Designing Multi-Attribute Procurement Mechanisms for Assortment Planning

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ABSTRACT

This research investigates how to design procurement mechanisms for assortment planning. We consider that a retailer buys directly from a manufacturer who possesses private information about the per-unit variable cost and per-variety setup cost. We first develop a screening model to assist the retailer in integrating assortment planning into supply chain contracting processes when only one manufacturer is available. We demonstrate that the screening mechanism is optimal among all feasible procurement strategies. When there are multiple competing manufacturers, we propose a supply contract auctioning mechanism and evaluate its performance. In this mechanism, the retailer announces a contract menu and the manufacturer that bids the highest upfront fee paid to the retailer wins the auction. The winner then chooses and executes a contract from the contract menu. We show that when the retailer uses the optimal screening contract menu as the object of the auction, it achieves the optimal procurement outcome if the screening contract menu does not pay rent to any manufacturer type. This finding sheds light on the connection between screening and auction mechanisms when there exists multi-dimensional private information.

[Keywords: Asymmetric Information, Assortment Planning, Auctions, Multi-attribute Procurement Contract]

INTRODUCTION

Because the assortment carried by a retailer could have a significant impact on the retailer's profit and sales, assortment planning has received a high level of priority from retailers, consultants, and software developers. Retailers recognize that more product variants help their business by catering to variety-seeking consumers (Coughlan et al., 2006). However, there are limits to the value of product variety. For instance, an additional variant sometimes adds little in the sense of "real" variety to the consumers and creates variety fatigue (Boatwright & Nunes, 2001; Chen et al., 2010). From an operational perspective, adding more variants inevitably increases the cost and complexity of the procurement and production processes. Therefore, the cost of carrying a wider assortment could eventually offset the benefits of adding more variants. When planning their assortments, retailers must address the trade-off between the width (the number of variants) and the depth (the inventory level of each variant) of their assortments.

The academic literature on assortment planning has grown rapidly over the past fifteen years. A variety of optimization models have been developed to assist retailers in choosing the optimal set of variants and the inventory level for each variant. Readers can refer to Kök et al. (2009) for a comprehensive review. An important prerequisite of the extant models is that the cost information is complete. However, many firms are reluctant to share sensitive information because of conflicting interests (e.g., Cachon & Zhang, 2006; Kurtulus & Nakkas, 2011; Kaya & Özer, 2012; Li & Sun, 2012). The informed firm often has an incentive to distort or hide information to gain strategic advantages such as information rent. The concern about asymmetric information has become an important issue for retailers in their procurement process. For example, Lockstrom et al. (2011) interviewed the purchasing manager, quality manager, or general manager of sampled companies and found that "there is a great deal of adversarial behavior where information asymmetry plays an important role, rendering full openness very difficult. This is reflected in the low degree of open book policies."

We aim to extend the assortment planning literature by considering information asymmetry. Our research is relevant and useful for retailers who are directly sourcing from foreign countries, where cultural differences and lack of trust (Zhao et al., 2007) could exacerbate the problem of information asymmetry. We build the procurement mechanism design model for assortment planning by incorporating the following three key features.

1. Direct Sourcing. Many retailers adopt the strategy of direct sourcing from overseas manufacturers. For example, when Guy Russo, the managing director of K-Mart Australia, took the helm in 2009 and tried to reverse the fortunes of a bankrupt Aussie icon, he found that “the majority of products that we got, 95 percent of them, all came from Asia. They already did come from Asia, but I was dealing with a local Australian agent. I was in shock when I saw the mark-up that our agents were putting on, that we could remove totally, and then just pass on those savings to customers.” By moving to a direct sourcing model, liaising with the same factories that supply global retailers such as Wal-Mart and Tesco, K-Mart Australia has managed to reduce the purchase cost of some goods by 50 percent and has passed the savings on to customers (Mitchell, 2012). Mr. Russo said, ”When we can match the quality of those famous brands, and then deal direct with the factory, we can deliver lowest prices.” Other retailers operating in Australia such as H&M (Robin, 2013) and Target (Lloyd, 2004) also adopted the direct sourcing model.

2. Multi-Dimensional Private Information. Because of the low degree of open book policies, Asian manufacturers are tight-lipped about their production costs, which are usually of a multi-dimensional nature. Take a garment factory as an example. The major steps of producing a garment include: 1) making the colored fabric from yarn, 2) fabric relaxing, 3) spreading, forming the layout and cutting, 4) embroidery and screen printing, 5) sewing, 6) spot cleaning and laundry, 7) ironing, and 8) packaging and shipping (www.textileschool.com). If the extra variant differs from others in color or fabric type, the factory must address the extra sorting and handling activities before each of the major steps. The per-unit cost of producing a garment of different colors may be the same, but the extra color could cost more. Because of information asymmetry (see, e.g., Lockstrom et al., 2011 and the references therein), the exact cost of producing an assortment is a closely guarded secret of many manufacturers. By following the previous literature in Marketing and Operations Management (e.g., Gaur & Honhon, 2006; Kurtulus & Nakkas, 2011), we assume that the cost of producing an assortment is linear with respect to the total quantity and the number of variants. We consider the use of screening and auction mechanisms to mitigate the effect of information asymmetry and contribute to the literature by building a connection between these two mechanisms.

3. Seasonal Retailing. Many retailers such as H&M use seasonal models (Robin, 2013). For example, stores are stocked with seasonal goods made by Asian manufacturers, and at the end of the season, unsold items are marked down for clearance. Agrawal and Smith (2009) presented a retail master calendar that describes the supply chain of two major American retailers that also use the seasonal model. The time line of our model is consistent with this retail master calendar. Because the planning horizon of each assortment contract is short, retailers must frequently manage the joint decisions on assortment planning and procurement contracting. Two other industrial characteristics are worth noting. The first characteristic is the supplier turnover, which is typical for retailers that buy directly from Asian manufacturers. In a field trip to the Wal-Mart Procurement Center in Southern China (organized by POMS Hong Kong Chapter), we learned that the supplier turnover rate is approximately $20-35\%$ annually. Wal-Mart is not the only company that deals with supplier turnover. Readers can refer to the references cited by Swinney and Netessine (2009) for more examples. The second characteristic is the uncertainty in the cost of raw materials attributed to the increasing volatility in commodity markets (Levy & Ferazani, 2006). The fluctuation in raw material costs could make the cost information obtained from the previous contracting cycle no longer valid. In summary, at the time of contracting with a new (or an existing) manufacturer, retailers usually do not possess all of the cost information but need to make joint decisions on assortment and procurement.

These three key industrial features form the basis of our model. Specifically, we consider a single-period newsvendor model in which the retailer jointly makes the assortment planning and procurement contracting decisions. The manufacturer possesses multi-dimensional private information regarding the volume-based production cost and the variant-based setup cost, which directly affect the retailer's decision on the width and depth of the assortment. We present the optimal screening mechanism to achieve the optimal procurement outcome for the single-manufacturer case. When there exist multiple competing manufacturers, we evaluate the performance of a supply contract-auctioning mechanism. To the best of our knowledge, we are the first to consider multi-dimensional private information in assortment planning.

RELATED LITERATURE

Our research is built upon two streams of literature: retail assortment planning and a multi-dimensional mechanism design. The literature on retail assortment planning generally focuses on how the consumers' choice of a favorite variant from an assortment could affect the assortment decision. It is typically assumed that the cost information is fully available when the retailer makes the assortment decision. One of the most important results on assortment planning was obtained by van Ryzin and Mahajan (1999). Under the assumption that all variants have an identical newsvendor ratio, van Ryzin and Mahajan demonstrated that the optimal assortment should consist of a certain number of the most popular variants. Subsequent research extended the analysis to study how the retailer's assortment decisions could be affected by component commonality (Bernstein et al., 2011), category captainship (Kurtulus & Nakkas, 2011), or endogenous pricing (Maddah & Bish, 2007; Aydin & Porteus, 2008). Several other studies took a different route by relaxing the assumption of an identical newsvendor ratio. For example, Li (2007) and Alptekinoğlu et al. (2009) showed that the optimal assortment does not always include the most popular variants. Instead of exhaustively reviewing the assortment planning literature, we refer readers to Kök et al. (2009) for a comprehensive review.

Mechanism design theory has been successfully applied to many supply chain problems with asymmetric information. However, the multi-dimensional problem is much more difficult than the one-dimensional problem. Readers can refer to Rochet and Stole (2003) for an excellent survey of the economics literature. The OM/SCM literature that uses mechanism design theory can be categorized into two groups. The first group considers multi-attribute screening mechanisms (e.g., the non-linear pricing mechanisms such as two-part tariffs) and one-dimensional information. The number of published articles in this group is so large that we refer readers to Kaya and

Özer (2012) for an updated literature review. The second group, which is the most relevant to our research, considers multi-dimensional private information. The articles in the second group are sparse. Chen-Ritzo et al. (2005) studied the use of multi-dimensional auctions in an experiment where the suppliers possess private information regarding quality and lead time. Asker and Cantillon (2010) evaluated the performance of scoring auctions and bargaining when the suppliers possess private information regarding the fixed and marginal costs associated with product quality. Whereas these two articles used additive and separable objective functions, we consider an objective function that is neither additive nor separable with respect to the number of varieties (an integer variable) and the order quantity (a continuous variable).

We also consider the use of auctions in assortment planning when there are multiple manufacturers available. Che (1993) proposed a scoring auction in which the buyer announces a scoring rule and suppliers are asked to bid on quality and price. The supplier who achieves the highest score is selected as the winner. Chen (2007) developed a supply contract auction where the object of the auction is a supply contract. The supplier who bids the highest price for the right to execute the supply contract is selected as the winner and is then delegated to make the inventory decision. In these two articles, the supplier's private information is one-dimensional, while we consider multi-dimensional private information.

THE SCREENING MODEL

We consider a single-period newsvendor model where a retailer jointly makes the assortment planning and procurement contracting decisions. The retail price of all the variants in the same assortment is assumed to be r. This assumption is consistent with the common practice of many retailers. For example, in a short visit to the two largest Australian retailers (Coles and Woolworths), we found that the same price is charged for clothes, ice cream, shampoos, shoes, detergents, and frozen foods in the same assortment. In the appendix, we provide a detailed discussion on relaxing the assumption of exogenous price.

We first consider the case that only one manufacturer is available. To facilitate the analysis, we assume that there are two manufacturer types, denoted by type 1 and type 2, respectively. The type of the manufacturer determines the cost of producing an assortment. We note that the variants in the same assortment may have different physical attributes (e.g., clothes in the same assortment differ in colors, patterns, or sizes) that do not affect quality or price. Hence, it is reasonable to assume that the per-unit cost is constant. Because we consider multiple manufacturer types, the per-unit cost should be manufacturer-dependent. Furthermore, when the retailer buys from the manufacturer, the retailer must buy at least one variant. Hence, the variant-based setup cost is assumed to be incurred for the second variant onward. To reflect these two features, the type-i manufacturer is assumed to incur a total cost of $c_iQ + k_i(n-1)^+$ when producing an assortment with n variants and total order quantity of Q units, where $(z)^+ = \max(0, z)$. This convex cost assumption is consistent with the previous literature in Marketing and Operations Management (e.g., Gaur & Honhon, 2006; Kurtulus & Nakkas, 2011). We also assume that $r > \max\{c_i\}$. The probability that the manufacturer is type-i is δ_i and ($\delta_1 + \delta_2 = 1$). Without a loss of generality, the reservation value of each type of manufacturer is normalized to be zero.

There are many different ways to model how a typical consumer makes a purchasing decision given an assortment. We adopt the multiplicative demand model used by Aydin and Porteus (2008) and several others. Specifically, the demand for variant j offered in the assortment is $D_jq_j(n)$, where D_j is independently and identically distributed with cumulative distribution function (cdf) $F(\cdot)$, and $q_j(n)$ is the proportion of consumers who buy variant j from an assortment with n variants. Because D_j is identically and independently distributed, we write D_j as D whenever convenient. We also assume that D is non-negative and has an upper bound $\bar{d}$. We do not consider stock-out based substitutions. In other words, a consumer who chooses variant j as the first choice and finds the variant to be out of stock does not make a second attempt. This assumption was used by van Ryzin and Mahajan (1999), Aydin and Porteus (2008), and many others.

Next, we derive the retailer's expected sales revenue. The following lemma demonstrates that it is optimal for the retailer to use the equal service level (ESL) policy, under which the probability of stock-out for each variant is equal.

Lemma 1 *The equal service level policy is optimal.*

This policy is easy to implement and is consistent with several previous results, such as those obtained by van Ryzin and Mahajan (1999). Under the ESL policy, we can greatly streamline the notation by using a single scalar x to determine the inventory level for each variant. Specifically, let $xq_j(n)$ be the order quantity of variant j. We see that the probability of stock-out is identical for each variant, but the order quantity for each variant is different and depends on the choice probability $q_j(n)$. The retailer's total expected sales revenue generated by an assortment with n variants is

$$r \sum_{j=1}^{n} E \min \left(xq_j(n), Dq_j(n)\right) = rE \min \left(x, D\right) \sum_{j=1}^{n} q_j(n) = rE \min \left(x, D\right) B(n), \tag{1}$$

where $B(n) = \sum_{j=1}^{n} q_j(n)$ is the proportion of consumers who attempt to buy from the assortment. We observe that the total order quantity is $Q = xB(n)$.

Note that instead of specifying the $q_j(n)$ function for each j, it is more convenient to specify the $B(n)$ function. We assume that $B(n)$ is a concave increasing function of n that satisfies $B(0) = 0$ and $B(n) \leq 1$ for any $n \geq 1$. Using this demand model, we can recover many demand models used in the previous research. For example, Kurtulus and Nakkas (2011) considered identical variants and the multinomial logit (MNL) model. They showed that $q_j(n) = v_r/(nv_r + 1)$, where the parameter v_r is the attractiveness of variant j relative to the no-purchase option, which is normalized to be 1. It is then easy to verify that $B(n) = nv_r/(nv_r + 1)$ satisfies our assumption. Now, suppose that the attractiveness is variant-specific in the MNL model. Similar to van Ryzin and Mahajan (1999), the retailer should first add variant j if it is more popular than variant i. We assume that $v_1 \geq v_2 \geq ... \geq v_n$, where v_j is the attractiveness of variant j relative to the no-purchase option. We can then obtain that

$$B(n)=\frac{\sum_{j=1}^{n} v_j}{\sum_{j=1}^{n} v_j+1}.$$

Using the monotone property of v_j, we can verify that $B(n)$ satisfies our assumption.

The MNL model is commonly used in the assortment planning literature, but it has an undesirable property of independence from irrelevant alternatives (IIA). We can avoid such problems by using a general form of the $B(n)$ function, which depends on the underlying choice process. For example, Gaur and Honhon (2006) considered the locational choice model and established that the first-choice intervals of all the products must be adjacent. In their choice model, the probability that a consumer buys from the assortment is $B(n)=G(l_n)-G(l_1)=G(l_1+2L(n-1))-G(l_1)$, where l_j denotes the location of product j on the preference spectrum, L represents the coverage distance of each product, and consumers are distributed over the spectrum with cdf $G(\cdot)$ and pdf $g\left(\cdot\right)$. If the $g(\cdot)$ is non-increasing (e.g., G follows the exponential or the uniform distribution), then $B(n)$ is a concave increasing function. Other models, which allow correlation over alternatives and overcome the problem of the IIA property, include mixed logit, conditional probit, generalized extreme values, and nested logit models (Train 2003). However, the choice probabilities of these models often have no closed forms and require simulations to evaluate the values.

The sequence of events is as follows. i) The retailer meets with a manufacturer whose type (or cost function) is private. ii) The retailer offers a menu of contracts and the manufacturer chooses a contract from the menu. iii) The products are delivered to the retailer before the selling season starts. iv) The retailer displays the assortment in the store. v) Consumers buy from the assortment. Unmet demand is lost, and unsold inventory is salvaged with zero value. The zero salvage value is introduced primarily to reduce the notational complexity; it does not qualitatively affect the final results. The above timeline is consistent with the standard newsvendor model (e.g., van Ryzin and Mahajan, 1999) and matches the retail master calendar presented by Agrawal and Smith (2009).

Using the multi-dimensional mechanism approach, it is sufficient to consider the following procurement mechanism. There are two contracts, denoted by contracts 1 and 2. Contract i specifies that the total payment to the manufacturer is P_i, the number of variants is n_i, and the total order quantity is $Q_i = x_i B(n_i)$. In the truth-revealing equilibrium, the type-i manufacturer takes contract i. While various contracts such as buy-back or revenue-sharing can be implemented, we note that different contracts require different sets of information and different levels of monitoring. Other factors such as transportation costs, tax, currency regulations, and legal considerations also play important roles in implementing them successfully (Kaya and Özer, 2012). For example, the substantial costs of returning merchandise limit the application of buy-back contracts. In the book and magazine markets, to reduce the cost of reserve logistics, only the covers are shipped to the publisher, and the cost of stripping the books and destroying the damaged books is the responsibility of the bookseller (Book Industry Study Group http://www.bisg.org). Another limitation of buy-back contracts is the chain-wide damage if the downstream retailer engages in signaling actions (Dai et al., 2012). The cost of verifying the sales revenue is another reason that often limits the application of revenue-sharing contracts. For example, the Walt Disney Company sued Blockbuster for cheating the video rental volume under a four-year revenue-sharing agreement (New York Times, January 4, 2003).

Because we consider that the retailer is buying directly from overseas countries, the above limitations (high transportation cost and high cost of verifying the sales of the foreign retailer or the amount of unsold inventory) exist in our problem setting. In addition, the currency and tax regulations in Asian countries (Zhao et al., 2007) could hinder the implementation of buy-back or revenue-sharing contracts. Finally, we note that the payment to the manufacturer under a buy-back or revenue-sharing contract is uncertain and depends on the realized demand. Because we consider risk-neutral players, by letting P_i equal the expected payment of a buy-back or revenue-sharing contract, we can induce the same outcome. Therefore, we focus our attention on the contract form that does not involve the information of sales or unsold inventory.

Assortment Contracting Problem

First, we derive the retailer's objective function. Let $\mathbb{N}$ be the collection of all non-negative integers. Suppose that the manufacturer is type-i. Under contract i, the total chain profit is given by

$$T_i(n_i, x_i) = rE\min(x_i, D)\,B(n_i) - c_i x_i B(n_i) - k_i(n_i - 1)^+. \tag{2}$$

We define the profit of the type-i manufacturer when contract i is chosen as follows:

$$w_i = P_i - c_i x_i B(n_i) - k_i(n_i - 1)^+.$$

Once we know the values of n_i, x_i, and w_i, we can identify the parameters (P_i, n_i, Q_i) of contract i. We also find that the retailer's profit is $rE\min(x_i, D)\,B(n_i) - P_i$, which equals $T_i(n_i, x_i) - w_i$. In other words, the retailer's profit is the total chain profit minus the manufacturer's profit. We observe that the retailer's objective is

$$\max_{n_i, x_i, w_i} Z = \delta_1\left[T_1(n_1, x_1) - w_1\right] + \delta_2\left[T_2(n_2, x_2) - w_2\right]. \tag{3}$$

Next, we derive the constraints. Consider that the type-i manufacturer accepts contract i. This manufacturer obtains a net profit of w_i. By switching to contract j ($i \neq j$), this manufacturer obtains a net profit of $P_j - c_i x_j B(n_j) - k_i(n_j - 1)^+$. To lure the type-$i$ manufacturer to take contract i, it must hold that $w_i \geq P_j - c_i x_j B(n_j) - k_i(n_j - 1)^+$, which is equivalent to

$$w_i \geq w_j + (c_j - c_i)x_j B(n_j) + (k_j - k_i)(n_j - 1)^+. \tag{4}$$

In summary, the retailer's constraints are the following:

$$w_1 \geq w_2 + (c_2 - c_1)x_2 B(n_2) + (k_2 - k_1)(n_2 - 1)^+; \tag{5}$$

$$w_2 \geq w_1 + (c_1 - c_2)x_1 B(n_1) + (k_1 - k_2)(n_1 - 1)^+; \tag{6}$$

$$w_i \geq 0; \tag{7}$$

$$n_i \in \mathbb{N} \text{ and } \bar{d} \geq x_i \geq 0. \tag{8}$$

Constraints (5) to (6) are the so-called incentive-compatibility (IC) constraints, constraint (7) is the individual rationality (IR) constraint, and constraint (8) is the feasibility constraint.

Our formulation allows us to do the following. First, we can easily verify whether the retailer's assortment decision (i.e., the breadth and the depth of the assortment) is chain-optimal. Second, we can easily identify how the chain profit is distributed between the retailer and each type of manufacturer. If $w_i = 0$, the type-i manufacturer does not make a profit that is greater than the reservation value. We say that the type-i manufacturer does not receive any information rent. If $w_i > 0$, the type-i manufacturer makes a profit that is strictly greater than the reservation value. We say that the type-i manufacturer receives an information rent. Third, constraints (5) to (6) reveal the difference between the multi-dimensional model and the classic one-dimensional model. Define

$$R_{ij}(n_j, x_j) = (c_j - c_i)x_j B(n_j) + (k_j - k_i)(n_j - 1)^+ \tag{9}$$

as the *incremental gain* of the type-i manufacturer if it switches to contract j. Whenever it does not cause any confusion, we write $R_{ij}(n_j, x_j)$ as R_{ij}. A positive R_{ij} implies that the type-i manufacturer is able to fulfill contract j with a lower total cost. If this is the case, the retailer must increase the payment in contract i to lure the type-i manufacturer. Note that the terms R_{12} and R_{21} will determine the rent paid to each type of manufacturer (see Lemma 3 in the next section). In the classic one-dimensional model (for example when $k_i = k_j$), the incremental gain R_{ij} only has one term and its sign is definite. However, in our model, R_{ij} in equation (9) has two terms. Therefore, its sign is indefinite when $k_i > k_j$ and $c_i < c_j$.

Chain Optimal Solution

The first benchmark is the chain optimal solution that maximizes $T_i(n_i, x_i)$ in equation (2). We need to introduce a few notations. Define $\pi_i(x) = rE\min(x, D) - c_i x$ as the newsvendor profit when the inventory scalar is x. Let $\bar{x}_i = F^{-1}\left(\frac{r-c_i}{r}\right)$ be the newsvendor solution. Let $\hat{n}_i$ be the solution of equation $B'(n) = k_i/\pi_i(\bar{x}_i)$, where $\hat{n}_i$ does not need to be an integer. Let $\lfloor n \rfloor$ be the largest integer that is not greater than n and $\lceil n \rceil$ be the smallest integer that is greater than n. In general, it holds that $\lfloor n \rfloor \leq n < \lceil n \rceil$ and $\lceil n \rceil - \lfloor n \rfloor = 1$. Now we provide the chain optimal solution and chain optimal profit as follows:

Lemma 2 *The chain optimal solution is*

$$\begin{aligned} \bar{x}_i &= F^{-1}\left(\frac{r - c_i}{r}\right), \\ \bar{n}_i &= \begin{cases} 1 & B'(1) < k_i/\pi_i(\bar{x}_i), \\ \lfloor \hat{n}_i \rfloor & B'(1) \geq k_i/\pi_i(\bar{x}_i) \text{ and } T_i\left(\lfloor \hat{n}_i \rfloor, \bar{x}_i\right) \geq T_i\left(\lceil \hat{n}_i \rceil, \bar{x}_i\right), \\ \lceil \hat{n}_i \rceil & B'(1) \geq k_i/\pi_i(\bar{x}_i) \text{ and } T_i\left(\lfloor \hat{n}_i \rfloor, \bar{x}_i\right) < T_i\left(\lceil \hat{n}_i \rceil, \bar{x}_i\right). \end{cases} \end{aligned}$$

The chain optimal profit is $\bar{Z} = \delta_1 T_1(\bar{n}_1, \bar{x}_1) + \delta_2 T_2(\bar{n}_2, \bar{x}_2)$.

OPTIMAL SCREENING SOLUTION

We solve the optimal multi-dimensional screening mechanism in two stages. First, we optimize the rent w_i paid to the manufacturer provided that the assortment parameters (n_i, x_i) are given. Second, using the expressions of the optimal rents, we find the assortment parameters (n_i^*, x_i^*) that maximize the retailer's expected profit.

Optimal Contract Menu

Suppose that the assortment parameters (n_i, x_i) are provided. Because the terms $T_i(n_i, x_i)$ and R_{ij} are known and given, the retailer's second-stage problem is to minimize the total expected rent, which equals $\delta_1 w_1 + \delta_2 w_2$, subject to constraints (5) to (6). Note that this is a linear programming (LP) problem with respect to w_1 and w_2, and the following Lemma summarizes the solution of this problem.

Lemma 3 *The second stage problem is feasible if and only if* $R_{12} + R_{21} \leq 0$*. Provided that* $R_{12} + R_{21} \leq 0$*, the optimal rent is* $w_i = \max(0, R_{ij})$ *for* $i = 1, 2$.

We now move our attention to solving the first-stage problem. Using Lemma 3, we can write the retailer's profit as follows:

$$\begin{aligned}
Z &= \delta_1 \left[T_1(n_1, x_1) - \max\left(0, R_{12}(n_2, x_2)\right)\right] + \delta_2 \left[T_2(n_2, x_2) - \max\left(0, R_{21}(n_1, x_1)\right)\right] \\
&= \delta_1 \left[T_1(n_1, x_1) - \frac{\delta_2}{\delta_1} \max\left(0, R_{21}(n_1, x_1)\right)\right] + \delta_2 \left[T_2(n_2, x_2) - \frac{\delta_1}{\delta_2} \max\left(0, R_{12}(n_2, x_2)\right)\right] \\
&= \delta_1 \Lambda_1(n_1, x_1) + \delta_2 \Lambda_2(n_2, x_2). \qquad (10)
\end{aligned}$$

The term $\Lambda_i(n_i, x_i)$ is called the virtual surplus, which equals the chain profit minus the information rent that must be paid to the manufacturer depending on the manufacturer's type. An immediate observation from equation (10) is that the optimal screening solution can be found by optimizing the virtual surplus $\Lambda_i(n_i, x_i)$ for each type i. Define

$$(n_i^*, x_i^*) = \arg \max_{n_i \in \mathbb{N}, x_i \geq 0} \left\{\Lambda_i(n_i, x_i)\right\}. \qquad (11)$$

Theorem 1 *The retailer's optimal screening mechanism is to offer contract i ($i = 1, 2$); if it satisfies that the number of variants is n_i^*, the total order quantity is $Q_i^* = x_i^* B(n_i^*)$, and the payment is*

$$P_i^* = c_i Q_i^* + k_i (n_i^* - 1)^+ + \max\left(0, R_{ij}(n_j^*, x_j^*)\right).$$

The retailer's optimal expected profit is $Z^* = \delta_1 \Lambda_1(n_1^*, x_1^*) + \delta_2 \Lambda_2(n_2^*, x_2^*)$.

Although Theorem 1 characterizes the optimal screening mechanism, it still requires us to optimize the virtual surplus.

Optimizing Virtual Surplus

Next, we investigate how to optimize the virtual surplus, which is given by

$$\begin{aligned}
\Lambda_i(n_i, x_i) &= T_i(n_i, x_i) - \frac{\delta_j}{\delta_i} \max(0, R_{ji}(n_i, x_i)) \\
&= rE \min\left(x_i, D\right) B(n_i) - c_i x_i B(n_i) - k_i (n_i - 1)^+ \\
&\quad - \frac{\delta_j}{\delta_i} \max\left(0, (c_i - c_j) x_i B(n_i) + (k_i - k_j)(n_i - 1)^+\right). \qquad (12)
\end{aligned}$$

We observe that the virtual surplus $\Lambda_i(n_i, x_i)$ is a complex piece-wise function. We define a useful identity as follows:

$$\bar{R}_{ij} = \bar{R}_{ij}(\bar{n}_j, \bar{x}_j) = (c_j - c_i)\bar{x}_j B(\bar{n}_j) + (k_j - k_i)(\bar{n}_j - 1)^+. \tag{13}$$

There is an important intermediate result in relation to $\bar{R}_{ij}$.

Lemma 4 *1) It satisfies that $\bar{R}_{12} + \bar{R}_{21} \leq 0$. 2) If $\bar{R}_{ij} > 0$, where $i \neq j$ and $\{i, j\} \in \{1, 2\}$, then $T_i(\bar{n}_i, \bar{x}_i) > T_j(\bar{n}_j, \bar{x}_j)$.*

Part 1) of Lemma 4 implies that at least one $\bar{R}_{ij}$ must be negative. Hence, regarding the optimization of the virtual surplus, we have two cases to consider: case 1) when both $\bar{R}_{12}$ and $\bar{R}_{21}$ are negative, and case 2) when $0 < \bar{R}_{ij} \leq -\bar{R}_{ji}$.

We first consider case 1) with both $\bar{R}_{12}$ and $\bar{R}_{21}$ being negative. Using the expression of the virtual surplus in equation (12), we find that $\Lambda_i(n_i, x_i) \leq T_i(n_i, x_i)$, where the equal sign holds if and only if $R_{ji}(n_i, x_i) \leq 0$. Because $(\bar{n}_i, \bar{x}_i)$ maximizes $T_i(n_i, x_i)$ and both $\bar{R}_{12}$ and $\bar{R}_{21}$ are negative, we conclude that the chain-optimal assortment parameters $(\bar{n}_i, \bar{x}_i)$ maximize the retailer's profit in equation (10). We summarize this result as follows:

Corollary 1 *If both $\bar{R}_{12}$ and $\bar{R}_{21}$ are non-positive, then the retailer's optimal screening mechanism is to offer contract i ($i = 1, 2$), which specifies that the number of variants is $n_i^* = \bar{n}_i$, the total order quantity is $Q_i^* = \bar{Q}_i = \bar{x}_i B(\bar{n}_i)$, and the payment is $P_i^* = c_i\bar{Q}_i + k_i(\bar{n}_i - 1)^+$. The retailer's optimal expected profit Z^* equals the optimal chain profit $\bar{Z}$.*

Corollary 1 identifies a situation such that the retailer takes the entire supply chain surplus, which is *maximized*. This outcome does not arise in the classic one-dimensional model. To see this point, suppose $k_i = k_j$, which implies that the two manufacturer types differ only in the per-unit cost. We see that the incremental gains $R_{ij}(n, x) = (c_j - c_i)xB(n)$ and $R_{ji}(n, x) = (c_i - c_j)xB(n)$ have opposite signs for any given (n, x). Because the optimal rents are $w_i = \max(0, R_{ij})$ and

$w_j = \max(0, R_{ji})$ according to Lemma 3, one of the manufacturer types must receive information rent. Specifically, part 2) of Lemma 4 demonstrates that the manufacturer type that generates a higher chain optimal profit receives information rent. In our two-dimensional model, recall that the incremental gain is $R_{ij}(n, x) = (c_j - c_i)xB(n) + (k_j - k_i)(n-1)^+$. When $c_j > c_i$ and $k_i < k_j$ (in this case, we say that the per-unit cost and per-variety cost are negatively correlated), the sign of $R_{ij}(n, x)$ is indefinite. As such, it is possible for the retailer to avoid paying any information rent. We also notice that the negative correlation between the volume-based cost and the variety-based cost exists in many factories (readers can refer to page 28 in Cachon & Terwiesch, 2012 for some practical examples).

It remains difficult to explicitly characterize the condition such that Corollary 1 holds. There are two main reasons: i) the number of varieties is an integer variable and ii) the incremental gain $\bar{R}_{ij}$ function given by equation (13) is a complex non-linear function involving 4 parameters (c_1, c_2, k_1, k_2). To obtain additional insights, we provide a numerical example to illustrate Corollary 1.

Example 1 *We assume that the retail price is $r = 10$ and that the demand D is uniformly distributed over the interval $(0, 40)$. The $B(n)$ function is assumed to be $\frac{n}{n+1}$, which can be viewed as the MNL model with $v_0 = \exp(0) = 1$ and $v_r = 1$. The probability that the manufacturer is type-1 is 0.5 (i.e., $\delta_1 = \delta_2 = 0.5$). The total production cost of the type-1 (type-2) manufacturer is assumed to be $2Q + 8(n-1)^+$ ($c_2Q + k_2(n-1)^+$, respectively), where Q is the total order quantity and n is the number of variants. This setting implies that we fix the cost parameters of the type-1 manufacturer at $(c_1 = 2, k_1 = 8)$ and vary that of the type-2. For any given pair of (c_2, k_2), we solve the optimal contract by using Theorem 1.*

Now consider that the type-2 manufacturer's production cost parameters are $(c_2 = 1.5, k_2 = 17)$, where the type-2 manufacturer has a lower volume-based cost (i.e., $c_2 < c_1$) and higher variety-based cost (i.e., $k_2 > k_1$). We find that $\bar{R}_{12} = -2.33$ and $\bar{R}_{21} = -6$, indicating that

Corollary 1 holds. Therefore, the optimal assortment is determined by $x_1^* = \bar{x}_1 = 0.8$*,* $n_1^* = \bar{n}_1 = 3$*,* $x_2^* = \bar{x}_2 = 0.85$*, and* $n_2^* = \bar{n}_2 = 2$*; and the retailer's expected profit is* 79.67*.*

We observe that the per-unit cost and per-variety cost are negatively correlated in Example 1. In our two-type model, the negative correlation implies that the type-i manufacturer has a low per-unit cost but a high per-variant cost, whereas the type-j manufacturer has a high per-unit cost but a low per-variant cost. Our results indicate that the optimal menu includes two contracts: a large-quantity-and-low-variety contract and a small-quantity-and-high-variety contract. The first contract matches the cost profile of the type-i manufacturer; whereas the second matches the cost profile of the type-j manufacturer.

The parameter space complementary to the condition specified in Corollary 1 is that either $\bar{R}_{12}$ or $\bar{R}_{21}$ is positive. If $\bar{R}_{ij}$ is positive, it means that $0 < \bar{R}_{ij} \leq -\bar{R}_{ji}$, which is the remaining case 2) of Lemma 4. Part 2) of Lemma 4 illustrates that the type-i manufacturer generates a larger optimal chain profit than the type-j manufacturer does. Hence, we call the type-i manufacturer the "top" agent and the type-j manufacturer the "bottom" agent. With $\bar{R}_{ji} \leq 0$, from equation (12) we observe that the assortment parameters for the top agent are set at the chain-optimal levels (i.e., the optimal assortment parameters defined by equation (11) satisfy that $n_i^* = \bar{n}_i$ and $x_i^* = \bar{x}_i$). Hence, our remaining task is to maximize the virtual surplus associated with the type-j manufacturer. Note that because $\bar{R}_{ij}$ is positive, the chain-optimal solution does not maximize the virtual surplus $\Lambda_j(n_j, x_j)$, which implies that the assortment parameters for the bottom agent are distorted (i.e., $(n_j^*, x_j^*) \neq (\bar{n}_j, \bar{x}_j)$). This finding extends the classic one-dimensional result. Specifically, an important characteristic of the optimal one-dimensional mechanism is no distortion at the top and downward distortion at the bottom (see Rochet & Stole, 2003). In the one-dimensional context, for example, when $k_1 = k_2$, the downward distortion would imply that $Q_j^* < \bar{Q}_j$. We will show later that the conjecture of downward distortion is not necessarily true in the multi-dimensional context.

Notice that the virtual surplus associated with type-j is

$$\Lambda_j(n_j, x_j) = \begin{cases} T_j(n_j, x_j) & \text{if } R_{ij}(n_j, x_j) \leq 0 \\ T_j(n_j, x_j) - \frac{\delta_i}{\delta_j} R_{ij}(n_j, x_j) & \text{if } R_{ij}(n_j, x_j) \geq 0. \end{cases}$$

Therefore, we need to consider two local optimal solutions. The first local optimal solution, $\left(n_j^a, x_j^a\right)$, is obtained by solving

$$\max_{n_j \in \mathbb{N}, x_j \geq 0} \left\{T_j(n_j, x_j)\right\}, \text{ subject to } R_{ij}(n_j, x_j) \leq 0; \tag{14}$$

whereas the second local optimal solution, $\left(n_j^b, x_j^b\right)$, is obtained by solving

$$\max_{n_j \in \mathbb{N}, x_j \geq 0} \left\{T_j(n_j, x_j) - \frac{\delta_i}{\delta_j} R_{ij}(n_j, x_j)\right\}, \text{ subject to } R_{ij}(n_j, x_j) \geq 0. \tag{15}$$

By comparing $\Lambda_j(n_j^a, x_j^a)$ with $\Lambda_j(n_j^b, x_j^b)$, we can find the global optimal solution (n_j^*, x_j^*), and the following Corollary characterizes the optimal mechanism for the remaining case 2) of Lemma 4.

Corollary 2 *Suppose that* $\bar{R}_{ij} > 0$ *where* $i \neq j$ *and* $\{i, j\} \in \{1, 2\}$*. There are two cases:*

a) *If* $\Lambda_j(n_j^a, x_j^a) \geq \Lambda_j(n_j^b, x_j^b)$*, the retailer's optimal screening mechanism is to offer contract-*i*, which satisfies that the number of variants is* $n_i^* = \bar{n}_i$*, the total order quantity is* $Q_i^* = \bar{Q}_i = \bar{x}_i B(\bar{n}_i)$*, and the payment is* $P_i^* = c_i \bar{Q}_i + k_i(\bar{n}_i - 1)^+$*; contract-*$j$ *satisfies that the number of variants is* $n_j^* = n_j^a$*, the total order quantity is* $Q_j^* = x_j^a B(n_j^a)$*, and the payment is* $P_j^* = c_j Q_j^* + k_j(n_j^a - 1)^+$*. The retailer's optimal expected profit is* $Z^* = \delta_i T_i(\bar{n}_i, \bar{x}_i) + \delta_j T_j(n_j^a, x_j^a)$*.*

b) *If* $\Lambda_j(n_j^a, x_j^a) < \Lambda_j(n_j^b, x_j^b)$*, the retailer's optimal screening mechanism is to offer contract-*i*, which satisfies that the number of variants is* $n_i^* = \bar{n}_i$*, the total order quantity is* $Q_i^* = \bar{Q}_i = \bar{x}_i B(\bar{n}_i)$*, and the payment is* $R_{ij}(n_j^b, x_j^b) + c_i Q_i^* + k_i(\bar{n}_i - 1)^+$*; contract-*$j$ *satisfies that the number of variants is* $n_j^* = n_j^b$*, the total order quantity is* $Q_j^* = x_j^b B(n_j^b)$*, and the payment is* $P_j^* = c_j Q_j^b + k_j(n_j^b - 1)^+$*. The retailer's optimal expected profit is* $Z^* = \delta_i T_i(\bar{n}_i, \bar{x}_i) + \delta_j \left[T_j(n_j^b, x_j^b) - \frac{\delta_i}{\delta_j} R_{ij}(n_j^b, x_j^b)\right]$*.*

Corollary 2 offers a few insights that are worth mentioning. The first insight is in relation to the rent paid to the manufacturer. In case a) of Corollary 2, neither manufacturer receives any information rent. The payment to the manufacturer just covers the total cost to produce the assortment. This outcome never arises in the one-dimensional context. In case b) of Corollary 2, the type-i manufacturer receives a positive rent, whereas the type-j manufacturer does not. The outcome of case b) is similar to the classic one-dimensional result. The second insight is in relation to the downward distortion conjecture. Consider case b) of Corollary 2 as an example. The first derivative of $T_j(n_j, x_j) - \frac{\delta_i}{\delta_j} R_{ij}(n_j, x_j)$ with respect to x_j is

$$-\frac{\delta_i}{\delta_j}(c_j - c_i) B(n_j) + B(n_j)\left[r - rF(x_j) - c_j\right].$$

Letting the above equation be 0, we find that the optimal inventory scalar is

$$x_j^b = F^{-1}\left(\frac{r - c_j}{r} - \frac{\delta_i(c_j - c_i)}{\delta_j r}\right).$$

Recall that $\frac{r-c_j}{r}$ is the chain-optimal newsvendor ratio. The term $\frac{\delta_i(c_j-c_i)}{\delta_j r}$ measures the magnitude of the distortion on the service level. Note that the sign of $c_j - c_i$ could be positive or negative, which determines the direction of distortions. For instance, it is possible that $c_j - c_i < 0$, $k_j - k_i > 0$, and $\bar{R}_{ij} > 0$. In this case, the type-i manufacturer has an advantage in the variety-based cost, which offsets the disadvantage in the volume-based cost so that the type-i manufacturer generates a larger optimal chain profit than the type-j manufacturer does. Because $c_j - c_i < 0$, the order quantity is distorted upward, which is in contrast to the downward distortion result in the classic one-dimensional context. The following example illustrates the main results of Corollary 2.

Example 2 *Consider another pair of cost parameters for the type-2 manufacturer. Let $c_2 = 1.5$ and $k_2 = 13$; and any other parameters in Example 1 remain unchanged. We find $\bar{R}_{12} = -6.33$ and $\bar{R}_{21} = 2$, indicating that Corollary 1 does not hold. However, $R_{12}(n_2^*, x_2^*) = -6.33$ and $R_{21}(n_1^*, x_1^*) = -2.2$ imply that both manufacturer types receive zero rent, which means that the precondition of Corollary 2a) is satisfied. The parameters of the optimal assortment are $x_1^* = \bar{x}_1 =$*

0.8, $n_1^* = 4 > \bar{n}_1 = 3$, $x_2^* = \bar{x}_2 = 0.85$, *and* $n_2^* = \bar{n}_2 = 2$*; the retailer's expected profit is* 81.67. *We observe that the width of the assortment for the type-*1 *manufacturer is distorted upward.*

Next, suppose that $c_2 = 2.5$ *and* $k_2 = 10$*, where the type-*1 *manufacturer dominates the type-*2 *manufacturer. We have* $R_{12}(n_2^*, x_2^*) = 11.33$ *and* $R_{21}(n_1^*, x_1^*) = -16$*, which satisfies the condition of Corollary 2b) because the type-*1 *manufacturer receives a rent of* 11.33*, and the type-*2 *manufacturer receives zero rent. We find that the parameters of the optimal assortment are* $x_1^* = \bar{x}_1 = 0.8$, $n_1^* = \bar{n}_1 = 3$, $x_2^* = 0.7 < \bar{x}_2 = 0.75$, *and* $n_2^* = \bar{n}_2 = 2$*; the retailer's expected profit is* 66.67. *We see that the service level of the assortment for the type-*2 *manufacturer is distorted downward.*

AUCTION MECHANISM

When there are multiple manufacturers, it is well known that the optimal procurement mechanism with multi-dimensional private information is difficult to characterize and implement (see the discussions by Chen-Ritzo, et al. 2005 and Asker and Cantillon, 2010). Next, we investigate the case with multiple competing manufacturers and present our findings.

Upper Bound

To facilitate the analysis, we continue to use the two-type model and assume that there are N competing manufacturers. The type of each manufacturer can be either type-1 with probability δ or type-2 with probability $1-\delta$. Without a loss of generality, we assume that $T_2(\bar{n}_2, \bar{x}_2) > T_1(\bar{n}_1, \bar{x}_1)$. Using the methodology presented by Asker and Cantillon (2010), we formulate the optimization model to find the upper bound of the optimal profit that the retailer can obtain among all feasible procurement strategies. We summarize the result as follows:

Theorem 2 *When there are* $N \geq 1$ *manufacturers available, the highest expected profit that the retailer can attain among all feasible procurement strategies is*

$$Z_U = \delta^N \Lambda_1(n_1^*, x_1^*) + (1 - \delta^N)\Lambda_2(n_2^*, x_2^*), \qquad (16)$$

where the virtual surplus $\Lambda_i(n_i, x_i)$ is given by equation (12) and the assortment parameter (n_i^*, x_i^*) is given by equation (11).

Several observations can be made from Theorem 2. First, when $N = 1$, Theorem 1 indicates that the optimal profit of the screening mechanism is $Z^* = \delta\Lambda_1(n_1^*, x_1^*) + (1-\delta)\Lambda_2(n_2^*, x_2^*)$, which equals Z_U. Hence, when there is only one available manufacturer, the mechanism derived in Theorem 1 is optimal among all feasible procurement strategies (rather than just being optimal among the screening mechanisms). Second, when N increases, the probability that there exists a type-2 manufacturer in the pool increases. Thus, the upper bound of the retailer's optimal profit increases. Third, if we can design a mechanism that gives the retailer an expected profit equal Z_U, then we know that this mechanism is indeed optimal.

Supply Contract Auction

Chen (2007) proposed a supply contract auction in which participating manufacturers are asked to bid for the right to execute a contract menu. If the retailer uses the optimal screening contract menu as the object of the auction, he shows that this supply contract auction delivers the optimal outcome to the retailer when the private information is represented by a one-dimensional and continuous scalar. An advantage of this auction mechanism is that the retailer does not need to know the number of manufacturers in advance because the object in the auction is independent of the number of manufacturers. By following the idea of Chen (2007), we present a similar auction mechanism with the following details. The retailer first announces a contract menu with two contracts. The contracts on the menu are specified in Theorem 1. All participating manufacturers are asked to submit a bid for the right to execute the contract menu. We adopt the Vickery auction format under which each manufacturer bids the true valuation and the winner pays the second highest price.

We first characterize the optimal bidding strategy for a manufacturer. For this supply contract auction, different manufacturers value the right to execute the contract menu differently, and the rent associated with contract-i is $w_i^* = R_{ji}(n_i^*, x_i^*)$. Thus, we see that the type-i manufacturer values this right for w_i^* because if it wins the auction, it will execute contract-i and attain a surplus of w_i^*. Next, we derive the expected profit of the auction. Recall that the type-2 manufacturer is the top type and Corollaries 1 and 2 suggest that $w_2^* \geq w_1^* = 0$. Therefore, we need to consider the following two events: i) When there are at least two type-2 manufacturers in the pool, the auction price is w_2^*. ii) When there is no more than one type-2 manufacturer in the pool, the auction price is $w_1^* = 0$. The first event occurs with probability $1 - \delta^N - N\delta^{N-1}(1-\delta)$, whereas the second event occurs with probability $\delta^N + N\delta^{N-1}(1-\delta)$. Hence, the expected revenue from the auction is $\left[1 - \delta^N - N\delta^{N-1}(1-\delta)\right] w_2^*$.

Next, we derive the total expected profit of the supply contract auction. We note that the chance that the winning manufacturer is type-2 is $1 - \delta^N$. Whenever a type-2 manufacturer wins the auction (if the bids are tied, we assume that the retailer uses a lottery to determine the winner), the retailer obtains an expected profit of $T_2(n_2^*, x_2^*) - w_2^*$. On the other hand, the chance that the winning manufacturer is type-1 is δ^N. If this event occurs, the retailer attains an expected profit of $T_1(n_1^*, x_1^*) - w_1^* = T_1(n_1^*, x_1^*)$ because $w_1^* = 0$. Thus, the retailer's total expected profit of this mechanism is

$$\begin{aligned} Z_A &= \delta^N T_1(n_1^*, x_1^*) + \left(1 - \delta^N\right)\left[T_2(n_2^*, x_2^*) - w_2^*\right] + \left[1 - \delta^N - N\delta^{N-1}(1-\delta)\right] w_2^* \\ &= \delta^N T_1(n_1^*, x_1^*) + \left(1 - \delta^N\right) T_2(n_2^*, x_2^*) - N\delta^{N-1}(1-\delta) w_2^*. \end{aligned}$$

Next, we investigate the performance of this supply contract auction. Using equation (12), we find that $\Lambda_1(n_1^*, x_1^*) = T_1(n_1^*, x_1^*) - \frac{1-\delta}{\delta} w_2^*$ and $\Lambda_2(n_2^*, x_2^*) = T_2(n_2^*, x_2^*)$. As such, we observe that equation (16) can be re-written as

$$Z_U = \delta^N T_1(n_1^*, x_1^*) + (1 - \delta^N) T_2(n_2^*, x_2^*) - \delta^{N-1}(1-\delta) w_2^*.$$

By comparing Z_U and Z_A, it yields that the difference between two terms is $(N-1)\,\delta^{N-1}(1-\delta)w_2^*$. Therefore, when $w_2^* = 0$ (which occurs if the conditions in Corollary 1 or case a) of Corollary 2 are satisfied), we see that the gap is zero and that the supply contract auction is optimal among all feasible procurement strategies.

Theorem 3 *Suppose that the screening contract menu specified in Theorem 1 gives zero rent to each manufacturer type. Using this contract menu as the object of the supply contract auction, the retailer achieves the optimal procurement outcome.*

Finally, we consider the remaining case when $w_2^* > 0$ (which arises if case b) of Corollary 2 occurs). Chen (2007) showed that with a continuous one-dimensional type, his supply contract auction implements the optimal procurement outcome. The gap between Z_U and Z_A is positive in our model when $w_2^* > 0$. However, we emphasize that this positive gap does not imply that our proposed auction cannot yield the optimal outcome because Z_U is an upper bound, and the discrete nature of our two-type model causes the positive gap.

CONCLUSION

Currently, many retailers in developed countries directly source from overseas manufacturers. At the time of signing a procurement contract, the production cost information is often incomplete because of various industrial characteristics such as manufacturer turnover, uncertainty in raw material costs, and a low degree of open book policies. To mitigate the adverse effect of information asymmetry, we apply the theory of multi-dimensional mechanism design to assortment planning. Specifically, we develop a procurement-assortment joint optimization model with three industrial features: direct sourcing, multi-dimensional private information, and seasonal retailing. When there is only one qualified manufacturer available, we demonstrate that the screening mechanism can achieve the optimal procurement outcome. When there are multiple competing manufacturers, we show that a supply contact auctioning mechanism can achieve the optimal procurement outcome under certain conditions.

In the classic one-dimensional model, the retailer cannot maximize the chain profit because of information asymmetry. In the multi-dimensional model, we find that the retailer can overcome information asymmetry and maximize the chain profit under certain conditions. For example, a manufacturer's production system could be either efficient (which has a low volume-based cost) or flexible (which has a low variety-based cost). Then, the retailer can offer two contracts on the menu: a high-volume-low-variety contract and a low-volume-high-variety contract. Facing these two contracts, the efficient (flexible) manufacturer that only excels in volume-based (variety-based) costs finds it disadvantageous to accept the low-volume-high-variety (high-volume-low-variety, respectively) contract. By optimizing the contract parameters, the retailer can substantially reduce information rent and increase the profit.

When there exist multiple competing manufacturers, we evaluate the performance of a supply contract auctioning mechanism. In this auction, the retailer announces a contract menu that determines the payment to the manufacturer according to the manufacturer's two-dimensional decision on variety and quantity. The key of the analysis is to design the "object" for the auction, which is the contract menu announced by the retailer. We find that the retailer can implement the optimal procurement outcome by using the optimal screening contract menu as the object in the auction if the optimal screening contract menu does not pay information rent to any manufacturer type. This result again underscores the importance for the retailer to leverage the negative correlation in a multi-dimensional environment.

Although our analysis is based on an assortment-planning setting, we believe that the managerial insights can be generalized to other problem contexts, for example, where a supplier possesses multi-dimensional private information regarding delivery time and quality. In addition, we note that our supply contract auctioning mechanism is consistent with the practice of vendor-manager-inventory (VMI) and an upfront fee (or slotting fee). Therefore, by applying our procurement strategies to the practice, we see that the winning supplier is the one who is willing to pay the highest upfront fee to manage the assortment on the retailer's behalf. Further, we note that the contract menu used in the auction process could be the same as that for the one-manufacturer case. In other words, if the retailer knows how to optimally write a procurement contract when only one manufacturer is available, then the same contract can be used to achieve the optimal procurement outcome even when there are multiple manufacturers.

APPENDIX

Proof of Lemma 1

We first prove an intermediate result as follows. Suppose that $g(y)$ is a strictly concave function and that $(e_1, e_2, ..., e_n)$ are positive constants. Consider the following optimization problem:

$$\max_{y_i} Y = \sum_{i=1}^{n} e_i g(y_i) \text{ subject to } \sum_{i=1}^{n} e_i y_i = Q.$$

The Lagrangian of the above contained optimization is

$$L = \sum_{i=1}^{n} e_i g(y_i) - \beta \left(\sum_{i=1}^{n} e_i y_i - Q \right),$$

where β is the Lagrangian multiplier. Solving the first order condition with respect to y_i, we find that

$$\begin{cases} y_1^* = y_2^* = ... = y_n^* = \frac{Q}{\sum_{i=1}^{n} e_i} = y^* \\ \beta^* = \frac{dg(y)}{dy}|_{y=y^*}. \end{cases}$$

Next, we use the contradiction method to prove Lemma 1. Suppose that the retailer does not adopt the ESL policy and that the assortment contains n varieties. Let x_j ($j = 1, 2..., n$) be the inventory scalar associated with variant j. The total order quantity is $Q_1 = \sum_{j=1}^{n} x_j q_j(n)$. The retailer's sales revenue is

$$\sum_{j=1}^{n} rE \min(x_j q_j(n), Dq_j(n)) = \sum_{j=1}^{n} q_j(n) rE \min(x_j, D).$$

Because $q_j(n)$ is given and $rE \min(x, D)$ is concave in x, we can apply the intermediate result. We conclude that the retailer's revenue can be improved by setting the inventory scalars x_j to be all equal and keeping the total order quantity unchanged. Therefore, the solution that uses unequal inventory scalars cannot be optimal.

Proof of Lemma 2

From equation (2), we observe that for any given n, the optimal inventory level must maximize the newsvendor profit function $\pi_i(x)$. The relevant first order condition leads to the newsvendor solution $\bar{x}_i = F^{-1}\left(\frac{r-c_i}{r}\right)$. Furthermore, the first derivative with respect to n is $B'(n)\bar{\pi}_i - k_i$, and the second derivative is $B''(n)\pi_i(\bar{x}_i) \leq 0$. Note that the variety-based setup cost is $k_i(n_i - 1)$. If $B'(1)\pi_i(\bar{x}_i) < k_i$, then it is optimal for the retailer to buy only one variety ($\bar{n}_i = 1$). If $B'(1)\pi_i(\bar{x}_i) \geq k_i$, then there exists a solution $\hat{n}_i$ such that $B'(\hat{n}_i)\pi_i(\bar{x}_i) = k_i$. Therefore, the optimal breadth of the assortment must be either $\lfloor \hat{n}_i \rfloor$ or $\lceil \hat{n}_i \rceil$.

Proof of Lemma 3

Figure 1 depicts the feasible region and the optimal solution of the LP problem. In the plane where the vertical axis is the w_1-axis and the horizontal axis is the w_2-axis, constraint (5) corresponds to the area above the straight line $w_1 = w_2 + R_{12}$ and constraint (6) corresponds to the area below the straight line $w_1 = w_2 - R_{21}$.

INSERT FIGURE 1 ABOVE HERE!

From constraints (5) to (6), we observe that $R_{12} + R_{21} \leq 0$, hence, the second stage problem is feasible if and only if $R_{12} + R_{21} \leq 0$. The optimal solution of the LP is the following (please refer to Figure 1). i) When both R_{12} and R_{21} are non-positive, $w_1 = w_2 = 0$ is feasible and minimizes the total expected rent. ii) When $R_{12} > 0$, then $R_{21} \leq -R_{12} < 0$. The solution to the LP is $w_1 = R_{12}$ and $w_2 = 0$. Because of symmetry, we find that if $R_{21} > 0$, then the optimal rents are $w_1 = 0$ and $w_2 = R_{21}$. In summary, the second stage problem is feasible if and only if $R_{12} + R_{21} \leq 0$ and the optimal rent is $w_i = \max(0, R_{ij})$.

Proof of Lemma 4

1) Using the definition of the chain-optimal solutions, we find that

$$T_1(\bar{n}_1, \bar{x}_1) + T_2(\bar{n}_2, \bar{x}_2) \geq T_1(\bar{n}_2, \bar{x}_2) + T_2(\bar{n}_1, \bar{x}_1).$$

By expanding the terms, we re-write the above inequality as follows:

$$\begin{aligned}
& B(\bar{n}_1)rE\min(\bar{x}_1,D) - c_1\bar{x}_1B(\bar{n}_1) - k_1(\bar{n}_1-1)^+ + B(\bar{n}_2)rE\min(\bar{x}_2,D) - c_2\bar{x}_2B(\bar{n}_2) - k_2(\bar{n}_2-1)^+ \\
\geq\ & B(\bar{n}_2)rE\min(\bar{x}_2,D) - c_1\bar{x}_2B(\bar{n}_2) - k_1(\bar{n}_2-1)^+ + B(\bar{n}_1)rE\min(\bar{x}_1,D) - c_2\bar{x}_1B(\bar{n}_1) - k_2(\bar{n}_1-1)^+.
\end{aligned}$$

With some algebra, we obtain

$$-c_1\bar{x}_1B(\bar{n}_1)-k_1(\bar{n}_1-1)^+-c_2\bar{x}_2B(\bar{n}_2)-k_2(\bar{n}_2-1)^+ \geq -c_1\bar{x}_2B(\bar{n}_2)-k_1(\bar{n}_2-1)^+-c_2\bar{x}_1B(\bar{n}_1)-k_2(\bar{n}_1-1)^+,$$

which is equivalent to

$$\begin{aligned}
0 &\geq c_1\bar{x}_1B(\bar{n}_1) - c_2\bar{x}_1B(\bar{n}_1) + k_1(\bar{n}_1-1)^+ - k_2(\bar{n}_1-1)^+ \\
&\quad + c_2\bar{x}_2B(\bar{n}_2) - c_1\bar{x}_2B(\bar{n}_2) + k_2(\bar{n}_2-1)^+ - k_1(\bar{n}_2-1)^+ \\
&= \bar{R}_{21} + \bar{R}_{12}.
\end{aligned}$$

2) Using the ranking that $T_i(\bar{n}_i,\bar{x}_i) \geq T_i(\bar{n}_j,\bar{x}_j)$ and the assumption that $\bar{R}_{ij} > 0$, we find

$$\begin{aligned}
T_i(\bar{n}_i,\bar{x}_i) &> T_i(\bar{n}_j,\bar{x}_j) - \bar{R}_{ij} \\
&= B(\bar{n}_j)rE\min(\bar{x}_j,D) - c_i\bar{x}_jB(\bar{n}_j) - k_i(\bar{n}_j-1)^+ - (c_j-c_i)\bar{x}_jB(\bar{n}_j) - (k_j-k_i)(\bar{n}_j-1)^+ \\
&= B(\bar{n}_j)rE\min(\bar{x}_j,D) - c_j\bar{x}_jB(\bar{n}_j) - k_j(\bar{n}_j-1)^+ \\
&= T_j(\bar{n}_j,\bar{x}_j).
\end{aligned}$$

Proof of Theorem 2

With risk-neutrality, the manufacturer's surplus depends on its expected payment and expected winning probability. Let y_i be the probability of winning the contract-i conditional on it being type i, and let P_i be the expected payment the manufacturer receives. Additionally, let w_i denote type-i manufacturer's equilibrium expected surplus. In other words, $w_i = P_i - y_i\left(c_ix_iB(n_i) + k_i(n_i-1)^+\right)$. The retailer's expected profit from a feasible procurement mechanism is

$$Z = N\sum_{i\in\{1,2\}} \delta_i\left(y_iT_i(n_i,x_i) - w_i\right).$$

The retailer seeks to maximize the above expression by choosing (n_i, x_i, w_i, y_i). Here, (n_i, x_i) determine the assortment characteristics, w_i determines the rent, and y_i determines how the mechanism operates. The IC constraint is

$$w_i \geq w_j + y_j R_{ij}(n_j, x_j), \text{ for } i \neq j \text{ and } \{i, j\} \in \{1, 2\}.$$

Note that the RHS of the above IC constraint is different from that in equation (4). When there are $N > 1$ manufacturers available, $y_j \leq 1$; whereas in the screening model, there is only one manufacturer, and $y_j = 1$ (i.e., no competition makes the winning probability 1). The IR constraint is $w_i \geq 0$, and the feasibility constraints on the assortment decision are $n_i \in \mathbb{N}$ and $0 \leq x_i \leq \bar{d}$.

There is another set of feasibility constraints on the winning probability y_i. We need to ensure that the probability of awarding the contract to a subset of the types is always less than or equal to the probability of such types in the population:

$$N \sum_{s \in \{1,2\}} \delta_i y_i \leq 1 - \left(1 - \sum_{s \in \{1,2\}} \delta_i\right)^N \text{ for all subsets } s \text{ in } \{1, 2\}.$$

Finally, the total probability must be 1, which implies

$$N \sum_{i \in \{1,2\}} \delta_i y_i = 1.$$

We first optimize the rent w_i when all the other parameters are given. Again, the insights of Lemma 3 carry over. The optimal rent is $w_i = y_j \max(0, R_{ij})$. Substituting the optimal rent into the retailer's profit function, we find that

$$\begin{aligned} Z &= N\left[\delta_1\left(y_1 T_1(n_1, x_1) - y_2 \max\left(0, R_{12}\left(n_2, x_2\right)\right)\right) + \delta_2\left(y_2 T_2(n_2, x_2) - y_1 \max\left(0, R_{21}(n_1, x_1)\right)\right)\right] \\ &= N\delta_1 y_1 \left[T_1(n_1, x_1) - \frac{\delta_2}{\delta_1} \max\left(0, R_{21}(n_1, x_1)\right)\right] + N\delta_2 y_2 \left[T_2(n_2, x_2) - \frac{\delta_1}{\delta_2} \max\left(0, R_{12}(n_2, x_2)\right)\right] \\ &= \sum_{i=\{1,2\}} N\delta_i y_i \Lambda_i(n_i, x_i), \end{aligned}$$

where $\Lambda_i(n_i, x_i)$ is the virtual surplus defined in equation (12). We see that the assortment parameters defined in equation (11) are optimal. Because the type-2 manufacturer generates a higher chain profit, the mechanism should select the type-2 manufacturer whenever there is a type-2 manufacturer available. Note that the probability that there is a type-2 manufacturer among N available manufacturers is $1-\delta^N$. Hence, the optimal winning probability y_2^* must satisfy $N\delta_2 y_2^* = 1-\delta^N$, which also implies that $N\delta_1 y_1^* = \delta^N$ because the total probability must be 1. We observe that the retailer's optimal profit is

$$Z_U = \delta^N \Lambda_1(n_1^*, x_1^*) + (1-\delta^N)\Lambda_2(n_2^*, x_2^*),$$

where the assortment parameters (n_i^*, x_i^*) are given by equation (11).

It is noteworthy that knowing the optimal parameters $(n_i^*, x_i^*, w_i^*, y_i^*)$ is still insufficient to characterize the mechanism that is optimal among all feasible strategies. The term P_i used in this formulation includes the payment for executing contract-i, subtracting any upfront fee that may be required for winning the contract. In the special case that $N = 1$, there is no upfront fee, and P_i is the contract payment for executing contract-i. When $N > 1$, the manufacturer may be asked to pay an upfront fee in the selection process. Therefore, knowing the value of (P_i, y_i) alone, we cannot fully describe how the optimal mechanism actually operates. Readers can refer to Asker and Cantillon (2010) and the references therein for more discussions about the difficulties in implementing the optimal mechanism.

Alternative Demand Model

The first possible direction to extend our research is to consider the demand model used in van Ryzin and Mahajan (1999). Specifically, we can assume that D_j, the demand for variant j, is a normal random variable with mean $\lambda q_j(n)$ and variance $\lambda q_j(n)$. The choice probability is $q_j(n) = v/(nv+1)$, which follows the MNL model and v is the attractiveness of identical variant j. The chain profit function becomes

$$
\begin{aligned}
T_i(n_i, x_i) &= -k_i(n_i - 1)^+ + \sum_{k=1}^{n_i} \left[rE \min\left(x_i, D_j\right) - c_i x_i \right] \\
&= -k_i(n_i - 1)^+ - n_i c_i x_i + n_i r \left[\frac{\lambda v}{nv+1} + \sqrt{\frac{\lambda v}{nv+1}} \left(-f(z) + z(1 - F(z))\right) \right],
\end{aligned}
$$

where $z = \frac{x_i - \lambda v_r/(nv_r+1)}{\sqrt{\lambda v_r/(nv_r+1)}}$ is the z-score, $f(z)$ is the PDF and $F(z)$ is the CDF of the standard normal distribution. The adoption of this alternative demand model does not change the structure of the constraints because the IC constraint is still

$$
w_i \geq w_j + (c_j - c_i)x_j n_j + (k_j - k_i)(n_j - 1)^+ = w_j + \tilde{R}_{ij}(n_j, x_j).
$$

Similar to Lemma 3, we find that the optimal rent is $\max\left(0, \tilde{R}_{ij}(n_j, x_j)\right)$ and hence, the virtual surplus is

$$
\Lambda_i(n_i, x_i) = T_i(n_i, x_i) - \frac{\delta_j}{\delta_i} \max\left(0, \tilde{R}_{ij}(n_j, x_j)\right).
$$

The retailer's profit is $\delta_1 \Lambda_1(n_1, x_1) + \delta_2 \Lambda_2(n_2, x_2)$. By optimizing the virtual surplus, we find the optimal mechanism.

Endogenous Price

It is possible to consider that the price of the assortment is endogenous. Note that the contract on the menu specifies the parameters (P_i, Q_i, n_i) but does not reveal the assortment price. We assume that a typical consumer attains a utility of $U_j = u - r + \epsilon_j$ by consuming one unit of variant j, where u is the maximum price that the consumer is willing to pay for the product, r is the retail price of the assortment (where $r \leq u$), and ϵ_j is an identically and independently distributed Gumbel random variable with mean 0 and the shape factor normalized to 1. The no-purchase option gives a typical consumer zero utility. The attractiveness of variant j equals $v_j = \exp(u - r)$ and the attractiveness of the no-purchase option equals $v_0 = \exp(0) = 1$. Hence, the typical consumer chooses variant j with probability

$$
q_j(n, r) = \frac{\exp(u - r)}{n \exp(u - r) + 1}.
$$

Using the multiplicative demand model, the price-sensitive demand for variant j is $D_j(r) = q_j(n,r)D$. The probability that a consumer makes an attempt to buy from the assortment with n variants is $B(n,r) = \frac{n\exp(u-r)}{n\exp(u-r)+1}$. Under the ESL policy, we can write the retailer's objective function as the following:

$$\max_{r_i,w_i,n_i,x_i} Z = \left\{ \begin{array}{l} \delta_1 \left[B(n_1,r_1)\left(r_1 E\min(x_1,D) - c_1x_1\right) - k_1(n_1-1)^+ - w_1\right] \\ +\delta_2 \left[B(n_2,r_2)\left(r_2 E\min(x_2,D) - c_2x_2\right) - k_2(n_2-1)^+ - w_2\right] \end{array} \right\}.$$

We define the virtual surplus as

$$\Lambda_i(n_i,x_i) = \pi_i(r_i,n_i,x_i) - k_i(n_i-1)^+ - \frac{\delta_j}{\delta_i}\max\left(0,R_{ij}\right),$$

where the newsvendor profit function $\pi_i(r,n,x)$ is given by

$$\pi_i(r,n,x) = \frac{n\exp(u-r)}{n\exp(u-r)+1}\left[rE\min(x,D) - c_ix\right]. \tag{17}$$

We find that the retailer's profit is $\delta_1\Lambda_1(n_1,x_1) + \delta_2\Lambda_2(n_2,x_2)$, which implies that by optimizing the virtual surplus, we can find the optimal mechanism.

To optimize the virtual surplus, we need to specify the condition such that $\pi_i(r,n,x)$ is jointly quasi-concave or jointly concave for any given n. Recently, Kocabıyıkŏglu and Popescu (2011) made important contributions toward identifying the condition that guarantees that $\pi_i(r,n,x)$ is jointly quasi-concave or jointly concave. They proposed the Lost Sales Rate (LSR) elasticity, which is given by

$$\varepsilon(r,x) = \frac{rF_r(r,x)}{1-F(r,x)},$$

where $F(r,x) = \Pr(D(r) \leq x)$ is the CDF of the price-dependent demand and $F_r(r,x)$ is the partial derivative with respect to r. If the LSR elasticity exceeds 0.5 (globally, respectively, path-wise), then $\pi_i(r,n,x)$ is jointly concave. If the LSR elasticity is increasing along with inventory or price, then $\pi_i(r,n,x)$ is jointly quasi-concave.

Lemma 5 *The newsvendor profit function $\pi_i(r,n,x)$ defined in equation (17) is jointly quasi-concave in (r,x) for any given $n \geq 1$ if the random noise D has an increasing failure rate.*

The quasi-concavity allows us to optimize the virtual surplus by using the first-order condition. Again, the three cases described in Corollaries 1 to 2 remain qualitatively unchanged. However, the retailer's profit increases because of two reasons. First, the retailer can now adjust the price optimally when matching with different manufacturer types. Second, the retailer distorts less significantly in the contract parameters.

Proof of Lemma 5

We apply Corollary 2 of Kocabiyikoğlu and Popescu (2011) to prove that the newsvendor profit function $\pi_i(r, n, x)$ defined in equation (17) is jointly quasi-concave for any given $n \geq 1$. Note that the price-dependent stochastic demand is $D(r) = q_j(n, r)D = \frac{D\exp(u-r)}{1+n\exp(u-r)}$. We need to first verify that $rd(r)$ is concave in r, which is a pre-requisite of Kocabiyikoğlu and Popescu (2011). Note that $rd(r) = \frac{r\lambda\exp(u-r)}{1+n\exp(u-r)}$. The second derivative with respect to r is

$$\frac{d^2}{dr^2}\left(\frac{r\lambda\exp(u-r)}{1+n\exp(u-r)}\right) = \frac{\lambda\left(re^{u-r} - 2e^{u-r} - 2ne^{2u-2r} - nre^{2u-2r}\right)}{3ne^{u-r} + 3n^2e^{2u-2r} + n^3e^{3u-3r} + 1}.$$

Because $n \geq 1$ and $r \leq u$, the numerator is negative (i.e., $re^{u-r} - nre^{2u-2r} < 0$ and $-2e^{u-r} - 2ne^{2u-2r} < 0$).

The second step is to verify whether $\frac{r\partial q_j(n,r)}{\partial r}$ is decreasing in r for any given $n \geq 1$. With some algebra, we find

$$\frac{r\partial q_j(n,r)}{\partial r} = r\frac{\partial}{\partial r}\left(\frac{\exp(u-r)}{n\exp(u-r)+1}\right) = \frac{-re^{u-r}}{2ne^{u-r} + n^2e^{2u-2r} + 1}$$

and

$$\frac{\partial}{\partial r}\left(\frac{-re^{u-r}}{2ne^{u-r} + n^2e^{2u-2r} + 1}\right) = \frac{-e^{u-r}\left[1 + 2ne^{u-r} + n^2e^{2u-2r} + p\left(n^2e^{2u-2r} - 1\right)\right]}{4ne^{u-r} + 6n^2e^{2u-2r} + n^4e^{4u-4r} + 4n^3e^{u-r}e^{2u-2r} + 1}.$$

Because $n^2e^{2u-2r} \geq 1$ for any $r \leq u$ and $n \geq 1$, we observe that the denominator is positive and the numerator is negative. Therefore, $\frac{r\partial q_j(n,r)}{\partial r}$ is decreasing in r for any given $n \geq 1$. If the random noise D has an increasing failure rate, then all the conditions stated in Kocabiyikoğlu and Popescu (2011) are satisfied; and $\pi_i(r, n, x)$ is jointly quasi-concave.

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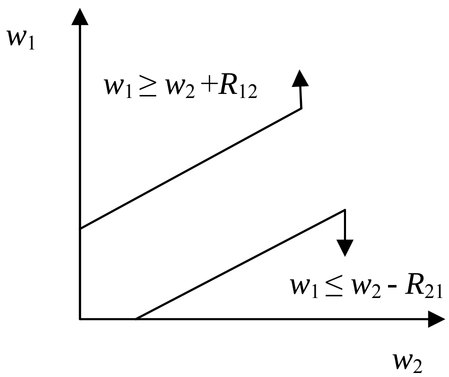
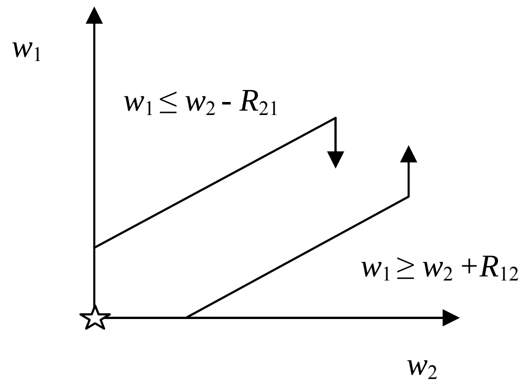
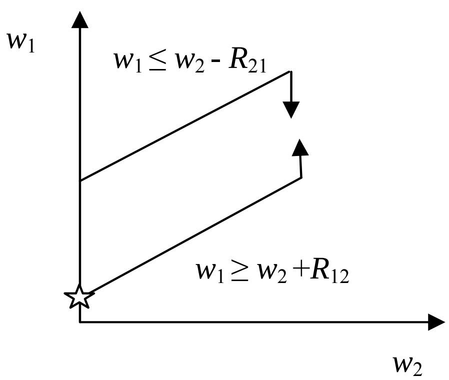
FIGURE



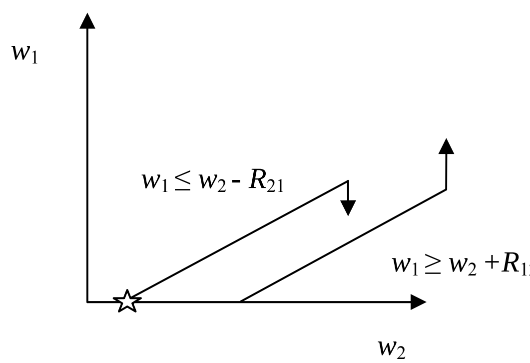
(a) The LP is infeasible if $R_{12} + R_{21} > 0$



(b) The LP solution is $w_1 = w_2 = 0$ if $R_{ij} < 0$



(c) $w_1 = R_{12}, w_2 = 0$ if $0 < R_{12} < -R_{21}$



(d) $w_1 = 0, w_2 = R_{21}$ if $0 < R_{21} < -R_{12}$

Figure 1: Feasible region and optimal rents

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