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A multi-period multi-criteria districting problem applied to primary care scheme with gradual assignment

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ABSTRACT

We present a modeling and optimization approach for the planning of primary healthcare services in order to efficiently direct patient admissions to general practitioners (GP) and to leverage the capacity of the healthcare system. We propose a multi-period multi-criteria districting model for the problem of designing GP districts in the presence of six criteria: workload balance, capacity, accessibility, compactness, income equity and district similarity. We combine the last three criteria into a single objective function and formulate the problem as a mixed integer program with binary location decision variables and relaxed allocation decision variables for gradual assignments. To assess the usefulness of the model, we test it on real-case scenarios of Istanbul, Turkey.

KEYWORDS: Health service, integer programming, multi-criteria, multi-period, location, districting

1. INTRODUCTION

The role of primary care that is provided by General Practitioners (GP) is the first level in a healthcare system where many patient needs can already be treated and the admission of patients to secondary care healthcare facilities (e.g., hospitals) is not required. As the demand for healthcare services is rapidly increasing due to factors such as aging societies, increasing life standards and expectations for healthy-living, an effective general practitioner scheme plays a crucial role in leveraging the capacity of the healthcare system (Buja et al., 2015).

The main premise of the GP scheme is to direct the admissions of patients to GPs instead of hospitals. While some countries enforce patients to visit a GP first, it is the ease of access to a GP that mainly determines the effectiveness of the scheme (Buja et al., 2015; Yiannakoulias et al., 2013). Either enforced or volunteered, the patient is motivated and more satisfied to visit a GP if the GP is easily reachable. On the other hand, doctors are also selective in locating their practices since their income is typically proportional to the number of patient visits as well as their age distributions.

In practice, the GP scheme is usually planned by traditional, judgmental and heuristic methods. Systematic approaches based on Operations Research techniques using the available data rather than the traditional regulatory approaches may result in significant efficiency improvements in a healthcare system. In a well-planned scheme, the primary care service should be consistently available and easily accessible over a geographical region with different population densities and characteristics, and should also respect equity for the patients and the doctors. Thus, the design of a GP scheme is closely related to location-based organizational planning and decision problems. There are two fundamentally different types of GP schemes. In the first, inhabitants are assigned to a GP in advance by a public healthcare administration unit. In the second, patients choose by themselves which GP to visit. Two examples for the first are Turkey and the United Kingdom. Although patients have in principle the freedom to visit a GP they are not assigned to, they have to ask the administration for permission to do that in Turkey (HMT 2010) or the non-designated GP in the UK may refuse to register an inhabitant on the grounds that the inhabitant is living outside their district (NHS Choices 2016). For these countries, the GP scheme design can be defined as a districting problem related to planning services and operations over a geographical region subject to various requirements. A solution to this problem will identify districts with inhabitants that are (expected) to visit a doctor's practice positioned at the center of the corresponding district. An example for a free of choice scheme is Germany. See Haase and Müller (2015) and Carello and Lanzarone (2014) for more details on the ensuing models and problems.

In this study, we focus on the first type of GP schemes where inhabitants are assigned to districts by the administration. We formulate the problem of designing a GP scheme first as a multi criteria districting model in the presence of the workload balance, capacity, accessibility, compactness, income equity requirements; of which the last two are combined into a single objective function. In this weighted sum objective function, "compactness" ensures the ease of travel for the patients by minimizing the distances to GP locations, as previously done by Steiner et al. (2015) and Datta et al. (2013), and "income equity" equalizes the attractiveness of each district for the doctors by balancing the income generated in all districts, which is a concept commonly used in the sales territory design applications (Lei et al., 2015; Zoltners and Sinha, 2005). Then, we extend this model into a multi-period model, which searches effective district plans for multiple periods considering the future values of the parameters. This model includes a third criterion, "district similarity", into the weighted sum objective function aiming to generate similar district plans between periods for achieving the continuity of care of patients with the same GP.

One issue in district planning for the GP scheme in Turkey and the UK is to incorporate to some extent the possibility that patients do not want to visit their designated GP. Nevertheless, one of the most influential factors affecting the patient's choice is the distance between the location of the patient and the doctor's practice. If all other factors are almost the same, it is very likely that a patient will still want to patronize a doctor's practice located very close to his/her own location. Hence, typically only patients located close to the border of a district may be inclined to visit neighboring doctors, as the marginal utility from patronizing the closest doctor diminishes. For this reason, we introduce in this paper the concept of gradual assignment where the demand may be partially split amongst a number of neighboring districts, and we incorporate this concept in our districting model. We also introduce a population-weighted distance limit for gradual assignment to ensure acceptable level of accessibility on foot and variable capacities for districts measured in terms of the number of GPs assigned to the GP centers. We formulate the problem as a multi-criteria mixed integer program with binary location decision variables and continuous allocation decision variables for the gradual assignment. Then we extend the problem formulation into a multi-period model. To assess the usefulness of the models, we test them on real-case scenarios of Istanbul, Turkey. The main contribution of our study is thus the multi-criteria as well as multi-period formulation with variable district capacities and the concept of gradual assignment.

The rest of the paper is organized as follows: in Section 2, we provide a literature review of districting problems. In Section 3, we describe the regional settings in Turkey that lead to the model we propose in this paper. In Section 4, we formally present our models, followed by a computational study in Section 5. Finally, we provide concluding remarks in Section 6.

2. LITERATURE REVIEW

The studies on district planning comprise an extensive literature under several titles for the problem such as districting, re-districting, territory design, territory alignment, zone design or sector design. Any of these terms refer to the problem of dividing a geographical area into districts to allocate demand in the region to the services offered within each district. The problem arises in various application domains with different requirements as in political districting, sales territory design, and districting for different kinds of services such as healthcare, schooling, police, etc. Since the literature is vast, in this section we would like to highlight some of the most recent and relevant studies.

A key component for districting that appears in almost all application domains is the balancing criterion, which is also relevant in our model. In political districting, the balancing criterion guarantees that all districts contain approximately the same number of voters. Two recent reviews on political districting by Ricca et al. (2011) and Webster (2013) as well as other studies including Ricca and Simone (2008), Bozkaya et al. (2011) consider this and other criteria in various districting models. In this context, the models are typically multi-objective and heuristic solution approaches such as tabu search, simulated annealing, old bachelor acceptance are implemented (Ricca and Simone, 2008; Bozkaya et al., 2011).

In sales territory design, the task is to assign a given set of (prospective) customer accounts, each with a fixed market potential, to the individual members of the sales force such that each customer is matched with a unique representative. The balancing criterion appears in this context as well in the form of equitable workload and travel time for each sales person to allow equal income opportunities in terms of incentive pay (Zoltners and Sinha, 2005). Sometimes, more than one performance measure such as workload, number of customers, product demand is used to balance sales territories. Rios-Mercado and Lopez-Perez (2013) present a mixed-integer linear programming model with disjoint assignment requirements and similarity with an existing plan, which is then solved by an iterative cut generation strategy within a branch-and-bound framework. Salazar-Aguilar et al. (2011) use the ε-constraint method for generating the optimal Pareto front and Salazar-Aguilar et al. (2013) use the GRASP methodology with and without the connectivity requirement for a bi-objective model incorporating compactness and balancing criteria.

Service districting encompasses many different contexts. In electrical power districting, one goal is to partition the power grid into economically viable districts to be assigned to different distribution companies. In this context, Bergey et al. (2003) propose a multi-criteria model that minimizes the compactness and the deviation of income potential from a target value to obtain non-overlapping and contiguous districts. Other applications include providing service to streets such as postal delivery, solid waste disposal or salt spreading (Lin and Kao, 2008; Butsch et al., 2014). The main concern here is not to exceed the working time of the service delivery person and to obtain non-overlapping and compact districts. School districting deal with assigning residential areas to schools while adhering to the capacity limitations and equal utilization of schools, maximal or average travel distances for students, and a good accessibility (Teixeira and Antunes, 2008).

Districting for the social services that include healthcare facilities and services is another application domain. The purpose of healthcare service districting is to identify for the inhabitants which facility to visit for a particular healthcare service (e.g. medical examinations), or to determine areas of responsibility of home-care visits by healthcare personnel, like nurses or physiotherapists. The design requirements are typically to obtain districts with good accessibility, equal workloads of service and travel time, and high capacity utilization of the social facility. Blais et al. (2003) solve a bi-objective home-care districting problem using tabu search. The districting criteria respected are the indivisibility of basic units, respect for borough boundaries, connectivity, visiting personnel mobility, and workload equilibrium. Benzarti et al. (2013) formulate mixed-integer programming models for balancing the personnel care workload and minimizing the travel distance to reach the patients in home healthcare domain.

In the specialized literature of healthcare districting, we can find only two studies where the districting problem of the healthcare system is formulated as a multi-objective model, each proposing their own version of a genetic algorithm as a heuristic solution approach. Datta et al. (2013) provide a multi-objective optimization model to reorganize geographical entities that they name as health authorities. The objectives are defined as follows: compactness, size homogeneity requiring that districts should be neither too big nor too small, and co-extensiveness ensuring that districts have common boundaries with a limited number of local authorities. The model is solved using a genetic algorithm. Steiner et al. (2015) aim to reduce the number of inter-district trips and the distances to be traveled by patients to design a decentralized healthcare system using three objectives: a) maximization of the homogeneity of population in the districts, b) maximization of the variety of medical procedures offered within a district, and c) minimization of the distances to be traveled by patients. They present a heuristic solution methodology that also uses a genetic algorithm.

All of the studies above consider districting settings with hard district boundaries where the demand (customers, patients, voters, etc.) is assigned to a single district. The model presented in our paper relaxes this assumption by implementing the gradual assignment concept in the healthcare domain to account for the real-world situation where patients might be indifferent between two or more GP practices if they are more or less the same distance from them. Furthermore, we introduce a multi-criteria model formulation for the overall design of the GP scheme with compactness and income balance for doctors, two criteria that have not been considered together in the healthcare districting literature before.

3. GENERAL PRACTITIONER SCHEME IN TURKEY

Over the last decade, Turkey, with its dynamic population and market, has transformed its healthcare system significantly. The adoption of a GP scheme that has been placed as the first level of care in the organization of the health system was one of the most important changes recently implemented. The second level of the system comprises services provided by the public and private hospitals. More complex services using advanced skills and equipment are provided by the third level, i.e. university hospitals.

Despite the increasing number of doctors in recent years, Turkey still has the lowest total healthcare spending and the number of physicians per capita among OECD countries, resulting in very large patient loads. In 2008, Turkey had 1.5 physicians per 1000 inhabitants whereas the average of the OECD was 3.1 physicians per 1000 inhabitants (PM Group, 2011). The recent establishment of the GP scheme is expected to provide personal healthcare support while controlling costs and maintaining lower patient loads for hospital-based specialists. Another important goal is to improve the coverage of the healthcare services and the equity of healthcare access among the regions.

The first initiative of the GP scheme has started in 2004 as a pilot study. The training of 5360 physicians was completed between 2006 and 2010 (IAH, 2015). Since 2011, the GP scheme has been introduced in every city of Turkey. Each city has a public health administration unit, which assigns the inhabitants to a GP center and a particular GP to that center. To maintain the freedom of choice, the patients can request a GP change by applying to the public health administration unit (HMT, 2010).

In Istanbul, the GP scheme began at the end of 2010. Istanbul is the biggest city of Turkey with respect to population. It has over 14 million inhabitants (18.5% of Turkey's total population) with a total area of 5.343 km² divided into 39 administrative districts (Wikipedia, 2015). The city is separated into two parts by a waterway, the Bosporus. The west side of the Bosporus belongs to the European continent with 25 districts and the east side to Asia with 14 districts as shown in Figure 1.

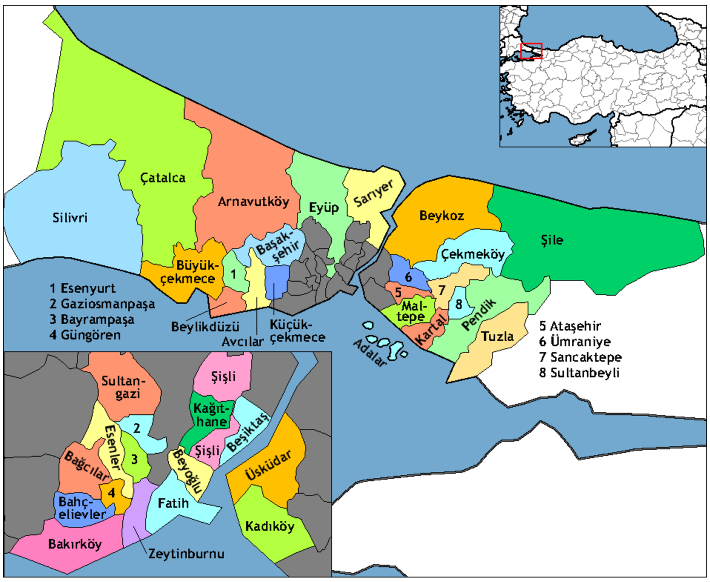


FIGURE 1: The map of Istanbul and its administrative districts.

As shown in Table 1, the GP services are provided in 762 GP centers by 3125 GPs in Istanbul since 2010 (IAH, 2015).

TABLE 1: The change of the capacity of the GP scheme in Istanbul

Before October 31, 2010	December 31, 2010
604 Primary healthcare centers	762 GP centers
2.412 Policlinic Rooms	3.125 GP units
2.100 Physicians	3.125 GPs
2.934 Nurses	3.090 GP center personnel

The public health administration unit's target in Istanbul is to increase the capacity and to partition the population so that a ratio of 1 GP per 3643 inhabitants is achieved. This would result in a daily workload of 40–45 patients per doctor. The partitioning of the population not only defines the workload of the GP but also the income of the doctors because their payments are based on the number of assigned inhabitants and their characteristics such as age and gender (HMT, 2010). An equal financial attractiveness of each GP center for the doctors will dissuade doctors from asking for more popular locations with higher income potential. It will also maintain equity among the doctors and avoid contention. Moreover, the assignments of inhabitants to centers will define the accessibility of the patients to GP services. So, it will also determine the quality level of the GP healthcare services provided to the population.

This study has been conducted in coordination with the Istanbul Public Health Unit, which is the planning and execution authority of the GP system in Istanbul. They provided us with data related to the current situation in Istanbul and the future projections of the GP system. They are currently using the GP centers from the old infrastructure of Public Health Centers and made the assignments of patients based on their place of residence in the subdistrict level. The two main problems of the current system are the following: (1) GPs are not accepting to work at certain GP centers due to a lower income potential and (2) the need to increase the number of GPs in the next 5-10 years to reach the desired target number of patients per GP.

In this study, we develop an advanced planning tool using an optimization model as presented in the following sections. Since the change of the infrastructure and constructing new centers require a high cost and time, changes in the GP scheme will be made gradually.

An effective assignment of the population to the GP healthcare centers will lead to equity among doctors and to higher level of service. As a final goal, a wellestablished efficient GP scheme will help to close the performance gap between the OECD countries in order to increase broad access to better medical services while decreasing the overall costs with Turkey's growing population. Thus, Turkey needs to focus on sustainable and efficient healthcare services, which will enable efficient use of resources and better coordination of healthcare services.

4. MODELING THE GENERAL PRACTITIONERS SCHEME DISTRICTING PROBLEM

The GP scheme districting problem comprises the set $J = \{1, \dots, m\}$ of basic units which represent polygonal neighborhood areas with demand for GP services. The centroid of each basic unit, with coordinates (x_j, y_j), represents the aggregate demand (i.e. population) to be assigned as well as a potential location for a GP center (i.e. district center). Each such location has a capacity c_j for the maximum number of GPs to work in the same center. We represent the number of GPs assigned to the GP center located in basic unit j by a decision variable. The Euclidean distance between two centroids i and j is denoted as d_{ij}.

The number of inhabitants, b_j, in each basic unit j as well as the distribution with respect to the specified population characteristics, $s \in S$, that are based on age groups and gender, are inputs of the problem. The target number of inhabitants to be assigned to a GP is known in advance and determined by the Public Health Management Unit (PHMU). Also the number of districts to design, p, is predetermined by the PHMU. Our goal is to search for the best district plan by grouping the basic units into p districts with respect to the specified criteria. A districting plan D is represented as $D = \{D_1, \dots, D_p\}$, where $J = \bigcup_{k=1}^{p} D_k$ and the set $D_k$is comprised of the basic units assigned to district D_k. One basic unit centroid in each district D_k is identified as the district center among all other basic units assigned to that district.

In typical cases of districting, each basic unit is assigned to exactly one district. In applications such as political districting and sales territory design, this requirement has practical necessities/advantages. However, for the districting GP scheme, we want to allow a patient to be possibly assigned to nearby GPs if the distances between the patient and these GPs are similar, which typically happens for patients close to the boundaries between districts. In order to represent this situation, we allow the gradual assignment of basic units to one or more districts by relaxing the binary assignment variable between basic units and district centers, leading to $D_i \cap D_k \neq \emptyset,\ i,k = 1,2,\dots,p$.

We use the following notation in the formulations:

Decision variables

x_{ij} = percentage of assignment of basic unit j to district center i

$$y_i = \begin{cases} 1 & \text{if basic unit } i \text{ is selected as a GP center (i.e. district center)} \\ 0 & \text{otherwise} \end{cases}$$

n_i = variable capacity (i.e. number of GPs) at the GP center i

r_i = expected income of the GP center i

Parameters

d_{ij} = distance between basic units i and j

b_j = number of inhabitants in basic unit j

b_{js} = number of inhabitants in basic unit j with age/gender characteristics

l_s = the income factor for age/gender characteristic s

c_i = maximum number of GPs at a GP center located in basic unit i

U = distance limit

p = number of districts

m = total number of basic units

N = total available number of GPs to be located

4.1. MODELING

Our first approach is to formulate the problem as a mixed integer linear program. We treat workload balance for GP centers as a hard constraint and the other two criteria as soft criteria to be minimized through a single weighted sum objective function:

$$\min_D F(D) = \alpha_{comp} * f_{comp}(D) + \alpha_{inc\_eq} * f_{inc\_eq}(D)$$

Where α_{comp} and α_{inc_eq} are the weights and $f_{comp}(D)$ and $f_{inc\_eq}(D)$ are the values of the compactness and income equity criterion, respectively, for any given solution D.

Compactness strives for short distances resulting in compact shapes of districts. Income equity among GPs is achieved by minimizing the total deviation of income potential in each district from the average income value.

Several formulations have been developed by various authors to define the measurements used in the objective functions and the constraints. We now describe how the measures are formulated in this study:

Compactness

In the districting for the GP scheme, our motivation is to reduce the travel distances. Hence, we employ a distance-based compactness measure in the objective function to be optimized. We use a formulation of the sum of distances between the center of the district and its basic units weighted with the number of (partially) assigned inhabitants of the basic units. We normalize this sum by the approximate total distance, which is calculated by multiplying the average distance d^{avr} with the average population b^{avr} and the number of assignments, $(m-p)$. Here we subtract p assignments because the assignment distances of p basic units coinciding with the district centers are 0.

$$f_{comp} = \frac{\sum_{i \in J} \sum_{j \in J} d_{ij} x_{ij} b_j}{(m-p) * d^{avr} * b^{avr}}$$

In addition to the compactness measure in the objective function, we also impose a population-weighted maximal distance constraint between a basic unit and its district center in order to prohibit excessively large travel distances for large groups of inhabitants.

Income Equity

We measure the income equity by minimizing the total deviation of the potential income in each district from the average income and include this term also in the objective function. The income of district is calculated as $R_i = \sum_{j \in J} \sum_{s \in S} l_s b_{js} x_{ij}$ and the total income as$R = \sum_{j \in J} \sum_{s \in S} b_{js} l_s$. The total deviation of income is then given by

$$f_{inc\_eq} = \frac{\sum_{i \in J} \left| R_i - \frac{R}{N} * n_i \right|}{R}$$

To include this term in the objective function, we first linearize it. We introduce $R_i^+$and R_i^-:

$$R_i^+ = \begin{cases} R_i - \frac{R}{N} * n_i, \text{if } R_i - \frac{R}{N} * n_i > 0 \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$R_i^- = \begin{cases} \frac{R}{N} * n_i - R_i, \text{if } R_i - \frac{R}{N} * n_i \leq 0 \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

Then f_{inc_eq} takes the following form:

$$f_{inc\_eq} = \frac{\sum_{i \in J} [R_i^+ + R_i^-]}{R}$$

subject to

$$R_i - \frac{R}{N} * n_i = R_i^+ - R_i^-$$

$$R_i^+, R_i^- \geq 0$$

Workload Balance

We assume that the workload is directly proportional to the number of inhabitants assigned to a GP and formulate the workload balance as a typical constraint used in the districting literature. A new component to this formulation is the inclusion of the capacity levels of the districts, n_i. The resulting equality constraint becomes:

$$\sum_{j \in J} b_j x_{ij} = \frac{\sum_{j \in J} b_j}{N} n_i, \forall\, i \in J$$

One side benefit of this constraint is that the resulting district workloads can be perfectly balanced due to the gradual assignment variable x_{ij} we have used, unlike the typical districting models where a tolerance factor for deviation from average must be used.

Capacity

Recently, more than one GP in one GP center has become common to provide more working hours for the patients and share the equipment, supporting healthcare personnel and fixed costs such as rents. In the GP scheme, the land costs could be quite high to locate a GP center, or some basic units may already have potential locations in place with different capacities of physicians to work together. In the former case, the capacity may be set to 0 and in the latter case the level of capacity (i.e. the number of

GPs) should be specified. In order to incorporate flexible capacities, the capacity constraint is formulated as follows:

$$n_i \leq c_i y_i \,, \forall\, i \in J$$

Distance Limit

Since we allow flexible capacities for the districts, this may result in districts with bigger areas than anticipated and the accessibility of the patients on foot will get worse. Therefore, we introduce a population-weighted assignment distance limit in order to avoid or limit the occurrence of excessively large distances between basic units and their associated GP center. The distance limit formulation is as follows:

$$d_{ij} x_{ij} \leq U, \qquad \forall\, i, j \in J$$

Note that even with this constraint, we may still incur large distances between basic units and their assigned GP centers due to the partial assignment variables. However, we observe in practice that this occurs very minimally (see Table 3) and this constraint further improves the overall compactness of the districts since the corresponding gradual assignment variable takes on a proportionally smaller value to satisfy this constraint. We test the significance of this constraint in our computational study and report improved compactness of districts.

Another modeling choice is to impose a maximum limit to the assignment distances as a hard constraint as follows:

$$d_{ij} z_{ij} \leq U, \qquad \forall\, i, j \in J$$

$$z_{ij} \geq x_{ij}, \qquad \forall\, i, j \in J$$

$$z_{ij} \in \{0,1\}, \quad \forall\, i, j \in J$$

where z_{ij} is a binary assignment variable. This approach will not allow any assignment to exceed the distance limit *U*. However, we choose to incorporate the constraint $d_{ij} x_{ij} \leq U$ instead to allow a -small- portion of the population to exceed the distance limit, adding to the flexibility of the gradual assignment solutions that can be produced.

Assignments

The assignment decision variable is relaxed and defined as follows:

$$0 \le x_{ij} \le 1, \quad \forall\, i, j \in J$$

As a result, the model solutions will include some partial assignments. This indicates that patients located in some districts will be assigned to two or more district centers, which is an expected outcome of the gradual assignment concept. We note that the split assignments resulting from this approach is at the aggregate level. If it is required that an authority further decide which individual patients should go to which center, a post treatment can be applied to these split basic units. An example would be to split a basic unit into subsets of street segments according to the required split percentage.

Using the above formulations, we propose the following mixed integer mathematical programming for the GP scheme districting problem:

$$\min \alpha_{comp} * \frac{\sum_{i\in J}\sum_{j\in J} d_{ij}x_{ij}b_j}{(m-p)*d^{avr}*b^{avr}} + \alpha_{inc\_eq} * \frac{\sum_{i\in J}[R_i^+ + R_i^-]}{R} \tag{1}$$

subject to

$$\sum_{j\in J} b_j x_{ij} = \frac{\sum_{j\in J} b_j}{N} n_i, \quad \forall\, i \in J \tag{2}$$

$$n_i \le c_i y_i\,, \quad \forall\, i \in J \tag{3}$$

$$d_{ij}x_{ij} \le U, \quad \forall\, i, j \in J \tag{4}$$

$$\sum_{i\in J} n_i = N \tag{5}$$

$$\sum_{i\in J} y_i = p \tag{6}$$

$$\sum_{i\in J} x_{ij} = 1, \quad \forall\, j \in J \tag{7}$$

$$\sum_{j\in J}\sum_{s\in S} l_s b_{js} x_{ij} - \frac{R}{N} * n_i = R_i^+ - R_i^-, \quad \forall\, i \in J \tag{8}$$

$$0 \le x_{ij} \le 1, \quad \forall\, i, j \in J \tag{9}$$

$$y_i \in \{0,1\}, \quad \forall\, i \in J \tag{10}$$

$$R_i^+, R_i^- \ge 0\,, \quad \forall\, i \in J \tag{11}$$

where, (1) is the weighted sum objective function formulated with the compactness and income equity measures, (2) is the work balance constraint, (3) is the capacity constraint, (4) imposes a population-weighted distance limit to gradual assignments for achieving an acceptable level of compactness, (5) and (6) set the available total number of GPs and the total number of districts to be generated, (7) assures that all inhabitants are assigned to a district, (8) is the linearization constraint for district incomes, and (9-11) are the definitions of the domains of the decision variables.

If the model can be solved to optimality, non-dominated solutions will be generated by considering various combinations of α_{comp} and α_{inc_eq}. However, this model represents an NP-hard multi-criteria optimization problem, so large problems such as real-world cases might not be solved to optimality. Therefore, we report near-optimal solutions in the sequel.

4.2. MULTI-PERIOD MODELING

Our second approach is to extend our multi-criteria single-period districting model into a multi-period districting model. The multi-period modeling approach enhances the efficiency of the healthcare planning as the changes on the demand side (i.e. population and demographics) as well as changes on the supply side (i.e. number of GPs, targeted number of patients per GP) imply adjustment of the districting plan. When these aforementioned changes are predicted and included in a multi-period modeling framework not only the efficiency is increased, service quality can also be improved.

In our modeling framework, we consider changes in the population and the targeted number of patients per GP between periods. As a result of these changes, the number of districts required to be generated as well as the population of each basic unit differ from one period to another. Let T be the number of periods in the planning horizon. In our notation, we include the time index $t = 1,...,T$ for all related parameters and variables.

When the model is extended to a multi-period horizon, an additional criterion, district similarity, is considered to be added in the objective function as a third criterion. The third criterion improves the planning efficiency and quality between periods as described below.

District Similarity

As a result of the multi-period model, district plans will differ from one period to another. Different district plans between periods mean that some of the patients should change their GPs in according to the revised district plan in each period. In a well-planned healthcare system, one of the goals is to ensure continuity of care to improve the patient-GP relationship. Thus, GP changes for the patients have to be avoided if possible. In our weighted sum objective function, we add a dissimilarity measure, which minimizes the number of patients changing their GPs. In various application domains, similar measures have been formulated in the literature (Bozkaya et al., 2003). Based on our healthcare districting modeling, we formulate dissimilarity between the district plans of different periods as follows;

$$\frac{\sum_{t=2}^{T}\sum_{j\in J}\sum_{i\in J}|X_{ijt}-X_{ijt-1}|*b_{jt}}{m*b_t^{avr}}$$

To include this term in the objective function, we first linearize it. We introduce X_{ijt}^{+} and X_{ijt}^{-}:

$$X_{ijt}^{+} = \begin{cases} X_{ijt} - X_{ijt-1}, \text{if } X_{ijt} - X_{ijt-1} > 0 \\ 0 \quad \text{otherwise} \end{cases}$$

$$X_{ijt}^{-} = \begin{cases} X_{ijt-1} - X_{ijt}, \text{if } X_{ijt} - X_{ijt-1} \leq 0 \\ 0 \quad \text{otherwise} \end{cases}$$

Then, the dissimilarity formulation takes the following form:

$$\frac{\sum_{t=2}^{T}\sum_{j\in J}\sum_{i\in J}[X_{ijt}^{+}+X_{ijt}^{-}]*b_{jt}}{m*b_t^{avr}} \quad (12)$$

Equation (12) is used to calculate the differences of number of people assigned from the same basic unit to the same district between different periods. Then these differences are summed up for all basic units, districts and periods to obtain a measure of change in terms of the number of people allocated to different districts between periods.

Finally, the dissimilarity criterion is included in the multi-period model as the third criterion. The resulting multi-period model is as follows:

$$\min C_1 * \sum_{t\in T}\frac{\sum_{i\in J}\sum_{j\in J} d_{ijt}x_{ijt}b_{jt}}{(m-p_t)*d_t^{avr}* b_t^{avr}} + C_2 * \sum_{t\in T}\frac{\sum_{i\in J}[R_{it}^{+}(x)+R_{it}^{-}(x)]}{R_t} + C_3 * \frac{\sum_{t=2}^{T}\sum_{j\in J}\sum_{i\in J}[X_{ijt}^{+}+X_{ijt}^{-}]*b_{jt}}{m*b_t^{avr}} \quad (13)$$

subject to

$$\sum_{j\in J} b_{jt}x_{ijt} = \frac{\sum_{j\in J} b_{jt}}{N_t} n_{it}, \forall\, i \in J, t \in T \quad (14)$$

$$n_{it} \leq c_{it}y_{it}\,,\ \ \forall\, i \in J, t \in T \quad (15)$$

$$d_{ijt}x_{ijt} \leq U, \ \forall \, i,j \in J, t \in T \tag{16}$$

$$\sum_{i \in J} n_{it} = N_t \,, \ \forall \, t \in T \tag{17}$$

$$\sum_{i \in J} y_{it} = p_t \, \forall \, i \in J, t \in T \tag{18}$$

$$\sum_{i \in J} x_{ijt} = 1, \forall \, j \in J \,, t \in T \tag{19}$$

$$R_{it}^{+} - R_{it}^{-} = \sum_{s \in S} \sum_{j \in J} b_{js} l_s X_{ijt} - \frac{R_t}{p_t}, \forall \, i \in J, t \in T \tag{20}$$

$$X_{ijt}^{+} - X_{ijt}^{-} = X_{ijt} - X_{ij(t-1)}, \forall \, i \in J, j \in J, t \in T \tag{21}$$

$$0 \leq x_{ijt} \leq 1, \forall \, i \in J, j \in J, t \in T \tag{22}$$

$$y_{it} \in \{0,1\}, \forall \, i \in J, t \in T \tag{23}$$

$$R_{it}^{+}, R_{it}^{-} \geq 0 \,, \ \ \forall \, i \in J, t \in T \tag{24}$$

$$X_{ijt}^{+}, X_{ijt}^{-} \geq 0, \forall \, i \in J, j \in J, t \in T \tag{25}$$

where, (13) is the weighted multi-period multi-criteria objective function defined with the compactness, income equity and dissimilarity measures, (14) is the work balance constraint in each period, (15) is the capacity constraint for each period, (16) imposes a population-weighted limit on the distance between the basic units and district centers, (17) and (18) set the available total number of GPs and the total number of districts to be generated in each period, (19) assures that all inhabitants are assigned to a district, (20) is the linearization constraint for district incomes, (21) is the linearization constraint for dissimilarity between assignments of successive periods, and (22-25) are the definitions of the domains of the decision variables.

With the multi-period model, we further note that the percent assignment (x_{ijt}) changes from one period to the next may render solutions undesirable from an implementation point of view. Even when x_{ijt} remains unchanged between periods, authorities may choose to re-assign groups of residents between districts. We view the latter case an operational issue that needs to be handled by the authorities to the best public interest, hence leave it outside the scope of our model. The former case is already handled in the third objective term that quantifies total district dissimilarity, but it can be handled in an alternative way in the form of a hard constraint that limits the amount of change in assignment from one period to the next:

$$|x_{ijt} - x_{ij,t+1}| \leq \gamma \qquad \forall t = 1, \dots, T-1$$

We have chosen the objective term approach to stay within the realm of multi-criteria optimization and not to impose further hard constraints on a problem that we think should be handled in a flexible way.

5. CASE STUDY: GP DISTRICTING IN ISTANBUL

We have generated the input data for our GP scheme districting problem using ArcGIS v10.2 and solved the corresponding mixed integer programming model using ILOG OPL Studio v12.6.In this section, we first present the data preparation steps for the case of GP scheme districting in Istanbul, Turkey. Then, we present the results for two scenarios in urban and rural parts of the city of Istanbul, and compare them under managerial viewpoints.

5.1 DATA PREPARATION AND VISUALIZATION

ArcGIS is a commercial off-the-shelf software system that allows storing and customizing a database of a region with its geographical features. We used its environment to manipulate the map of Istanbul with its database and to extract the required datasets for our optimization process. We used the same platform to visualize the obtained solutions we present. In what follows, we provide further details on the underlying GIS framework in terms of the input layers of the geographic data used, the manipulation of the data for optimization purposes, and the visualization of optimization results.

For our optimization, we started out with an input polygon data layer of the set of administrative regions that are stored in ArcGIS' File Geodatabase format. Stored in this input data layer are various data characteristics that are needed for solving the districting problem such as population distribution and urban/rural characteristic of the administrative regions. However, we found that the resolution of the available administrative polygons was not high enough to be defined as the basic units of our districting problem. This was due to the fact that the number of the inhabitants living in the administrative regions is much higher than the requested number of inhabitants to be assigned to GP districts. Therefore, we applied the grid feature of ArcGIS to subdivide each region into smaller areas, and obtained a final set of basic units for our problem with higher resolution. Then, we generated the population distribution of the basic units proportional to the area of the basic units after clipping out the nonresidential areas such as woods, parks, etc. Finally, we extracted the centroid points of the resulting basic units and calculated the distances between them.

The results of the optimization include the district centers and the assignments of all basic units to these centers. Since we allow a gradual assignment of a basic unit to more than one district, we also need the tools available in the ArcGIS platform for the visualization of the districts. In our visualizations, we represent a fixed number of inhabitants of a basic unit with a dot that is colored based on the GP center the dot is assigned to. This gives us dot density maps of the resulting district plans with dots showing the color of the assigned GP districts.

5.2 COMPUTATIONAL RESULTS FOR THE SINGLE-PERIOD MODEL

We have chosen two administrative regions in Istanbul to generate the GP districts using our proposed mathematical program. One of the regions, Kadikoy, is centrally located where the population is densely distributed all over the area. The other region, Tuzla, is a peripheral one that consists of high and low density population areas.

Scenario 1: Kadikoy, a Central Region

After splitting the area by grids, we have obtained 747 basic units in Kadikoy with a total of 482,562 inhabitants. We have set the number of GP districts to create as 100 and the total number of GPs to assign to the districts as 150. Accordingly, the number of inhabitants per GP is 3,217.

To consider scenarios with different weights on compactness versus income equity, we varied the coefficients of the objective function with a step size of 0.1. We set the distance limit to1.25 km, since Kadikoy is a densely populated area where GP services are typically accessible within a walking distance. Then, we solved the model using the CPLEX solver of ILOG OPL optimization studio on an Intel Core i3 4150 3.5GHz computer with 8 GB RAM. The values of the two criteria of the weighted sum objective function and diagram obtained with these values are given in Table 2 and Figure 2, respectively. The columns Gap and Runtime specify the remaining CPLEX gap and the computation time upon termination, respectively.

Note that some of the solutions are dominated by others. This is due to the fact that we could not find a proven optimal solution within the designated time limit for any of the weight combinations, and only optimal solutions are guaranteed to be non-dominated. We have marked the solution ID’s with an asterisk (*) that are non-dominated with respect to our solution set. We also show these points in the diagram as a square.

TABLE 2: Computational Results for the Kadikoy Dataset

Sol. ID	Coefficient of Compactness	Coefficient of Income Equity	Compactness	Income Equity	Gap	Runtime (seconds)
1*	0.1	0.9	0.07189	0.00016	3.41 %	20000
2*	0.2	0.8	0.07093	0.00020	2.58 %	20000
3	0.3	0.7	0.07058	0.00025	2.31 %	20000
4*	0.4	0.6	0.07055	0.00024	2.28 %	20000
5	0.5	0.5	0.07047	0.00030	2.20 %	60000
6*	0.6	0.4	0.07037	0.00034	2.15 %	20000
7	0.7	0.3	0.07039	0.00030	2.13 %	50000
8*	0.8	0.2	0.07039	0.00033	2.12 %	50000
10	0.9	0.1	0.07043	0.00034	2.13 %	50000

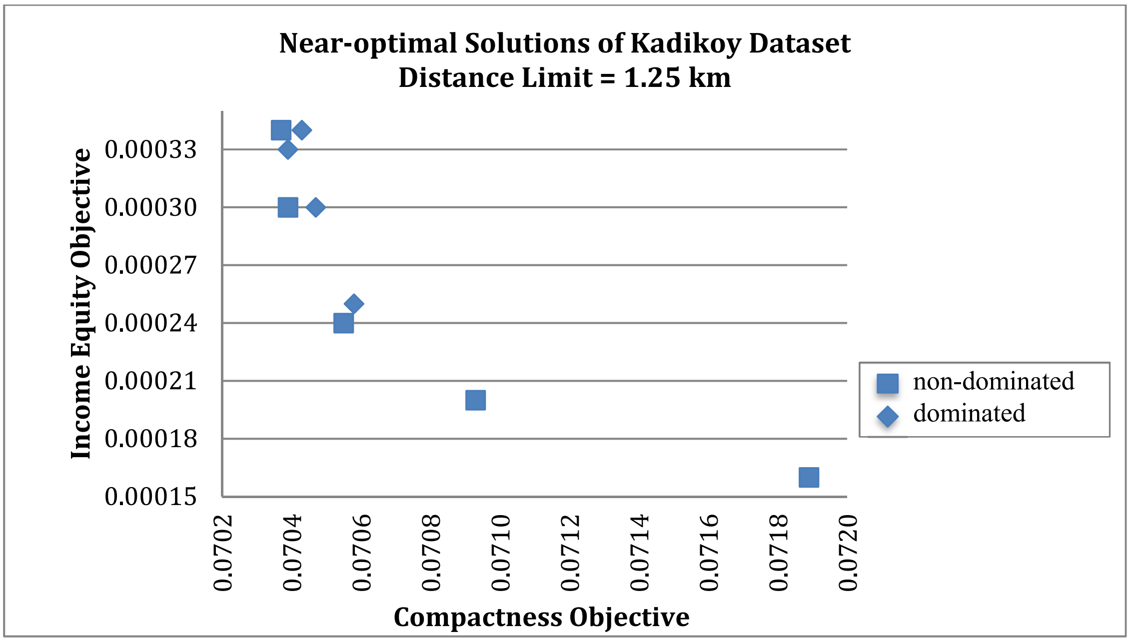


FIGURE 2: Diagram of the near-optimal solutions for the Kadikoy Dataset.

Looking at Figure 2, we observe that an improvement on income equity can be obtained quite “cheaply” – i.e. with virtually no cost in compactness up to a certain point, but thereafter getting more gains in income equity is quite expensive. A solution to suggest to a decision maker therefore would beat the elbow point of the curve. Income equity will ensure the equity among doctors, which will provide the work satisfaction of doctors, and it will also prevent doctors from competing for some “high-income” providing areas. On the other hand, the income equity will also implicitly ensure the satisfaction of patients. The income equity depends on the number of patients at each segment based on the age groups. Each patient segment has a different earning factor for the doctors identified with respect to the average required service time for that segment. Thus, the total service time of the doctors are also balanced indirectly, which will result in similar workloads for the doctors and similar waiting times for the patients at each GP.

The Significance of the Distance Limit

Since we already have the compactness criterion in the weighted sum objective function, we may rely on this measure for geographical tightness of the resulting districts, and thus can test if it really benefits to use a population-weighted distance limit. As seen in Table 3, the distance limit constraint provides significant improvements. Therefore, the accessibility of the patients to the GP centers is improved when a distance limit is used.

TABLE 3: The results with and without the distance limit

Distance limit	**Assignments exceeding distance limit**	**Nr. of people exceeding distance limit**	**Maximum distance of assignments**	**Average distance of assignments**
No	61	519 people	9.53 km	0.53 km
Yes	22	30 people	3.97 km	0.32 km

Out of 747 assignments, 22 still exceed the distance limit even though a distance limit constraint of 1.25 km is incorporated into the model. This is because of the way the gradual assignment variables $0 \leq x_{ij} \leq 1$ are used in constraint (4) of the model.

We could avoid these assignments by setting the corresponding variables to 0 for which the distance is greater than 1.25. However, we have chosen to leave these variables unchanged since the distance limit has been exceeded only for 30 people out of 482,562.

Finally, the district plan for the Kadikoy dataset with the (0.4-0.6) coefficients for compactness and income equity, respectively is visualized by ArcGIS in Figure 3. Each color on the map represents a dot density visualization of the associated GP district and each GP center features the number of GPs assigned to the center.

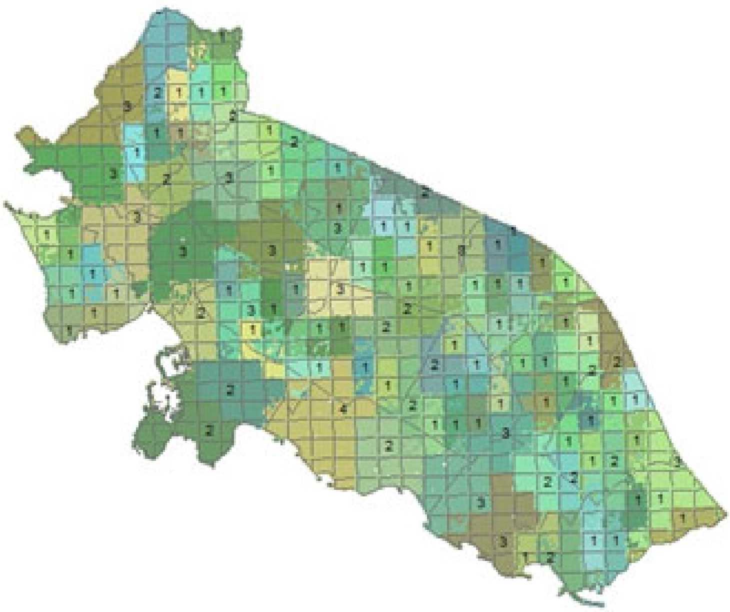


FIGURE 3: The district plan for the Kadikoy dataset.

Scenario 2: Tuzla, a Peripheral Region

Tuzla is a peripheral region that consists of large areas with very low density of population in contrast to the high density of population all over the region of Kadikoy. In order to observe the effect of different population distributions on the districting plan, we present both regions in our case study.

After splitting the area of Tuzla by grids, we have obtained 472 basic units with a total of 442,264 inhabitants. We have set the number of GP districts to create as 50 and the total number of GPs to assign to the districts as 75. Accordingly, the number of inhabitants per GP is 2,955.

We have varied the coefficients of the weighted sum objective function with a step size of 0.2, resulting in the coefficients of (0.2-0.8), (0.4-0.6), (0.6-0.4) and (0.8-0.2). Since Tuzla is not a very central district of Istanbul and has many suburban parts, patients in this area may accept to travel longer distances. In order to decrease the gap, this time we have chosen to relax the distance limit and changed it to 2.5 km. We solved the problem using the CPLEX solver of ILOG OPL optimization studio on an Intel Core i3 4150 computer with 8 GB RAM with the maximum computing time of 5,000 seconds. The values of the two terms in the objective function and the Pareto curve obtained with these values are given in Table 4 and Figure 4.

TABLE 4: Computational Results for the Tuzla Dataset with Distance Limit=2.5 km.

Coefficient of Compactness	**Coefficient of Income Equity**	**Compactness**	**Income Equity**	**Gap**
0.2	0.8	0.03284	0.00033	7.46%
0.4	0.6	0.03253	0.00038	6.75%
0.6	0.4	0.03223	0.00044	5.77%
0.8	0.2	0.03206	0.00052	5.37%

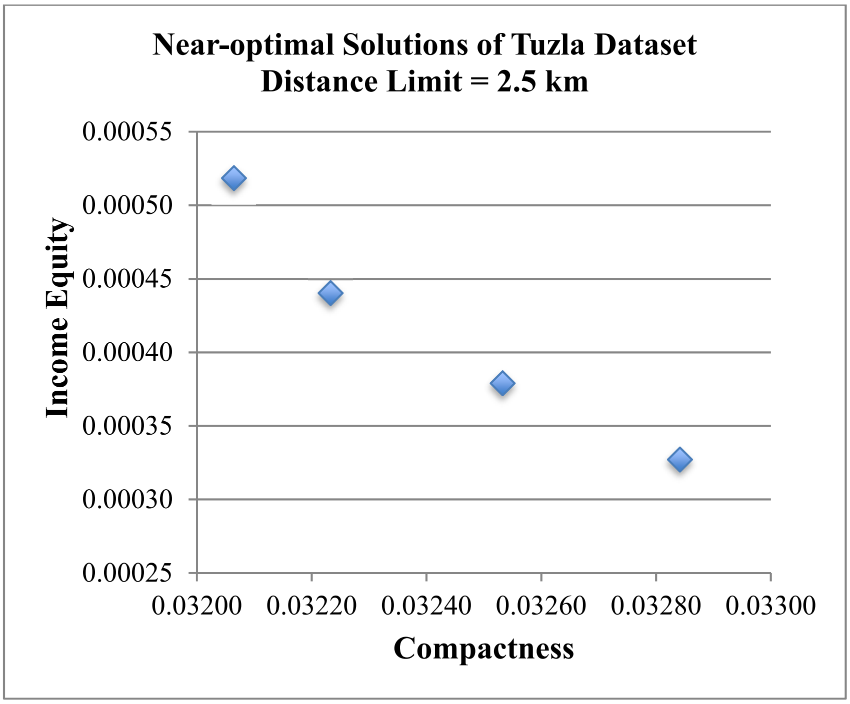


FIGURE 4: Diagram of the near-optimal solutions for the Tuzla Dataset.

For the case of Tuzla, we do not get a similar "elbow" on the curve that would be a natural solution for the decision maker. This means that when the area is not densely populated, there exists a sharp trade-off between the compactness and balance criteria. However, this trade-off can be by-passed when the area is densely populated as observed in Kadikoy region.

The district plan for the Tuzla dataset with the (0.6-0.4) coefficients for compactness and income equity is visualized by ArcGIS in Figure 5. As before, each color on the map represents a dot density visualization of the associated GP district and each GP center features the number of GPs assigned to the center.

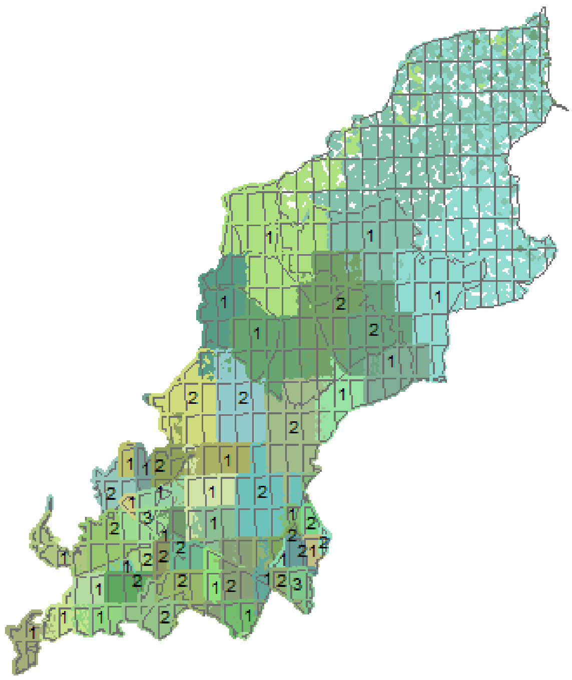


FIGURE 5: The district plan for the Tuzla dataset.

Note that, as the dot-density views and the model solutions indicate, patients located in some districts will have split assignments between two or more district centers, which is an expected outcome of the gradual assignment concept. As we discussed previously, from an implementation aspect, an authority could apply a post treatment (e.g. splitting basic units on the basis of street segments) to decide which particular patients should be (re-)assigned to which center. This would have a positive impact on the applicability, hence the overall acceptability, of the resulting solutions.

5.3 COMPUTATIONAL RESULTS FOR THE MULTI-PERIOD MODEL

We set the number of periods as two for the dataset of Kadikoy, where each period is 3 years. Parameter projections for this two-period problem are given in Table 5.

TABLE 5: Future Parameter Projections for a Two-Period Problem.

	Number of GPs (districts)	Total Population	Population per GP
1st Period	100	482,562	4826
2nd Period	130	508,707	3913

It was not possible to solve the capacitated multi-period districting model with the capacity of our workstation due to high memory load. So, we simplified the model by excluding the idea of assigning multiple GPs to each district (constraints 15 and 17) and the distance limit constraint (16). As a result, each GP center in each district has only one GP as result of this model. We believe the increased complexity in the multi-period model with constraints (15)-(17) is due to the addition of new n_{it} decision variables over multiple periods and the associated inter-linking constraints, as the exclusion of these variables and constraints allow us to generate solutions in allocated computational times. Allowing more than one GP per center considerably increases the solution space and, thus, also the combinatorial complexity of the problem.

When we solve this multi-period model with two criteria namely compactness and the income equity combined into a single weighted objective function with a gap of 0.31% within a computational time of 1000 seconds, the two component values of the objective function are as follows:

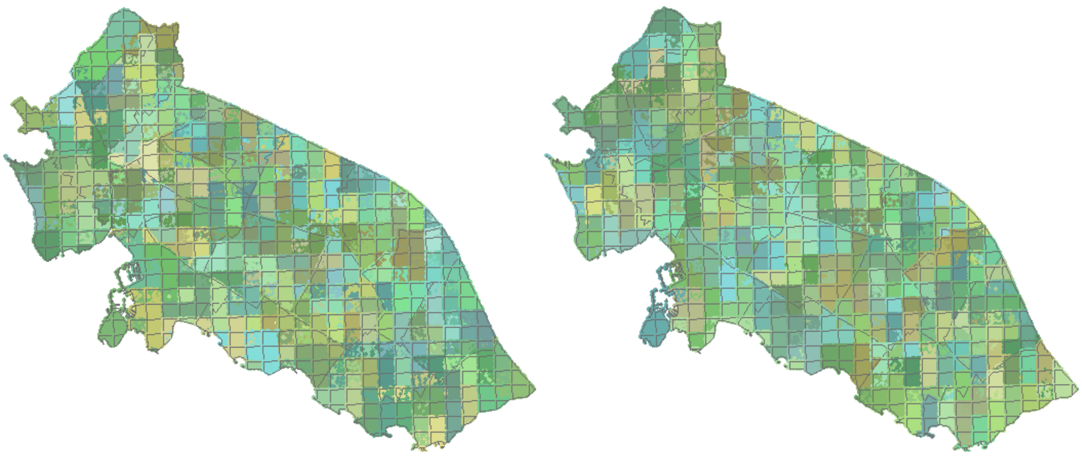
- Compactness = 0.127718
- Income equity = 6.71637

Then we add the district similarity criterion into the objective function seeking for similar districts in different periods. When this model is solved, we obtain the three component values of the objective function as follows (with a gap of 13.65%, runtime: 50.000 seconds):

- Compactness = 0.161115
- Income equity = 5.71600

- Similarity of districts= 1.00033

As seen, there is loss of compactness when we add the district similarity criterion. To assess whether there is an improvement in the similarity of the districts when the third criterion is added in the model, we checked the number of people assigned to different districts in different periods. The number of people assigned to different districts in different periods without adding the dissimilarity criterion to the objective function is 346,251 whereas the same number with the addition of the dissimilarity criterion is 119,724. So we can see that there is a significant improvement in the similarity of the districts when we add the third district similarity criterion into the objective function. The districts obtained for the two periods of the multi-period multi-criteria model are given in Figure 6(a) and (b).



(a) 1st period's district plan (b) 2nd period's district plan

FIGURE 6: The multi-period district plan for the Kadikoy dataset.

6. CONCLUSION

In this paper, we have developed and proposed a planning tool including an optimization model for the design of the General Practitioner districts. This tool is helpful for making the location and territorial decisions for GP practices and hence providing efficient services to potential patients in practice. In our modeling approach, we consider the access of patients to the GP centers and the income equity and workload balance of the GPs.

We proposed a multi-period multi-criteria mixed integer programming model and we obtained near-optimal solutions with reasonable gaps. The main contribution of our model is to eliminate hard boundaries for districts imposed by the borders of basic units, and allow partial assignment while at the same time keeping compactness at an acceptable level. To facilitate this approach, we have subdivided an area into smaller basic units to ensure we can use high resolution basic units for districting. We have then tested the model on two regions of the city of Istanbul, Turkey. The results show that an improvement on income equity can be obtained quite "cheaply" – i.e. with virtually no cost in compactness up to a certain point in the efficient frontier. Besides, the results confirm that the multi-period model with an additional district similarity criterion and anticipating on the future conditions will improve the continuity of care principle of the healthcare system ensuring a better patient-service. As for future work, we aim to decrease the gap and the runtime of the optimization process by searching via more efficient formulations and also to deal with bigger cases. We also plan to develop a heuristic solution approach that may help with solving the complete multi-period model and generating near-optimal solutions.

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