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A Relay-Coding Matrix for Multi-user Cooperative Communications

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***Abstract*—In this paper, we consider a scenario that multiple users transmit data to a common destination node with the assistance of one relay station. We present one type of coding matrices that are invertible in the finite field and have maximum information spreading. We analyze the performance based on the input-output weight enumerate function (IOWEF) of the relay coding matrix and show a close match with the results of the log-likelihood ratio decoding. The method gives a way to readily determine when relay coding is preferred. Further, since data from each user is generally coded with channel codes, using relay coding also enables turbo decoding that alternates between a multi-user relay decoder and a number of single-user decoders at the destination. We show via simulations that the turbo decoding converges within only two or three iterations, yet provides significant gains compared with a reference relay system without the multi-user cooperation.**

***Index Terms*—Relay, Multi-user cooperative communications, input-output weight enumerate functions, turbo decoding.**

I. Introduction

In this paper, we consider a wireless communication scenario when multi-users are sending information to a common destination with the help of single relay. This scenario has wide applications. For example, the IMT-advanced, the latest 4G wireless communication effort from ITU [1], requires uplink peak spectrum efficiency of 15 bps/Hz and throughput as high as possible in the cell edge. Among the enabling techniques being considered, a consensus from different companies is the use of relay stations (RS) where the cooperative communications can be applied to improve the link performance. Another example is the wireless sensor networks where multiple nodes broadcast their information back to a central sink through intermediate nodes.

In cooperative communications, a popular approach is that the cooperation, considering multiple paths, is for one user at a time. Recently, there were some work being proposed to consider data aggregation for two users. In [2], a scenario was studied where the RS with decode-and-forward (DF) ability relays "$a \oplus b$" instead of symbols "a" and "b" individually, where $\oplus$ is the operation of "XOR". It was shown that the performance can be improved given the same spectrum efficiency. The work was extended in [3] where the soft information was relayed with analog modulation. Discussion on equivalent BSC channels with the same setup was presented in [4]. Basically, these work were part of the effort of shifting the application of network coding [5] in wired network into wireless communications. Despite interesting results reported in these and other papers, it is recognized that a broad range of issues still need further research. For example, the comparison with a reference system equipped with full optimal maximal ratio combining (MRC) diversity may be desired, which is not in [2]–[4]. The direct link is often weaker than the relay link. Therefore, it might also be desired that the source data of different users should be completely recovered from the relayed data in the case where the relay link is good enough but the direct links do not function well. This situation is evidenced in the current uplink of IEEE 802.16j where the links between mobile station (MS) and base station (BS) are not considered when a relay is used. This requires that the coding matrix must be invertible. Further, under the assumption of mobiles being uniformly distributed in a cell, there are in fact many mobiles near the cell edge. Therefore, multiple users may use one relay simultaneously.

Taking the above issues into account, we present a relay-coding-based multi-user cooperative communication design for the targeted scenario. In section II, a type of relay coding matrices is described with the requirements of full rank in the finite field and maximum information spreading. In section III, symbol detection based on MRC and log-likelihood ratio (LLR) is discussed. Particularly, we analyze the performance of relay-coding based on the input-output weight enumerate function (IOWEF) of the code. The analysis shows a close match with LLR detection and thus can be used to check when the relay coding is preferred and how much the gain is. In section IV, we further show that a turbo decoding process can be exploited when the source data is also channel coded. Simulation shows that this decoding converges very fast with additional gain. Section V concludes this paper.

II. System and Relay Coding Matrix

Without loss of generality, the proposed system is shown in Fig. 1 where there are four users joining the cooperation. Instead of transmitting original information a, b, c, d, we transmit their combinations p_1, p_2, p_3, p_4 shown in the lower part of Fig. 1. For example, $p_1 = a \oplus b \oplus c$. Once the data being coded at the relay have the same modulation constellation, this coding operation may be the "addition" operation in an appropriate finite field $GF(2^m)$ where m is the number of bits per symbol. If users use different types of modulation, the coding at the relay can always be reduced to the "addition" in

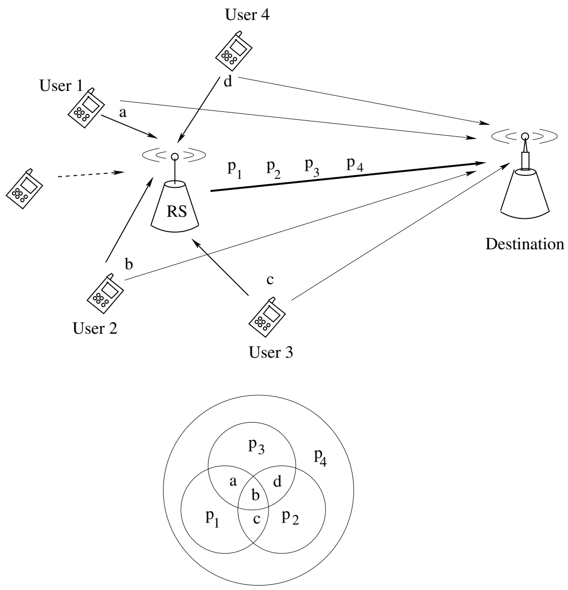

Fig. 1. An example of proposed system

the binary finite field $GF(2)$. For simplicity, we assume the BPSK modulation for all MSs in this paper.

A. Relay-Coding Matrix

There are many ways, such as random network coding and linear block codes, can be employed for the coding at RS. In our system, we consider this coding based on the requirements in rank and information spreading. First, we require that the relay outputs are linearly independent. Therefore, these coded data, if perfectly decoded at the BS, can recover the original information of different users completely, without information from the direct links. As a result, a coding matrix should be of full rank in the finite field. The number of this type of matrices with size of $K \times K$ is $\prod_{i=0}^{K-1}(2^K - 2^i)$, which can be obtained by sequentially selecting a non-zero row vector that is not the linear combination of previous rows. This number should be further divided by a factor $K!$ if we do not distinguish matrices with row-permutations. Second, we expect to spread information of one user to multiple output symbols after coding. Therefore, when data are received from both direct and relay links, it may provide a gain via iterative decoding.

Let $\mathbf{U} = [u_1^t, u_2^t, \ldots, u_K^t]^T$ be symbols from K users that are received and decoded at the relay at time t. The relay output will be $\mathbf{P} = \mathbf{AU}$, where $\mathbf{A} = \{a_{i,j}\}$ is a $K \times K$ coding matrix and $a_{i,j} \in \{0, 1\}$. In order to give the maximum spreading, we can set each row of $\mathbf{A}$ with all "1"s except one "0", which results in K different rows. In addition, "$\mathbf{A}$" needs to be invertible (i.e., of full rank) in the finite field. From mathematical induction, it can be easily seen that A designed as above is in full rank when K is an even number. To make A invertible for odd values of K, we can simply set one row, such as the 1st row, to be all "1"'s. Verification, again, can be readily obtained from the mathematical induction. Therefore, the matrices used are as follows.

$$A_{Keven} = \begin{bmatrix} 1 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ 1 & \cdots & 0 & 1 \end{bmatrix}, A_{Kodd} = \begin{bmatrix} 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ 1 & \cdots & 0 & 1 \end{bmatrix}.$$

It is interesting to recognize that the above design for $K = 4$ used in Fig. 1 is exactly the extended-Hamming (8,4,4) code. It also needs to note that $K = 2$ is a special case which has two "1"s in one row and one "1" in the other, in order to have both information spreading and full rank.

In this paper, we assume that the relay correctly decodes the data from sources; direct links and relay links are separated in either time slots or channel frequency so that synchronization is not considered as an issue here.

TABLE I
IOWEF, $A_{w,j}W^wX^j$

	n even	n odd
$w = 1$	$\binom{n}{1}\ W^1X^n$	$(n-1)W^1X^n + W^1X^{n+1}$
w odd	$\binom{n}{w}W^wX^n$	$\binom{n-1}{w}W^wX^n + \binom{n-1}{w-1}W^wX^{n+1}$
w even	$\binom{n}{w}W^wX^{2w}$	$\binom{n-1}{w}W^wX^{2w} + \binom{n-1}{w-1}W^wX^{2w-1}$
$w = n$	W^nX^{2n}	W^nX^{n+1}

B. Input-Output Weight Enumerate Functions (IOWEF)

The IOWEF of a linear code is defined as

$$A(W, X) = \sum_{w,j} A_{w,j}W^wZ^j$$

where $A_{w,j}$ denotes the number of codewords with Hamming weight j generated by input information words with Hamming distance w. Therefore, the Hamming weight in the coded parity part (data from relay) is $j - w$. For the proposed relay coding matrices, $A(W, X)$ can be found as shown in Table I.

For example, when $n = 4$, it can be found that

$$A(W, X) = 4WX^4 + 6W^2X^4 + 4W^3X^4 + W^4X^8$$

III. Information Spreading and Diversity

We set the reference system as a typical DF-based relay system that the relay decodes data of each user and then re-encodes with the same code and send to the destination. In such scenario, there are two copies of data at the destination for each user, from direct and relay links respectively. The optimal symbol detection technique is the MRC diversity.

For the proposed system shown in Fig. 1 with four users using one relay simultaneously, the data from multiple users are coded in the relay using the extended-Hamming (8,4,4) code, i.e., simple multiplication through the matrix A with $K = 4$. It has the same spectrum efficiency with the reference system so that we can focus on the error performance only. It is also possible to code different number of users with different linear codes and transmit either all or part of the coded data in order to obtain various trade-off between the performance and bandwidth.

For the system example considered here, intuitively, each symbol of one particular user will be transmitted four times (in different forms) instead of twice in the reference system. For example, "a" is embedded in p_1, p_3, p_4 in addition to the one through direction link. However, data from some different users are "mixed" together in the relay to use the same transmission power which is used by a single user symbol in the reference system. The former feature is beneficial in terms of information spreading but the latter characteristic appears a minus due to the share of transmit power. Therefore, the first issue to be clarified is whether the net gain is positive for the proposed multi-user cooperative communication. We consider this through the log-likelihood ratio of each symbol conditioned on received data.

A. LLR of the Reference System

For a transmitted BPSK symbol u_k from the k-th user, $k = 1, \ldots, K$, in the reference system, each u_k will be sent to the destination through two paths experiencing different fading. Suppose that we have received two copies $y_{k,i} = a_{k,i}u_k + n_{k,i}$, $i = 1, 2$, where $a_{k,i}$ is the independent fading factor and $n_{k,i} \sim \mathcal{N}(0, \sigma_{k,i}^2)$ determines the average SNR at BS from each link. Then, we can calculate the conditional LLR

$$
\begin{aligned}
L(u_k|y_{k,1}, y_2) &= \ln \frac{P(u_k = +1|y_{k,1}, y_{k,2})}{P(u_k = -1|y_{k,1}, y_{k,2})} \\
= L(u_k) + \ln \frac{P(y_{k,1}|u_k = +1)}{P(y_{k,1}|u_k = -1)} &+ \ln \frac{P(y_{k,2}|u_k = +1)}{P(y_{k,2}|u_k = -1)} \\
&= L(u_k) + L_c^{(k,1)} y_{k,1} + L_c^{(k,2)} y_{k,2} \qquad (1)
\end{aligned}
$$

where

$$L(u_k) = \ln \frac{P(u_k = +1)}{P(u_k = -1)} \qquad (2)$$

which is often termed as the *a priori* information of $u_k, k = 1, \ldots, K$; $L_c^{(k,i)} = 2a_{k,i}/\sigma_{k,i}^2, \quad i = 1, 2$. Then $u_k = +1$ (or -1) when $L(u_k|y_{k,1}, y_{k,2}) \geq 0$ (or < 0). $L(u_k) = 0$ when the *a priori* information of u_k is not available at the BS. It should be noted that equation (1) when $L(u_k) = 0$ is exactly the MRC which is the optimal diversity process with two receiving copies.

B. LLR of the Proposed System

This is in fact to find the LLR values of the systematic symbols in a block code but with different fading for different symbols. Let C be the set of all possible codewords generated in the relay and $\mathbf{u}$ is a specific codeword with u_k at position k. (N, K) is the code used in the relay. $\mathbf{y}$ is the received data. Similar to the derivation in AWGN [6], We can find

$$
\begin{aligned}
L(u_k|\mathbf{y}) &= \ln \frac{\sum_{\mathbf{u} \in C, u_k = +1} P(\mathbf{u}|\mathbf{y})}{\sum_{\mathbf{u} \in C, u_k = -1} P(\mathbf{u}|\mathbf{y})} \\
&= \ln \frac{\sum_{\mathbf{u} \in C, u_k = +1} \left(\prod_{j=1}^N p(y_j|u_j) \cdot \prod_{j=1}^N p(u_j)\right)}{\sum_{\mathbf{u} \in C, u_k = -1} \left(\prod_{j=1}^N p(y_j|u_j) \cdot \prod_{j=1}^N p(u_j)\right)} \\
&= \ln \frac{p(u_k = +1; y_k) \cdot \sum_{\mathbf{u} \in C, u_k = +1} \prod_{j=1, j \neq k}^N p(u_j, y_j)}{p(u_k = -1; y_k) \cdot \sum_{\mathbf{u} \in C, u_k = -1} \prod_{j=1, j \neq k}^N p(u_j, y_j)} \qquad (3) \\
&= L(u_k) + L_c^{(k)} y_k \\
&+ \ln \frac{\sum_{\mathbf{u} \in C, u_k = +1} \Pi_{j=1, j \neq k}^N \exp(L(u_j; y_j) u_j / 2)}{\sum_{\mathbf{u} \in C, u_k = -1} \Pi_{j=1, j \neq k}^N \exp(L(u_j; y_j) u_j / 2)} \qquad (4)
\end{aligned}
$$

where

$$L(u_j; y_j) = L_c^{(j)} y_j + L(u_j), \quad 1 \leq j \leq N \qquad (5)$$

In the example shown in Fig. 1, $K = 4$, $N = 8$ and $\mathbf{u} = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8] = [a, b, c, d, p_1, p_2, p_3, p_4]$. $L_c^{(j)} = 2a_j/\sigma_j^2, j = 1, \ldots, N$. In the above equations, the general case also considers the *a priori* information of relay coded symbols $u_K, u_{K+1}, \ldots, u_{2K}$. However, if this part of information is not available or based on the source symbols, we can simply set $L(u_j) = 0$ for $j = K, \ldots, 2K$.

As a result, we can simply compare the reference system and the proposed system by calculating the LLR of each information symbol based on equations (1)(4) and then conduct the threshold detection. For simplicity, we set all direct links from sources to the destination with the same SNR (SNR_d) and vary the SNR between the relay and the destination (SNR_r) within a range. Fig. 3 shows the simulation results for the detection BER in Rayleigh fading channels. It can be observed that for most SNR scenarios the relay coding provides much better performance compared to the direct DF method.

C. Performance Analysis of the Relay Coding

While the performance of MRC can be determined readily, detection based on the simulation for the relay-coding is very time consuming, which makes it unpractical to check up the performance. In the following, we analyze the performance of relay-coding via the IOWEF.

First, suppose M-branch MRC is considered. Let $\boldsymbol{\gamma} = \{\bar{\gamma}_1, \cdots, \bar{\gamma}_M\}$ be the average SNRs of those branches. With BPSK assumption, the average bit error rate $\bar{P}_b$ after MRC diversity is [7]

$$\bar{P}_b(\boldsymbol{\gamma}) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^M M_{\gamma_i}\left(-\frac{1}{\sin^2 \phi}\right) d\phi \qquad (6)$$

where

$$M_{\gamma_i}(s) = \int_0^\infty p_{\gamma_i}(\gamma) e^{s\gamma} d\gamma \qquad (7)$$

is the Moment generating function (MGF) for nonnegative random variable γ_i with pdf $p_{\gamma_i}(\gamma)$. When Rayleigh distribution is considered, then

$$M_{\gamma_i}(s) = (1 - s\bar{\gamma}_i)^{-1} \qquad (8)$$

where $\bar{\gamma}_i$ is the average SNR of the received signal in the ith branch.

Second, when coding is used, the performance can be determined by the Union bound. From $A(W, X)$, bit level

IOWEF can be obtained as

$$B(W,X)=\frac{1}{K}\sum_{w,j} wA_{w,j}W^{w}X^{j} \tag{9}$$

where K is the length of information bits, i.e., the number of source nodes in cooperation here. With Union bound, the bit error rate of the relay coding is

$$\tilde{P}_b \leq \frac{w}{k}\sum_{w,j} A_{w,j}P(w,j) \tag{10}$$

Where $P(w,j)$ is the error probability that the decoder selects, instead of the codeword $\mathbf{0}$, a codeword with Hamming distance j in which w is the weight in the source word and $j-w$ is the weight of the coded parity. In the relay coding here, symbols in the source word are transmitted from the direct links and coded parity symbols are transmitted over the relay link. As a result, $P(w,j)$ is equivalent to the MRC error probability that combines w bits from different direct links and $j-w$ bits from the relay link. Let $\boldsymbol{\zeta}=\{\alpha_1,\cdots,\alpha_w,\beta_1,\cdots,\beta_{j-w}\}$ be the average SNR corresponding to the w direct links and the relay link, where $\beta_1=\cdots=\beta_{j-w}$ as these symbols are all transmitted via the relay link. Then, $P(w,j)=\bar{P}_b(\boldsymbol{\zeta})$ which can be calculated by eqn (6).

In this work, for simplicity, we assume all direct links have the same SNR. If this assumption is not valid, the IOWEF needs to be expanded to include the positions of those w bits. The same analysis is still applicable though more complex.

The results of the this analytic work can be readily obtained via direct numerical evaluation and are also plotted in Figs. 3 and 4. It can be noted that the results match the simulation results of LLR detection very well. As a result, for a given channel scenario it can be easily detected whether the relay coding should be used and how much gain may be obtained.

IV. Turbo Decoding

A. Multi-user Decoding and Single-user Decoding

It is known that the channel coding is always employed in wireless communications. That it, symbols to be transmitted from each user have already been coded. When the additional coding is introduced in the relay, the overall coding structure is in fact a product coding system. Let $\mathbf{u_i}, i=1,\ldots,K$ be the source information of the ith user. Then the coded data of K users are

$$Y=(\mathbf{y_1},\mathbf{y_2},\ldots,\mathbf{y_K})^T \tag{11}$$

where

$$\mathbf{y_i}=\mathbf{G_i}\mathbf{u_i}. \tag{12}$$

The relay coded data are

$$\mathbf{R}=(\mathbf{r_1},\mathbf{r_2},\ldots,\mathbf{r_K})^T=\mathbf{AY} \tag{13}$$

where sizes of $\mathbf{r_i}$, $\mathbf{y_i}$ $\mathbf{u_i}$ and $\mathbf{G_i}$ are $n\times 1$, $n\times 1$, $k_i\times 1$ and $n\times k_i$ respectively. In the horizontal direction of the product code, (n,k_i) is the code $\mathbf{G_i}$ of the ith MS which can be in any type defined in the standard specifications. In the vertical direction, it is the relay coding across multiple users. Both $\mathbf{Y}$ and $\mathbf{R}$ will be received by the destination. The final objective of the maximum likelihood decoding is to find an estimate of the information data $\hat{\mathbf{u}}_i, i=1,\ldots,K$ which, after coding in both directions, have the shortest distance from the received data at the destination. This problem can be near-optimally solved by applying the turbo decoding principle.

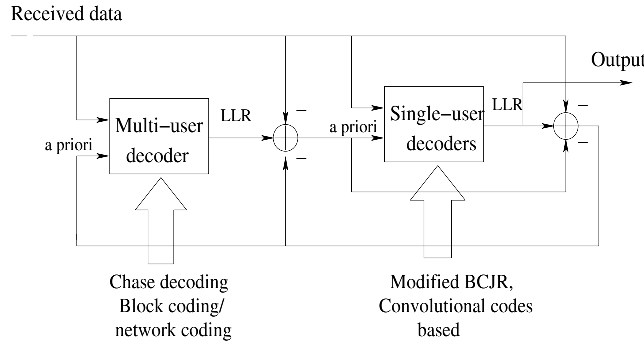

Fig. 2. Decoding structure for single users and across multiple users

The decoding structure is illustrated in Fig. 2. This process bears the decoding principle of turbo codes but it should be noted that the information is exchanged between the decoding of a group of single users and the decoding of multiple users.

B. Extrinsic Information for Parity Symbols

One important issue in the decoding implementation of this system is the processing of the extrinsic information for parity symbols. This is because the relay coding is applied to all data in the horizontal direction including both systematic and parity information of each participating user. The general idea of providing extrinsic information for both systematic and parity bits was first presented in [8] but without any implementation details. A specific example was presented in [9]. We have also modified [10] the conventional BCJR maximum *a posteriori* probability (MAP) algorithm [11] for the calculation of LLR of the parity symbols, which is used here for the turbo decoding.

V. Simulation Results

First, the symbol detection of the diversity and coding is compared in Fig. 3. We use $K=4$ and (8,4,4) extended Hamming code at the relay. We fix the SNR of direct links (SNR_D) as -3, 1.5, 6 dBs, respectively, and then change the SNR of the relay link. It shows that when SNR_D is very low (-3dB) MRC (red, dashed curves) performs slightly better than the relay coding (blue, dashed curves). However, when SNR_D increases, relay coding is always preferred to the repetition transmission with large gains. The numerical results of the theoretical analysis (black, solid curves) have shown very close match (actually being a tight up-bound) with the time consuming simulation results. As a result, the theoretic method can be used to readily determine when the relay coding is needed and how much gain it will be in fading channels. Fig. 4 shows the performance comparison of MRC and relay coding for different $K (K=3,4,6)$.

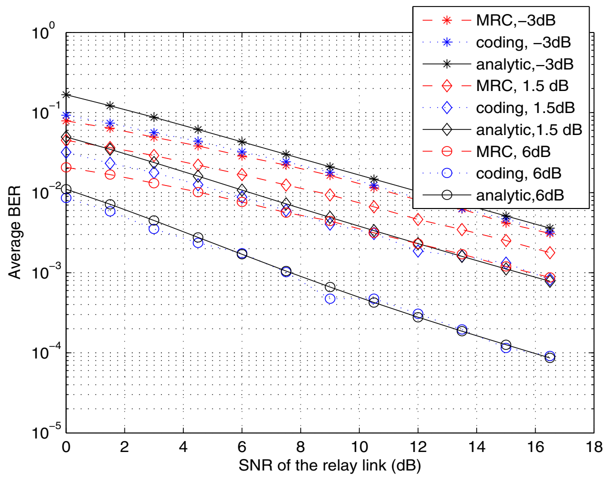

Fig. 3. Performance comparison, fading channels.

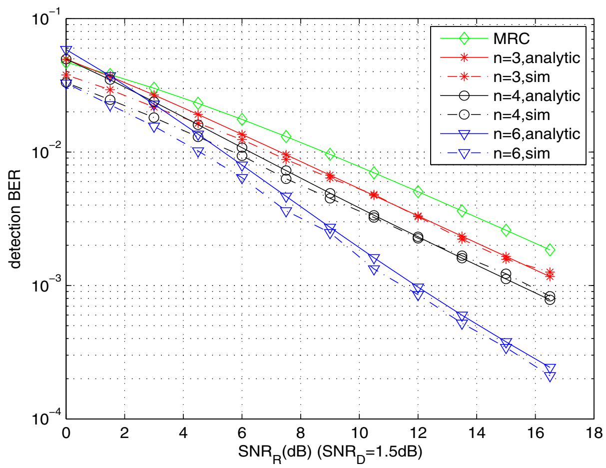

Fig. 4. Detection Performance for $K = 3, 4, 6$ (SNR_D=1.5dB).

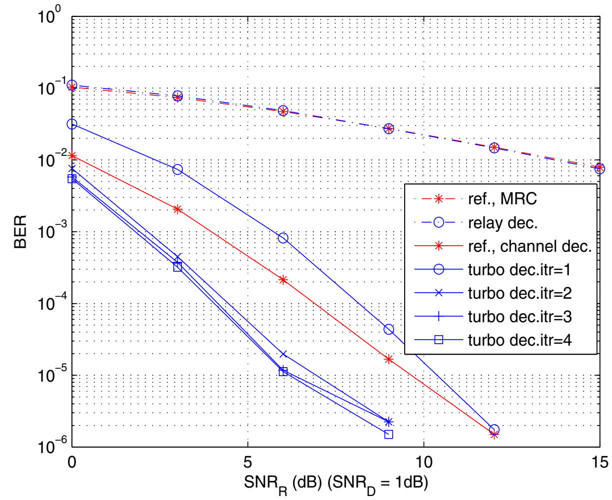

Fig. 5. BER performance, SNR_D = 1dB, $K = 4$.

When channel coding is considered together, theoretic BER analysis is too much involved. We use the simulation to check the performance. We use RSC code with generator $(7, 5)_{oct}$ and rate 1/2 as the code for each user. Each link may experiences different Rayleigh block fading. The comparison with the reference system where diversity is followed with channel decoding is shown in Fig. 5. That $SNR_D = 1$dB is equivalent to the case when direct link is -2 dB in direct symbol detection without coding. In this case, the MRC detection and relay coding performs almost the same. However, with the turbo decoding, the performance of the proposed system exceeds the reference system quickly, with a gain above 2 dB at 10^{-5}. It can be recognized when the quality of direct link is increased, the overall gain will be even higher. In addition, it can be noted that the decoding generally convergences within 3 iterations. This is because the coding in the vertical direction (i.e., at the relay) is for a small value of K. Hence the correlation between the extrinsic information in this direction occurs quickly.

VI. Discussions and Conclusions

In this paper, a relay coding matrix is discussed and analyzed for multi-user cooperative communications. The analytic results shows close match with the LLR detection, and can check up the performance in advance. Then, the turbo decoding is implemented, which shows a fast convergence with additional gains.

Acknowledgements

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References

[1] in *http://www.3gpp.org/ftp/Specs/archive/36_series/36.913/36913-800.zip*.

[2] C. Hausl, F. Schreckenbach, I. Oikonomidis, and G. Bauch, "Iterative network and channel decoding on a Tanner graph," in *Proceedings of Allerton Conference on Communication, Control and Computing*, Urbana Champaign, September 2005.

[3] S. Yang and R. Koetter, "Network coding over a noisy relay: a belief propagation approach," in *arXiv:cs/0701062v1*, 2007.

[4] Y. Chen, S. Kishore, and J. Li, "Wireless diversity through network coding," in *IEEE Wireless Communications and Networking Conferenc*, vol. 3, Las Vegas, NV, 1997, pp. 1681–1686.

[5] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, July 2000.

[6] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 429–445, March 1996.

[7] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[8] S. Benedetto, D. Divsalar, Montorsi, and F. Pollara, "A soft-input soft-ouput APP module for iterative decoding of concatenated codes," *IEEE Communications Letters*, vol. 1, no. 1, pp. 22–24, January 1997.

[9] A. Ambroze, G. Wade, and M. Tomlinson, "Iterative map decoding for serial concatenated convolutional codes," *IEE Proceedings on Communications*, vol. 145, no. 2, pp. 53–59, April 1998.

[10] L. Cao, J. Zhang, and N. Kanno, "Multiuser cooperative communications with relay-coding for uplink IMT-advanced 4G systems," in *Proc. IEEE GlobeCom 2009*, Hawaii, USA, Nov.-Dec. 2009.

[11] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Transactions on Information Theory*, vol. IT-20, no. 2, pp. 284–287, March 1974.