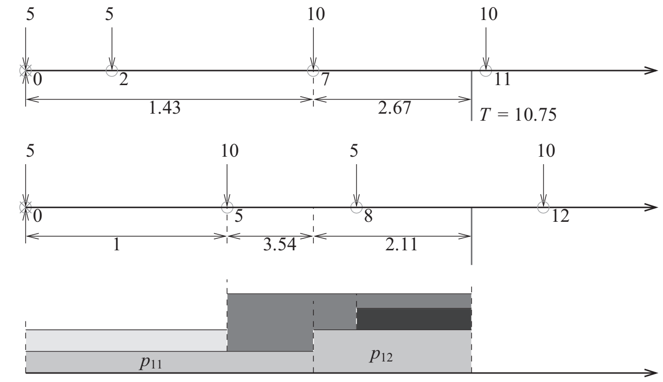

Fig. 11. Optimal transmit powers $\mathbf{p}_1 = [1.43, 1.43, 2.67]$ mW, $\mathbf{p}_2 = [1, 3.54, 2.11]$ mW, with durations $\mathbf{l} = [5, 2, 3.75]$ s.

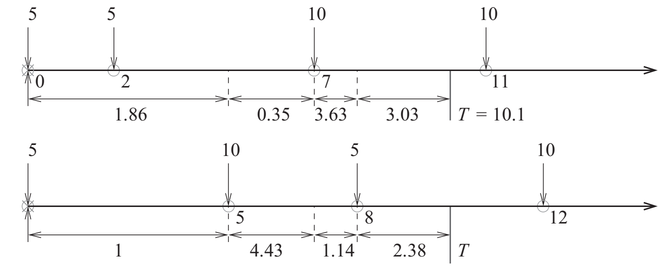

Fig. 12. Optimal transmit powers $\mathbf{p}_1 = [1.86, 0.35, 3.63, 3.03]$ mW, $\mathbf{p}_2 = [1, 4.43, 1.14, 2.38]$ mW, with durations $\mathbf{l} = [5, 2, 1, 2.1]$ s.

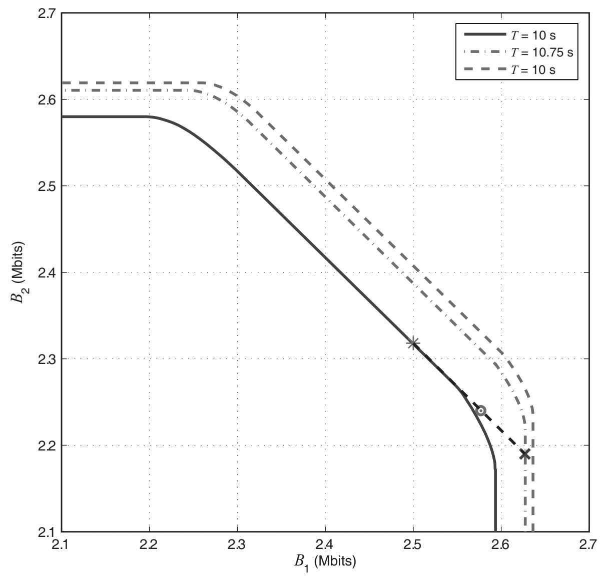

Fig. 13. The maximum departure region of the multiple access channel for various T.

VI. CONCLUSIONS

In this paper, we investigated the transmission completion time minimization problem in an energy harvesting multiple access communication system. We assumed that the packets have already arrived and are ready to be transmitted at the transmitters before the transmission starts. We first proposed a *generalized iterative backward waterfilling* algorithm and characterized the maximum departure region for any given deadline constraint T. Then, based on these findings, we simplified the transmission completion time minimization problem into convex optimization problems, and solved the overall problem efficiently.

REFERENCES

[1] E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Trans. Netw.*, vol. 10, pp. 487–499, Aug. 2002.

[2] M. A. Zafer and E. Modiano, "A calculus approach to energy-efficient data transmission with quality of service constraints," *IEEE/ACM Trans. Netw.*, vol. 17, pp. 898–911, June 2009.

[3] ——, "Delay-constrained energy efficient data transmission over a wireless fading channel," in *Proc. Information Theory and Applications Workshop*, Jan. 2007, pp. 289–298.

[4] W. Chen, U. Mitra, and M. Neely, "Energy-efficient scheduling with individual delay constraints over a fading channel," in *Proc. WiOpt*, Apr. 2007, pp. 1–10.

[5] A. El Gamal, C. Nair, B. Prabhakar, E. Uysal-Biyikoglu, and S. Zahedi, "Energy-efficient scheduling of packet transmissions over wireless networks," in *Proc. IEEE INFOCOM*, vol. 3, Nov. 2002, pp. 1773–1782.

[6] E. Uysal-Biyikoglu and A. El Gamal, "On adaptive transmission for energy efficiency in wireless data networks," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3081–3094, Dec. 2004.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with rechargeable nodes," in *Proc. IEEE ICC*, June, 2011.

[8] J. Yang and S. Ulukus, "Transmission completion time minimization in an energy harvesting system," in *Proc. CISS*, Mar. 2010.

[9] ——, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, pp. 220–230, Jan. 2012.

[10] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting systems," submitted, Sept. 2010. Also available at http://wcan.ee.psu.edu.

[11] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun* vol. 29, no. 8, pp. 1732–1743, Sept. 2011.

[12] M. Antepli and E. Uysal-Biyikoglu, and H. Erkal, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE J. Sel. Areas Commun*, submitted, Oct. 2010.

[13] O. Ozel, J. Yang, and S. Ulukus, "Broadcasting with a battery limited energy harvesting rechargeable transmitter," in *Proc. WiOpt*, May 2011.

[14] O. Kaya and S. Ulukus, "Achieving the capacity region boundary of fading CDMA channels via generalized iterative waterfilling," *IEEE Trans. Wireless Commun*, vol. 5, no.11, pp. 3215–3223, Nov. 2006.

[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley and Sons, Inc, 1991.

[16] D. Tse and S. Hanly, "Multiaccess fading channels–Part I: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Inf. Theory*, vol. 7, pp. 2796–2815, 1998.

[17] D. Bertsekas and J. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, 1997.

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