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Publication details:
Computer-Aided Civil and Infrastructure Engineering
v. 34
Chapter No. 12
pp. 1100 - 1118
1093-9687 (ISSN); 1467-8667 (ISSN)

Publication Date:
2019-07-30

Publisher DOI:
https://doi.org/10.1111/mice.12486

A network traffic assignment model for autonomous vehicles with parking choices

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Abstract: *This article is the first in the literature to investigate the network traffic equilibrium for travelling and parking with autonomous vehicles (AVs) under a fully automated traffic environment. Given that AVs can drop off the travellers at their destinations and then drive to the parking spaces by themselves, we introduce the joint equilibrium of AV route choice and parking location choice, and develop a variational inequality (VI) based formulation for the proposed equilibrium. We prove the equivalence between the proposed VI model and the defined equilibrium conditions. We also show that the link flow solution at equilibrium is unique, even though both the route choices and parking choices are endogenous when human-occupied AV trips (from origin to destination) and empty AV trips (from destination to parking) are interacting with each other on the same network. We then develop a solution methodology based on the parking-route choice structure, where we adjust parking choices in the upper level and route choices in the lower level. Numerical analysis is conducted to explore insights from the introduced modelling framework for AV network equilibrium. The results reveal the significant difference in network equilibrium flows between the AV and non-AV situations. The results also indicate the sensitivity of the AV traffic pattern to different factors, such as value of time, parking pricing and supply. The proposed approach provides a critical modelling device for studying the traffic equilibrium under AV behaviour patterns, which can be used for the assessment of parking policies and infrastructure development in the future era of AVs.*

1 BACKGROUNDS

In recent years, automobile and technology industries bring the potential of computerisation into driving, leading to the anticipation of a new version of future transportation - autonomous vehicles (AVs). The advancements in communication technology provide such a new vehicle mode with the capability of intelligent motion and action, including adaptive steering control and autonomous parking assistance systems (Burns, 2013). The emergence of AVs has the potential to reduce crashes, ease congestion and increase energy efficiency, particularly considering car sharing. In particular, AVs could completely change mobility in the coming decades (Soteropoulos et al., 2019). As AVs can drive themselves on roads and automatically navigate multiple types of traffic environment contexts, direct human inputs are no longer needed to complete daily trips. This means that former drivers could engage in other activities in an AV, such as having meals. Such a dramatic transformation not only increases the utility of commutes, but also provides mobility to vulnerable groups, such as the disabled. Although AVs are not yet available to a mass market, the rapid development of the technology in automation and robotics will break down the barriers to implementation and market penetration, which indicates revolutionization in motoring in the foreseeable future (Qu, 2009; Fagnant and Kockelman, 2015).To prepare a healthy transportation system for the era of autonomous vehicles, transportation scientists are obliged to first give an answer to the important question – how will the AVs potentially change the network traffic performance? This is because a reliable estimation of network traffic pattern serves as the foundation of system assessment and governmental policymaking for infrastructure development. One methodological approach through which the network traffic pattern can be examined is the traffic assignment under the user equilibrium (UE) principle, which is also recognised as the network equilibrium. In the area of traffic assignment, the notion of user equilibrium was initially proposed by Wardrop (1952), followed by intensive research efforts in transport network modelling (Evans, 1976; Friesz, 1983; Hamdouch et al., 2004; Ban et al., 2006; Unnikrishnan et al., 2009; Duthie et al., 2010; Bar-Gera, 2010; Jiang et al., 2012; Ferguson et al., 2012; Meng et al., 2014; Patriksson, 2015; Yang et al., 2017; Zhang and Waller, 2018; Xie et al., 2019). The Wardropian UE principle holds the selfish route-choice assumption, which states that every traveller minimises his or her own travel cost, and no one can further reduce the individual cost by unilaterally changing their routes at equilibrium.

Previously, a diversity of network equilibrium models was developed to capture travellers' behavioural realism and system-level traffic performance (Yang and Huang, 2004; Ziliaskopoulos et al, 2004; Unnikrishnan et al., 2009). For example, He et al. (2015), Xie and Jiang (2016), Chen et al. (2017b), Cen et al. (2018), and Zhang et al. (2018) modelled the network equilibrium considering driving ranges of electric vehicles and deployment of recharging facilities. Chen et al. (2017a) developed a mathematical framework for the optimal design of AV zones in a general network. Wen et al. (2018a, 2018b) and Duell et al. (2018) introduced the strategic user equilibrium accounting for demand volatility, which can replicate the behaviour of observed link travel time variability.

In the scientific literature, transportation system analysis considering parking behaviour, which incorporates parking demand and cruising to find an appropriate vacant space, has attracted increasing attention. This is because the parking activity is an essential component in the travelling process, which could significantly influence the transportation system performance (Zhang et al., 2005; Lam et al., 2006; Li et al., 2007, 2008; Zhang et al., 2008). For instance, Jiang and Xie (2014) addressed a network equilibrium problem with combined destination, route and parking choices under mixed-vehicular traffic consisting of both gasoline vehicles and electric vehicles. Boyles et al. (2015) presented a modelling framework which incorporated parking search into traffic assignment under the network equilibrium conditions. Liu et al. (2018) formulated the network traffic equilibrium with park-and-ride facilities. All of the above-referenced studies explored the extent to which parking behaviour and relevant policies could impact the equilibrium traffic pattern with traditional non-autonomous vehicles.

Although significant research efforts have been made to the network equilibrium modelling, analysis on the joint equilibrium of route and parking choice for autonomous vehicles is still lacking. This gap is to be properly addressed, and AVs may significantly reshape the parking-related problems. In central business districts of cities where the level of human activity is high, the space available for parking purposes is costly and limited, while, on the other side, a great deal of parking demand is generated by commuters every day (Van Ommeren et al., 2012). These traffic-related externalities incur a huge social parking cost considering the value of land at city centres, and meanwhile cause inconvenience or losses to travellers (Arnott et al., 1991). For example, given a non-AV environment, travellers may have to drive their cars to search for a vacant parking space, which can be very costly to travellers. To mitigate these issues, academic studies related to parking have been experiencing a surge in popularity, particularly with the focuses on cruising for parking (Shoup 2006; Arnott and Inci, 2006; Inci and Lindsey, 2015; Liu and Geroliminis, 2016), parking reservation or permit schemes (Yang et al., 2013; Liu et al., 2014; Shao et al., 2016; Chen et al., 2019), and parking pricing (Arnott et al., 1991; Qian and Rajagopal, 2014; Zheng and Geroliminis, 2016; Nourinejad and Roorda, 2017). However, all of these parking-related studies propose approaches for non-AV transportation systems. By contrast, in the context of AVs, the new behaviour patterns are threefold: (i) AVs can drop off commuters at the workplace and then find a parking spot via self-driving, which means that commuters are no longer need to spend time in cruising for parking by themselves; (ii) since AVs can drop off commuters before finding a parking spot via self-driving, the parking location is not necessarily close to the city centre, resulting in savings in land use at city centres and reductions in social parking costs; (iii) the time spent in walking between the parking lot and the workplace and that in searching for an available space within a parking facility can be completely eliminated with AV commutes.

Motivated to incorporate the abovementioned new behaviour patterns associated with AVs, this study develops a modelling framework for network equilibrium with parking when commuters travel with AVs. Under such a framework, parking location choice is determined jointly with the route choice. Moreover, the parking flows (from travellers' destinations to the chosen parking locations) and the commuting flows (from travellers' origins to the destinations) are interacting with each other on the same network, which adds complexity to the modelling of the joint route and parking location choice problem. Recently, Liu (2018) has modelled the equilibrium of commuting and parking with AVs in a linear city. However, Liu (2018) is based on a highly stylized city setting. This study extends the literature by investigating the AV route-and-parking choice equilibrium notion at the network-wide level and exploring insights into autonomous system performance. For the sake of smart planning policies for future automated transportation, the achievements of this study are to provide answers to the following two important questions: (i) What are the essentials of network equilibrium with AV commuting and parking with regards to the non-AV situation? (ii) How do different influencing factors modify the equilibrium network performance in a fully autonomous transportation system? These factors including, e.g., the potential reduced value of in-vehicle time for AV trips.

We differentiate our research work from the existing scientific literature on traffic assignment modelling by three dimensions and summarise the novelty of this study as follows. First, we originally develop a rigorous modelling framework for network equilibrium of commuting and parking under a fully AV environment. Second, we explore insights into the comparison between the equilibrium traffic pattern under the AV situation and that under the non-AV situation. Third, we investigate the sensitivity of autonomous vehicular traffic equilibria to multiple influencing factors, such as value of time for AVs, parking pricing and supply.

In the era of AVs, there exist multiple possible scenarios: (i) privately-owned AVs; (ii) shared AVs, including ride-hailing and car-sharing (Haboucha et al., 2017). This study investigates the AV network equilibrium under Scenario (i), while follow-up studies will cover both options. Consistent with the opinion in the literature, we believe that Scenario (i) is a possibility in the foreseeable future. Some of the reasons are summarised as follows. On the producer's side, most major car manufacturers are racing to develop fully autonomous vehicles (Guerra, 2016). This implies that the supply of AVs is expected to satisfy the need of AV ownership. In addition, most developed AVs are electrified, and the cost of electric vehicles (EVs) have been falling. This is because of the rapid development of battery technology (Nykvist and Nilsson, 2015) and the government incentive policy to lower the upfront purchasing cost of EVs for consumers (Zhou et al., 2015; Noori and Tatari, 2016). Meanwhile, with the improvements in driverless technology, there will be a significant reduction in the production cost for AVs (Sergeenkov, 2019). Based on the above analysis, the mass-market penetration of AVs at an acceptable price can be anticipated. On the consumer's side, people are motivated to own personal AVs, in consideration of convenience, reliability, costs and status (Litman, 2019). Particularly, private ownership of AVs will benefit people on high incomes, due to their larger travel distances and higher perceived value of time (Wadud, 2017). The existing studies also claim that individuals who currently own vehicles are likely to prefer private ownership of AVs to the mode of shared use (Lavieri et al., 2017).

The rest of this article is structured as follows. Section 2 introduces the concept of the network equilibrium with AV travelling and parking. Section 3 presents the modelling framework for the defined equilibrium by using the variational inequality formulation. Section 4 develops the methodological approach to solve the proposed model. Numerical analysis is conducted in Section 5. Finally, concluding remarks are provided in Section 6.

2 NETWORK EQUILIBRIUM WITH AV

In this section, we define a new mathematical concept of the network equilibrium with both route choice and parking location choice under autonomous transportation systems. Notations used in this article are summarised in Table 1 unless otherwise specified.

Table 1
Mathematical notations

Notation	Description
G	Network graph
O	Set of origin node: $\{r\}$
D	Set of destination node: $\{s\}$
A	Set of link: $\{a\}$
P	Set of parking node: $\{p\}$
Ω	Set of origin-destination (OD) pair:$\{(r,s)\} \subseteq O \times D$
t_a	Travel time on link a
d_{rs}	Travel demand between OD pair (r,s)
$\boldsymbol{d}$	Travel demand vector, $[d_{rs}]$
$q_{rs,p}$	The travel demand between OD pair (r,s) ending up with parking location p
$\boldsymbol{q}$	Vector of demand by parking location, $\left[q_{rs,p}\right]$
$\boldsymbol{q}^*$	Vector of demand by parking location at equilibrium, $\left[q^*_{rs,p}\right]$
D_{tot}	Total travel demand for the network
π	Path index
Π	Set of paths
Π_{rs}	Set of paths for OD pair (r,s)
$T_{rs,\pi}$	The travel time from r to s through path π, $\pi \in \Pi_{rs}$
$h_{rs,\pi}$	Flow on path π from origin r to destination s, $\pi \in \Pi_{rs}$
$\boldsymbol{h}$	Path flow vector for the network, $\left[h_{rs,\pi}\right]$
$\boldsymbol{h}^*$	Equilibrium path flow vector, $\left[h^*_{rs,\pi}\right]$
f_a	Flow on link $a \in A$
$\boldsymbol{f}$	Link flow vector $[f_a]$
$\boldsymbol{f}^*$	Equilibrium link flow vector $[f^*_a]$
α	The value of time for driving AVs
β	The value of time for AV self-driving
Z_p	Parking fee at the parking lot p
Ψ_p	Capacity of parking lot p
$C_{rs,p}$	Travel disutility for a trip from origin r to destination s, ending up with parking location p
$C^*_{rs,p}$	Travel disutility at equilibrium for a trip from origin r to destination s, ending up with parking location p
U_{rs}	Set of feasible parking locations for OD pair (r,s)

2.1 AV travel process and disutility

We first describe the AV travel process. For commutes with AVs, the travel process follows the procedure: Depart from the origin → Arrive at the destination (e.g. workplace, school, shopping centre) and drop off commuters → Find a parking place by AV self-driving. This process has been discussed in Liu (2018). It has multiple significant differences from travels with non-AVs, particularly: (i) with AVs, commuters first reach the final destination, and then find an appropriate parking space, while with non-AVs, the parking location has to be determined prior to arrival to the destination; (ii) Walking from the parking spot to the final destination is eliminated for AV commutes, so that AVs can be parked far away from the destination with no concern for an unacceptable walking distance; (iii) Value of time (VoT) for driving AVs and AV self-driving is considered to be different from that for driving non-AVs, which would have impacts on travellers' route choice behaviour.

We now describe the decision-making on travel and parking choices for AV commuters. The AVs select an appropriate parking location prior to a trip, which will minimise the total individual travel disutility from the origin to the final parking location via the destination. In line with the parking choice and the willingness to minimise individual travel disutility, they will choose two shortest paths with minimal travel time, which includes one connecting from the origin to the destination, and the other one connecting from the destination to the selected parking location. This parking-route choice structure is later on utilised to develop the solving approach for the equilibrium model. Note that in reality, these choices (route and parking location) might be made simultaneously.

We denote $C_{rs,p}$ as the travel disutility for AVs that depart from the origin r and travel to destination s, dropping off commuters, and finally arrive at the parking location p via self-driving. We assume that for each OD pair, the feasible parking locations can be predetermined prior to a trip. We then formulate $C_{rs,p}$ measured in cost unit as follows

$$C_{rs,p} = \alpha \cdot T_{rs} + \beta \cdot T_{sp} + Z_p, \forall r \in O, s \in D, p \in U_{rs}. \quad (1)$$

where T_{rs} and T_{sp} are respectively the travel time from the origin node r to the destination node s and that from the destination node s to the parking facility p; the physical meanings of the other terms can be found in Table 1.

It is expected that α and β are less than the value of time for driving a conventional vehicle, as people can get some benefit (such as relaxation) that results from driving automation in an AV environment. However, this does not mean that AV commuters do not care about the travel time. This is because the travel time directly contributes to the travel disutility with AVs, which is reflected in Equation (1). There are many reasons behind this. For example, even if commuters can do some work other than driving in their AVs, they are not absolutely free to do anything (for instance, having sports, doing housework). This mean that an increase in the in-vehicle travel time will cause a sacrifice of time to be spent on some other activities (or at least less flexibility). For another example, a travel-time increase usually means an increase in the travel distance or a decrease in the vehicle speed, which could incur a greater cost of energy consumption (Zhang et al., 2019). Hence, each AV commuter is motivated to reduce his or her own travel time and travel disutility, resulting in a user-equilibrium traffic pattern. Next section will discuss the mathematical representation of the network equilibrium with AVs.

2.2 Equilibrium of travel and parking with AVs

The equilibrium conditions of parking location choice can be written as follows.

$$\left(C^*_{rs,p} - \widetilde{C_{rs}}\right) \cdot q^*_{rs,p} = 0 \text{ , } \forall r \in O, s \in D, p \in U_{rs}, \quad (2)$$

$$C^*_{rs,p} \geq \widetilde{C_{rs}}, \forall r \in O, s \in D, p \in U_{rs}, \quad (3)$$

$$q^*_{rs,p} \geq 0, \forall r \in O, s \in D, p \in U_{rs}, \quad (4)$$

where $\widetilde{C_{rs}}$ is the minimal overall travel disutility for trips between OD pair (r, s) ; $C^*_{rs,p}$ and $q^*_{rs,p}$ respectively represent the corresponding values of $C_{rs,p}$ and $q_{rs,p}$ at equilibrium. In terms of travel demand, $\left[q_{rs,p}\right]$ can be transformed into $[d_{rs}]$ combined with $\left[d_{sp}\right]$, where $[d_{rs}]$ represents the travel demand vector including each OD pair (r, s), and $\left[d_{sp}\right]$ is the vector of travel demand from the destination node s to the parking location p. With $\boldsymbol{q} = \left[q_{rs,p}\right]$ considered given, we can determine the values of d_{rs} and d_{sp} as follows

$$d_{rs} = \sum_{p \in U_{rs}} q_{rs,p}, \forall r \in O, s \in D, \quad (5)$$

$$d_{sp} = \sum_{r \in O} q_{rs,p}, \forall s \in D, p \in P. \quad (6)$$

Furthermore, the following demand conservation relationships between d_{rs} and d_{sp} must hold.

$$\sum_{r \in O} d_{rs} = \sum_{p \in P} d_{sp}, \forall s \in D, \quad (7)$$

$$\sum_{r \in O, s \in D} d_{rs} = \sum_{s \in D, p \in P} d_{sp} = D_{tot}. \quad (8)$$

$\left[d_{sp}\right]$ can be considered as the newly derived travel demand $\bar{\boldsymbol{d}} = \left[\bar{d}_{\bar{r}\bar{s}}\right]$ due to parking activities, wherein we have

$$\bar{d}_{\bar{r}\bar{s}} = d_{sp}, \bar{r} = s \in \bar{O}, \bar{s} = p \in \bar{D},. \quad (9)$$

where $\bar{O} = \{s\}$, $\bar{D} = \{p\}$. Also, for the trips from s to p, we define the OD set $\bar{\Omega}$ as $\bar{\Omega} = \{s, p\} = \{(\bar{r}, \bar{s})\} \subseteq \bar{O} \times \bar{D}$, and the set of paths connecting each $(\bar{r}, \bar{s})$ as $\Pi_{\overline{rs}}$.

Equation (9) indicates that the traveller's destination can be treated as a new origin for the AVs serving them. This is because once travellers are dropped at their destinations, their AVs will start with this destination and complete another trip for the parking purpose. Then, we define the transformed OD set as $\Omega' = \{(r', s')\} = \Omega \cup \bar{\Omega} \subseteq O' \times D'$, with the path set for each OD pair (r', s') as $\Pi_{r's'} = \Pi_{rs} \cup \Pi_{\overline{rs}}$. We can then determine the corresponding travel demand as $\boldsymbol{d}' = \left[d'_{r's'}\right]$, where we have

$$d'_{r's'} = \begin{cases} d_{rs} + \bar{d}_{\bar{r}\bar{s}}, & if\ (r',s') = (r,s) = (\bar{r},\bar{s}) \\ d_{rs}, & if\ (r',s') = (r,s) \neq (\bar{r},\bar{s}) \\ \bar{d}_{\bar{r}\bar{s}}, & if\ (r',s') = (\bar{r},\bar{s}) \neq (r,s) \\ 0, & if\ (r',s') \neq (\bar{r},\bar{s}), (r',s') \neq (r,s) \end{cases} \quad (10)$$

Given the parking choice, the AVs will always choose the shortest route with the minimal travel time, for the travel with AVs from the origin to the destination and from the destination to the selected parking location. This means that the network traffic pattern will be formed by assigning the equivalent demand $\boldsymbol{d'}$ under user equilibrium, at which no one can further reduce the individual travel time by unilaterally changing his or her route. Different from $\boldsymbol{d}$, the transformed OD demand vector $\boldsymbol{d'}$ is variable in this context. This is because $\boldsymbol{d}'$ contains the demand $\bar{d}_{\bar{r}\bar{s}}$ from the destination to the parking location, while the parking location choice is associated with the decision vector $\boldsymbol{q}$, which is not known a priori. Mathematically, as per Equations (6) and (9), the variable $\bar{d}_{\bar{r}\bar{s}}$ (a component of $d_{r's'}$ as shown in Equation (10)) is calculated by using $[q_{rs,p}]$, which is yet to be solved in the proposed equilibrium problem.

With the transformed OD demand, the equilibrium condition of route choice by AVs can then be written as follows.

$$\left(T^*_{r's',\pi} - \widetilde{T_{r's'}}\right) \cdot h^*_{r's',\pi} = 0, \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}, \quad (11)$$

$$T^*_{r's',\pi} \geq \widetilde{T_{r's'}}, \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}, \quad (12)$$

$$h^*_{r's',\pi} \geq 0, \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}. \quad (13)$$

$\widetilde{T_{r's'}}$ represents the minimum travel time from the origin r' to the destination s', and we have $\widetilde{T_{r's'}} = \min_{\pi \in \Pi_{r's'}} T_{r's',\pi}$. Also, we let $\boldsymbol{h}' = [h_{r's',\pi}]$ and $\boldsymbol{h}'^* = [h^*_{r's',\pi}]$. One of the benefits of the OD transformation is that the traffic attributes (e.g. flow, demand) for both the trip from r to s and that from s to p are considered with the derived OD set Ω' in our equilibrium analysis. For example, $\boldsymbol{h}'$ contains path flow between r and s and that between s and p.

Note that, for simplicity, many existing studies on network equilibrium with parking have employed special links (e.g. fictitious links) connecting the destination and the parking place. By contrast, we assign the demand that results from the cruise-for-parking via AV-self driving to the real network. By doing this, we can investigate the significance of AV parking events to the equilibrium traffic pattern, which will be further discussed in Section 5.

Based on the above analysis, we can define the joint user equilibrium of parking location choice and route choice in the fully AV environment as follows. In the next section, we will present the modelling framework for the defined network equilibrium.

Definition 1. *A traffic pattern ($\boldsymbol{h}'^*$, $\boldsymbol{q}^*$) is formed at user equilibrium of parking location and route choice under a fully AV environment, if it satisfies the conditions (2) through (4), and (11) through (13) simultaneously.*

3 MODEL FORMULATION

In this section, we develop the modelling framework for the proposed user-equilibrium traffic assignment problem for AVs, and then investigate the problem equivalency and solution properties. In this section, we use the variational inequality (VI) as the modelling medium, which was initially introduced by Smith (1979) and Dafermos (1980). Different from previous works, this study develops a VI model formulation for the joint equilibrium of AV commuting and parking on the network level.

3.1 VI formulation for network equilibrium with AVs

We first denote Φ as the set of combinations of feasible route flow $h_{r's',\pi}$ and travel demand by parking locations $q_{rs,p}$. Then, $h_{r's',\pi}$ and $q_{rs,p}$ must satisfy Equations (5) through (10), as well as the following constraints.

$$q_{rs,p} \geq 0, \forall r \in O, s \in D, p \in U_{rs}, \quad (14)$$

$$d_{rs} - \sum_{p \in U_{rs}} q_{rs,p} = 0, \forall r \in O, s \in D, \quad (15)$$

$$h_{r's',\pi} \geq 0, \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}, \quad (16)$$

$$d'_{r's'} - \sum_{\pi \in \Pi_{r's'}} h_{r's',\pi} = 0, \quad (17)$$

$$f_a = \sum_{(r',s') \in \Omega'} \sum_{\pi \in \Pi_{r's'}} h_{r's',\pi} \cdot \delta^{r's'}_{a,\pi}, \quad (18)$$

$$T_{r's',\pi} = \sum_a t_a \cdot \delta^{r's'}_{a,\pi}, \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}, \quad (19)$$

$$\sum_{r \in O, s \in D} q_{rs,p} \leq \Psi_p. \quad (20)$$

Equation (14) is the nonnegativity constraint on travel demand by parking location. Equation (15) states that the sum of travel demand by all the feasible parking locations for OD pair (r,s) is equal to the travel demand of the OD pair. Equations (16)-(19) are constraints associated with the traffic pattern obtained by assigning the equivalent demand $\boldsymbol{d'}$ to the network. Equation (16) is the nonnegativity constraint on path flows, while (17) states that the sum of flows of the paths connecting OD pair (r',s') is equal to the demand between (r',s') . Equations (18) and (19) are the definitional constraints on link attributes and path attributes, where $\delta^{r's'}_{a,\pi}$ is the link-subpath incidence coefficient, which equals one if link a is contained in the path π connecting OD pair (r',s'), and zero otherwise. In addition, Equation (19) describes the conservation constraint on travel time. In this study, we consider flow-dependent link travel time, which is assumed to be monotonically increasing with the flow:

$$t_a = t_a(f_a). \quad (21)$$

Inequality (20) represents the parking capacity constraint, which ensures that the total number of AVs parked at the location p should not exceed the capacity of that parking lot. In a static assignment model, we investigate the traffic pattern for a given time unit. In this context, the parking capacity Ψ_p

represents the number of parking spaces available at p for that time unit considered. With the Constraint (20), $C^*_{rs,p}$ and $\widetilde{C_{rs}}$ in parking location choice conditions (2) through (4) are corresponding to the generalised travel disutility. Particularly, we have

$$C^*_{rs,p} = {C^*_{rs,p}}' + v_{rs,p}. \quad (22)$$

In Equation (22), ${C^*_{rs,p}}'$ is the travel disutility, which is defined by Equation (1). Similar to $\widetilde{C_{rs}}$ within the parking location choice conditions, $v_{rs,p}$ is not known a priori. $v_{rs,p}$ represents the values of multipliers associated with the parking capacity. We have

$$v_{rs,p}\left(\Psi_p - \sum_{r\in O, s\in D} q_{rs,p}\right) = 0 \quad \textit{subject to} \quad (20). \quad (23)$$

i.e.

$$\begin{cases} v_{rs,p} \geq 0 \;\rightarrow\; \Psi_p = \sum_{r\in O, s\in D} q_{rs,p} \\ v_{rs,p} = 0 \;\rightarrow\; \Psi_p > \sum_{r\in O, s\in D} q_{rs,p} \end{cases}. \quad (24)$$

It can be seen that $v_{rs,p}$ could become positive only if the parking location p has been saturated at capacity. With $v_{rs,p}$ incorporated, the travel disutility across all the used parking locations for each OD pair will be equal to the same minimum value.

We present the equivalent variational inequality (VI) formulation for the network equilibrium described in **Theorem 1** as follows.

Theorem 1. *The network traffic pattern* $\boldsymbol{z}^* = (\boldsymbol{h'}^*, \boldsymbol{q}^*)$ *reaches the joint user-equilibrium of parking location choice and route choice, if and only if it satisfies the variational inequality (VI) problem as follows.*

P1:

$$\sum_{(r,s)} \sum_p C^*_{rs,p} \cdot \left(q_{rs,p} - q^*_{rs,p}\right) + \sum_{(r',s')} \sum_\pi T^*_{r's',\pi} \cdot \left(h_{r's',\pi} - h^*_{r's',\pi}\right) \geq 0, \quad (25)$$

subject to $\left(h_{r's',\pi}, q_{rs,p}\right) \in \Phi$.

When we substitute (18) and (19) into (25), the VI problem can be re-written by specifying the combination of link flows and travel demands by parking locations, *i.e.* $\boldsymbol{z}^*=(\boldsymbol{f}^*, \boldsymbol{q}^*)$, as follows.

P2:

$$\sum_{(r,s)} \sum_p C^*_{rs,p} \cdot \left(q_{rs,p} - q^*_{rs,p}\right) + \sum_a t_a(f^*_a) \cdot (f_a - f^*_a) \geq 0, \quad (26)$$

subject to $\left(h_{r's',\pi}, q_{rs,p}\right) \in \Phi$.

In this section, the developed VI model describes the defined network equilibrium with AVs. It is worth mentioning that although AVs can realise driving automation, each AV is assumed to work for the sake of their owners, i.e. individual commuters. Given this, the traffic pattern obtained by the user-equilibrium traffic assignment model can better reflect choice and behaviour patterns of AVs than that by the system-optimal one. If we consider a centralised and coordinated AV system, adopting system-optimal traffic assignment will be more appropriate. For the system-optimal assignment, on the modelling side, an additional term representing the marginal cost is to be added to the travel disutility function. The mathematical framework of the system-optimal traffic assignment for AV commuting and parking will be further investigated in a follow-up study.

In Section 3.2, we will prove that the VI model formulation is the necessary and sufficient condition of the user-equilibrium traffic pattern determined by the parking location choice conditions, i.e. (2) through (4), and the route choice conditions, i.e. (11) through (13).

3.2 Problem equivalency

In this part, we will demonstrate that the developed VI model formulation is equivalent to the notion of joint network equilibrium of parking location choice and route choice of AVs as proposed by Definition 1. The proof of equivalency is to show that the transformation of the proposed AV network equilibrium problem to the mathematical program is justified.

3.2.1 Proof of necessity

First, we discuss the equilibrium parking choice conditions (2) through (4), from which we obtain

$$\left(C^*_{rs,p} - \widetilde{C_{rs}}\right) \cdot \left(q_{rs,p} - q^*_{rs,p}\right) \geq 0 \quad , \quad \forall r \in O, s \in D, p \in U_{rs}. \quad (27)$$

Summing inequality (27) across all the feasible parking locations $p \in U_{rs}$ and using constraint (15) yields

$$\sum_p C^*_{rs,p} \cdot \left(q_{rs,p} - q^*_{rs,p}\right) \geq 0, \forall r \in O, s \in D. \quad (28)$$

Summing inequality (28) for all OD pairs $(r,s) \in \Omega$ yields

$$\sum_{(r,s)} \sum_p C^*_{rs,p} \cdot \left(q_{rs,p} - q^*_{rs,p}\right) \geq 0 \quad (29)$$

Hence, from the equilibrium conditions of parking location choice, inequality (29) can be derived.

Second, we discuss the equilibrium route choice conditions (11) through (13). These conditions imply that

$$\left(T^*_{r's',\pi} - \widetilde{T_{r's'}}\right) \cdot \left(h_{r's',\pi} - h^*_{r's',\pi}\right) \geq 0, \quad \forall r' \in O', s' \in D', \pi \in \Pi_{r's'}. \quad (30)$$

Summing inequality (30) over all paths $\pi \in \Pi_{r's'}$ and using constraint (17) yields

$$\sum_{\pi\in\Pi_{r's'}} T^*_{r's',\pi} \cdot \left(h_{r's',\pi} - h^*_{r's',\pi}\right) \geq 0. \quad (31)$$

Summing inequality (31) for all OD pairs $(r', s') \in \Omega'$, and using (18) and (21), we obtain

$$\sum_a t_a(f^*_a) \cdot (f_a - f^*_a) \geq 0. \quad (32)$$

This means that inequality (32) can be derived from the route choice conditions (11) through (13). By combining (29) and (32), we obtain the VI formulation (26). This completes the proof of necessity.

3.2.2 Proof of sufficiency

For sufficiency, we need to prove that any solution $\boldsymbol{z}^* = (\boldsymbol{f}^*, \boldsymbol{q}^*)$ to the VI formulation (26) satisfies the user-equilibrium conditions of parking location choice and route choice. First, we prove that the equilibrium route choice conditions (11) to (13) always hold for any solution ($\boldsymbol{f}^*$, $\boldsymbol{q}^*$). When the OD demand by parking location is considered given, in other words, we let $\boldsymbol{q}$ be equal to the optimal solution $\boldsymbol{q}^*$, the first term in (26) will vanish. We can then rewrite VI (26) as follows

$$\sum_a t_a(f_a^*) \cdot (f_a - f_a^*) \geq 0\,, \tag{33}$$

subject to $\left(h_{r's',\pi}, q_{rs,p}\right) \in \Phi$.

As discussed early, (33) can be written by specifying the path flows

$$\sum_{(r',s')} \sum_\pi T^*_{r's',\pi} \cdot \left(h_{r's',\pi} - h^*_{r's',\pi}\right). \tag{34}$$

For any OD pair (r_1', s_1') among the network, we can derive a feasible traffic pattern in terms of path flows as follows. We let $h_{r's',\pi} = h^*_{r's',\pi}$, $\forall\, (r_1', s_1') \in \Omega' \backslash \{(r_1', s_1')\}, \forall \pi \in \Pi_{r's'}$. We select a path $\vartheta_1 \in \Pi_{r_1's_1'}$ such that $h^*_{r_1's_1',\vartheta_1} > 0$. We can always find at least one positive path flow between (r_1', s_1'), simply because the travel demand for each OD pair considered is positive. We then select any path $\vartheta_2 \in \Pi_{r_1's_1'} \backslash \{\vartheta_1\}$, and reassign the traffic flow: Let $h_{r_1's_1',\vartheta_1} = h^*_{r_1's_1',\vartheta_1} - \tau$, $h_{r_1's_1',\vartheta_2} = h^*_{r_1's_1',\vartheta_2} + \tau$, where τ is positive but sufficiently small, i.e. $0 < \tau \ll h^*_{r_1's_1',\vartheta_1}$; Let all the other path flows be equal to $h^*_{r_1's_1',\vartheta}, \forall \vartheta \in \Pi_{r_1's_1'} \backslash \{\vartheta_1, \vartheta_2\}$. When we substitute the newly constructed traffic pattern into inequality (34), we can get

$$\left(T^*_{r_1's_1',\vartheta_2} - T^*_{r_1's_1',\vartheta_1}\right) \cdot \tau \geq 0. \tag{35}$$

We now discuss the following two situations:
(i) If $h^*_{r_1's_1',\vartheta_2} = 0$, according to (35), $T^*_{r_1's_1',\vartheta_2} \geq T^*_{r_1's_1',\vartheta_1}$ must hold.
(ii) If $h^*_{r_1's_1',\vartheta_2} > 0$, we can use the same method to construct another feasible network traffic pattern by reallocating the flows between paths ϑ_1 and ϑ_2, which yields

$$\left(T^*_{r_1's_1',\vartheta_1} - T^*_{r_1's_1',\vartheta_2}\right) \cdot \tau \geq 0. \tag{36}$$

From (35) and (36), it can be easily verified that $T^*_{r_1's_1',\vartheta_1} = T^*_{r_1's_1',\vartheta_2}$ if both $h^*_{r_1's_1',\vartheta_1} > 0$ and $h^*_{r_1's_1',\vartheta_2} > 0$ hold.

By summarising the two situations: (i) (i.e., $h^*_{r_1's_1',\vartheta_2} = 0$) and (ii) (i.e., $h^*_{r_1's_1',\vartheta_2} > 0$), we can conclude that for OD pair (r_1', s_1'), all the used paths have the same minimal travel time. In other words, the travel time of any unused path is no smaller than that of any used path. Given that the OD pair (r_1', s_1') is selected in a random manner, the above conclusion can be generalised to all the OD pairs among a network. This means that a solution to the VI formulation (26) satisfies the equilibrium route choice conditions that are formulated by (11) to (13).

Second, we prove that the equilibrium parking location choice conditions (2) to (4) always hold for any solution $\boldsymbol{z}^* = (\boldsymbol{f}^*, \boldsymbol{q}^*)$ to the VI formulation (26). For any given parking pattern $\boldsymbol{q}$, the travel disutility $C_{rs,p}$ represents the minimum total cost of the whole travel process, which is calculated based on the equilibrium flow pattern, i.e. $C_{rs,p} = C_{rs,p}(\boldsymbol{f}), \forall r \in O, s \in D, p \in U_{rs}$. We have $f \equiv f^*$, because: (i) the transformed OD demands by AVs $\boldsymbol{d}'$ can be uniquely determined when any $\boldsymbol{q}$ is given, which can be easily verified by Equations (5) through (10); (ii) when assigning $\boldsymbol{d}'$ to the network under user equilibrium conditions, the solution $\boldsymbol{f}$ is unique (Sheffi, 1985). Hence, when considering the parking location choice, we can re-express the VI formulation (26) as follows

$$\sum_{(r,s)} \sum_p C^*_{rs,p} \cdot \left(q_{rs,p} - q^*_{rs,p}\right) \geq 0. \tag{37}$$

Similar to the proof of satisfaction of the equilibrium route choice conditions, we present the proof for parking location choice as follows. We select an OD pair (r_1, s_1) from Ω in a random manner, and construct a travel demand pattern by parking locations with a reallocation of ϵ from parking location φ_1 to φ_2 such that $q_{r_1s_1,\varphi_1} = q^*_{r_1s_1,\varphi_1} - \epsilon$, $q_{r_1s_1,\varphi_2} = q^*_{r_1s_1,\varphi_2} + \epsilon$. We assume $q^*_{r_1s_1,\varphi_1} > 0$, given that such a positive travel demand by parking location must exist because of $\sum_p q^*_{r_1s_1,p} = d_{r_1s_1} > 0$. Substituting the constructed demand into (37), we obtain

$$\left(C^*_{r_1s_1,\varphi_2} - C^*_{r_1s_1,\varphi_1}\right) \cdot \epsilon \geq 0. \tag{38}$$

If $q^*_{r_1s_1,\varphi_2} = 0$, we have $C^*_{r_1s_1,\varphi_2} \geq C^*_{r_1s_1,\varphi_1}$; If $q^*_{r_1s_1,\varphi_2} > 0$, by reconstructing the demand pattern in the same way, we obtain

$$\left(C^*_{r_1s_1,\varphi_1} - C^*_{r_1s_1,\varphi_2}\right) \cdot \epsilon \geq 0, \tag{39}$$

which results in $C^*_{r_1s_1,\varphi_2} = C^*_{r_1s_1,\varphi_1}$. Hence, all the selected parking locations are corresponding to the same minimal travel disutility. This means that the VI formulation (26) satisfies not only the equilibrium condition of route choice, but also the equilibrium condition of the parking location choice. This completes the proof of sufficiency.

To summarise, Section 3.2.1 and Section 3.2.2 provide the proof of equivalency between the VI formulation (25) and (26) and the proposed network equilibrium for AV commuting and parking defined in Section 2.

3.3 Solution property (existence and uniqueness/non-uniqueness)

For the property of the proposed VI formulation, a solution to the model must exist as all the terms and functions within the model are continuous and the set of feasible flows Φ is nonempty and compact (Facchinei and Pang, 2003).

In terms of convexity, since the travel disutility and route travel time are not necessarily monotonically increasing with the corresponding travel demand and route flows respectively, the model cannot be guaranteed to strict convexity with respect to the travel disutility or route travel time. Hence, multiple local optimal solutions might exist. To illustrate, we present a demonstrating example on the synthetic network shown in Figure 1.

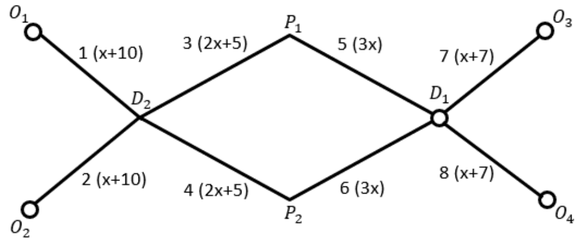


Figure 1 Synthetic network

In this demonstration, four OD pairs are considered, i.e. (O_1, D_1), (O_2, D_1), (O_3, D_2) and (O_4, D_2), with the travel demand assumed to be $d_{O_1D_1} = d_{O_2D_1} = d_{O_3D_2} = d_{O_4D_2} = 50$. For the value of time, we let $\alpha = 2$ and $\beta = 1$ in this case. Also, the parking fee is assumed to be 20, and the parking capacity to be 150 for each of the two public parking places P_1 and P_2. The parking spots in the public parking places can be used by commuters travelling between any OD pair. The link number is labelled along the corresponding link on the network graph, followed by the link travel time function in the parenthesis where x refers to the link flow. We assume that each link has two symmetric directions. By solving the defined network equilibrium problem for AV travelling and parking, we can obtain multiple solutions of the traffic pattern ($\boldsymbol{h'}^*$, $\boldsymbol{q}^*$), where some are listed in Table 2.

Table 2
Example of AV network equilibrium solutions

Example No.	Solution	
	$\boldsymbol{h'}^*$	$\boldsymbol{q}^*$
1	$h_{O_1D_1,1-3-5} = 50, h_{O_1D_1,1-4-6} = 0,$ $h_{O_2D_1,2-3-5} = 0, h_{O_2D_1,2-4-6} = 50,$ $h_{O_3D_2,7-5-3} = 50, h_{O_3D_2,7-6-4} = 0,$ $h_{O_4D_2,8-5-3} = 0, h_{O_4D_2,8-6-4} = 50,$ $h_{D_1P_1,5} = 50, h_{D_1P_2,6} = 50,$ $h_{D_2P_1,3} = 50, h_{D_2P_2,4} = 50.$	$q_{O_1D_1,P_1} = 50, q_{O_1D_1,P_2} = 0,$ $q_{O_2D_1,P_1} = 0, q_{O_2D_1,P_2} = 50,$ $q_{O_3D_2,P_1} = 50, q_{O_3D_2,P_2} = 0,$ $q_{O_4D_2,P_1} = 0, q_{O_4D_2,P_2} = 50.$
2	$h_{O_1D_1,1-3-5} = 0, h_{O_1D_1,1-4-6} = 50,$ $h_{O_2D_1,2-3-5} = 50, h_{O_2D_1,2-4-6} = 0,$ $h_{O_3D_2,7-5-3} = 0, h_{O_3D_2,7-6-4} = 50,$ $h_{O_4D_2,8-5-3} = 50, h_{O_4D_2,8-6-4} = 0,$ $h_{D_1P_1,5} = 50, h_{D_1P_2,6} = 50,$ $h_{D_2P_1,3} = 50, h_{D_2P_2,4} = 50.$	$q_{O_1D_1,P_1} = 0, q_{O_1D_1,P_2} = 50,$ $q_{O_2D_1,P_1} = 50, q_{O_2D_1,P_2} = 0,$ $q_{O_3D_2,P_1} = 0, q_{O_3D_2,P_2} = 50,$ $q_{O_4D_2,P_1} = 50, q_{O_4D_2,P_2} = 0.$
3	$h_{O_1D_1,1-3-5} = 20, h_{O_1D_1,1-4-6} = 30,$ $h_{O_2D_1,2-3-5} = 30, h_{O_2D_1,2-4-6} = 20,$ $h_{O_3D_2,7-5-3} = 20, h_{O_3D_2,7-6-4} = 30,$ $h_{O_4D_2,8-5-3} = 30, h_{O_4D_2,8-6-4} = 20,$ $h_{D_1P_1,5} = 50, h_{D_1P_2,6} = 50,$ $h_{D_2P_1,3} = 50, h_{D_2P_2,4} = 50.$	$q_{O_1D_1,P_1} = 20, q_{O_1D_1,P_2} = 30,$ $q_{O_2D_1,P_1} = 30, q_{O_2D_1,P_2} = 20,$ $q_{O_3D_2,P_1} = 20, q_{O_3D_2,P_2} = 30,$ $q_{O_4D_2,P_1} = 30, q_{O_4D_2,P_2} = 20.$
4	$h_{O_1D_1,1-3-5} = 30, h_{O_1D_1,1-4-6} = 20,$ $h_{O_2D_1,2-3-5} = 20, h_{O_2D_1,2-4-6} = 30,$ $h_{O_3D_2,7-5-3} = 30, h_{O_3D_2,7-6-4} = 20,$ $h_{O_4D_2,8-5-3} = 20, h_{O_4D_2,8-6-4} = 30,$ $h_{D_1P_1,5} = 50, h_{D_1P_2,6} = 50,$ $h_{D_2P_1,3} = 50, h_{D_2P_2,4} = 50.$	$q_{O_1D_1,P_1} = 30, q_{O_1D_1,P_2} = 20,$ $q_{O_2D_1,P_1} = 20, q_{O_2D_1,P_2} = 30,$ $q_{O_3D_2,P_1} = 30, q_{O_3D_2,P_2} = 20,$ $q_{O_4D_2,P_1} = 20, q_{O_4D_2,P_2} = 30.$

The example traffic patterns described in Table 2 are equilibrium solutions, because it can be easily verified that every network user for each OD pair turns out to have the identical travel disutility, and no one can further reduce the individual travel cost by unilaterally changing his or her own path choice or parking choice. This reveals that at the defined AV network equilibrium, the solution ($\boldsymbol{h'}^*$, $\boldsymbol{q}^*$) is not necessarily unique. The equilibrium solutions in Table 2 also show that $h_{D_1P_1,5}, h_{D_1P_2,6}, h_{D_2P_1,3}$ and $h_{D_2P_2,4}$ are positive, which means that the empty trips to parking lots have caused additional flows on Links 3,4,5, and 6, resulting in worse traffic congestion.

Despite the non-uniqueness property of ($\boldsymbol{h'}^*$, $\boldsymbol{q}^*$), we can establish the uniqueness of the equilibrium link flow pattern $\boldsymbol{f}^*$, which is discussed in the following.

Proposition 1. *Given any equilibrium pattern of* $\boldsymbol{q}^*$*, the corresponding link flow pattern* $\boldsymbol{f}^*$ *at equilibrium is unique.*

Proof. With $\boldsymbol{q}^* = [q_{rs,p}{}^*]$ considered given, we can uniquely determine the transformed OD demand $\boldsymbol{d'} = [d'_{r's'}]$, $\forall\,(r',s') \in \Omega'$, which can be calculated by Equations (5) through (10). The solution $\boldsymbol{f}^*$ can be obtained by assigning the demand $\boldsymbol{d'}$ to the network under the deterministic user-equilibrium conditions in this context. Such a traffic assignment problem with homogeneous traffic is strictly convex if the link travel time function is monotonically increasing, i.e. Equation (21), which leads to a unique global optimal solution in terms of link flows (Sheffi, 1985).

Proposition 2. *Given any two different equilibrium patterns of* $\boldsymbol{q_1}^*$ *and* $\boldsymbol{q_2}^*$*, the following relationship between the two corresponding equilibrium link flow patterns must hold:* $\boldsymbol{f_1}^* = \boldsymbol{f_2}^*$*, where* $\boldsymbol{f_1}^* = [f_{a(1)}]$ *and* $\boldsymbol{f_2}^* = [f_{a(2)}]$.

Proof. First, as per **Proposition 1**, we confirm that with $\boldsymbol{q_1}^*$ and $\boldsymbol{q_2}^*$ given, $\boldsymbol{f_1}^*$ and $\boldsymbol{f_2}^*$ can be uniquely determined, respectively. Second, based on Equation (26), we have the following relationship under the defined network equilibrium:

$$\sum_{(r,s)}\sum_p C_{rs,p(1)}\cdot\left(q_{rs,p(2)}-q_{rs,p(1)}\right)+\sum_a t_{a(1)}\left(f_{a(1)}\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)\geq 0, \tag{40}$$

$$\sum_{(r,s)}\sum_p C_{rs,p(2)}\cdot\left(q_{rs,p(1)}-q_{rs,p(2)}\right)+\sum_a t_{a(2)}\left(f_{a(2)}\right)\cdot\left(f_{a(1)}-f_{a(2)}\right)\geq 0. \tag{41}$$

By combining (40) and (41), we have:

$$\left[\sum_{(r,s)}\sum_p\left(C_{rs,p(1)}-C_{rs,p(2)}\right)\cdot\left(q_{rs,p(2)}-q_{rs,p(1)}\right)\right]+\left[\sum_a\left(t_{a(1)}\left(f_{a(1)}\right)-t_{a(2)}\left(f_{a(2)}\right)\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)\right]\geq 0. \tag{42}$$

As per the defined equilibrium conditions of AV parking choice, we have $C_{rs,p(1)}=C_{rs,p(2)}=C^*_{rs,p}$, which causes the value of the term in the first square bracket within Equation (42) to be zero. We can then simplify Equation (42) as follows:

$$\sum_a\left(t_{a(1)}\left(f_{a(1)}\right)-t_{a(2)}\left(f_{a(2)}\right)\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)\geq 0. \tag{43}$$

Third, under the assumption of the monotonically increasing link travel time function, it holds that

$$\left(t_{a(1)}\left(f_{a(1)}\right)-t_{a(2)}\left(f_{a(2)}\right)\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)\leq 0, \forall a\in A, \tag{44}$$

and the equality is satisfied only when $f_{a(1)}=f_{a(2)}$. Summing (44) over all links $a\in A$ yields

$$\sum_{a\in A}\left(t_{a(1)}\left(f_{a(1)}\right)-t_{a(2)}\left(f_{a(2)}\right)\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)\leq 0, \tag{45}$$

and similarly, the equality holds only when $\boldsymbol{f_1}^*=\boldsymbol{f_2}^*$. In other words, if $\boldsymbol{f_1}^*\neq\boldsymbol{f_2}^*$, we then have

$$\sum_{a\in A}\left(t_{a(1)}\left(f_{a(1)}\right)-t_{a(2)}\left(f_{a(2)}\right)\right)\cdot\left(f_{a(2)}-f_{a(1)}\right)< 0, \tag{46}$$

which obviously contradicts with inequality (43). Therefore, $\boldsymbol{f_1}^*$ and $\boldsymbol{f_2}^*$ must be identical.

In the above analysis, we have proved the uniqueness of the link flow pattern under the defined network equilibrium for AVs. One implication of the unique link flow pattern is that the equilibrium parking demand at each parking lot, i.e., parking occupancy rate, can also be uniquely determined, since it can be calculated as the summation of all incoming link flows. Another important implication of the uniqueness is that the evaluation of network system performance and the assessment of transport policies can be conducted in a consistent manner when compared to the case with multiple equilibrium solutions.

4 SOLUTION METHODOLOGY

We now discuss the solution approach for the VI formulation. The proposed VI formulation for network equilibrium with AVs is indeed the outcome of integrating two sequential correlated sub-problems (the route choice problem and the parking choice problem). Specifically, with the OD demand and the parking supply considered given, the equilibrium conditions of parking location choice, i.e. (2) through (4), are equivalent to the following VI sub-problem (i):

P3: VI sub-problem (i):

$$\sum_{(r,s)}\sum_p C^*_{rs,p}\cdot\left(q_{rs,p}-q^*_{rs,p}\right)\geq 0, \tag{47}$$

subject to (14) (15) and (20).

With $\boldsymbol{q}^*=\left[q_{rs,p}\right]$ determined by VI sub-problem (i), the equilibrium conditions of route choice, i.e. (11) through (13), are equivalent to the following VI sub-problem (ii):

P4: VI sub-problem (ii):

$$\sum_a t_a(f^*_a)\cdot(f_a-f^*_a)\geq 0, \tag{48}$$

subject to (5)-(10) and (16)-(19).

Indeed, the parking choice process and the route choice process are interrelated with each other. Under an initial parking choice pattern $\boldsymbol{q}$, the path flow pattern $\boldsymbol{h'}$ will be formed with each AV commuter pursuing the minimum individual travel cost, resulting in an update on the link flow pattern $\boldsymbol{f}$. The updated link flows will modify the travel disutility $C_{rs,p}$, leading to the reformation of $\boldsymbol{q}$ under the equilibrium conditions. This loop will continue until the convergence is reached. Based on the above analysis, we develop the solution approach for our proposed network equilibrium problem with AVs. The solution procedure as depicted in Figure 2 is summarised below.

Step 1 Initialising parameters

- Initialise iteration count $n=0$.
- Initialise decision vectors $\boldsymbol{f}=\boldsymbol{f}_{(n)}=\boldsymbol{0}$, $\boldsymbol{q}=\boldsymbol{q}_{(n)}=\boldsymbol{0}$ and $\boldsymbol{d'}=\boldsymbol{d'}_{(n)}=\boldsymbol{0}$

Step 2 Determining the initial OD demand by parking choices

- Calculating the travel disutility $C_{rs,p(0)}$ based on the free-flow traffic pattern.
- Selecting p among U_{rs} that minimises $C_{rs,p(0)}$, assigning d_{rs} to $q_{rs,p}$ by using the All-or-Nothing algorithm, and updating $\boldsymbol{q}_{(0)}$.

Step 3 Solving VI sub-problem (ii)

- Determining the travel demand $\boldsymbol{d'}_{(n+1)}$ based on $\boldsymbol{q}_{(n)}$ by using Equations (5)-(10).
- Calculating the user-equilibrium traffic pattern $\boldsymbol{f}_{(n+1)}$ under $\boldsymbol{d'}_{(n+1)}$ by using the Frank-Wolfe algorithm.

Step 4 Modifying the solution to VI sub-problem (i)

- Calculating the travel disutility $C_{rs,p(n+1)}$ based on $\boldsymbol{f}_{(n+1)}$.
- Imposing the All-or-Nothing algorithm to obtain the auxiliary OD travel demand by parking choices, denoted as $\boldsymbol{j}=\boldsymbol{j}_{(n+1)}=\left[j_{rs,p(n+1)}\right]$.
- Updating the OD travel demand by parking choices

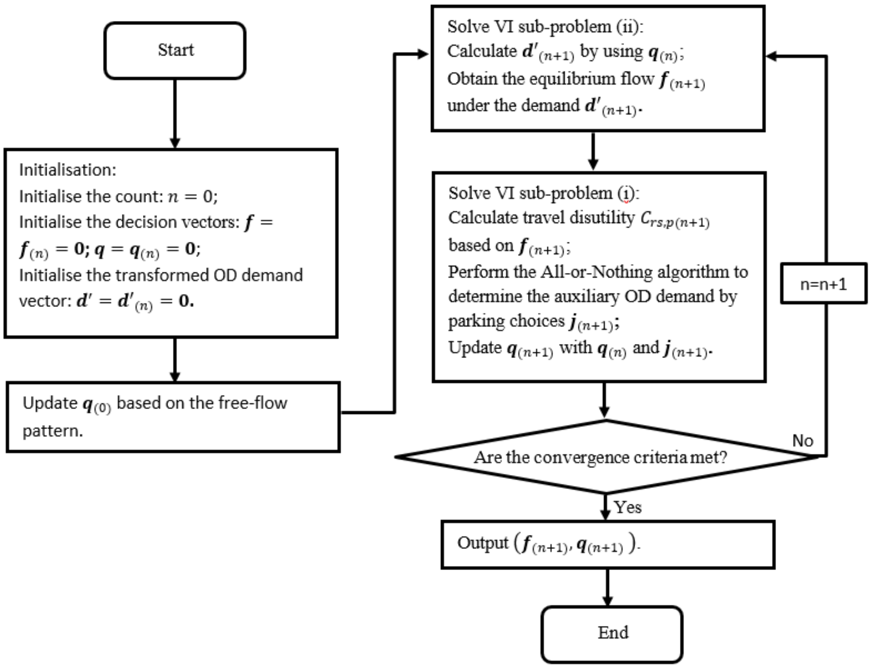


Figure 2 Flowchart of the solution procedure

- by using the Method of Successive Averages (MSA) algorithm: $\boldsymbol{q}_{(n+1)} = \left[q_{rs,p(n+1)}\right]$, where $q_{rs,p(n+1)} = \frac{n}{n+1} \cdot q_{rs,p(n)} + \frac{1}{n+1} \cdot j_{rs,p(n+1)}$.

Step 5 Examining convergence

- The algorithm will converge provided that the distance between $\boldsymbol{z}_{(n+1)}$ and $\boldsymbol{z}_{(n)}$ is sufficiently small. If so, let $\boldsymbol{z}^* = (\boldsymbol{f}^*, \boldsymbol{q}^*) = \left(\boldsymbol{f}_{(n+1)}, \boldsymbol{q}_{(n+1)}\right)$, and terminate. Otherwise, increment $n = n + 1$ and return to *Step 3*.

Remark 4.1. In *Step* 3, a number of existing solution approaches can be used to solve the VI sub-problem (ii), such as the Gradient Projection Method, the Frank-Wolfe Method. In this study, we adopt the Frank-Wolfe algorithm, considering its computational efficiency without the need of path enumeration (LeBlanc et al., 1985; Ukkusuri et al., 2007; Unnikrishnan and Waller, 2009). In *Step* 4, some other methods, such as the solution algorithm for a combined distribution and assignment model (Evans, 1976; Friesz, 1983; Patriksson, 2015), could also be used for updating the demand by parking locations. In this study, we adopt the MSA algorithm, given that it has been justified by the existing literature on network equilibrium with parking (Tong et al., 2004; Lam et al., 2006; Leurent and Boujnah, 2014).

Remark 4.2. In *Step 2* and *Step 4*, the All-or-Nothing algorithm is implemented, subject to the parking capacity constraint, i.e. Equation (20). To obtain the results of $\boldsymbol{q}_{(0)}$ (in *Step 2*) and $\boldsymbol{j}_{(n+1)}$ (in *Step 4*), we solve the following optimisation problem (we use $\boldsymbol{j}_{(n+1)}$ in the formulation below, which can be replaced by $\boldsymbol{q}_{(0)}$ when *Step 2* is executed):

P5:

$$min \sum_{rs} \sum_{p} C_{rs,p(n+1)} \cdot j_{rs,p(n+1)}, \tag{49}$$

subject to

$$\sum_{r\in O, s\in D} j_{rs,p(n+1)} \leq \Psi_p, \tag{50}$$

$$\sum_{p\in U_{rs}} j_{rs,p(n+1)} = d_{rs}. \tag{51}$$

P5 is a linear programming problem, which can be solved by using the CPLEX optimiser (CPLEX, 2009). With the optimisation problem *P5* solved in *Step 2* and *Step 4*, we ensure that AV commuters always choose the parking location that results in the minimal travel disutility, which is consistent with the equilibrium conditions of AV parking choice.

Remark 4.3. For the convergence criterion in *Step 5*, a relative gap, which incorporates the sum of functions by component in $\boldsymbol{z}$, is appropriate for use (Sheffi, 1985). In this study, the total relative gap on each vector for iteration n is calculated with the formulae as follows:

$$RG(\boldsymbol{f}) = \frac{\sqrt{\sum_a \left(f_{a(n+1)} - f_{a(n)}\right)^2}}{\sum_a f_{a(n)}}, \tag{52}$$

$$RG(\boldsymbol{q}) = \frac{\sqrt{\sum_{(r,s)} \sum_p \left(q_{rs,p(n+1)} - q_{rs,p(n)}\right)^2}}{\sum_{(r,s)} \sum_p q_{rs,p(n)}}. \tag{53}$$

Then, the selection criterion can be written as:

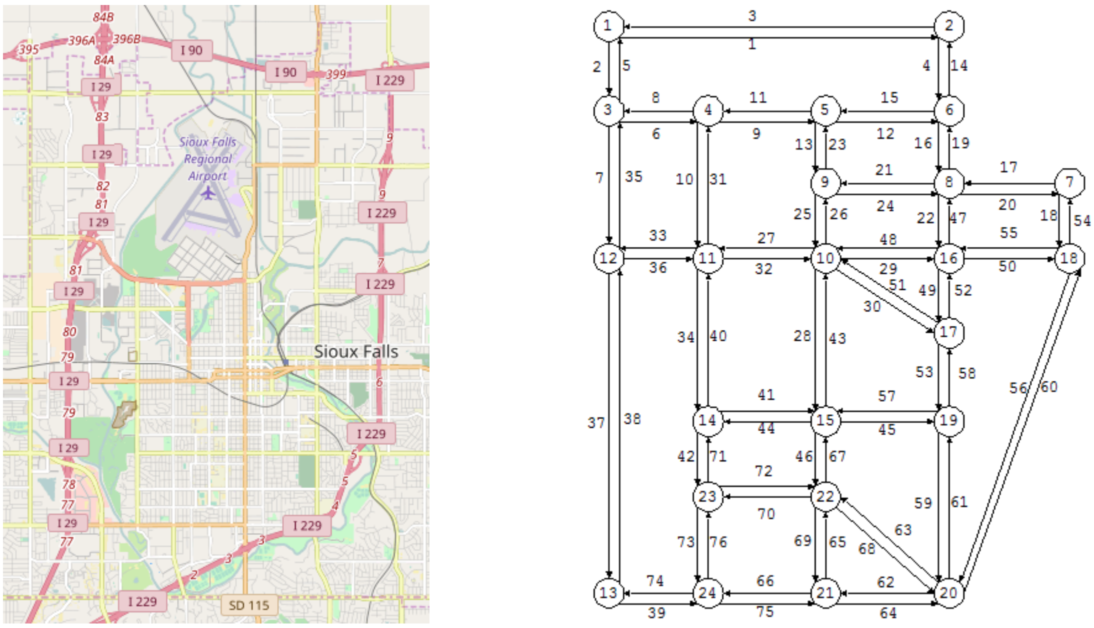
$$RG(\boldsymbol{f}) < Tol_1 \; and \; RG(\boldsymbol{q}) < Tol_2, \tag{54}$$

where Tol_1 and Tol_2 are the predetermined tolerance thresholds.

5 COMPUTATIONAL RESULTS AND DISCUSSIONS

In this section, we detail the implementation, summarise the computational results, and explore insights into the introduced network equilibrium with AVs. We apply the proposed model to the Sioux-Falls city network. The network data can be obtained from GitHub (2016). Figure 3 shows the real Sioux-Falls city network in (a) (from https://www.openstreetmap.org), and the Sioux-Falls grid network in (b), which is used as a test problem. The data for travel demands and parking attributes are summarised in Table 3. There are three AV parking options considered in our case study: (i) parking at the workplace in the central business district (CBD); (ii) parking near the CBD; and (iii) parking at home.

This section is organised as follows. Section 5.1 investigates the difference in network equilibrium between the AV and non-AV situations. Section 5.2 and 5.3 respectively explore the insights into the impacts of VoT and parking supply on the AV traffic pattern under the defined equilibrium. In this study, the experiments are implemented on a Windows 7 platform with an Intel Core i7-4770 processor at 3.40 GHz and 16.0 GB of RAM. We report that the computational cost of solving the proposed network equilibrium model is around 10 to 20 seconds, under the convergence criterion with $Tol_1 = 0.1\%$ and $Tol_2 = 0.1\%$.



(a) real network (b) test network

Figure 3 Topology of Sioux-Falls city network

5.1 Demonstration (I)

In this demonstration, we present some numerical results which are obtained from applying the proposed model to the Sioux-Falls city network. The purpose of this demonstration is twofold: (i) to verify the developed modelling framework for the AV network equilibrium with parking and compare the derived traffic pattern with that under the non-AV situation; (ii) to assess the impacts on network flow results by the parking fee. We solve the developed VI model for AV network equilibrium on the Sioux-Falls city network, and in the meantime, solve the user-equilibrium traffic assignment problem under the non-AV environment by using the same network data. For commutes with non-AVs, the travel process follows the procedure: Depart from home → Drive to the parking lot → Walk to the destination. In this demonstration, we assume that people will choose to park at their destination, due to the unacceptable walking distance between the destination and the parking lots (except those at Node 10 and Node 15). In the implementation, we adopt the parking fee at public parking lots given in Table 3 as a benchmark, and then gradually decrease the value. For the value of time, we let $\alpha = 7.0$ and $\beta = 3.0$ for demonstration purposes. We denote the percentage of decrease in parking fee as ∂, and investigate the system-level travel costs against ∂ both in AV and non-AV environments. The results are summarised in Figure 4.

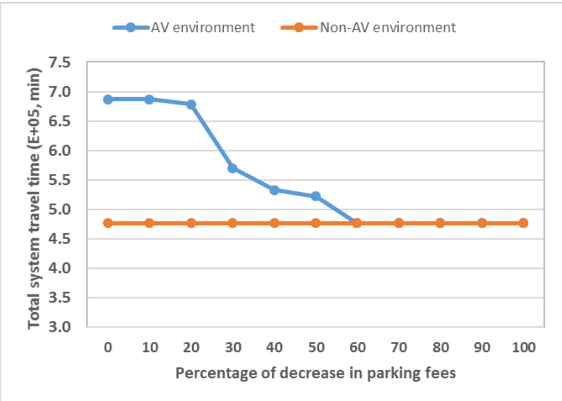
Table 3
Summary of traffic data for Sioux-Falls city network

(a) OD demand:

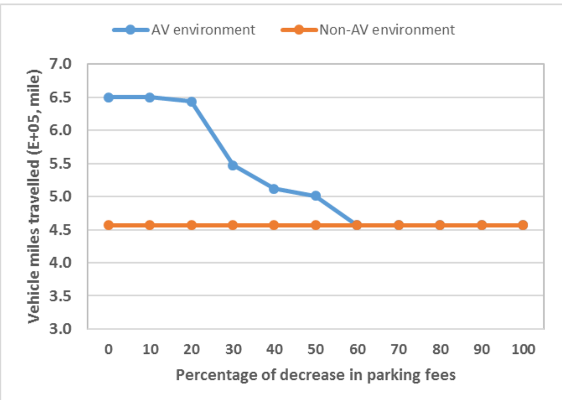
OD pair	Travel demand	OD pair	Travel demand
(1,10)	2,000	(7,10)	1,200
(1,15)	3,000	(7,15)	3,000
(2,10)	2,000	(13,10)	3,600
(2,15)	2,400	(13,15)	2,400
(3,10)	1,600	(20,10)	4,000
(3,15)	3,200	(20,15)	2,000

(b) Parking options:

Parking location (Centroid No.)	Description	Parking fee (dollar)	Parking capacity (veh)
10	Public parking in CBD	50	14400
15	Public parking in CBD	50	16000
9	Public parking near CBD	30	10000
11	Public parking near CBD	30	8000
14	Public parking near CBD	30	8000
1	Private parking at home	0	5000
2	Private parking at home	0	4400
3	Private parking at home	0	4800
7	Private parking at home	0	4200
13	Private parking at home	0	6000
20	Private parking at home	0	6000



(a)



(b)

Figure 4 Comparison between AV and non-AV environments

It can be observed from Figure 4 that, for an AV environment, the system-level travel costs, including both $TSTT$ and VMT, become smaller with the decrease in the parking fee at public parking lots. This is primarily because when the parking fee decreases, more AV commuters would prefer to park their cars in the city centre, in order to avoid the travel cost of cruising for a cheaper parking lot or driving themselves back home for the parking purpose. This can be reflected by the ratio of use (i.e. the parking demand divided by the parking capacity) for the parking lots at the workplace in the CBD, as depicted in Figure 5. By contrast, the system-level travel costs remain unchanged under the non-AV environment. This is because, in this case, conventional vehicles have to be parked at the workplace regardless of the parking fee, as the walking distance between the workplace and the other public parking lots are unacceptable (also, non-AVs are unlikely to be parked at home).

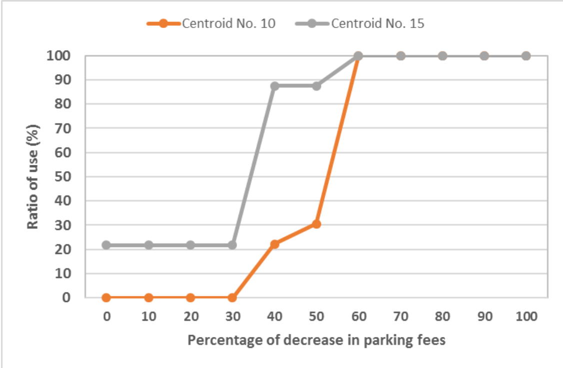


Figure 5 Ratio of use for the parking lots at the workplace

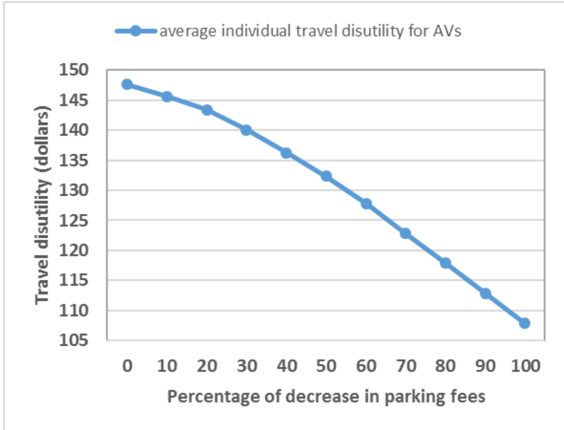
Another important insight from Figure 4 is that when the parking fee at the public parking place is high, the system-level travel costs for the AV situation are larger than those for the non-AV situation. This is caused by the empty trips by AV self-driving, the purpose of which is to find an appropriate parking lot (or parking at home) after dropping off commuters at their destinations, in order to achieve the minimal individual travel disutility. When the parking fee at the public parking lots is no longer costly, AVs will choose to park at the workplace, as discussed above. This means that the difference in the travel behaviour and the resultant network-wide travel cost between AV and non-AV situations would become less significant, as shown in Figure 4. Figure 4 also reveals that for the AV and non-AV situations, the values of $TSTT$ and VMT tend to change in a consistent manner. This is primarily because both metrics indicate the system-level travel cost. We adopt these two metrics to describe travel costs from different aspects: $TSTT$ represents the time spent on trips, while VMT is associated with the travel distance (Zhang et al., 2019).

In addition, Figure 5 provides the insight into the inbound and outbound flows, with regards to the travel destinations, Node 10 and Node 15. For these two nodes, the inbound AV flow resulting from the travel-related trip remains unchanged, which equals the sum of travel demands ending up with each destination node. By contrast, the outbound AV flow for the parking-related trip can be calculated as the abovementioned inbound flow multiplied by a percentage. The percentage is equal to 100% minus the ratio shown in Figure 5. Therefore, Figure 5 implies that the outbound congestion caused by trips for the parking purpose is reduced with the decrease in the parking fee.

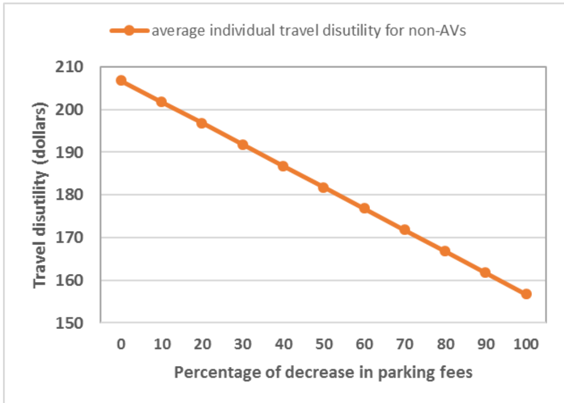
In this demonstration, we also investigate the average travel cost for individual trips, i.e. individual travel disutility, in both AV and non-AV situations. Given that conventional non-AVs have to be completely operated by commuters, the value of unit time cost for driving non-AVs, denoted as α', is higher than that for driving AVs, i.e. $\alpha' > \alpha$, which is assumed to be $\alpha' = 10$ dollars per unit time in this demonstration. We denote C_{rs}' as the travel disutility for non-AVs travelling from r to s and ending up with s for parking purposes. Different from AVs, the travel disutility for non-AVs can be determined as follows

$$C_{rs}' = \alpha' \cdot T_{rs} + Z_s, \forall r \in O, s \in D. \qquad (55)$$

where Z_s is the parking fee at the parking lot s. The average individual travel disutility can be calculated by using Equations (1) and (55) when the network equilibrium has been reached. The computational results are shown in Figure 6.



(a)



(b)

Figure 6 Results of individual travel disutility

It can be seen in Figure 6 that for both AV and non-AV situations, the average individual travel disutility decreases when the parking fee drops. For both situations, the common reason behind this phenomenon is that the parking fee is one of the important contributors to the travel disutility, which is indicated by (1) and (55). Another reason for the AV situation only is that a lower parking fee could incentivise more commuters to park at public parking lots, which decreases the road congestion caused by empty AV trips after dropping commuters at their workplace. Furthermore, when looking into Figure 4, we can find that under the AV environment, the individual travel cost is smaller than that in the non-AV situations, which indicates an increase in the utility of trips. To summarise the insights into Figure 4 and Figure 6, we can see that the autonomous vehicle will achieve high market penetration as it can lead to benefit for network users. However, on the other hand, the advent of an AV era means a new challenge to transport planners as it reshapes the network pattern and results in the increased traffic congestion.

5.2 Demonstration (II)

In this section, we vary the value of unit time cost for AV self-driving (i.e. β), and conduct sensitivity analysis. Specifically, we proceed to examine how the network traffic pattern shift due to the change of cost for AV self-driving in a fully AV environment. The results for the total system travel time and vehicle miles travelled at equilibrium are given in Figure 7, where seven representative values, namely, $\beta = 4.0, 3.5, 3.0, \ldots, 1.0$ dollars per unit time are adopted. The results of the percentage of commuters who go back home for the parking purpose are shown in Figure 8.

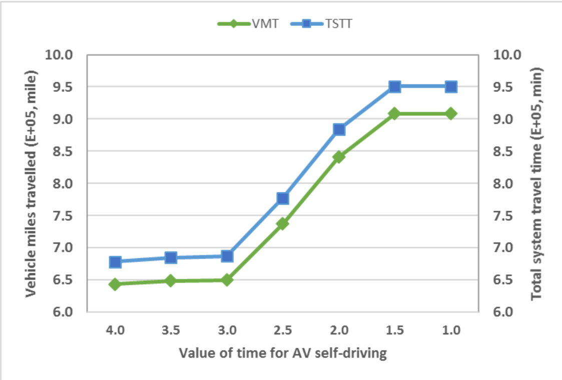


Figure 7 Variation of system performance metrics against VoT for AV self-driving

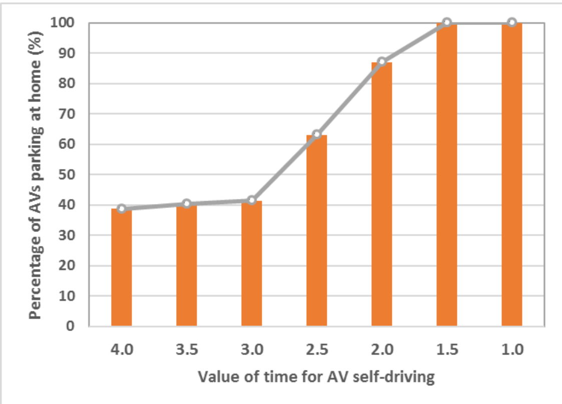
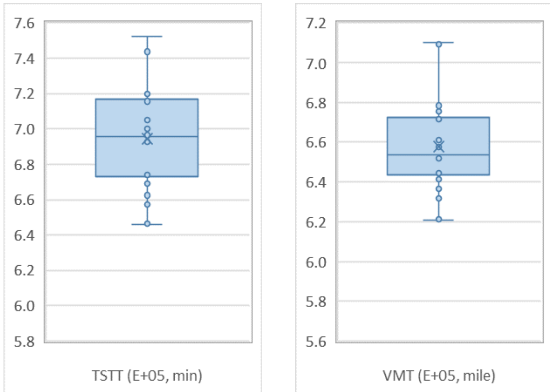


Figure 8 Variation of parking choice against VoT for AV self-driving

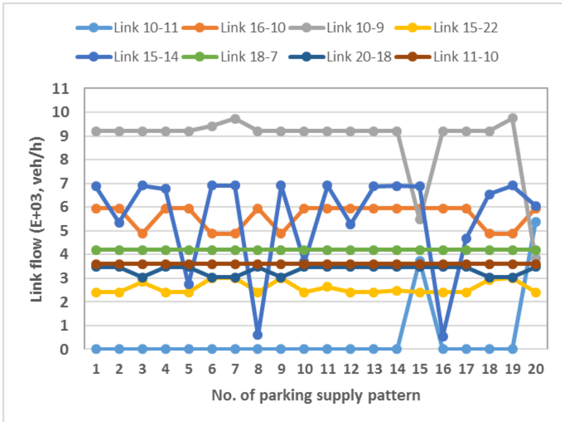
Figure 7 shows that both $TSTT$ and VMT go up with the decrease in the value of time for empty AV trips. This is because the parking fee at public parking lots becomes relatively greater with regards to the decreased value of β. Hence, an increasing number of AVs are motivated to drive themselves back home after dropping off commuters at the workplace, which can be observed in Figure 8. Such empty trips via AV-self driving bring about additional flows among the network, which could increase the road congestion and, in the meantime, negatively affect the traffic condition for trips from the residential area to the workplace. After the value of β decreases to 1.5, both network metrics remain unchanged. This is because such a VoT is sufficiently small, so that all AVs haven chosen to go back home to park after dropping off commuters. This means that a further reduction in the value of β can no longer have impacts on the equilibrium traffic pattern under this circumstance. Another insight into Figure 7 is that the way the $TSTT$ and VMT vary over β is nonlinear. This implies that when a change in the VoT for AV self-driving, it is difficult to quantify the extent to which the system performance metric will change without employing a modelling framework such as the one proposed in this study.

5.3 Demonstration (III)

In Demonstration (III), we evaluate the network-wide impacts of parking supply strategies, by varying the distribution of parking spaces among all the public parking lots. We assume the total number of parking spaces to be consistent with that given in Table 3. Then, we generate the random proportions of parking spaces allocated to each of the public parking lots. We test the network performance against twenty different parking space distribution patterns, numbered as 1, 2, …, 20. The computational results are summarised in Figure 9: Figure 9(a) displays box plots which illustrate the total system costs at equilibrium; Figure 9(b) records the results of equilibrium flow on some selected links.



(a)



(b)

Figure 9 Variation of equilibrium flows against parking supply

The results shown in Figure 9 reveal that the parking space distribution among different parking lots serves as a significant contributor to equilibrium results at both the network-wide level and the link-based level. From Figure 9(b), we notice that the equilibrium flow changes considerably on some links (for instance, Link 15-14), but fluctuates slightly or remains unchanged on some other links (for instance, Link 18-7). It is important to note that there is no consistent relationship between the link flow and the parking supply, or between changes in different link flows over the parking space distribution. For example, when looking into patterns No.6 and No.7, we find that the equilibrium flow on Link 10-9 changes while that on Link 16-10 stays the same. By contrast, when the parking supply pattern switches from No. 9 to No. 10, it can be observed that the change in the equilibrium flow happens to Link 16-10, instead of Link 10-9. Hence, Figure 9 indicates that the proposed equilibrium approach in this study is a critical modelling device, which can be used to assess and to rank the parking supply policies in the context of infrastructure investment for future AV commutes. Specifically, the main objective of transportation planning is to maximise the overall benefit for the community subject to the limited amount of capital available. Under most circumstances, a set of comprehensive planning schemes are initially proposed, with each of them including measures such as network capacity expansion, parking facility deployment, parking pricing, and so forth. To evaluate the improvement in the performance of autonomous transportation systems, the modelling framework proposed by this study is needed in order to understand the reaction of AV commuters and estimate the traffic pattern under different schemes. Based on the evaluation result and the project budget, the governmental department can then rank the planning options and determine the optimal scheme for the capital investment in transport facility development.

6 CONCLUSIONS

This article introduces a novel modelling framework for the joint network equilibrium of route choice and parking location choice of autonomous vehicles (AVs). This is the first attempt in the literature to quantify the network equilibrium pattern under the fully autonomous traffic environment with new parking behaviour associated with AVs. We formulate the AV network equilibrium model as a variational inequality (VI) problem based on the hierarchical choice structure, and mathematically prove the equivalency between the VI model and the AV user-equilibrium conditions. Properties of the equilibrium solution are discussed, and the solution approach is developed. We conduct a series of computational experiments to validate the efficacy of the proposed approach. In particular, we compare the equilibrium traffic patterns under the AV and non-AV situations. We find that the application of AVs can lead to a lower individual travel disutility as compared to conventional vehicles, while on the other hand, it could worsen traffic congestion due to the empty trip of finding an appropriate parking place via self-driving. The results also indicate that in a fully AV environment, the system-level travel cost tends to become greater, when the charge for public parking increases or the value of time for AV self-driving decreases. Moreover, the results show the spatial interactions between the AV equilibrium flow and the parking space distribution, and indicate that the latter has significant impacts on equilibrium flow both at the network level and at the link level. However, the way that the equilibrium result varies with the parking facility deployment is shown to be irregular. This means that it is difficult to quantify the change in AV traffic patterns under different facility provision options without solving the proposed model. Given the application prospect, the developed model can be employed to evaluate the network performance with AV operations and assess infrastructure development schemes for future transportation planning.

This study can be extended in multiple directions. First, given the developed equilibrium model, we can optimise the parking planning and parking pricing strategy depending on different socioeconomic objectives. Second, as the first step to model network traffic patterns with both route and parking choices for AVs, we adopt a static equilibrium assignment approach. Future studies will extend it to the time-dependent case, where the accumulation of AV parking demand versus the park capacity should be taken into account. In the context of dynamic traffic assignment, the tactical AV parking choice with real-time information provision will be considered, in which case AVs might also have an altruistic route choice. Third, we will incorporate the car-sharing strategy and the ride-hailing service to the established framework for AV network equilibrium in the follow-up study. Fourth, future studies may model the network equilibrium problem for heterogeneous traffic consisting of both AVs and non-AVs in the transitional period, wherein the mode choice and its influencing factors (for instance, value of time for AVs) will be investigated. The combinations of flows both on links and at intersections will also be modelled in this area. Last but not least, human factors may be considered in the context of AV commutes, and their impacts on both individual travel behaviour and network-wide performance can be studied.

ACKNOWLEDGEMENTS

The authors would like to thank the anonymous referees for their thoughtful comments, which helped improve both the exposition and technical quality of the paper. This research is partially supported by a grant from the Australian Research Council (LP160101021) and is partially supported by funding from the UNSW Digital Grid Futures Institute, UNSW, Sydney, under a cross disciplinary fund scheme.

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