# Optimal portfolio choice to split orders during supply disruptions: An application of sport's principle for routine sourcing

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## Abstract

Sourcing in the face of supply chain disruptions has been one of the most challenging tasks in supply chain management, particularly when such disruptions occur due to natural calamities, such as flood, fire, and earthquake, affecting both the primary and the backup suppliers. Invariably, such disruptions lead to reduced supply from the primary supplier, encouraging the supplier to place fresh orders with the backup suppliers. In order to mitigate the adverse effect of supply disruption, in this paper we use the concepts underlying the well-known Duckworth–Lewis–Stern method, used in cricket, to revise the supply target of the primary supplier and to decide a target for the backup supplier. We simulated the supply disruption scenarios in an experimental setting by conducting a two-round questionnaire survey among 300 purchase managers. The means and variances of the participants' estimates of probabilities of meeting the revised targets within the scheduled time for various model-generated supply scenarios were used to find the participants' risk preferences. In the second-round survey, the participants, clustered in groups of 10, ranked their own risk preferences. These ranks were used to find the optimal portfolio choices. Finally, we validated the theoretical predictions for the risk options using two approaches—one, at the group level by estimating the within- and the between-group risk preferences of buyers, and, two, at the aggregate level, by considering all the participants, fitting quantile regression model to the experimental results, and estimating the risk preference structures for different quantiles of the relative risk-return trade-off distributions.



***Subject Areas: Supply disruption, Duckworth–Lewis–Stern method, Mean-variance decision-theoretic model, and Portfolio of risky options, Risk preference structures***


## INTRODUCTION

Supply disruption refers to an extreme event with an adverse operational impact on the supply chains (Parker & Ameen, 2018). It lowers a firm's long-term economic and social performance, and the recovery process can be slow (Hendricks & Singhal, 2005; Torabi, Baghersad, & Mansouri, 2015). When these disruptions occur due to natural calamities, such as flood, fire, and earthquake, they can be of long duration and can significantly, and adversely, impact the financial performance of the buying firms. We cite below a few cases of supply disruption on account of flood, fire, and earthquake.

A major flood devastated the Indian states of Maharastra and Karnataka in the year 2019, caused immense loss to the leather firms (Kolhapuri slippers) operating in these states, and led to shutting down of all tanneries in the city of Kolhapuri causing an estimated economic loss of 1.35 billion USD[1]. The August 2018 flood in Chennai disrupted the supply of motorcycle parts and components and brought the Chennai-based Eicher Motor's production of motorcycles to a standstill, despite the multi sourcing strategy followed by the company for different components[2]. During the 2018 Kerala flood, one of India's largest tyre manufacturers suffered a significant supply disruption and incurred a loss of approximately 12% on the net income because the supply of natural rubber, the key material for tyre production, stopped for a long time due to the flood. As Kerala (a south Indian state) produces about 85% of domestic natural rubber, the suppliers were very badly impacted during the catastrophe[2]. The backup suppliers located in Thailand also failed to supply, owing to the outbreak of a fungal disease on rubber plantation[3] during the same time. Toyota had to temporarily shut down 20 of its 30 Japanese assembly lines because the supply of P-valves (used in the car braking system) from Aisin Seiki Co., the primary P-valve supplier, was badly hit due to a fire at its plant site (Sheffi, 2007). Motorola was sourcing chips from TSMC and UMC in Taiwan. When both the suppliers were hit by a Taiwan earthquake, it caused a severe supply disruption for Motorola for weeks together (Sheffi, 2007).

In a recent review of management science models for supply chain disruption, Snyder et al. (2016) identified routine sourcing and contingent rerouting as the two major flexible sourcing strategies to counter the effect of such disruptions. Of these two, routine sourcing is more popular as a strategy to ameliorate the effect of catastrophe-caused disruptions and is well studied by several authors (e.g., Dada et al., 2007; Gurnani et al., 2014; Li, 2017; Demirel et al., 2018; Zhao & Freeman, 2019). In routine sourcing, the buyer splits orders among the pre-selected suppliers to overcome the shortage arising out of uncertainty of supply from the catastrophe-affected suppliers. But because these pre-selected suppliers have different resource endowments and because they are also badly affected by the catastrophe, the natural question is "how to split the orders among these suppliers?" Snyder et al. (2016, p. 95) have recognized the optimal splitting of orders among suppliers as the most challenging task during supply disruption.

Several authors (e.g., Sawik, 2011; Hu, Gurnani, & Wang, 2013; Hu & Kostamis, 2015) have addressed the problem of optimal portfolio choice of a buyer during supply disruption. He, Huang, & Yuan (2016) have identified three distinct approaches used by these authors to address the problem. The first approach models the number of units delivered by the supplier as a random fraction of the total units ordered (Güler, 2015). The second approach models the supply disruption risk where the supplier can deliver either the total amount ordered or nothing (Gupta, He, & Sethi, 2015). The third approach models supply risk with either uncertain capacity (Li et al., 2013) or uncertain lead time (Kouvelis & Li, 2012).

The optimal portfolio choice problem is addressed in the literature from a variety of perspectives. These perspectives take the form of optimal strategy based on sourcing cost and service level (Dada et al., 2007; Gurnani et al., 2014), profit-oriented approach for risk management (Zhao & Freeman, 2019), optimal strategy to handle demand uncertainties (Tomlin, 2009; Hu & Kostamis, 2015), handling lead-time uncertainties (Kouvelis & Li, 2012), capacity restoration (Li et al., 2013; Hu et al., 2013; Guo, Zhao, & Xu, 2016), and inventory management (Hou et al., 2019). These perspectives consider objective, quantitative aspects of supply disruption. Notably, none of these studies have considered the behavioural aspects of a buyer for optimal portfolio choice during supply disruption.

Gurnani et al. (2014) have recognized the importance of a buyer's preferences towards the perceived risks of splitting the order as they are influenced by the buyer's behavioural aspects. In a review of literature on behavioural operations management, Fahimnia et al. (2019) recognize the buying behaviour under supply disruption as an important research area but admit that this area is scarcely researched. No study, however, has explicitly considered the problem of optimal portfolio choice of a risk-averse buyer with risk preferences, to split the orders among the catastrophe-affected, differently endowed suppliers.

As indicated above, studies addressing behavioural aspects of decision maker in the context of supply disruption are limited in number. A few notable studies are by Schweitzer and Cachon (2000) and Castañeda and Gonçalves (2018), who have used experiments to study the effect of behaviour of decision makers under uncertainty and have advocated experiments as means for conducting such studies. In behavioural sciences, gaming, simulation, and questionnaire surveys are used as experimental platforms to explore the influence of behaviour of decision makers on their decisions. Such experiments help to verify prevailing hypotheses, validate previous results, and suggest new hypotheses and relationships. But studies involving use of *experimental inquiry* to explore the influence of buyer behaviour on optimal portfolio choice are few.

To sum up, we can say that catastrophe-induced supply disruption forces a buyer to split order between the primary supplier and the backup suppliers (routine sourcing). When all these suppliers are affected by the catastrophe, target setting for these suppliers becomes a challenging task. Past knowledge about the supply uncertainties provides the buyer a guide to decide a portfolio of risky options based on the estimated relative trade-offs between risks and returns. An important dimension of research in the field of portfolio choice during supply disruption is the effect of buyer behaviour on the target setting process. The small number of studies that have been reported in this under-studied area indicate the use of experiments to study the buyer behavioural aspects for target setting.

## Scope, Objective, and Research Questions

This paper takes the case of a buyer who switches to routine sourcing and who considers both capacity and lead time uncertainties (the third approach) for optimal portfolio choice to address the issue of shortage during supply disruption. The paper also assumes that there are only two suppliers (a primary supplier and a backup supplier) who are endowed with different resource positions and are situated in close geographical proximity so that both are adversely affected when a natural calamity strikes the region, these assumptions being the same as those made by Li (2017) and Zhao and Freeman (2019). We also assume that price does not play a role in the buyer's optimal portfolio choice either because the price is regulated by the government or because the resource scarcity causes product price rise in similar ways for both the suppliers. We further assume that the suppliers cannot make any differentiated product to take advantage of the resource scarcity position. The last two assumptions hold for supply of a wide range of commodities, such as tea, sugar, and meat. In effect, the model assumes that the depleted capacity, the presence of a backup supplier, and their history of lead time uncertainties are the only factors that influence the optimal portfolio choice of the buyer.

The broad objective of this paper is to optimally split orders between a primary supplier and a backup supplier when natural disasters disrupt their supply potentials so that the buyer's orders are fulfilled within the specified period to the highest extent possible.

The specific research questions that the paper seeks to answer are the following:

- *Q1: How to revise the supply target of a primary supplier and set fresh target for a backup supplier when the supply capacities and supply chain are badly affected due to the occurrence of natural disasters?*
- *Q2: How to refine the revised targets by using the past knowledge of the buyer on supply and lead time uncertainties of the suppliers?*
- *Q3: How to generate the optimal supplier portfolio during supply disruption and how to test its sensitivity to the risk preference of the buyer?*
- *Q4: How to demonstrate the application of the model in conditions of supply disruption in an experimental setting?*

## Overview of the Study

Supply disruption has a striking similarity with the disruption of play in a widely popular sport, namely cricket, in case of natural calamities. The Duckworth–Lewis–Stern method (DLS) method is used in cricket to set revised target scores for the two playing teams. The method considers the remaining time of the match and the number of wickets (resource capacity) left with the playing teams as the only factors to set the revised targets. It does not consider match rewards promised to the teams by the International Cricket Council as an influencing factor, an assumption which is similar to our assumption that no supplier can have any price premium on account of resource scarcity. Furthermore, the DLS method considers both the teams to be almost equally competitive, which is similar to our assumption that the supplier cannot make any differentiated product at times of depleted resource positions. Naturally, the DLS method provides a high potential to be adopted to solve the problem of optimal portfolio choice during supply disruption. The paper answers the first research question by adjusting supply lead times and setting supply targets for the two suppliers according to their resource positions in ways similar to the DLS method.

The second and third questions are answered by developing a two-moment decision-theoretic model following the approaches used by Epstein (2005), Eichner (2008), and Eichner & Wagener (2003, 2009, 2011, 2012). The model considers the risks perceived by a risk-averse buyer due to the uncertainty associated with supply disruptions, for making an optimal portfolio choice to split orders between the primary supplier and the backup supplier. With depleted and uncertain resource endowments, the realised supply quantities from both the suppliers are likely to differ from the revised targets set by the DLS principle. The likely shortfalls require a rationalization of the revised targets. Such a rationalization is done by considering the preferences of the buyer towards the perceived risks of obtaining the supplies under supply uncertainty that helps in finding the buyer's optimal portfolio choice. Thus, our approach not only revises the original target for the primary supplier and sets new target for the backup supplier using the DLS method, but subsequently refines these targets based on the preferences of the risk-averse buyer towards the supply uncertainties experienced by the buyer in the past.

To answer the fourth question, we conducted a two-round questionnaire survey on 300 purchase managers. In the first-round survey, we provided the DLS method-generated revised supply targets to the participants in the survey and presented them with 10 scenarios, each scenario representing the amount of order outstanding with the primary supplier at a particular time point in the scheduled supply period and the fractions of capacity of the primary supplier and the backup supplier which are unaffected by the natural calamity. The participants were asked to estimate the probabilities of fulfilling the orders by the two suppliers in the remaining supply lead time. In the second-round survey, groups of ten participants were provided with mean and standard deviation of probability estimates for the ten scenarios made by each participant in the group and each participant was asked to rank the scenarios of means and standard deviations. Ranking so done, helped to estimate the risk preferences of the respondents.

### Contribution of the Paper

The paper contributes to the supply chain research arena in many ways. It demonstrates the use of the DLS method used in cricket to set revised supply targets. When a one-day (50-over) cricket match gets interrupted by bad weather, floodlight failure, or undesired crowd behaviour, the well-accepted DLS method (Duckworth & Lewis, 2004) is applied to revise the target scores for the two playing teams. The cue provided by this method is used in the present study to set a revised supply target for the primary supplier and new target for the backup supplier. The second contribution the paper makes is the consideration of the behavioural aspects of a risk-averse buyer modelled as her risk preferences in order to refine the revised supply targets. The third contribution is the use of design of experiments using human subjects to demonstrate the application of the model to supply disruption problem in an experimental setting. Two other noteworthy contributions of the paper are the way the risk preference structure of a buyer can be measured in terms of elasticities (i.e., relative willingness to pay for an incremental perturbation in risk) and confirmation of the fact that the risk preference structure would generally follow both decreasing absolute risk aversion and variance vulnerability of preferences, both of which are supportive of the existence of "properness" in the risk aversion behaviour.

### Organization of the Paper

This paper is organized as follows: The next section makes a literature review pertaining to optimal portfolio choice to split the order and behavioural investigations using experimental approaches. Followed by the modelling framework comprising the DLS method for target fixing and the two-moment decision model of the buyer. Then we discuss the comparative statics for changes in the distribution of primary and backup requisitions and the buyer's optimum portfolio choice owing to changes in the dependence structure between these two random variables. Subsequently, we present the experimental procedure, within- and between-group analysis of experimental results, and an econometric analysis of these results. Finally, we interpret the results, state the conclusions reached, and discuss the various connotations of the modelling approach and the conclusions reached.

## LITERATURE REVIEW

Several authors (e.g., Tang, 2006; Tomlin, 2009; Sawik, 2011; Li, 2017; Torabi et al., 2015; Namdar et al., 2018, Dada et al., 2007; Gurnani et al., 2014; Li, 2017; Demirel et al., 2018; Zhao & Freeman, 2019) have extensively addressed the issues concerning supply portfolio under supply disruptions. The most common setting considered by them is that of routine sourcing, where a single buyer faces uncertainty of supply due to catastrophe-affected suppliers. In such a situation the difference in resource endowments of these suppliers leads to differing supply uncertainties (Snyder et al., 2016). The buyer's preference towards perceived risks of splitting the order among these suppliers is influenced by behavioural aspects of the buyer, particularly when the buyer is risk-averse (Gurnani et al., 2014).

However, the problem of optimal portfolio choice of a risk-averse buyer whose preferences towards perceived risks of splitting the orders among catastrophe-affected suppliers having different resource endowments has not been addressed in the literature.

The seminal work of Schweitzer and Cachon (2000) in the field of behavioural operations management (BOM) addresses the problem of routine sourcing and suggests that an optimal solution to a routine sourcing problem based on analytical findings deviates under behavioural influence of individuals in practice, due to varied decision-making capacities of individuals. Furthermore, the study identifies that individual decisions to solve the problem is occasionally correct. However, their deviation from the optimal decisions are systematic and predictable. Research on BOM include both analytical and behavioural disciplines. The central idea is to build decision-making models that explain, predict and improve the analytical models in operations management domain (Becker-Peth & Thonemann, 2019). Even though literature identifies an exhaustive review paper on BOM, discussing the buying behaviour under supply risk (Fahimnia et al., 2019). Gurnani et al. (2014) have also emphasized studying behavioural aspects of buying decisions through integrating analytical and experimental approaches.

Many studies have used an *expected utility* – based approach to derive optimal portfolio choice of a buyer. For instance, Li (2017) developed, for a deterministic demand scenario, an optimal portfolio choice of a buyer who procures from two uncertain sources associated with random disruption risk and random yield risk. Gurnani et al. (2014) and He et al. (2017) derived optimal conditions based on cost and risk parameters of a buyer while sourcing either from a single source (more certain but costly) or from a backup supplier (more risky but cheaper). Zhao and Freeman (2019) used a profit-oriented correlation structure to optimally split order between uncertain sources. A few authors have also considered a risk-averse buyer facing uncertainty of supply, while making an optimal portfolio choice. For instance, Shu et al. (2015) developed a single-period model and obtained a unique optimal order quantity for the effective control of supply risk under stochastic demand scenario. Later, considering different cases of capacity and probability of disruption, Dupont et al. (2018) studied a similar type of scenario for a risk-averse buyer.

The above-mentioned studies have modelled the buyer's risk preference using the expected utility approach, whereas we have used a two-moment decision-theoretic model to determine the optimal portfolio choice of a risk-averse buyer seeking supply required quantity from a primary supplier (RQ) within a certain period. But a catastrophe-affected supplier may fail to supply the required quantity during the supply period. Hence, the buyer will place order for the remaining quantity (BQ) with another supplier—defined as the 'backup supplier'—to ensure smooth operations by receiving the supply within the remaining supply time. However, the catastrophe has adversely affected the representative backup supplier as well. Here, the perceived risk of getting the required quantity from each supplier is measured in terms of the mean ($\mu$) and standard deviation ($\sigma$) of the uncertain total quantity demanded by the buyer from each supplier; and the preference of a representative buyer dependent on suppliers is based on a "well-behaved" utility function, defined over these two moments.

The two-moment model applied in this study is understood to be an intuitive instrument in the analysis of decision-making under uncertainty. The preference framework for the model is a perfect substitute for the classical expected utility (EU) approach, given that all feasible distributions belong to a location–scale family (Meyer, 1987). In such contexts, risk attitudes (such as risk aversion, prudence) of a resource-dependent buyer, originally formulated in the EU approach as reported earlier, have convenient analogues in terms of two-moment decision models (or, in other words, $(\mu, \sigma)$-preferences). Meyer (1987) has converted the measures of absolute and relative risk aversion and their monotonicity properties from the EU approach into the $(\mu, \sigma)$ decision-theoretic framework. Lajeri & Nielsen (2000) and Eichner & Wagener (2003; 2005) have developed a $(\mu, \sigma)$-equivalent for Kimball's (1990) notion of decreasing absolute prudence. Contributions towards modelling risk preferences, (e.g., Epstein, 1985; Ormiston & Schlee, 2001; Eichner, 2008; Eichner & Wagener, 2009; 2011; 2012; Guo et al., 2018) have demonstrated analogues of the EU properties like 'risk vulnerability', 'temperance', 'properness', 'standardness' etc. in terms of the relative willingness-to-pay for a change in risk, which falls under the ambit of mean-variance preference-theoretic analysis.

In response to the above stated need we have used the DLS method to set targets for the suppliers after disruption i.e., based on differing both lead time and resource constraint uncertainties of suppliers. Further, we have integrated it with a two-moment model to study the risk preferences of the buyer. Subsequently, we used the DLS method as a proxy in an experimental setup to collect responses and analyse the risk preference of buyers. Based on what we observe in our experimental evidence, the preferences defined over the optimal portfolio of the representative buyer's choices follow upward sloped and strictly convex indifference curves in the risk-return plane, supporting our analytical modelling framework of the *a priori* assumption of the buyer being risk-averse in nature. Thus, we model the risk preferences of a risk-averse buyer to split orders between risky suppliers under time and resource constraints in a market, where the suppliers producing generic inputs can neither sell at any price different from the market-determined price nor can they sell any differentiated product at that price i.e., the "marginal cost pricing equilibrium scenario" (Wu & Zhou, 2016).

## MODELING FRAMEWORK

In this section we have addressed the optimal portfolio choice of the risk averse buyer under routine sourcing using a two-step approach i.e., splitting the order between two suppliers (primary and backup suppliers) affected by catastrophe using the DLS method, followed by a two-moment decision theoretic approach to analytically derive the risk preference behaviour of the buyer.

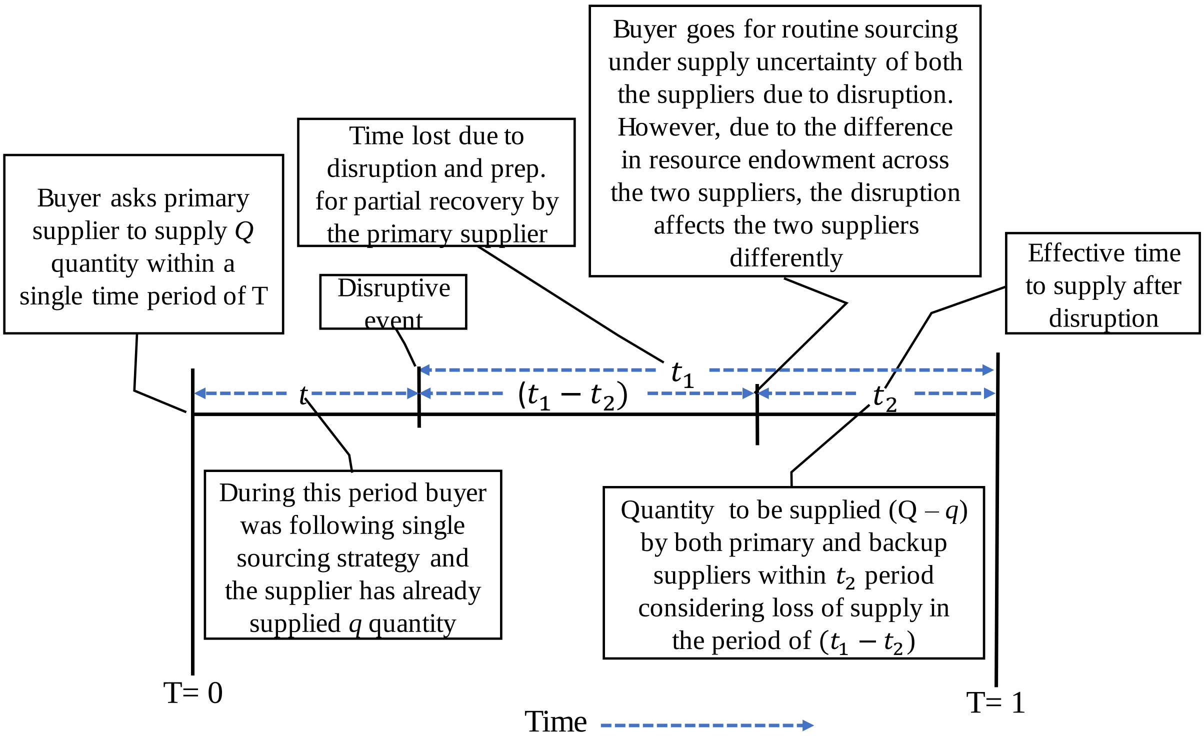


**Fig 1: Sequential decision making in a single period**

The framework used in this model is depicted in Fig. 1, where we consider a single period model encompassing four dates: $Time\ (T) = 0; T = t; T = (1 - t_2)$; and $T = 1$. Here, $T = 0$ is the date of ordering and $T = 1$ is final date of receiving the quantity $Q$ from the supplier(s). At $T = t$, the unprecedented disruption takes place. It is assumed that during $T = 0$ and $T = t$, the buyer received $q$ units against the order placed with the primary supplier. It is assumed that, due to the natural calamity, the supplier is unable to deliver any item for a period $(1 - t - t_2 = t_1 - t_2)$, where $t_2$ is the time left in the supply lead time to make the delivery. Realizing the lost time and the depleted resource position of the primary supplier, the buyer decides at time $T = 1 - t_2$ to opt for routine sourcing and places a fresh order with a back-up supplier and revises the quantity for the primary supplier in order to get the supply of $(Q - q)$ during a period $t_2$.

## TARGET SETTING USING SPORT'S PRINCIPLES

Originating in England in the 18th century, limited over Cricket has now become one of the most popular and widely followed sports globally (Shah, Sampat, Savla, & Bhowmick, 2015; Duckworth & Lewis, 2004). This sport is played between two teams each consisting of eleven players on a cricket field. Each phase of play is called an inning, during which one team bats, attempting to score as many runs as possible within a limited over (e.g., 50 overs) at the expense of 10 wickets, followed by the other team that chases the runs in their given inning. However, due to unwanted disruptions (as mentioned earlier) a definite result is not obtained. In such cases, the DLS method has been used to revise the target scores and/or declare a winner.

The DL (Duckworth–Lewis) principle and the subsequently revised DLS method are nothing but a statistical method used to predict the target score of the team batting in the second innings of a limited over (50-over long) cricket match, interrupted due to unavoidable circumstances (Duckworth & Lewis, 2004; Shah et al., 2015). When a few overs are lost, setting an adjusted target is not as simple as proportionally reducing the batting team's target, because a team batting second with a few wickets in hand is expected to play more aggressively (than when it has the full 50 overs to play) and achieve a higher run rate. In order to eliminate this anomaly, the DLS method considers the most common situation where two teams play a full-length cricket match with each side having 100% of its resources (Duckworth & Lewis, 2004; Stern, 2009; Shah et al., 2015). The central focus of the DLS method is to adjust "remaining time left in the sport" based on the remaining resources available (i.e., the remaining number of overs a team must face and the remaining number of wickets in hand). As one can observe in Fig. 2, there is an exponential increase in the target to be scored by the team batting second, with less number of wickets and overs in hand (http://www.-boltoncricket.co.uk/DLcalc.html accessed on 24 June, 2018). The model underlying the graph is derived in Appendix A. We identify an analogy between cricket and suppliers in supply chain with respect to the process of target fixing during catastrophe-caused disruptions.

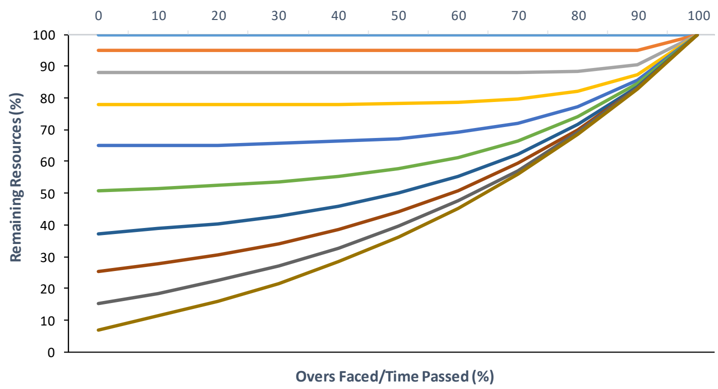


**Fig 2: Target setting of teams/suppliers using DLS method**

*(Source: https://www.wikiwand.com/en/Duckworth-Lewis-Stern_method)*

### *Application and Interpretation of the DLS method*

Suppose a buyer asks a supplier to supply $Q$ units within a time-length from date $T = 0$ to the date $T = t$ (before a catastrophe strikes). However, this supplier had been able to supply only $q$ units ($q < Q$) within the time-length $t$ ($t < T$), since at date $T = t$, a catastrophe erodes the supplier's resource position (capacity) to bring it down to (100 – $c$) per cent of its full capacity ($0 < c < 100$), resulting in disruption of the supply to the buyer. We further assume that the supplier cannot supply a single unit within a time length $t_1 - t_2$ after the catastrophe strikes. With the depleted capacity, the supplier can at best supply $D$ ($D < Q - q$) number of items in the remaining time $t_2$. Apprehending the reduced supply from the primary supplier, the buyer looks for, and places fresh order of BQ ($= Q – q – D$) units, with a back-up supplier with $t_2$ $(0 < t_2 < t_1)$ of the supply lead time remaining.

Since no supply took place for a time period $(t_1 - t_2)$, the supply target for the primary supplier (RQ) needs to be adjusted to compensate for the loss of time. The proportion of resources (capacity and lead-time) lost due to the catastrophe in the period $t_1$ is $\{Prob(t_1, c) - Prob(t_2, c)\}$.

So the proportion of supply resources available with the primary supplier is $R_{RQ} = [1 - Prob(t_1, c) + Prob(t_2, c)]$. Thus, the primary supplier's supply target is reduced to become $TQ = QR_{RQ}$ and the remaining quantity needed to be supplied by the primary supplier is $TR_{RQ} = (TQ - q)$.

The target can be revised and set by considering the values of $Prob(t_1, c)$ and $Prob(t_2, c)$ from the DLS based tabulated values (see Appendix A, Table A). The target score is the next higher integer. Hypothetical examples of three cases for the same are reported in Table 1, where buyer's required quantity to supply $Q$ = 300 units.

**Table 1: Target setting for both Primary and backup suppliers**

| **Parameter** | **Case 1** | **Case 2** | **Case 3** |
|---|---|---|---|
| Already supplied (out of $Q$ = 300) is $q$ | 150 | 100 | 80 |
| Capacity loss, $c$ | 60% | 40% | 20% |
| Remaining time to supply, $t_1$ | 40% | 60% | 80% |
| Effective time to supply, $t_2$ | 20% | 20% | 20% |
| $Prob(t_1, c)$ | 0.308 | 0.541 | 0.778 |
| $Prob(t_2, c)$ | 0.228 | 0.283 | 0.308 |
| $Prob(t_1, c) - Prob(t_2, c)$ | 0.080 | 0.258 | 0.470 |
| $R_{RQ} = [1 - Prob(t_1, c) + Prob(t_2, c)]$ | 0.920 | 0.742 | 0.530 |
| $TQ$(Revised Target) $= QR_{RQ}$ | 276 | 223 | 159 |
| Based on the revised target, remaining quantity to be supplied ($TR_{RQ} = TQ - q$) by the primary supplier within $t_2$ time and remaining $(100 - c)$% capacity | 126 (= 276-150) | 123 (= 223-100) | 79 (= 159-80) |
| Quantity to be supplied by the backup supplier(s) within $t_2$ time is $TR_{BQ} = Q - TQ$ | 24 (= 300-276) | 77 (= 300 -223) | 141 (= 300 -159) |

Using the DLS method, the buyer revises the target for the primary supplier so that it can supply the amount within the remaining time-frame $t_2$. However, whether the supplier, with its limited capacity, would be able to supply the revised quantity depends on its own input choices. Ordinarily, the optimum input choice should be such that the actual quantity being supplied by the supplier converges to the DLS-specified target. But the supplier also faces the so-called "lead time uncertainties", which affects its optimum input choice. Hence, the primary supplier may fail to supply even the DLS-specified revised quantity.

Because both the primary supplier and the backup supplier are affected by the natural calamity, the supply from the backup supplier is also equally risky. In other words, the input choice of the backup supplier is a random variable. We have applied the mean-variance decision-theoretic modelling approach to consider the risk preference of the buyer while addressing her optimal portfolio choice decision.

## TWO-MOMENT DECISION-THEORETIC MODEL

In this section, we are proposing a two-moment decision-theoretic model to analyse the buyer's sourcing decision under uncertainty, based on the sequence of events as shown in Fig. 1.

At date $T = (1 - t_2)$, the perceived probabilities $\alpha_k^{RQ} > 0$ are associated with supplying the remaining quantity $TR_{RQ}$ (after the DLS-specified revision of target) by the primary supplier for $k$ different possible effective capacities, having probabilities $(c_1, \dots, c_k)$, with $0 < c_i < 1$, will lead to the realised probability $p = (p_1, p_2, \dots, p_k)$, (wherein $p_i = \alpha_i^{RQ}.c_i$, for any $i = 1, \dots, k$) of obtaining the DLS-specified revised quantity from the primary supplier at date $T = 1$.

Similarly, at date $T = (1 - t_2)$, the perceived probabilities $\alpha_k^{BQ} > 0$ are associated with supplying the freshly targeted quantity $TR_{BQ}$ by the backup supplier for $k$ different possible effective capacities, having probabilities $(c_1, \dots, c_k)$, with $0 < c_i < 1$, will lead to the realised probability $q = (q_1, q_2, \dots, q_k)$, (wherein $q_i = \alpha_i^{BQ}.c_i$, for any $i = 1, \dots, k$) of obtaining the remaining quantity from the backup supplier at date $T = 1$. By construction, $p_i + q_i = 1$ for any $i$.

We are going to examine the risk preferences of a risk-averse buyer in a perfectly competitive scenario, for which it is sufficient to consider that the primary and the backup suppliers are facing any one of the $k$ different possible effective capacities. This simplification has been made for analytical simplicity and hence the subscript $i$ has been dropped. Consequently, we consider $p$ and $(1 - p)$ as the reduced probabilities of supplying the DLS-specified quantity and the remaining quantity by the primary and the backup supplier, respectively. In other words, $p$ is the endogenous variable of the model denoting the buyer's realised probability of obtaining the DLS-specified required quantity from the primary supplier at date $T = 1$; and $(1 - p)$ is the realised probability of obtaining the remaining quantity from the backup supplier at date $T = 1$.

First, we denote the buyer's preferences by a two-parameter utility function:

$$U = U(\sigma_Y, \mu_Y) \qquad (1)$$

where

$$\tilde{Y} = p(TR_{RQ} - \widetilde{D}_{RQ}) + (1-p)\big(TR_{BQ} - \widetilde{D}_{BQ}\big) = p\widetilde{RQ} + (1-p)\widetilde{BQ} \qquad (1.1)$$

and $\widetilde{D}_{RQ}$ and $\widetilde{D}_{BQ}$ are the random deviations from $TR_{RQ}$ and $TR_{BQ}$ (where the two latter order quantities are obtained from the DLS method, as explained in Table 1) respectively, with $\widetilde{D}_{RQ} \in (-\underline{D}_{RQ}, \overline{D}_{RQ})$ and $\widetilde{D}_{BQ} \in (-\underline{D}_{BQ}, \overline{D}_{BQ})$. For the sake of notational simplicity, we are dealing with the random 'net' quantities perceived by the buyer to be received from the primary and backup suppliers respectively as $\widetilde{RQ}$ and $\widetilde{BQ}$.

$$\mu_Y = E(\tilde{Y}) = p\mu_{RQ} + (1-p)\mu_{BQ} \qquad (2)$$

$$\sigma_Y = \sqrt{p^2\sigma_{RQ}^2 + (1-p)^2\sigma_{BQ}^2 + 2p(1-p)\mathrm{cov}\big(\widetilde{RQ}, \widetilde{BQ}\big)} \qquad (3)$$

For any random variable $\widetilde{W}, (\sigma_W, \mu_W)$ are the standard deviation and mean parameters. Subsequently, the covariance between any pair of random variables $\widetilde{W}, \tilde{Z}$ is denoted as $\mathrm{cov}\big(\widetilde{W}, \tilde{Z}\big)$. Since, the $\tilde{Y}(p)$ is a linear function of the two random variables $(\widetilde{RQ}, \widetilde{BQ})$, correlations (or covariances) serve as the most appropriate parameter to characterize the dependence structures between them (Eichner & Wagener, 2011). We can rewrite $\mathrm{cov}\big(\widetilde{RQ}, \widetilde{BQ}\big)$ as $\rho\sigma_{RQ}\sigma_{BQ}$, where $\rho \in (-1, +1)$ is the Pearson correlation coefficient between $\widetilde{RQ}$ and $\widetilde{BQ}$.

For this case the random variables are: (1) 'net' quantities perceived by the buyer to be received from the primary supplier at date $T = 1$; and (2) 'net' quantities perceived by the buyer to be received from the backup supplier at date $T = 1$, both of which would affect $\tilde{Y}(p)$ through their distributions and dependence structure (Cf. Embrechts et al., 2002).

We are making the following assumption regarding the buyer's preference function defined over risk and return, where the preference function means the buyer's utility function that maps the buyer's choice over risk of the portfolio $(\sigma_Y)$ versus return $(\mu_Y)$ in $\mathbb{R}_+$. Here our preference function is $U(\sigma_Y, \mu_Y)$, which follows the assumptions (1)-(4) mentioned below.

(1) For any random objective function $\widetilde{W}$ (where $\widetilde{W} \in \{\widetilde{RQ}, \widetilde{BQ}, \tilde{Y}\}$), the utility function $U(\sigma_W, \mu_W)$ is, at least four times continuously differentiable.
(2) We have, the marginal utility with respect to (w.r.t. hereafter) $\mu_W$ as positive while the marginal utility w.r.t. $\sigma_W$ as negative i.e., $U_\mu(\sigma_W, \mu_W) > 0$, $U_\sigma(\sigma_W, \mu_W) < 0$. In other words, we are assuming that the buyer's preferences towards risk satisfy non-satiation (increasing in $\mu_W$) and the buyer is risk-averse (decreasing in $\sigma_W$).
(3) The indifference curves (ICs hereafter) in $(\sigma_W, \mu_W)$-plane are positively sloped and strictly convex.

(4) The ICs enter the $\mu_W$-axis with zero slope i.e., exhibiting risk-neutrality for very small risks.

The above-mentioned assumptions restrict this study to a risk-averse buyer only, with monotonic and strictly quasi-concave preferences. This implies that the buyer is worse off receiving an additional source of risk because of the backup supplier's uncertain supply prospect, starting from an already risky situation. In other words, the compensation that is required for facing uncertainties in backup supplier's supply prospect, in addition to the risk emanating from the uncertain supply prospect of the primary supplier, is higher than the compensation required for facing only the risk owing to the uncertain supply prospect of the primary supplier alone.

The marginal rate of substitution (MRS) between risk and return for $\tilde{Y}(p)$ is defined by

$$S(\sigma_Y, \mu_Y) = -\left(\frac{U_\sigma(\sigma_Y, \mu_Y)}{U_\mu(\sigma_Y, \mu_Y)}\right). \tag{D1}$$

$S > 0$ is the two-parameter equivalent to Arrow–Pratt measure (Arrow, 1970; Pratt, 1964) of absolute risk aversion (or, equivalently, risk attitude).

## The decision dilemma

The risk-averse buyer chooses to maximise

$$\max_{(0 \le p \le 1)} U(\sigma_Y, \mu_Y) \text{ s.t., (2) and (3)}$$

***Scenario 1: $\mu_{RQ} = \mu_{BQ}$***

First, let us consider the scenario with $\mu_{RQ} = \mu_{BQ}$. In that case, the buyer would optimally select the portfolio that minimizes the variance of suppliers. Considering $p = \bar{p}$ as the optimum choice of portfolio when $\mu_{RQ} = \mu_{BQ}$, the optimization problem boils down to

$$0.U_\mu\left(\sigma_Y(\bar{p}, \sigma_{RQ}, \sigma_{BQ}, \rho), \mu_Y(\bar{p}, \mu_{RQ}, \mu_{BQ})\right) + \left(\partial\sigma_Y(\bar{p}, \sigma_{RQ}, \sigma_{BQ}, \rho)/\partial p\right).U_\sigma\left(\sigma_Y(\bar{p}, \sigma_{RQ}, \sigma_{BQ}, \rho), \mu_Y(\bar{p}, \mu_{RQ}, \mu_{BQ})\right) = 0$$

Or,

$$\left(\partial\sigma_Y(\bar{p}, \sigma_{RQ}, \sigma_{BQ}, \rho)/\partial p\right).S(\sigma_Y, \mu_Y) = 0,$$

As from equation D1, $S(\sigma_Y, \mu_Y) = -\frac{U_\sigma(\sigma_Y, \mu_Y)}{U_\mu(\sigma_Y, \mu_Y)}$ is the marginal rate of substitution between risk and return (equivalently, the relative willingness-to-pay for change in risk). Since, choosing optimal portfolio of supply sources is a risky venture under disruptions, and the buyer is risk-averse, $S(\sigma_Y(.),\mu_Y(.))$ is positive.

Hence, the problem boils down to choosing $\bar{p}$ such that $\left(\partial\sigma_Y(\bar{p},\sigma_{RQ},\sigma_{BQ},\rho)/\partial p\right)=0$, which is effectively the variance ($\sigma_Y^2$) minimization problem: choosing $\bar{p}$ as the minimum variance portfolio, such that $\bar{p} \coloneqq \arg\min_p \sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)$. Moreover, we assume that the moments $\sigma_{RQ},\sigma_{BQ}$, and $\rho$ are such that the buyer is receiving supply from both primary and backup suppliers simultaneously to meet the required input supply even under this minimum-variance portfolio. In other words, $(\partial\sigma_Y/\partial p)_{p=0} < 0 < (\partial\sigma_Y/\partial p)_{p=1}$. Wright (1987) and Eichner & Wagener (2011) argued that this condition is satisfied provided,

$$\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right) < \min\{\sigma_{RQ}^2,\sigma_{BQ}^2\} \text{ holds true.} \tag{4}$$

See Appendix B for a formal proof of eq. (4).

The inequality in (4) implies $\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right) < \left(\sigma_{RQ}^2+\sigma_{BQ}^2\right)/2$. Hence, $\sigma_Y(p)$ is strictly convex. Therefore, the minimization of buyer's preferences function following equation (3) w.r.t. $p$ yields,

$$\left(\partial\sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)/\partial p\right) = (1/\sigma_Y)\left[p\left(\sigma_{RQ}^2+\sigma_{BQ}^2-2\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)\right)-\sigma_{BQ}^2+\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)\right]=0$$

That yields, the minimum variance portfolio, $\bar{p}$ (as defined before), such that

$$\bar{p} = \frac{\sigma_{BQ}^2-\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)}{\sigma_{RQ}^2+\sigma_{BQ}^2-2\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)} \in (0,1). \tag{5}$$

From here, it can be shown that more than half of the ordering quantity will be allocated to the primary supplier.

***Scenario 2: $\mu_{RQ} \neq \mu_{BQ}$***

Now, let us consider more general scenario with $\mu_{RQ} \neq \mu_{BQ}$, but obeying condition (4). In other words, it is always possible that the primary supplier fails to even supply the DLS-specified quantity and therefore, the buyer must not depend entirely only on the primary supplier rather simultaneously ask the backup supplier to supply the remaining quantity (although the buyer has not been dependent on the backup supplier so far). Condition (4) ensures this precise fact for which $p^*$(optimum $p$) $\in (0,1)$.[1]

[1] The motivation for the risk-averse buyer to go for both primary and backup suppliers under routine sourcing has also been discussed by Sawik (2011), where he has demonstrated that for a risk-averse buyer, the impact of disruption risks is mitigated by diversification of the supply portfolio. Hence, $p^*$(optimum $p$) must lie between 0 and 1.

Focusing only on the interior solution of the decision problem, the First Order Condition (F.O.C. hereafter) yields,

$$\frac{\mu_{RQ} - \mu_{BQ}}{(\partial \sigma_Y(.)/\partial p)_{p=p^*}} = \{S(\sigma_Y(.), \mu_Y(.))\}_{p=p^*} \qquad (6)$$

Wherein

$$\left(\sigma_Y(p, \sigma_{RQ}, \sigma_{BQ}, \rho)\right)_{p=p^*} \left(\frac{\partial \sigma_Y}{\partial p}\right)_{p=p^*}$$
$$= p^* \sigma_{RQ}^2 - (1 - p^*)\sigma_{BQ}^2 + (1 - 2p^*)\mathrm{cov}\left(\widetilde{RQ}, \widetilde{BQ}\right) \qquad (7)$$

It can easily be demonstrated that the Second-Order Condition for maximum always holds true due to (i) quasi-concavity of $U(\sigma_Y, \mu_Y)$, (ii) the risk-averse nature of the buyer, and (iii) convexity of $(\partial \sigma_Y(.)/\partial p)$ in $p$ (See Appendix-B for the explicit proof of the Second-Order Condition satisfaction). F.O.C. in (6) then defines the marginal condition where the slope of a $(\sigma_Y, \mu_Y)$-indifference curve (denoted by the LHS) or the marginal willingness to pay (in terms of expected quantity foregone) for a reduction in the risk associated with the overall supply prospects of the two suppliers, is equal to the slope of the so-called "efficiency frontier" as sown in Fig. 3 (i.e., at point 0 of the diagram below).

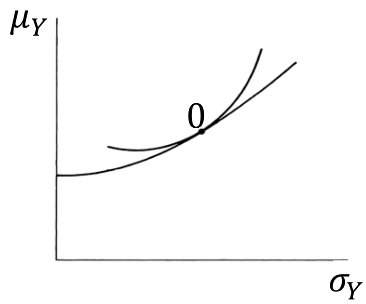


**Fig. 3: Optimum choice under uncertainty**

If $(\mu_{RQ} - \mu_{BQ}) > 0$, $(\partial \sigma_Y/\partial p)$ is always positive at the optimum. In other words, as the buyer is becoming more inclined towards the primary supplier for receiving the DLS-specified quantity, the overall risk i.e., the randomness associated with the final realised supply quantity increases at an increasing rate at $p = p^*$. In other words, $\left(\frac{\partial \sigma_y}{\partial p}\right) > 0$, at $p = p^*$. At *any* solution to (7), the second-order derivative is negative, following the monotonicity and quasi-concavity properties of the preference function, risk-aversion nature of the buyer and the convexity of $\sigma_Y(p^*)$. Therefore, (6) ensures the existence of a unique solution of $p^*$.

Let us specifically take up the case of $\mu_{RQ} > \mu_{BQ}$, in order to demonstrate under what conditions (in terms of relative sensitivity towards risks), a risk-averse buyer maintains the "solidarity network" by keeping her trust upon the primary supplier than upon the backup supplier, even when it is highly possible for the primary supplier to fail in supplying the DLS-specified quantity. Additionally, we need to satisfy the criteria, for which the increases in standard deviations of $\widetilde{RQ}$ and $\widetilde{BQ}$ (such that their correlation is held fixed but covariance varies) would also increase the overall risks, we need the following criteria to be satisfied:

$$\left(\frac{\partial\sigma_Y\left(p,\rho,\sigma_{RQ},\sigma_{BQ}\right)_{p=p^*}}{\partial\sigma_{RQ}}\right) > 0, \tag{8$a$}$$

and

$$\left(\frac{\partial\sigma_Y\left(p,\rho,\sigma_{RQ},\sigma_{BQ}\right)_{p=p^*}}{\partial\sigma_{BQ}}\right) > 0 \tag{8$b$}$$

Note that,

$$\left(\frac{\partial\sigma_y}{\partial\sigma_{RQ}}\right)_{p=p^*} = \frac{2p^*}{\sigma_Y}\left[p^*\sigma_{RQ} + (1-p^*)\rho\sigma_{BQ}\right]$$

While,

$$\left(\frac{\partial\sigma_y}{\partial(1-p)}\right)_{p=p^*} = \frac{2\sigma_{BQ}}{\sigma_Y}\left[p^*\sigma_{RQ} + (1-p^*)\rho\sigma_{BQ}\right]$$

Since, for the risk-averse buyer, approaching the backup supplier alone for any supply request itself is a risky venture and should magnify the higher overall riskiness of the total supply prospect, $\left(\frac{\partial\sigma_y}{\partial(1-p)}\right)_{p=p^*} > 0$, and therefore, condition (8$a$) i.e., $\left(\frac{\partial\sigma_Y\left(p,\rho,\sigma_{RQ},\sigma_{BQ}\right)_{p=p^*}}{\partial\sigma_{RQ}}\right) > 0$, is automatically satisfied.

Similarly, condition (8$b$) requires

$$\left[(1-p^*)\sigma_{BQ} + \rho p^*\sigma_{RQ}\right] > 0 \tag{8c}$$

– to hold at the optimum.

Thus, none of these conditions pre-imposes any criterion on the sign of the correlation-coefficient, ρ. For instance, even with $\rho \in (-1,0)$, condition (8c) is automatically satisfied as long as $\rho > -\left[\frac{(1-p^*)\sigma_{BQ}}{p^*\sigma_{RQ}}\right]$ holds.

Below we define two concepts used in the study further. This definition will lay foundation before proceeding to the comparative static analyses.

Let us define $\varepsilon_{\sigma}(\mu_Y, \sigma_Y)$ as the elasticity of the marginal rate of substitution between risk and return w.r.t. the standard deviation of the final realised supply quantity obtained by the buyer, ceteris paribus. Algebraically,

$$\varepsilon_{\sigma}(\mu_Y, \sigma_Y) = [\{\partial S(\sigma_Y, \mu_Y)/\partial \sigma_Y\}\{\sigma_Y/S(\sigma_Y, \mu_Y)\}] \qquad \text{(D2)}$$

Similarly, we denote $\varepsilon_{\mu}(\sigma_Y, \mu_Y)$ as the elasticity of the marginal rate of substitution between risk and return w.r.t. the expected value of the final realised supply quantity obtained by the buyer, ceteris paribus. Algebraically,

$$\varepsilon_{\mu}(\mu_Y, \sigma_Y) = [\{\partial S(\sigma_Y, \mu_Y)/\partial \mu_Y\}\{\mu_Y/S(\sigma_Y, \mu_Y)\}] \qquad \text{(D3)}$$

## COMPARATIVE STATICS

### Changes in the distribution of uncertain net quantity obtainable from the primary supplier

In order to analyse the changes in the distribution of the random *net* quantity receivable from the primary supplier, which is due to the random deviation of the DLS specified revised quantity due to resource uncertainty induced random supply prospect of the primary supplier. To be precise, this sub-section explores how the buyer's relative decision of sourcing from the primary supplier (vis-à-vis the backup supplier) at the margin is affected respectively

(a) for a perturbation in the standard deviation of the random *net* quantity receivable from the primary supplier ($\sigma_{RQ}$), *ceteris paribus*;

and

(b) for a perturbation in the mean of the random *net* quantity receivable from the primary supplier ($\mu_{RQ}$), *ceteris paribus*.

In this comparative static analysis, we have derived the propositions using the implicit partial differentiation technique following the works of Eichner and Wagener (2009, 2011, 2012), Broll & Mukherjee, (2017), Broll et al. (2020), Mukherjee et al. (2020). The implicit partial differentiation technique has been used to trace the partial impact of parametric perturbation (*ceteris paribus*) to analyze only the comparative static response of the endogenous decision variable (the risk perception of the buyer, $p$) around its optimum (maximum) owing to the changes in the distributions of $\widetilde{RQ}$ and

$\widetilde{BQ}$ and the dependence structure between them i.e., in $\text{cov}(\widetilde{RQ}, \widetilde{BQ})$. Consequently, we are using the implicit partial differentiation technique because of: (a) the involvement of more than one parameter like the endogenous variable ($p$) and the other parameters $(\rho, \mu_{RQ}, \mu_{BQ}, \sigma_{RQ}, \sigma_{BQ})$; (b) the F.O.C. characterizes the equation as being a function of $(p, \rho, \mu_{RQ}, \mu_{BQ}, \sigma_{RQ}, \sigma_{BQ})$, from where it is impossible to isolate $p$, and express it neatly in terms of any one of the moments $(\mu_{RQ}, \mu_{BQ}, \sigma_{RQ}, \sigma_{BQ})$. On the top of that, all our comparative static effects are expressed in terms of relative trade-offs between risks and returns, likewise in Eichner & Wagener (2009; 2011); Mukherjee et al. (2020).

Our first comparative static result traces out the impact of a perturbation in the standard deviation of $\widetilde{RQ}$ on the optimum responsibility to be allocated to the primary supplier. As argued above, the risk-averse buyer in our decision problem always maintains the "solidarity network" with her trust upon the primary supplier (i.e., $\mu_{RQ} > \mu_{BQ}$). Then,

**Proposition 1(a)**. *The buyer will always reduce the optimal dependence upon the primary supplier (i.e., lower $p^*$) in response to an increase in the standard deviation of the final realizations of the supply from the primary supplier, if and only if the elasticity of risk aversion w.r.t the standard deviation of $\tilde{Y}$ is greater than $-1$, ceteris paribus.*

**Proof.** See Appendix B.

A small rise in $\sigma_{RQ}$ results in lower revelation to the risk from primary supplier (and, thus, to a lower $\mu_Y$), provided the slope of the indifference curve becomes more sensitive to an increase in $\sigma_{RQ}$ than the slope of the efficiency frontier (which, at the optimum, is locally proportional to the value of risk aversion, $S$). Algebraically,

$$S_\sigma \frac{\partial \sigma_Y}{\partial \sigma_{RQ}} > -\left( \frac{S}{\left(\frac{\partial \sigma_Y(.)}{\partial p}\right)_{p=p^*}} \right) \times \frac{\partial^2 \sigma_Y(.)_{p=p^*}}{\partial p \partial \sigma_{RQ}}.$$

In order to ensure the reliability of results, we consider the "extreme case" where $\sigma_{BQ} = 0 = \text{Cov}(\widetilde{RQ}, \widetilde{BQ})$ i.e., the backup supplier will certainly supply, which directly yields $\varepsilon_\sigma > -1$. In other words, the degree of risk aversion must not significantly worsen with increase in riskiness of the supply prospect from the primary supplier.

Under our four assumptions regarding the buyer's preference function mentioned earlier, risk aversion behaviour derived in Proposition 1(a) in terms of the relative trade-off between risk and return (i.e. the elasticity condition) can be transformed to the properties of vNM expected utility representation.

Eichner & Wagener (2009) showed that for any $\beta > 1$, the sufficiency condition, $\varepsilon_\sigma > 1 - \beta$, is equivalent to the fact that the corresponding "index of relative prudence" $\left(= -U_{\mu\sigma}/U_{\mu\mu}\right)$ is smaller than $\beta$ in the EU-framework. Hence, putting $\beta = 2$, Proposition 1(a) states that the *index of relative prudence is smaller than 2* for a risk-averse buyer. This result also resembles Hadar & Seo (1990) for independent risks and Meyer & Ormiston (1994) for interconnected risk cases.

Moving on to evaluating the impact of a small increase in $\mu_{RQ}$, ceteris paribus, we can state the following proposition.

**Proposition 1(b)**. *The buyer will always increase the optimal dependence upon the primary supplier (i.e., higher $p^*$) in response to the mean of the final realizations of supply from the primary supplier, if and only if the elasticity of risk aversion with respect to the mean of $\tilde{Y}$ is less than 1, ceteris paribus.*

**Proof**. See Appendix B.

Increasing $\mu_{RQ}$ will lead to a higher dependence on the primary supplier, implying a higher overall risk, $\sigma_Y$, provided the consequential change in the slope of the indifference curve (which is proportional to $S_\mu$) is smaller than the subsequent change in the slope of the efficiency frontier (locally proportional to $S$) i.e.,

$$S_\mu p^* < S/\left(\mu_{RQ} - \mu_{BQ}\right)$$

This will always hold whenever the degree of risk aversion does not increase intensively in $\mu_Y$, w.r.t. its initial value.

Eichner & Wagener (2009) shows that $\varepsilon_\mu < 1$ is equivalent to stating that the "index of relative risk aversion" being smaller than one in an EU-framework, provided the four assumptions outlined previously are satisfied.

## Changes in the distribution of uncertain net quantity obtainable from the backup supplier

Let us now consider the fluctuations in the distribution of the random *net* quantity receivable from the backup supplier, which is due to the random deviation of the DLS specified order quantity due to the backup supplier's random supply prospect. This sub-section explores how the buyer's relative decision of sourcing from the primary supplier (vis-à-vis the backup supplier) at the margin is affected respectively:

(a) for a perturbation in the standard deviation of the random *net* quantity receivable from the backup supplier ($\sigma_{BQ}$), *ceteris paribus*;

and,

(b) for a perturbation in the mean of the random *net* quantity receivable from the backup supplier ($\mu_{BQ}$), *ceteris paribus*.

Given the above, let us first explore (a) i.e., the impact of a small increase in $\sigma_{BQ}$, *ceteris paribus*.

**Proposition 2a**. *The buyer will increase the optimal dependence upon the risky primary supplier (i.e., higher $p^*$) in response to an increase in the standard deviation of the final realizations of the supply from the lesser risky backup supplier, if and only if the elasticity of risk aversion with respect to the standard deviation of $\tilde{Y}$ is less than $-1$, ceteris paribus*.

**Proof**. See Appendix B.

The condition $\varepsilon_\sigma(\sigma_Y, \mu_Y) < -1$ necessitates $S_\sigma < 0$. Given that we have reasonably assumed the buyer's preference $U(.)$ is monotonic and strictly quasi-concave, as Eichner (2008) showed, $S.S_\mu + S_\sigma > 0$ holds true. Therefore, $S_\sigma < 0$ implies $S_\mu$ must be strictly positive. $S_\mu > 0$ indicates that the buyer's willingness to pay for a reduction in risk increases in $\mu_Y$. This signifies the Arrow-Pratt notion of increasing absolute risk aversion (IARA). Hence, IARA is also a necessary condition for the result stated in Proposition 2(a).

Looking back at the above proof of Proposition 2(a), it is easy to infer that the buyer would increase the optimal dependence upon the risky primary supplier (i.e., lower $p^*$) in response to an increase in $\sigma_{BQ}$, if and only if $\varepsilon_\sigma(\sigma_Y, \mu_Y) \to -\infty$.[2]

Now let us move to trace out the impact of a small increase in $\mu_{BQ}$ on optimum $p$.

**Proposition 2(b)**. *The buyer will always decrease the optimal dependence upon the primary supplier (i.e., lower $p^*$) in response to the mean of the final realizations of supply from the primary supplier, ceteris paribus, if and only if preferences follow IARA.*

**Proof**. See Appendix B.

[2] Eichner & Wagener (2011) discussed in detail about this. However, since, in the present context, this seems to be pointed towards an improbable result, we refrain to discuss in detail about this.

In other words, the risk-averse buyer will always reduce dependence upon the riskier primary supplier if the backup supplier is expected to fulfil her requirement, provided the buyer's preferences satisfy IARA.

## Optimum portfolio choice owing to changes in the dependence structure

**Proposition 3**. *If $p^* > (<)\, 1/2$, the buyer will increase (reduce) her dependence on the supply of required quantity from the primary supplier (which is a vulnerable option), in response to the increase in the concordance between these two sources of risks, if and only if the elasticity of risk aversion w.r.t. the standard deviation of $\tilde{Y}$ is less than (greater than) 1.*

**Proof**. See Appendix B.

To understand the implication of this proposition, let us coin the famous terminology of "variance-vulnerability" (or "variance-affinity") according to Eichner & Wagener (2003; 2009; 2011; 2012). Here the risk-averse buyer can be diagnosed as variance-vulnerable (variance-affine) if she reduces (or increases) her optimal dependence on the riskier bait of receiving the required quantity from the primary supplier, owing to an increase in an erraticism in $\tilde{Y}$, which is the result of the increase in the concordance between the two sources of risks (since $\left(\partial\sigma_Y/\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)\right) > 0$).

Whenever $p^* > (<)\, 1/2$, an increase (a decrease) in $p$ would induce the buyer to opt for more inclination towards one of the two supply sources. Therefore, Proposition 3 tells us under what condition(s) the buyer responds to the increased concordance between the two sources of risks by choosing more inclination towards one of the volatile supply sources.

As mentioned earlier, Eichner & Wagener (2009) demonstrated that $\varepsilon_\sigma > 1 - \beta$ is equivalent to the "index of relative prudence" being smaller than $\beta$ in the EU-approach. Hence, putting $\beta = 0$, we obtain the results that if $p^* > (<)\, 1/2$ and for positive (negative) "index of relative prudence", $\left(\partial\sigma_Y/\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)\right) > (<) 0$, which is similar in spirit to Epstein & Tanny (1980), although their contribution was only true for non-positively correlated asset returns, wherein our results hold for both positively and non-positively correlated random variables.

According to Tomlin (2009), for a risk-averse buyer, as correlation between the riskiness in supply requirements from the primary and backup suppliers increases, the dual-sourcing strategy would be more preferred with a greater degree of risk-aversion (in our case, with $p^* > 0.5$). This is because a risk-averse buyer would be more concerned about meeting the demand and would be more concerned about recuperating the remaining supply from the backup supplier. Hence, in such context of $p^* > 0.5$, the buyer would be reluctant to rely more on the backup supplier and rather would be inclined more towards the primary supplier. This is exactly what we have demonstrated in Proposition 3.

To address this optimal portfolio choice decision of a buyer to get the required quantity from suppliers under lead time and capacity uncertainties, we have used an experimental setup to generate data using the DLS method. Subsequently, we analysed the experimental data to compare with the mean-variance decision-theoretic modelling approach to understand the buyer's risk preferences and optimum allocation of supply portfolio. Specifically, our experiment results demonstrate the nature of risk aversion behaviour of the decision-maker that conforms with our Propositions (1) through (3). However, for the given respondents, the results only demonstrate positive correlations between two sources of risk under supply disruptions due to catastrophes. At the same time, our analytical framework captures more generic and universal results, remaining valid for positively or negatively correlated supply risks.

## EXPERIMENTAL SETUP AND ANALYSIS

To test our model, we have conducted four (repetitive) experiments to understand potential buyers' risk attitude in choosing between a primary and a backup supplier. For conducting the experiments, we contacted the respondents willing to participate in the experiment. In the first round, we aimed to find respondents with adequate experience in handling manufacturing firms' routine sourcing activities.

In the last quarter of 2018, we contacted 669 respondents engaged across 40 manufacturing firms in India. We have selectively considered those 40 manufacturing (buying) firms who used supply rating (or vendor rating) scorecards to classify their suppliers. The list of suppliers rated higher than the minimum desired value is treated as qualifiers. Among these qualifying suppliers, the best graded supplier is classified as the primary supplier, and the next best one is classified as the backup supplier. Thus, to be consistent with our analytical setting we have considered two suppliers throughout the experiments. Furthermore, to measure potential buyers' risk attitude while going for optimal portfolio choices, we provide them various scenarios, wherein for each scenario, the resource endowment (i.e., lead time and capacity uncertainties) is different for each of the two suppliers. We sent a simple questionnaire having a set of two questions, namely:

- Professional details: Your job title; your organizational function; years of experience in purchasing; industry type.
- Experience details: Have you ever experienced supply disruption related issues while engaged in purchasing activities i.e., when your suppliers were not able to supply the required quantity on time because of disruption at their end? If yes, what percentage of times?

After shortlisting the respondents i.e., based on experience (more than 5 years) and handling supply disruption (more than 20%), the final number of respondents narrowed down to 300, representing 44.8%

of the contacted respondents. Appendix D, Table D1 provides a detailed break-down of the shortlisted respondents sample. Approximately 72.7% of respondents were managers, mainly involved in purchasing (59%) and handling supply disruption issues for years. These respondents were having an average of 12.5 years of experience and are likely to possess an overarching, boundary-spanning view of their firms' purchasing activities.

**EXPERIMENTAL ANALYSIS**

Step 1: *Select purchase managers as respondents based on their purchasing experience*. For this experiment, we have selected 300 purchase managers with ten years of experience. At least 20% of the time, they have handled supply disruption during their purchasing of raw materials (more of a homogeneous sample in terms of experience).

Step 2: *The respondents were grouped randomly into 30 groups, each comprising of 10 respondents. Here, we coded each group and its members for our further analysis. Meanwhile, we provided a detailed explanation of the DLS method to all the respondents. After sensitization, we put the idea of an industrial set-up where the same catastrophe (like flood, earthquake, and Tsunami) could happen and the portfolio choice of allocating your supply between the primary and the backup suppliers becomes realistic, rather than depending on single source of supply.*

Step 3: *Design survey template by controlling the effective capacity and remaining supply time as the manipulation variables to obtain the target quantity (using DLS method as proxy) to be supplied by both the suppliers individually, as treatment variables. Based on which perceived risk, as a direct measure, of the respondent were measured (Green, Tull, & Albaum, 2009). Moreover, it has been observed that such manipulation did not produce changes in measures of related but different constructs.*

Step 4: *Create two sets of sequential treatments each having 10 scenarios for primary (Appendix D, Table A2) and backup (Appendix D, Table A3) suppliers, respectively.*

Step 5: *In Experiment 1, we ask each of the respondents from each of the groups to estimate the perceived probability (dependent variable as a direct measure) of supplying the target quantity (DLS based) by both the suppliers, this is based on the treatment variables. Subsequently, as a part of blocking the respondents' biasness, the Randomized Complete Block Design (RCBD) is performed (Following Montgomery, 2001) during the experimental data collection procedure (see Analysis of RCBD in appendix D, Table-D4 and -D5). where each group is considered as a block and DLS based target quantities are given as treatments to collect the data. Next, we obtain the random net realization i.e., by multiplying perceived probability with the DLS target for each respondent.*

Step 6: *Now we have 10 responses from each group based on given treatments, for both RQ and BQ. Next, we calculate the mean and standard deviation of the random net realization for each group.*

Thus, based on these 10 observations of random net realization values of each group, we calculate the two moments for the primary $(\mu_{RQ}, \sigma_{RQ})$ and backup suppliers $(\mu_{BQ}, \sigma_{BQ})$ in **Experiment 1**.

*Step 7:* *In experiment 2, the mean and standard deviations obtained in Step 6 were now ranked on a scale of 1 to 10 by the respondents. Having hidden the identity, the mean and standard deviation were the only source for the ranking activity.*

*Step 8:* *The summative value of ranks, for each of the choices is calculated*

*Step 9:* *The minimum rank sum for each group for both RQ and BQ was identified and the corresponding probability of making the optimal choice is also calculated i.e., Minimum [(1-(rank sum/100)), ...)], probability of optimal choice = Minimum [(1-(14/100)), (1- (22/100)), ..., (1-(73/100))] which is 0.86.*

*Step 10:* *We obtain 30 optimal choices (p*) for each of RQ and BQ and arrange the same in descending order of p* value and plot the same against corresponding optimal values of $\mu$ and $\sigma$ across groups in a $\mu - \sigma$ plane as reported in Fig. 4.*

**Experiment 2** enables us to identify the intra-group probability of selecting RQ optimally (i.e., $p^*$). Choosing a backup supplier is just a complementary event.

Fig. 4, where the vertical-axis measures mean (i.e., return) and the horizontal-axis measures the standard deviation (i.e., risk), demonstrates the decision-maker's choice to take more risk (i.e., higher S.D.) if and only if, she is compensated with even higher return (i.e., her willingness-to-pay for more return increases with increase in risk). But this locus is strictly quasi-concave implying the increase in willingness-to-pay at a decreasing rate, with increase in risk. This reflects the risk vulnerability[3] of preferences and appropriateness of risk aversion behaviour as suggested by Lajeri-Chaherli (2002) and Eichner (2008). Hence, the premium required for uncertainties on account of the supply prospects of both types of suppliers is certainly higher than that required to pay if the backup supplier is trustworthy i.e., supplying on time. Furthermore, from Figure 3 it is evident that all the optimal choices (i.e., corresponding $p^*$values) lie on a positively sloped strictly quasi-concave locus, which establishes the fact that group $i$ $(i = 1, \ldots, 30)$ is willing to give up more than the $(i + 1)^{\text{th}}$ group in terms of expected return for reduction in risk.

[3] Please see Lajeri-Chaherli (2002, 27, Figure 4, pp. 55) and Eichner (2008, 54(3), Figure 1, pp. 590) and the explanation also given in Appendix B.

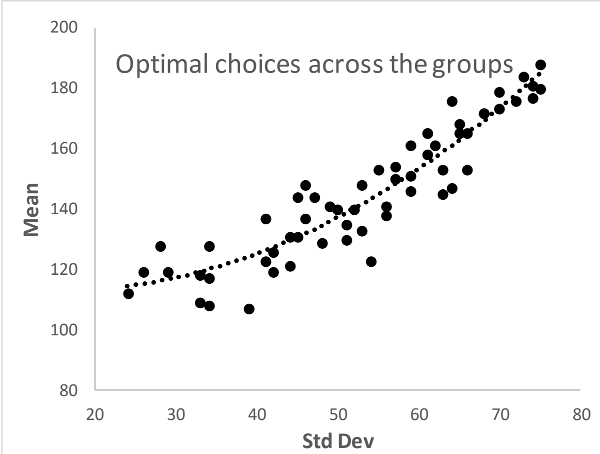


**Fig. 4: Optimal choices across groups for selecting RQ and BQ based on their offerings**

*<u>Step 11:</u> In experiment 3, considering the 30 optimal choices ($\mu_{RQ}^{*}$) individually (for RQ and BQ separately) we divide the mean optimal choices into four quartiles. Further, we choose the range of second and third quartile to generate new data whereas we maintain the original range of standard deviation i.e. we compress the mean w.r.t the standard deviation. Subsequently, we follow Step 7 through 9 to obtain the new optimal choice. Figure 4 reports the indifference curve of relative change in standard deviation w.r.t their means for both RQ and BQ, based on similar exercise.*

In experiment 3, we try to find the intra-group optimal probability ($p^{*}$) of going to RQ when the relative change in the standard deviation happened w.r.t their means. We perform this by using the new dataset generated by *compressing the mean w.r.t. the standard deviation. Subsequently, we plot the corresponding* ($\mu_{RQ}$) and $\sigma_{RQ}$ against the newly obtained $p^{*}$. The same procedure (as followed in step 7 through 9) is repeated for tracing out the change in $p^{*}$ owing to changes only in ($\mu_{BQ}$) and ($\sigma_{BQ}$). Lastly, based on the selection of 30 optimal primary choices and 30 corresponding secondary choices, we find the covariance of the two optimal choices in each set.

Fig. 5 reports the horizontal movement of buyers' preferences keeping the vertical axis almost constant (i.e., small change in return), w.r.t. significant variation in risk. This is a scenario of relative increase in risk (w.r.t. return) in panels I and II, depicting increased steepness in the slope (i.e. higher "risk vulnerability" of the buyer) of the new indifference-curve present in the $(\mu - \sigma)$ plane. This reflects that the buyer will be more vulnerable towards choosing between the relative increase in risk w.r.t. return and the optimal choice at a fixed return in case of RQ. Therefore, the buyer will prefer to choose less risky return than optimal return under high risk conditions. Similar observations were also drawn in case of BQ. Moreover, these observations capture the analogous comparative static response as reported by Eichner (2008), also the results of this study is reported in Appendix C.

Additionally, Fig. 5 (in Panels I and II) explains the impacts on $p^*$ owing to the relative increase in standard deviations w.r.t. the means for RQ and BQ, respectively. As depicted in Figure 4, when the standard deviations of the realized quantities supplied for the primary and backup suppliers are increasing, w.r.t. their means, the loci of the newer choices become steeper than the original loci of optimal choices. These outcomes bolster the predictions as derived in propositions 1(a) and 2(a) together.

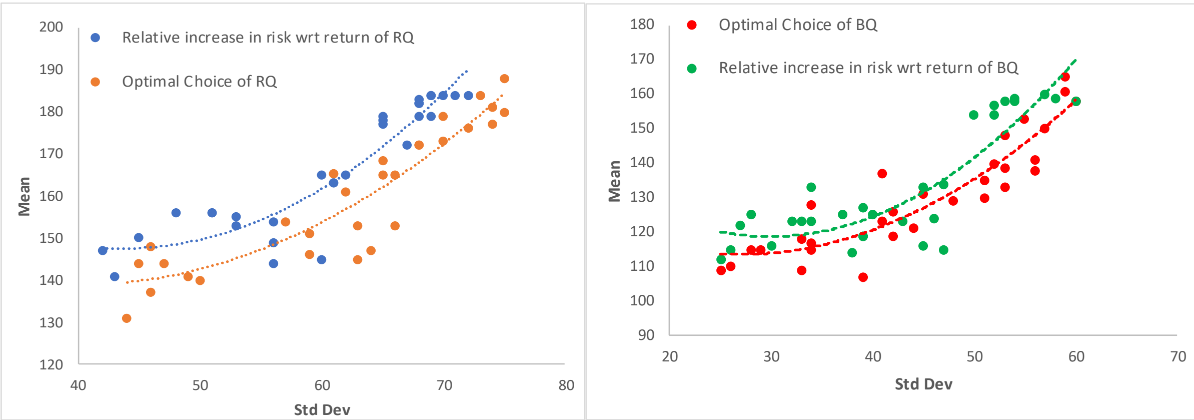


**Fig. 5: Indifference curves representing the relative change in standard deviations *w.r.t.* their means for RQ and BQ**

<u>*Step 12:*</u> *In experiment 4, we compress the standard deviation with respect to mean following a similar procedure as to Step 11. Fig. 6 reports the indifference curve of relative change in means with respect to their standard deviations for both the suppliers.*

Figure 5 reflects the scenario of relative increase in mean (w.r.t. risk) in panels I and II, depicting the reduction in the slope of the new indifference curve at the new $p^*$ corresponding to its optimal $\mu$ and $\sigma$ choices, which reproduces similar comparative static response to the one captured in Eichner (2008) also stated in Appendix C.

Figure 5 (in Panels I and II) explains the changes in $p^*$ following the relative increase in mean w.r.t. standard deviation for RQ and BQ, respectively. As depicted in Figure 5, when the expectations of the realised quantities supplied by the primary and backup suppliers are increasing, w.r.t. their standard deviations, the loci of the new optimal choices become flatter than the loci of the initial optimal choices. These outcomes, therefore, strengthen the predictions derived in propositions 1(b) and 2(b) together.

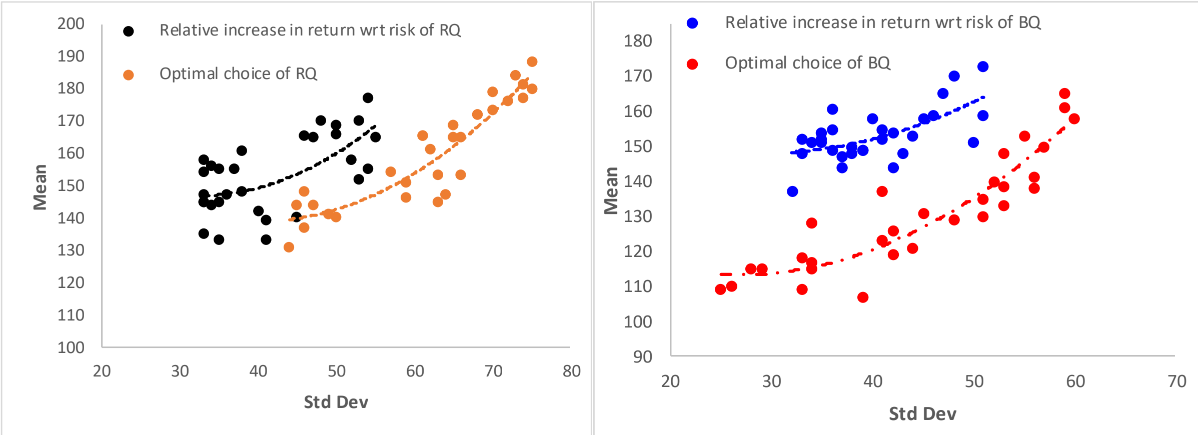


**Fig. 6: Indifference curves representing the relative change in means *w.r.t.* their standard deviations of supply for RQ and BQ**

After this, we also compute the correlation between these two sources of risk ($\sigma_{RQ}\ and\ \sigma_{BQ}$) to check whether both sources of risks are indeed positively correlated among the 30 groups of respondents. The correlation coefficient is 0.86 at a 5% level of significance, indicating strong positive correlation, under the optimal choices. Albeit these experimental results only demonstrate positive correlations between two sources of risk under supply disruptions due to catastrophe, our analytical framework in the previous section yields more generic and universal results. Hence, our analytical framework remains robust for positively or negatively correlated supply risks. Therefore, any group of respondents would be categorized as variance-vulnerable (variance-affine) if that group reduces (increases) its optimal dependence on the riskier bait of receiving the required quantity from the primary supplier with increased concordance between these two sources of risks.

## ECONOMETRIC ESTIMATION

So far, we have done experimental study to understand the risk preferences across 30 groups (using RCBD), but did not consider the intragroup data for all respondents (30 groups of 10 respondents each), at different points of the entire Marginal Rate of Substitution (i.e., for the entire distribution of $S$). We assume the following flexible parametric buyer's preferences that nest all possible risk preference structures:

$$U = U(\sigma_Y, \mu_Y) = \mu_Y^a - \sigma_Y^b \tag{15}$$

Where $\mu_Y$ and $\sigma_Y$ are already defined in Section 3, while $a$ and $b$ are parameters constituting risk-preferences and we assume that $b > 0$, for a risk-averse decision-maker. One can see Broll & Mukherjee (2017), Broll et al. (2020); Mukherjee et al. (2020) in this context.

The MRS is now given by $S(\sigma_Y, \mu_Y) = -\frac{U_\sigma(\sigma_Y,\mu_Y)}{U_\mu(\sigma_Y,\mu_Y)} = \frac{b}{a}\mu_Y^{1-a}\sigma_Y^{b-1}$.

Focusing only on the interior solution of the decision problem, the first order condition (F.O.C. hereafter) yields,

$$\frac{\mu_{RQ} - \mu_{BQ}}{(\partial \sigma_Y(p^*)/\partial p)} = S\big(\sigma_Y(p^*), \mu_Y(p^*)\big) = \frac{b}{a}\mu_Y^{1-a}\sigma_Y^{b-1} \tag{16}$$

Therefore, it is easy to infer

$$\ln S = \ln\frac{b}{a} + (1-a)\ln\mu_Y + (b-1)\ln\sigma_Y \tag{17}$$

Where $\varepsilon_\sigma(\mu_Y, \sigma_Y) = (b-1)$ and $\varepsilon_\mu(\mu_Y, \sigma_Y) = (1-a)$.

To quantitatively test our predictions, we use (17) as our unique structurally estimable equation. Secondly, to capture the differences across respondents in terms of the structure of their risk preferences, we need to generate the coefficient estimates at different points of the entire risk distribution (i.e., of $S$). To meet these two objectives, we utilize the quantile regression method (Koenker, 2005). This is an extensively used estimation technique used to investigate the relation between the dependent variable and a set of explanatory variables in specific quantiles.

Standard Ordinary Least Square techniques focus on estimating the average response of the dependent variable to the changes in values of the explanatory variables. However, in the present context, we need to estimate the coefficient of the explanatory variables at different points of distribution of the dependent variable. Thus, we have used the quantile regression technique, which can give us separate coefficient estimates (the risk aversion elasticities) for different quantiles of the dependent variable (participants' attitude towards risks). Another advantage of this method is its robustness concerning the outlier values of the dependent variable. Once the coefficients are estimated, bootstrap replications are used to generate standard errors to avoid imposing distributional assumptions, which is also an advantage of using this method.

Table 2A reports the estimation results. Subsequently, the F-tests reported in Table 2B shows that most of the coefficients significantly differ across consecutive quantiles, rationalizing the use of quantile regression in the present context.[4] The risk appetite of the participants shows the following patterns.

**Table 2A: Regression Results**

| **Dependent variable: ln $S$** | **10%** | **20%** | **30%** | **40%** | **50%** | **60%** | **70%** | **80%** | **90%** |
|---|---|---|---|---|---|---|---|---|---|

[4] There exists significant literature that have used quantile regression in the context of "small" sample comprising only 113 observations or less. One such example is of Gomanee et al. (2005).

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\ln \mu_Y$ | -0.161 | -0.225* | -0.201** | -0.438** | -0.378** | -0.265** | -0.345** | -0.231** | -0.177* |
| | (0.185) | (0.109) | (0.084) | (0.061) | (0.056) | (0.051) | (0.072) | (0.084) | (0.101) |
| $\ln \sigma_Y$ | 0.385+ | 0.187* | 0.246* | 0.308** | 0.257** | 0.387* | 0.417** | 0.279* | 0.128+ |
| | (0.149) | (0.102) | (0.094) | (0.077) | (0.082) | (0.091) | (0.079) | (0.109) | (0.129) |
| $\ln \frac{b}{a}$ | -2.936** | -1.531* | 0.107* | -1.916** | -1.658** | -1.368** | -1.003** | -0.774* | 0.688** |
| | (0.399) | (0.229) | (0.193) | (0.160) | (0.150) | (0.201) | (0.186) | (0.078) | (0.048) |
| Observations | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 |
| Pseudo $R^2$ | 0.093 | 0.086 | 0.092 | 0.098 | 0.103 | 0.120 | 0.157 | 0.203 | 0.267 |

Note: **, *, + are respectively denoting levels of statistical significance at 1%, 5% and 10% levels; standard errors are in parentheses.

We have performed a quantile regression for 300 cross-sectional observations by collecting the data for all the groups without any time-effect. It is widely known that R-square is generally low in cross-sectional data as compared to time-series data. Wooldridge (2002) also supports the same stating that in any regression using cross-sectional data, one should rely more on the significance of the individual coefficients (likewise we have found in our regression results) rather than the magnitude and significance of R-square. Hence, we are emphasizing more on the significance of individual coefficients.

Starting with the intercept term, we can see that $\beta_1$ i.e., $\ln \frac{b}{a}$ is statistically significant for all quantiles. Consequently, the antilog of this term is also non-zero across all the quantiles. Therefore, none of the participants exhibit risk neutrality. After that looking at the coefficient of $\ln \sigma_Y$ (which corresponds to $\varepsilon_\sigma$, which is also equal to $b-1$) is statistically significant across all quantiles. However, given that this term is positive across all the quantiles, $b$ is significantly greater than 1 or $S_\sigma > 0$, which implies all participants in these quantiles are "variance vulnerable". On the other hand, since $\varepsilon_\sigma$ is greater than -1 across all the quantiles, we can infer (i) $b > 0$, implying risk aversion, and (ii) $\frac{\partial p^*}{\partial \sigma_{RQ}} < 0$ i.e., the participants in these quantiles reduced their optimal dependence upon the primary supplier (i.e., lower $p^*$) in response to the greater uncertainty in its supply prospects. This is directly following Proposition 1(a). In fact, since $(b-1)$ is less than 1 in all cases, we can state that $p^*$ is more likely to be less than ½ and the participants are going to exhibit negative relative prudence w.r.t. the increase in the concordance between the two sources of risks. This is directly following our theoretical prediction stated in Proposition 3. Furthermore, because $\frac{b}{a}$ is less than 1 in all quantiles barring the 30th and 90th quantiles only, the preference structure for these participants are not characterized by constant relative risk aversion (CRRA).

On the other hand, the coefficient of $\ln \mu_Y$, which is related to $(1-a)$, is statistically significant from the 20th quantile up to the 90th quantile of the risk distribution. However, the point to be noted that $(1-a)$, or equivalently, $\varepsilon_\mu$, is negative across all the quantiles (with statistical significance from the 20th quantile) where $b > 1$ (i.e. $S_\sigma > 0$). Given the fact that $S_\mu = (b/a)(1-a)\left(\sigma_Y^{b-1}/\mu_Y^a\right) < (>) 0$ with $a > (<) 1$, we can safely infer that the participants falling in these quantiles are exhibiting "decreasing absolute risk aversion" or DARA (with $a > 1$) and simultaneously "variance vulnerability" with $b > 1$. At the same time, since, $\varepsilon_\mu = (1-a) < 1$, we also have $\frac{\partial p^*}{\partial \mu_{RQ}} > 0$. This has been reported in Proposition 1(b).

**Table 2B: F-test for equality of coefficients across different quantiles**

| Dependent Variable S | 20% | | 30% | | 40% | | 50% | | 60% | | 70% | | 80% | | 90% | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ | ln μ | ln σ |
| 10% | 5.58** | 3.05* | | | | | | | | | | | | | | |
| 20% | | | 1.97* | 0.09 | | | | | | | | | | | | |
| 30% | | | | | 0.16 | 7.83** | | | | | | | | | | |
| 40% | | | | | | | 2.14* | 0.23 | | | | | | | | |
| 50% | | | | | | | | | 8.35** | 0.08 | | | | | | |
| 60% | | | | | | | | | | | 0.59 | 0.78 | | | | |
| 70% | | | | | | | | | | | | | 4.66** | 0.79 | | |
| 80% | | | | | | | | | | | | | | | 0.06 | 5.15** |

Thus, this study provides an understanding of the buyer's portfolio choice under uncertainties of supply. For example, based on experiment 1, mean and standard deviation of primary and backup supplier's supply quantity are (147 ± 99) and (68 ± 44), respectively (see Appendix D). Now considering the quantile regression output for the case of 50% where $\ln\frac{b}{a} = -1.658$, (1-a) i.e., coefficient of $\ln \mu_Y$ is $-0.378$ and $(b-1)$ i.e., coefficient of $\ln \sigma_Y$ is 0.257. Based on these figures the computed value of $\ln S$ using equation (17) is $-2.22$. These observations imply that under no catastrophe, when the buyer is confronting the risk emanated from the uncertain supply-prospect of the primary supplier only, the buyer would have to be compensated by 15.72% more for him to diversify between both sources of supply. In other words, the buyer would demand 15.72% more from the backup supplier, albeit the total requirement is of 300 units.

## CONCLUSIONS

Under supply disruption due to catastrophe, when the primary supplier exhausts a certain proportion of her resources as well as some of her stipulated time to supply, the risk-averse buyer would prefer to split the order by giving the responsibility of supplying certain proportion of the remaining order to some backup supplier. However, maintaining a primary supplier and getting the desired quantity from backup suppliers under uncertainty of supply is challenging for any resource-dependent buyer. This task is understood to be complex, considering the stipulation of time (Snyder et al., 2016), capacity of the supplier disrupted by the catastrophe like global pandemic (or due to close geographical proximity to the site of catastrophe) and maintaining a long-term relationship with the primary supplier at the same time by revising the targets for both the suppliers. This revision is to aid them meet the newly set targets. This study uses the popular DLS method to split the orders.

Because disrupted cricket matches provide a strong parallel with disrupted supplies in supply chains, we thought it fit to draw upon the nuances of the method of DLS, which is applied in the cricket matches to revised the targets for the competing teams, to revise supply targets for the primary and back-up suppliers. But, whereas the DLS method relies on the data available in hundreds of previously conducted, disrupted international cricket matches to estimate the model parameters, there are no such available past data in the context of supply chains.

Subsequently, a two-moment model is considered to analyse the comparative static responses of perturbation under each of the stochastic parameters in the buyer's portfolio of risky options. This is based on his/her relative trade-offs between risks and returns. The key determinants for the risk-preferences of buyer's choice turn out to be elasticities of risk aversion w.r.t. the mean and the standard deviation of the total random supply of both the suppliers taken together.

The two moments approach yields all the comparative static responses in terms of the relative trade-offs between returns and risks, without taking recourse to the higher-order or cross-derivatives of the utility functions and their composites. This is the primary advantage over the EU framework. We further undertake an experiment using the RCBD, considering 300 respondents to exemplify and test the robustness of the theoretical predictions across 30 different groups (blocks) of respondents. We analysed the data using a two-prong approach to study the risk preference of buyer at both the group level and the individual level. The risk preference of buyers at group level is analysed considering the supply prospect risk of timely supply, associated with both the suppliers. It is observed that the buyers need to be compensated more for managing the risk for both primary and backup suppliers than in case of primary supplier only. Thus, the buyer's perceived risk aversion towards uncertain supply prospects of the primary supplier is never enhanced considering unaltered precautionary premium even when the buyer diversifies the supply portfolio between the primary supplier and backup supplier, given the fact that the backup supplier is also disrupted by the catastrophe. Considering propositions 1(a) and 2(a) together, the experimental outcome (experiment 2) bolsters the observations of Guo et al. (2016) that with unexpected enhancement in the riskiness of supply prospects w.r.t. returns for each of the two suppliers, the optimal choice of the buyers always inclines towards the relatively less risky supplier. However, with increase in the expectations of high returns w.r.t. risks, the buyers would optimally hinge over the relatively riskier supplier. These outcomes strengthen the predictions derived in propositions 1(b) and 2(b) together.

Although our analytical modelling framework suggests more generic and universal results for positively or negatively correlated risks associated with uncertain supply-prospects of the disrupted primary and backup suppliers, for the given sample, our experimental study exhibits strong positive correlation of perceived risks associated with the uncertain supply prospects from both sources, under the optimal choices. Nevertheless, the generic conclusion of proposition 3 from our analytical model, viz., owing to the increased concordance of the two sources of risks, the buyers reduce (or increase) their dependence on the riskier supplier, for receiving the required quantity from the primary supplier, also holds true in the experiment. This phenomenon is known as variance-vulnerable (variance-affine) by Zhao & Freeman (2019) because of the global nature of the disruption (as in our case) of affecting the disrupted suppliers due to their geographical proximity

Subsequently, at the individual level the data are analysed using quantile regression. It is observed that all the respondents are variance vulnerable, implying a buyer risk aversion behaviour. The respondents in all quantiles reduced their optimal dependence on the primary supplier in response to the enhanced riskiness in its supply prospects as reported in Proposition 1(a). Moreover, based on the quantile regression the buyer would have to be compensated by 15.72% more in order to diversify between both the sources. The buyers exhibit negative relative prudence w.r.t. the increase in the concordance between the two sources of risks as reported in Proposition 3. The preference structure for these participants cannot be characterized by constant relative risk aversion (CRRA) because of the inconsistency (not being $< 1$) in the constant term of the quantile regression. However, considering $20^{th}$ through $90^{th}$ quantile of the risk distribution we infer that the participants in these quantiles exhibit "decreasing absolute risk aversion" and variance vulnerability of preferences, which are supportive of the existence of "properness" in the risk aversion behaviour, as reported in Proposition 1(b). Thus, this paper contributes to the BOM literature by mapping the risk preference behaviour of risk averse buyers at group and individual level. Where the buyers are dependent on supply resources under supply time and capacity uncertainty.

## Appendix A; Appendix B; Appendix C; Appendix D

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**APPENDIX – A**

We use the well-established DL-method to set the supply target reflecting the resource availability of the suppliers, keeping note of the buyer's supply requirement. Following DLS method we establish a two-factor relationship to determine the proportion of tentative quantity to be supplied by the supplier using two resources namely remaining supply time and in-hand capacity. To obtain this relationship we apply a mathematical expression using DLS method. For instance, a buyer could obtain $G(t)$ units of supply on an average in $t$ days from a supplier (say RQ) which may be labelled by the following exponential equation (Duckworth & Lewis, 1998)

$$G(t) = G_0[1 - \exp(-\gamma t)] \quad \text{(A1)}$$

$G_t$ will be $G_0$ when $t \to \infty$, thus we can interpret $G_0$ as the asymptotic average of total supply without any supply lead-time constraint and $\gamma$ is the exponential decay constant obtained from the DLS method. Considering the loss of capacity with $t$ days remaining to supply to be $c$, the asymptote becomes lower, and the decay constant becomes higher. Both asymptotes being functions of $c$. Based on Duckworth & Lewis (1998) the revised relationship is of the form:

$$G(t,c) = G_0(c)[1 - \exp\{-\gamma(c)t\}] \quad \text{(A2)}$$

Similarly, $G_0(c)$ is defined as the asymptotic average of total supply based on the remaining capacity without considering the lead-time constraint as previously mentioned and $\gamma(c)$ being the exponential decay constant. Both the expressions depend on the portion of resources already lost.

Considering equation (2) at the beginning of the supply (with no capacity lost) where ($t = T$ and $c = 0$), we get

$$G(T,0) = G_0(c)[1 - \exp\{-\gamma T\}]$$

and the ratio

$$Pro(t,c) = \frac{G(t,c)}{G(T,0)} \quad (3)$$

Equation (3) represents the average proportion of the tentative supply target with a lead-time of $t$ period remaining and $c$ percent capacity lost. Unlike an interrupted cricket match, where we can rely on the data observed in hundred previously conducted international matches the absence of data for the suppliers urges the use of DLS parameter setting as a proxy to revise the target of the suppliers. Table A1 provides a snapshot from Fig 1 exhibiting the DLS based target, under time remaining and capacity lost parameter settings.

**Table A1: Snapshot of the resource-based parameters value**

| | Capacity lost (1-Remaining Resources) in % | | | | |
|---|---|---|---|---|---|
| Time Remaining (Total time- time passed) | 0 | 20 | 40 | 60 | 80 |
| 50 (100%) | 1.000 | 0.851 | 0.627 | 0.349 | 0.119 |
| 40 (80%) | 0.893 | 0.778 | 0.595 | 0.346 | 0.119 |
| 30 (60%) | 0.751 | 0.673 | 0.541 | 0.336 | 0.119 |
| 20 (40%) | 0.566 | 0.524 | 0.446 | 0.308 | 0.119 |
| 10 (20%) | 0.321 | 0.308 | 0.283 | 0.228 | 0.114 |

## Appendix - B

### Proof of Claim (4)

Let us explicitly prove this below.

Differentiating $\sigma_Y$ in Eq. (3) with respect to (w.r.t. hereafter) $p$,

$$(\partial\sigma_Y/\partial p) = (1/\sigma_Y)\left[p\sigma_{RQ}^2 - (1-p)\sigma_{BQ}^2 + (1-2p)\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)\right]$$

Therefore, at $p = 0$,

$$(\partial\sigma_Y/\partial p) = (1/\sigma_Y)\left[\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right) - \sigma_{BQ}^2\right];$$

While, at $p = 1$,

$$(\partial\sigma_Y/\partial p) = (1/\sigma_Y)\left[\sigma_{RQ}^2 - \mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right)\right].$$

Hence, for $(\partial\sigma_Y/\partial p)_{p=0} < 0 < (\partial\sigma_Y/\partial p)_{p=1}$, to hold simultaneously, we need the following three criteria to be satisfied simultaneously as well:

(i) $\mathrm{cov}\left(\widetilde{RQ},\widetilde{BQ}\right) < \sigma_{RQ}^2$;

(ii) $\quad \mathrm{cov}(\widetilde{RQ}, \widetilde{BQ}) < \sigma_{BQ}^2$; and

(iii) $\quad \mathrm{cov}(\widetilde{RQ}, \widetilde{BQ}) < (\sigma_{RQ}^2 + \sigma_{BQ}^2)/2$.

It is easy to deduce that all of these above three inequalities are satisfied if $\mathrm{cov}(\widetilde{RQ}, \widetilde{BQ}) < \min\{\sigma_{RQ}^2, \sigma_{BQ}^2\}$. Hence,

$$\mathrm{cov}(\widetilde{RQ}, \widetilde{BQ}) < \min\{\sigma_{RQ}^2, \sigma_{BQ}^2\}. \qquad \text{(Q.E.D.)}$$

**<u>Proof of the Second-order Condition for the Interior Solution of</u>**

**$\max_{(0\le p\le 1)} U(\sigma_Y(p,\ldots), \mu_Y(p,\ldots))$ s.t. (2) and (3)**

Let us rearrange the F.O.C. in (6) as

$$\omega: \\ = (\mu_{RQ} - \mu_{BQ})U_\mu\left(\sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)_{p=p^*}, \mu_Y(p,\mu_{RQ},\mu_{BQ})_{p=p^*}\right) \\ + \left(\partial\sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)_{p=p^*}/\partial p\right)U_\sigma\left(\sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)_{p=p^*}, \mu_Y(p,\mu_{RQ},\mu_{BQ})_{p=p^*}\right) = 0.$$

Therefore, we have,

$$\omega_p(p^*,..) = -\left(\partial\sigma_Y(.)_{p=p^*}/\partial p\right)^2(1/U_\mu^2)(-U_\sigma^2 U_{\mu\mu} + 2U_\sigma U_\mu U_{\sigma\mu} - U_\mu^2 U_{\sigma\sigma}) + \\ (\partial^2\sigma_Y(.)_{p=p^*}/\partial p^2)U_\sigma\left(\sigma_Y(p,\sigma_{RQ},\sigma_{BQ},\rho)_{p=p^*}, \mu_Y(p,\mu_{RQ},\mu_{BQ})_{p=p^*}\right) \qquad \text{(B1)}$$

Given the strict quasi-concavity of $U(\sigma_Y, \mu_Y)$, the bordered Hessian determinant of $U(.)$ must be strictly positive, i.e.

$$\Lambda = \begin{vmatrix} 0 & U_\sigma & U_\mu \\ U_\sigma & U_{\sigma\sigma} & U_{\sigma\mu} \\ U_\mu & U_{\mu\sigma} & U_{\mu\mu} \end{vmatrix} = (-U_\sigma^2 U_{\mu\mu} + 2U_\sigma U_\mu U_{\sigma\mu} - U_\mu^2 U_{\sigma\sigma}) > 0. \qquad \text{(B1.1)}$$

Given (B1.1), risk-aversion nature of the buyer (owing to which, $U_\sigma(.) < 0$), and strict convexity of $\sigma_Y(.)$ in $p$ (owing to which, $(\partial^2\sigma_Y(.)/\partial p^2) > 0$), from (B1) we have $\omega_p(p^*,..) < 0$. Hence, the S.O.C. for maximization is satisfied. (Q.E.D.)

**<u>Proof of Proposition 1(a)</u>**.

**Proof**. Implicit partial differentiation[1] of (6) w.r.t. $\sigma_{RQ}$ yields,

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \sigma_{RQ}}\right) = \operatorname{sgn}\left[S\big(\sigma_Y(.)_{p=p^*}, \mu_Y(.)_{p=p^*}\big)\frac{\partial^2 \sigma_Y(.)_{p=p^*}}{\partial p \partial \sigma_{RQ}} + \frac{\partial \sigma_Y(.)_{p=p^*}}{\partial p}\frac{\partial S}{\partial \sigma_Y}\frac{\partial \sigma_Y}{\partial \sigma_{RQ}}\right]$$

Since, differentiating (3) and (7) partially w.r.t. $\sigma_{RQ}$, we obtain

$$\frac{\partial \sigma_Y}{\partial \sigma_{RQ}} = \frac{{p^*}^2 \sigma_{RQ}}{\sigma_Y(.)_{p=p^*}}$$

and,

$$\frac{\partial^2 \sigma_Y(.)}{\partial p \partial \sigma_{RQ}} = \frac{p^* \sigma_{RQ}}{\sigma_Y(.)_{p=p^*}}\left[2 - K_1(.)_{p=p*}\right] \text{ respectively.}$$

Where,

$$K_1(.)_{p=p^*} = \frac{p^*}{\sigma_y}\left(\frac{\partial \sigma_y}{\partial p}\right)_{p=p^*} > 0.$$

This is because the buyer is risk averse. Therefore, in order to keep the risk-premium, $(\mu_{RQ} - \mu_{BQ})$, positive at the equilibrium, from the F.O.C. it is easy to see that $\left(\frac{\partial \sigma_y}{\partial p}\right) > 0$, at $p = p^*$.

Substituting values, it is easy to obtain

$$K_1(.)_{p=p^*} = 1 - \sigma_{BQ}\left(\frac{(1-p^*)\sigma_{BQ} + \rho p^* \sigma_{RQ}}{\sigma_Y^2}\right).$$

Using condition (8c) we have,

---

[1] We use the Signum function (sgn) because of implicit partial differentiation (e.g., Broll & Mukherjee, 2017; Broll et al., 2020).

$$(1-p^*)\sigma_{BQ} + \rho p^* \sigma_{RQ} > 0, \forall \rho \in \left(-\frac{(1-p^*)\sigma_{BQ}}{p^*\sigma_{RQ}}, 1\right)$$

Therefore, $\forall \rho \in \left(-\frac{(1-p^*)\sigma_{BQ}}{p^*\sigma_{RQ}}, 1\right)$; & $\forall \sigma_{BQ} > 0, \sigma_{RQ} > 0,$ *we always have* $0 < K_1(.)_{p=p^*} < 1.$ This demonstrates the robustness of our results for both positive and negative correlation between $\widetilde{RQ}$ and $\widetilde{BQ}$.

After some simple manipulations

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \sigma_{RQ}}\right) = \operatorname{sgn}\left(\frac{2}{K_1(.)_{p=p^*}} - 1 + \varepsilon_\sigma\big(\sigma_Y(.), \mu_Y(.)\big)_{p=p^*}\right) \tag{B2}$$

Where, $\varepsilon_\sigma(\mu_Y, \sigma_Y)$ is already defined in D1.

Therefore, we have,

$$\left(\frac{\partial p^*}{\partial \sigma_{RQ}}\right) \leq 0, \text{if and only if } \varepsilon_\sigma(\sigma_Y, \mu_Y) \leq 1 - \frac{2}{K_1(.)}.$$

Since, $\left\{1 - \frac{2}{K_1(p^*)}\right\} \leq -1$, $\frac{\partial p^*}{\partial \sigma_{RQ}} < 0$, whenever $\varepsilon_\sigma(\sigma_Y, \mu_Y) > -1, \forall(\sigma_Y, \mu_Y)$. (Q.E.D.)

Note that the above result holds $\forall \rho \in \left(-\frac{(1-p^*)\sigma_{BQ}}{p^*\sigma_{RQ}}, 1\right)$.

**<u>Proof of Proposition 1(b)</u>**.

Implicit partial differentiation of (6) w.r.t. $\mu_{RQ}$ yields

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \mu_{RQ}}\right) = \operatorname{sgn}\left[1 - \frac{\partial \sigma_Y(.)_{p=p^*}}{\partial p}\frac{\partial S}{\partial \mu_Y}\frac{\partial \mu_Y}{\partial \mu_{RQ}}\right]$$

$$= \operatorname{sgn}\left[1-\frac{\mu_Y(p,\mu_{RQ},\mu_{BQ})_{p=p^*}}{S(\sigma_Y(.),\mu_Y(.))_{p=p^*}}\frac{\partial S}{\partial \mu_Y}\left(\frac{1}{\left\{1+\frac{1}{p^*\left((\mu_{RQ}/\mu_{BQ})-1\right)}\right\}}\right)\right]$$

$$= \operatorname{sgn}\left[1-\left(\frac{1}{\left\{1+\frac{1}{L(.)_{p=p^*}}\right\}}\right)\varepsilon_\mu(\sigma_Y(.),\mu_Y(.))_{p=p^*}\right]$$

(B3)

Since, $L(p^*) = p^*\left((\mu_{RQ}/\mu_{BQ})-1\right) < 1$, as we have $\mu_{RQ} > \mu_{BQ} > 0$. Therefore, $\{1 + 1/L(.)_{p=p^*}\} > 1$. From (10), it is easy to infer $\frac{\partial p^*}{\partial \mu_{RQ}} > 0$, if and only if $\varepsilon_\mu(\sigma_Y, \mu_Y) < \{1 + 1/L(.)\}$. Since $\{1 + 1/L(.)\} > 1$, it can safely be concluded $\frac{\partial p^*}{\partial \mu_{RQ}} > 0$ always holds true, whenever $\varepsilon_\mu(\sigma_Y, \mu_Y)_{p=p^*} < 1$ holds true.[2] (Q.E.D.)

**<u>Proof of Proposition 2(a)</u>**.

Implicitly differentiating (6) w.r.t. $\sigma_{BQ}$ yields,

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \sigma_{BQ}}\right) = \operatorname{sgn}\left[S(\sigma_Y(.),\mu_Y(.))_{p=p^*}\left(\frac{\partial^2\sigma_Y(.)}{\partial p\partial\sigma_{BQ}}\right)_{p=p^*} + \left(\frac{\partial\sigma_Y(.)}{\partial p}\right)_{p=p^*}\frac{\partial S}{\partial\sigma_Y}\frac{\partial\sigma_Y}{\partial\sigma_{BQ}}\right]$$

Since, partial differentiation of (3) and (7) w.r.t. $\sigma_{BQ}$ yields respectively

$$\frac{\partial\sigma_Y}{\partial\sigma_{BQ}} = \frac{(1-p^*)^2\sigma_{BQ}}{\sigma_Y(p^*)}$$

and,

[2] This result is, also, evidently independent of the sign of $\rho$.

$$\left(\frac{\partial^2\sigma_Y(.)}{\partial p\partial\sigma_{BQ}}\right)_{p=p^*} = \frac{(1-p^*)\sigma_{RQ}}{\sigma_Y(p^*)}[K_2(p^*)-2]$$

Where,

$$K_2(p^*) = -\frac{(1-p^*)}{\sigma_y}\left(\frac{\partial\sigma_y}{\partial p}\right)_{p=p^*}.$$

After some rearrangements

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial\sigma_{BQ}}\right) = -\operatorname{sgn}\left(\frac{2}{K_2(p^*)} - 1 + \varepsilon_\sigma\big(\sigma_Y(.),\mu_Y(.)\big)_{p=p^*}\right) \quad \text{(B4)}$$

Since, $p$ denotes the probability associated with a risky activity and the buyer is risk-averse, $(\partial\sigma_y/\partial p) > 0$. If $p^* \to 0$, i.e. the riskiness associated with the primary supplier's supply prospect is highly uncertain, and $S$ is bounded from above, it can be inferred $K_2(p^*) \in (-\infty, 0]$.[3] Therefore, $\frac{2}{K_2(p^*)} - 1 \in (-\infty, 1]$ and subsequently $\left(\frac{\partial p^*}{\partial\sigma_{BQ}}\right) > 0$ for all parameter values, if and only if $\varepsilon_\sigma(\sigma_Y,\mu_Y) < -1\ \forall\ (\sigma_Y,\mu_Y)$. (Q.E.D.)

**<u>Proof of Proposition 2(b)</u>**.

Implicit partial differentiation of (6) w.r.t. $\mu_{BQ}$ yields

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial\mu_{BQ}}\right) = -\operatorname{sgn}\left[1 + \left(\frac{\partial\sigma_Y(.)}{\partial p}\right)_{p=p^*}\frac{\partial S}{\partial\mu_Y}\frac{\partial\mu_Y}{\partial\mu_{BQ}}\right] \quad \text{(B5)}$$

Note that

$$\frac{\partial\mu_Y}{\partial\mu_{BQ}} = (1-p^*) > 0;\ \left(\frac{\partial\sigma_Y(.)}{\partial p}\right)_{p=p^*} > 0.$$

[3] Therefore, for this result to hold true, we do not need any *a priori* assumption regarding the sign of $\rho$.

Therefore, $\left(\frac{\partial p^*}{\partial \mu_{BQ}}\right) < 0$, if and only if $\frac{\partial S}{\partial \mu_Y} = S_\mu > 0 \; \forall \; (\sigma_Y, \mu_Y)$. (Q.E.D.)

**<u>Proof of Proposition 3</u>**.

Implicit partial differentiation of (6) w.r.t. $\mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)$ yields

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}\right) = -\operatorname{sgn}\left[\frac{S \frac{\partial^2 \sigma_Y}{\partial p \partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}}{\frac{\partial \sigma_Y}{\partial p}} + S_\sigma \frac{\partial \sigma_Y}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}\right]$$

After marginal simplification, we obtain

$$\operatorname{sgn}\left(\frac{\partial p^*}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}\right) = -\operatorname{sgn}\left[\Lambda(p^*) + \varepsilon_\sigma\right] \tag{B6}$$

Where,

$$\Lambda(p^*) = \frac{\sigma_Y \frac{\partial^2 \sigma_Y}{\partial p \partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}}{\frac{\partial \sigma_Y}{\partial p} \frac{\partial \sigma_Y}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)}}.$$

Since,

$$\frac{\partial \sigma_Y}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)} = \frac{p^*(1-p^*)}{\sigma_Y} > 0,$$

and,

$$\frac{\partial^2 \sigma_Y}{\partial p \partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)} = \frac{\partial^2 \sigma_Y}{\partial \mathrm{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right) \partial p} = \frac{1-2p^*}{\sigma_Y} - \frac{p^*(1-p^*) \frac{\partial \sigma_Y}{\partial p}}{\sigma_Y^2},$$

one can therefore obtain

$$\Lambda(p^*) = \sigma_Y \frac{1-2p^*}{p^*(1-p^*) \frac{\partial \sigma_Y}{\partial p}} - 1. \tag{B7}$$

From (B7), one can easily see $\left(\partial p^*/\partial \text{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)\right) < 0$, if and only if $\varepsilon_\sigma(\sigma_Y, \mu_Y) > -\Lambda(p^*)$. However, from (B7) one can understand that we get $\Lambda(p^*) \in (-\infty, -1)$ provided $p^* > 1/2$ (whereas $\Lambda \to -\infty$ would only materialize if $\mu_{RQ} \cong \mu_{BQ}$ takes place), and $\Lambda(p^*) > -1$, if and only if $p^* < 1/2$.[4]

Therefore,

(i) Whenever $p^* > 1/2$, $\left(\partial p^*/\partial \text{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)\right) > 0$ only if $\varepsilon_\sigma(\sigma_Y, \mu_Y) > 1$.

(ii) Whenever $p^* < 1/2$, $\left(\partial p^*/\partial \text{Cov}\left(\widetilde{RQ}, \widetilde{BQ}\right)\right) < 0$ only if $\varepsilon_\sigma(\sigma_Y, \mu_Y) < 1$.

(Q.E.D.)

## APPENDIX - C

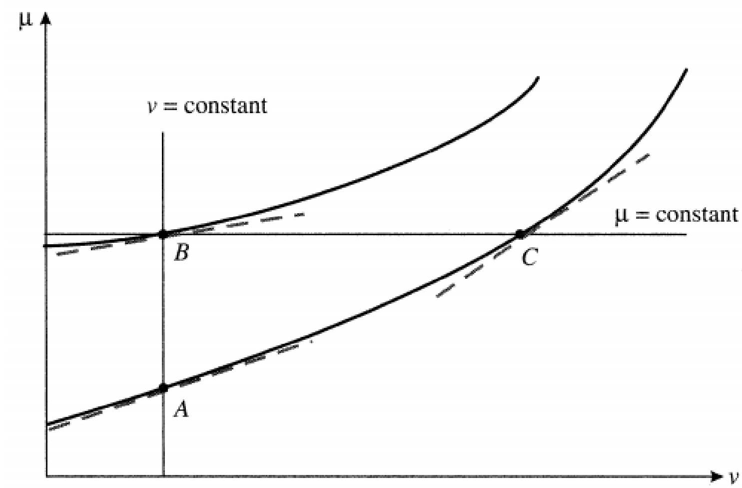


**Figure B: ($v$, $\mu$) Indifference curves**

(*Source: Eichner (2008, 54(3), p. 590; Figure 1*)

In the above figure, Eichner (2008) measured variance or risk (v) in the horizontal axis and mean (μ) in the vertical axis. He represented point A as the initial optimum choice, point B as the change in the optimal choice from point A after an increase in mean (given risk, i.e. variance), and point C as the change in the optimum choice from point B after an increase in variance (given mean). Hence, in his paper, the comparative static response of the increase in risk, given mean, is reflected

[4] However, the proof of this result doesn't require any assumption regarding the direction of correlation between $\widetilde{RQ}$ and $\widetilde{BQ}$.

by the movement from point B to point C, and the comparative static response of an increase in mean, given risk, is reflected by the movement from point A to point B. Also, as Eichner (2008) proved, movement from point A to point B leads to reduction in the slope of the tangent line to the indifference curve in the (Risk, Return)-plane. Since this slope is nothing else but the marginal rate of substitution between risk and return, or, equivalently, the mean-variance analogue of the Arrow-Pratt measure of the absolute risk aversion, reduction in the slope with w.r.t. mean (keeping variance constant) reflects the notion of "decreasing absolute risk aversion" (DARA). However, increase in the slope w.r.t. variance (keeping mean constant) makes the indifference curve more "convex", and thereby reflects notion of "variance vulnerability" of preferences.

However, in our paper, we have used mean–standard deviation $(\mu, \sigma)$ preferences, which is equivalent to the mean-variance preferences considered in Eichner (2008). In our experiment, Figure 4 in our paper, reflects precisely the scenario of relative increase in risk (w.r.t. return) in panels I & II, depicting increased steepness in the slope of the new indifference-curve in the $(\sigma, \mu)$-plane. Since this slope is nothing else but the marginal rate of substitution between risk (standard deviation) and return, or, equivalently, the mean – standard deviation analogue of the Arrow-Pratt measure of the absolute risk aversion, increase in this slope also reflects the notion of variance vulnerability of preferences.

Figure 5 reflects the scenario of relative increase in mean (w.r.t. risk) in panels I & II, depicting the reduction in the slope of the new indifference curve at the new $p^*$. This reduction in the slope, by the same argument as above, reflects DARA.

## Appendix - D

### Data Collection and Experimental Setup

**Table D1. Sample**

| Industry | N | % |
|---|---|---|
| Automotive and Parts | 57 | 19 |
| Chemical | 96 | 32 |
| Pharmaceutical | 60 | 20 |
| Construction and Building Materials | 45 | 15 |
| Electronic and Electrical Equipment | 42 | 14 |
| Respondent job title | | |

| | | |
|---|---|---|
| CxO/Vice President | 14 | 4.7 |
| Director/Department Head | 68 | 34.6 |
| Manager | 218 | 72.7 |
| Respondent function | | |
| Supply Chain Management | 71 | 23.7 |
| Production/Manufacturing | 52 | 17.3 |
| Purchasing | 177 | 59.0 |
| Experience in purchasing | | |
| 5 to 9 years | 32 | 10.7 |
| 10 to 14 years | 183 | 61.0 |
| 15 years and above | 85 | 28.3 |

To verify that the sample respondents were having adequate experience for experimental setup. We test the mean experience of the respondents (as 10 years) where the variance is unknown, and the sample follows normal distribution because of large sample (300 respondents).

Null hypothesis, $H_0$: Average Experience of respondents $= 10$ years against alternative hypothesis, $H_a$: Average Experience of respondents $< 10$ years.

We accept the null hypothesis at 5% level of significance as $t_{0.05,299}^{observe} (= 1.451) < t_{0.05,299}^{tabulated}$ (= 1.645). Hence, the sample is well experienced having 10 years of experience and at least 20% of the time they have handled supply disruption during materials purchasing.

***Data Collection templates using the treatment variables***

Sample data collected from a respondent given in Table D2 and D3 against primary and backup supplier respectively.

**Table D2: Data collection Set-up for Primary Supplier**

| Control variables | | Treatment | Dependent | Computed |
|---|---|---|---|---|
| Available Effective Capacity | Time remaining | DLS Target | Perceived Probability of supplying the DLS target | Random net realization $\widetilde{RQ} = (TR_{RQ} - \widetilde{D}_{RQ})$ |
| 10% | 10% | 33 | 0.3 | 10 |
| 20% | 20% | 67 | 0.5 | 34 |
| 30% | 30% | 103 | 0.6 | 62 |
| 40% | 40% | 138 | 0.6 | 83 |

| | | | | |
|---|---|---|---|---|
| 50% | 50% | 174 | 0.7 | 122 |
| 60% | 60% | 208 | 0.8 | 166 |
| 70% | 70% | 240 | 0.85 | 204 |
| 80% | 80% | 266 | 0.92 | 239 |
| 90% | 90% | 287 | 0.95 | 264 |
| 100% | 100% | 300 | 0.99 | 285 |

*For example, one of the sample respondent's Mean = 147 and Standard Deviation = 99 can be calculated based on the primary supplier selection set-up*

**Table D3: Data collection Set-up for Backup Supplier**

| Control variables | | Treatment | Dependent | Computed |
|---|---|---|---|---|
| Available Effective Capacity | Time remaining | Remaining quantity | Perceived Probability of supplying the remaining quantity | Random net realization $\widetilde{BQ} = \left(TR_{BQ} - \widetilde{D}_{BQ}\right)$ |
| 10% | 10% | 267 | 0.4 | 107 |
| 20% | 20% | 233 | 0.5 | 117 |
| 30% | 30% | 197 | 0.55 | 108 |
| 40% | 40% | 162 | 0.65 | 105 |
| 50% | 50% | 126 | 0.7 | 88 |
| 60% | 60% | 92 | 0.75 | 69 |
| 70% | 70% | 60 | 0.8 | 48 |
| 80% | 80% | 34 | 0.85 | 29 |
| 90% | 90% | 13 | 0.9 | 12 |
| 100% | 100% | 0 | 0.95 | 0 |

*For example, same sample respondent's Mean = 68 and Standard Deviation = 44 can be calculated based on the backup supplier selection set-up*

To remove the biasness in the experimental design we have followed Randomized Complete Block Design (RCBD) approach (Montgomery, 2001). Here, we have one observation per treatment in each block (group of 10 respondents); while the order, in which the treatments run within each block, is determined randomly. In this experiment, we are interested to test for the equality of the treatment and block means. Table-D4 and -D5 report the test statistics for treatments and blocks.

From these two tables, we can confirm that the differences between groups (blocks) are not statistically significant at 5% level. Hence, we can really on the data.

**Table D4: Analysis of Variance for The Perceived Probability of Supplying the DLS Target by The Primary Supplier**

| Source of variation | Sum of square | Degree of freedom | Mean square | $F_0$ | Observation |
|---|---|---|---|---|---|
| Treatments (DLS based Targets for the primary supplier) | 23.30 | 9 | 2.59 | 1.61 | $F_0(1.61) < F_{0.05,9,261}(1.88)$ |
| Blocks (Groups of respondents) | 61.50 | 29 | 2.12 | 1.33 | $F_0(1.33) < F_{0.05,29,261}(1.47)$ |
| Error | 417.80 | 261 | 1.60 | | --- |
| Total | 502.6 | 299 | | | --- |

**Table D5: Analysis of variance for the Perceived Probability of supplying the remaining quantity**

**By the backup Supplier**

| Source of variation | Sum of square | Degree of freedom | Mean square | $F_0$ | Observation |
|---|---|---|---|---|---|
| Treatments (Supplying the remaining quantity by the backup supplier) | 31.5 | 9 | 3.5 | 1.72 | $F_0(1.72) < F_{0.05,9,261}(1.88)$ |
| Blocks (Groups of respondents) | 74.1 | 29 | 2.56 | 1.25 | $F_0(1.25) < F_{0.05,29,261}(1.47)$ |
| Error | 532.7 | 261 | 2.04 | | --- |
| Total | 638.3 | 299 | | | --- |

**Appendix References**:


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