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A population-based approach to the bi-level multi-follower problem: an application to the electricity retail market

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Abstract

Dynamic tariffs are expected to be implemented as commercial offers of electricity retailers in smart grids, conveying price signals aimed at shaping usage patterns, with potential benefits to enhance grid efficiency and reduce end-users' costs. Retailers and consumers have divergent goals. While the objective of the retailer is maximizing profits, the purpose of consumers is minimizing their electricity bill. The interaction between the retailer and consumers can be modeled by means of bi-level (BL) programming: the retailer sets the prices to be charged to consumers and these react by scheduling flexible appliances according to those prices and their comfort requirements.

In this work, two hybrid BL optimization approaches are proposed to solve this problem, considering one leader (retailer) and multiple followers (consumers). Two population-based approaches were developed, a genetic algorithm (GA) and a particle swarm optimization (PSO) algorithm, to deal with the upper level problem, both encompassing an exact mixed integer linear programming solver to address the lower level optimization problem. Different scenarios were generated, comprising one leader (retailer imposing different price schemes) and three followers (with different consumer profiles). Typical residential appliances were considered, with different operation cycles. Also, diverse tariff structures set by the retailer were analyzed. The performance of the two algorithms was compared. Results revealed a consistent superiority of PSO over GA.

Keywords: Bi-level multi-follower optimization; Hybrid algorithms; Genetic algorithms; Particle swarm optimization; Dynamic tariffs; Demand response.

1. Introduction

Dynamic tariffs are a pricing strategy in which the supplier sets time-differentiated prices for a product or service according to certain conditions that may include congestion, market demands, etc. Dynamic tariffs have already been applied in several business areas (e.g. tourism industry, public transport, etc.) and it is expected that they may become a common practice in the electricity retail sector in the realm of smart grids. In the electricity sector, the retailers buy energy in the liberalized electricity markets and make selling contracts with their clients. The energy is typically charged to residential consumers in a flat tariff (i.e., considering the same price for an entire day) or in dual time-of-use tariffs (i.e., considering two distinct periods of prices within one day, such as the peak period – where the electricity price is higher – and the off-peak period – where the electricity price is lower). Consumers buy the amount of energy necessary to satisfy their needs, adjusting the consumption according to their own budget constraints and comfort requirements. With the implementation of dynamic tariffs, the retailer defines distinct ranges of energy prices more or less variable within the day, thus encouraging consumers to adopt different patterns of consumption with potential savings. Once the consumers have received the information of the prices to be charged, they may react by scheduling the operation cycle of their controllable appliances according to their willingness to pay and the flexibility they have in changing habitual operation time slots. While the retailer aims to maximize profit, the purpose of consumers is to minimize their electricity bill considering preferences and requirements regarding the quality of the energy services (electric vehicle charging, laundry, hot water, etc.). The interaction between the retailer and consumers can be hierarchically modeled as a bi-level (BL) programming problem, where the retailer is the leader (upper level decision maker) and the consumer is the follower (lower level decision maker). The retailer decides first, but the consumers' reaction will affect the retailer's objectives.

In this work, we formulate the interaction between an electricity retailer and several clusters of consumers with different consumption profiles as a BL optimization model with multiple followers. The maximization of the retailer's profit is the upper level objective function and the minimization of the consumers' costs are the lower level objective functions.

When multiple followers are involved in a BL decision-making process, for each feasible solution generated at the upper level, different reactions of the followers could arise at the lower level. Also, different relationships among the followers could lead to different decisions of the leader. Lu et al. (Lu et al. 2006) generalized a framework for the BL optimization problem with multiple followers by describing three different situations that result in nine different types of relationships among the followers. The most usual situation is described as *uncooperative*, where the followers do not share decision variables, objectives and constraints among them. In the *cooperative* situation, there is a complete sharing of decision variables among the followers. The third situation is defined as *partially cooperative*, where the followers partially share the decision variables in their objectives and/or constraints. Each of these latter two situations can be split into four sub-cases defined by the relationships among the objectives and constrains of the followers. Lu et al. (Lu et al. 2006) proposed BL multi-follower (BLMF) decision models for the nine cases, which require different approaches to derive an optimal solution. In each model, the authors assumed that the leader has full knowledge about the objectives and constraints of the followers as well as the relationships among them. Based on such relationships, the leader must be able to anticipate the followers' reactions to his decisions.

In this work, we are particularly interested in multi-follower decision problems and how they can be tackled. In the following, a brief review of BLMF approaches in several domains is presented.

Calvete et al. (Calvete and Galé 2007) proved that a linear BL programming problem with a single leader and multiple independent followers (i.e., the objective function and the set of constraints of each follower only include the leader's variables and his own variables) can be reformulated as a linear BL problem with a single follower. This is accomplished by defining the lower level objective function as the sum of the followers' objective functions subject to the whole set of constraints of all followers.

Wang et al. (Wang et al. 2009) proposed a fuzzy interactive decision making algorithm to address the BLMF model with partially shared variables among followers and separate objective functions and constraints.

Ansari et al. (Ansari and Rezai 2011) presented a new method to solve multi-objective linear BLMF programming problems, in which there is no information shared among followers, using fuzzy programming and the *K*th-best algorithm (Bialas and Karwan 1984). This approach considers multiple followers with multi-objective problems at the lower level and also multiple objective functions at the upper level.

Zhou et al. (Zhou et al. 2016) developed a BLMF model with uncertain multiple objectives at the upper level to support decision making in low-carbon power dispatch and generation. The leader is the regional power grid corporation while the followers are the power generation groups. The leader allocates quotas to each follower, while the followers decide on their respective power generation plans and prices. The BL problem is then transformed into solvable single level models.

Islam et al. (Islam et al. 2016) proposed a memetic algorithm for solving BLMF optimization problems. The methodology uses a combination of global and local search to consistently reach competitive solutions with less function evaluations in comparison with other approaches. The approach was tested with both dependent and independent followers using benchmark problems.

In the context of electricity retail markets, BLMF models are scarce. However, some contributions can be found in the literature considering just a single follower.

Alves et al. (Alves et al. 2016) proposed a BL model for the interaction of a retailer (who aims to maximize profit) with a cluster of consumers with similar consumption and demand response profiles (who want to minimize costs). A hybrid approach was developed to handle the problem, which consists of a genetic algorithm (GA) at the upper level and an exact mixed-integer linear programming (MILP) solver at the lower level to solve the consumer's problem for each retailer's decision. This work was extended to a semivectorial BL optimization model (Alves and Antunes 2018), in which two consumer's objective functions are considered: minimize electricity bill and minimize the dissatisfaction associated with rescheduling the appliance operation. In this case, nondominated solutions must be calculated to the lower level problem. The paper deals with the uncertainty about the reaction of the consumer, who has to choose a solution among many different tradeoffs for his objective functions, and the impact of the consumer's choice on the retailer's profit. Different types of solutions to the semivectoral BL problem are computed, which result from more optimistic or pessimistic retailer's stances regarding the consumer's option.

Carrasqueira et al. (Carrasqueira et al. 2017) proposed two population-based approaches for both upper and lower levels, one based on a GA and another on a PSO algorithm, to address the same problem as the one proposed in (Alves et al. 2016) but dealing with a different model, which is not a mathematical programming model. Therefore, the lower level problem is also addressed by a metaheuristic approach. Soares et al. (Soares et al. 2019) developed a different lower level model including shiftable, thermostatic and interruptible loads. The inclusion of a thermostatic load in the lower level problem imposes a higher computation effort, which impairs obtaining the optimal solution by a state-of-the-art solver. Therefore, the focus of this work is to compute good quality estimates for the retailer's profit.

Meng and Zeng (Meng and Zeng 2013) proposed a BL model to determine the real-time energy prices that maximize the retailer's profit, while the consumer attempts to minimize the electricity bill for such prices. The model is then converted into a single-level problem using the Karush-Kuhn-Tucker conditions and it is solved using a branch-and-bound algorithm. The authors further developed BLMF models (Meng and Zeng 2013, Meng and Zeng 2014, Meng and Zeng 2015, Meng and Zeng 2016) with the same objective functions. Hybrid algorithms were used considering a GA at the upper level and a linear programming solver for the consumers' problems at the lower level.

Sekizaki et al. (Sekizaki et al. 2016) proposed a BL model in which different types of consumers (i.e., not only residential consumers) are considered and flexible responses to the prices set by the retailer are allowed. The objective function of each type of consumer is a weighted sum of the cost of power purchasing and "disutility" caused by the reduction of the load served. A GA is developed to tackle this model.

With the exception of (Alves et al. 2016, Carrasqueira et al. 2017, Alves and Antunes 2018), all BL models described above in the context of the electricity retail market do not incorporate information associated with the operation cycles of typical household appliances. However, the inclusion of these data into the model leads to a more realistic load characterization, also enabling a more reliable representation of the consumers' goals. Typical appliances used by residential consumers and the corresponding operation cycles are characterized by Soares et al. (Soares et al. 2014).

The BLMF model proposed in this work to deal with the interaction between a retailer and multiple consumers also incorporates detailed household appliance operational characteristics, including the information associated with the operation cycles of shiftable loads. The resulting lower level problems are mixed-integer non-linear programming problems, which become MILP problems after the instantiation of the upper level decision variables (electricity prices). The characteristics of the BLMF model prevent the use of algorithms that convert the problem into one level using the KKT conditions, or algorithms that use linear programming solvers for the lower level problems (for instance, as the ones used by (Meng and Zeng 2013) or (Meng and Zeng 2016)).

We have developed two hybrid algorithms to deal with the BLMF problem, each one combining a population-based algorithm (a GA and a PSO) for the upper level problem and an exact MILP solver to the lower level problems. A comparison of these algorithms is presented considering different scenarios with a retailer instance and three residential consumers' profiles. The consumer profiles of each scenario differ in the base (uncontrollable) load, the household controllable appliances and their operation cycles, the power requested to the grid and the time slots allowed for load operation. Also, different tariff structures set by the retailer were analyzed.

The main contributions of the current paper are: modelling a more realistic situation involving different clusters of residential consumers with different appliance ownership and different consumption patterns (multiple followers); considering power constraints at the upper level to avoid undesirable consumption peaks due to grid management reasons, which may result from technical restrictions of power transformers and lines as established by the distribution network operator, and therefore the solutions obtained by each follower at the lower level must be aggregated to verify the power constraints at the upper level; performing a comparative analysis of the two hybrid BL approaches in order to assess their capability to explore the upper level search space aiming to improve the retailer's profit; developing a *repair routine* for (electricity prices) variables with a fixed number of decimals, since it is a common practice in commercial offers by electricity retailers to set energy prices with a given number of decimal places. The performance of the two hybrid algorithms developed was compared under realistic conditions. This study also enables to assess the interest to consumers to adopt dynamic vs. flat tariff schemes.

The manuscript is organized as follows. In section 2 the main concepts of BL optimization are summarized, in particular considering multiple followers. The new formulation to model the interaction between a retailer and multiple consumer clusters in the retail electricity market is presented in section 3. Section 4 describes two hybrid population-based algorithmic approaches, combining either a GA or a PSO with a MILP exact solver. In section 5, the results of the study are presented and discussed. The conclusions are drawn in section 6.

2. Bi-level optimization with multiple followers

BL programming is a special kind of optimization models involving two optimization problems being one problem embedded into the other. The problems are hierarchically related being the upper level problem usually referred to as the leader problem, while the lower level optimization problem is commonly referred to as the follower problem. The two optimization problems involve different types of decision variables and have their individual objective functions subject to specific constraints. The leader needs to anticipate the optimal response of the followers. Succinctly, the leader starts by setting his decision variable values to which the followers react by choosing the values for their decision variables in the set of solutions restricted by the values of the upper level decision variables fixed by the leader. The resolution of BL optimization problems remains a great challenge due to their inherent non-convexity, and even the linear BL problem is NP-hard (Dempe 2002).

The general BL programming problem with a single leader and multiple independent followers, which do not share variables, constraints and objective functions (*uncooperative* followers as referred to above) can be stated as follows:

$$\begin{aligned} \max_{x \in X} \quad & F(x, y_1, \dots, y_N) \\ s.t. \quad & G(x, y_1, \dots, y_N) \leq 0 \\ & y_k \in \underset{y'_k \in Y_k}{\arg\min} \left\{ f_k\left(x, y'_k\right) : g_k\left(x, y'_k\right) \leq 0 \right\}, k = 1, \dots, N \end{aligned}$$

where $X \subset \mathbb{R}^n$ and $Y_k \subset \mathbb{R}^{m_k}$ are closed sets, n is the number of upper level variables and m_k is the number of lower level variables of the follower $k = 1, \dots, N$, with N being the number of followers. The decision variables x are controlled by the leader while the follower k controls decision variables y_k. $F(x, y_1, \dots, y_N)$ is the leader's objective function and $f_k(x, y_k)$ is the objective function of the follower $k \in \{1, \dots, N\}$. Variables x are kept constant while the lower level objective functions are optimized since each follower k optimizes his objective function $f_k(x, y_k)$ after decision variables x are selected by the leader. $G(x, y_1, \dots, y_N)$ and $g_k\left(x, y'_k\right)$, $k = 1, \dots, N$, represent the set of multiple constraint functions at the upper and lower levels, respectively.

Let $x \in X$ be a vector of decision variables fixed by the leader. The *feasible* and the *rational reaction* sets of the follower $k \in \{1, \dots, N\}$ are the sets $Y_k(x) = \{y_k \in Y_k : g_k(x, y_k) \leq 0\}$ and $\Psi_k(x) = \left\{ y_k \in Y_k : y_k \in \underset{y'_k \in Y_k(x)}{\arg\min}\, f_k(x, y'_k) \right\}$, respectively. The feasible set of the BL problem described above, also called *inducible region*, is $IR = \left\{(x, y_1, \dots, y_N) : x \in X, G(x, y_1, \dots, y_N) \leq 0, y_k \in \Psi_k(x), k \in \{1, \dots, N\}\right\}$.

In bilevel programming, if $\Psi_k(x)$ is not uniquely determined for all x, then either an *optimistic* or a *pessimistic* approach is usually adopted. The optimistic approach assumes that the follower always takes an optimal solution that is the best for the leader, while the pessimistic approach assumes that the solution chosen by the follower is the worst for the leader. In the present study, a general MILP solver is used (*cplex*) to compute optimal solutions to the lower level problems in the framework of the hybrid approaches; the solutions found are possible optimal reactions of the consumers to the electricity prices set by the retailer. Due to the computation effort associated with the calculation of all alternative optimal solutions to each lower level MILP problem instantiated for each x, we accept the first optimal solution given by the solver, which it is not known whether it is the best, the worst or an intermediate solution to the retailer. Therefore, neither an optimistic nor a pessimistic perspective is underlying in this study.

3. A bi-level model for residential electricity retail market problems with multiple followers

In the electricity retail market, the retailer buys energy in the wholesale market to sell to consumers. The retailer defines dynamic tariffs, while the consumers react by adjusting the energy consumption through the scheduling of load operation. The aim of the retailer is to maximize the profit while the goal of consumers is to minimize their electricity bill, satisfying comfort requirements.

The planning period $\bar{T} = \{1, \ldots, T\}$, with T being the number of time intervals, is divided into I sub-periods $P_i = [P_{1_i}, P_{2_i}] \subset \bar{T},\ i \in \{1, \ldots, I\}$, such that $\bigcup_{i=1}^{I} P_i = \bar{T}$ and $\overline{P_i} = P_{2_i} - P_{1_i} + 1$ is the amplitude of P_i. The retailer defines the values for the I upper level decision variables, the prices $x = (x_1, \ldots, x_I)$ to be charged to the consumers. x_i (in €/kW$\hbar$) is the price of electricity in the pre-defined sub-period P_i, $i \in \{1, \ldots, I\}$, where $\hbar$ is the unit of time the planning period is discretized into (hour, minute, quarter of hour, etc.).

The model considers N different clusters of consumers, grouping consumers with similar consumption and demand response profiles. For each consumer cluster $k \in \{1, \ldots, N\}$ there is a base load b_{kt} (in kW) for each time t in the planning period $\bar{T}$, which corresponds to non-controllable loads, and J_k different shiftable appliances (whose operation cycle cannot be interrupted once initiated), such that each load $j \in \{1, \ldots, J_k\}$ requires from the grid a power p_{kjt} (in kW) at time t of the planning period $\bar{T}$.

The objective function of the upper level problem (equation (1) in the BLMF model presented below) corresponds to the maximization of the retailer's profit, being defined as the difference between the revenue with the sale of energy to consumers and the cost of buying energy in the wholesale market. Coefficients π_t in equation (1) are the prices of energy incurred by retailer at time $t \in \{1, \ldots, T\}$. In this model, θ_k, $k \in \{1, \ldots, N\}$, is the fraction of consumer cluster k in the set of all consumers.

The electricity prices set by the retailer are limited to a minimum and maximum values, $\underline{x_i}$ and $\overline{x_i}$ respectively, for each sub-period P_i, i.e. $\underline{x_i} \le x_i \le \overline{x_i},\ i \in \{1, \ldots, I\}$, (constraints (2) and (3) in the BL model presented below). Additionally, an average electricity price for the whole planning period $\bar{T}$ is imposed, $x^{AVG} = \frac{1}{T}\sum_{i=1}^{I}\overline{P_i}\, x_i$ (constraint (4) in the model, similar to the one considered by (Zugno et al. 2013)).

An upper level power constraint imposes the overall power requested from the grid by non-controllable and flexible loads for each time $t \in \{1, \dots, T\}$ from all consumer clusters does not exceed C_t (constraint (5) in the model).

BLMF MODEL

$$\max_x F = \sum_{k=1}^{N}\theta_k\left(\sum_{i=1}^{I}\sum_{t\in P_i} x_i\left(b_{kt} + \sum_{j=1}^{J_k} p_{kjt}\right) - \sum_{t=1}^{T}\pi_t\left(b_{kt} + \sum_{j=1}^{J_k} p_{kjt}\right)\right) \tag{1}$$

s.t.

$$x_i \le \overline{x_i}\,, \quad i = 1, \dots, I \tag{2}$$

$$x_i \ge \underline{x_i}\,, \quad i = 1, \dots, I \tag{3}$$

$$\frac{1}{T}\sum_{i=1}^{I}\overline{P_i}x_i = x^{AVG} \tag{4}$$

$$\sum_{k=1}^{N}\theta_k\left(b_{kt} + \sum_{j=1}^{J_k} p_{kjt}\right) \le C_t\,, \quad t = 1, \dots, T \tag{5}$$

$$\left.\begin{array}{ll}
\min\limits_{p_k, w_k} f_k = \sum_{i=1}^{I}\sum_{t\in P_i} x_i\left(b_{kt} + \sum_{j=1}^{J_k} p_{kjt}\right) & (6) \\
\text{s.t.} & \\
p_{kjt} = \sum_{r=1}^{d_{kj}} f_{kjr} w_{kjrt}\,,\ j = 1, \dots, J_k\,, t = T_{1_{kj}}, \dots, T_{2_{kj}} & (7) \\
p_{kjt} = 0\,,\ j = 1, \dots, J_k\,, t < T_{1_{kj}} \vee t > T_{2_{kj}} & (8) \\
b_{kt} + \sum_{j=1}^{J_k} p_{kjt} \le C_{kt}\,, \quad t = 1, \dots, T & (9) \\
\sum_{r=1}^{d_{kj}} w_{kjrt} \le 1\,,\ j = 1, \dots, J_k\,, t = T_{1_{kj}}, \dots, T_{2_{kj}} & (10) \\
w_{kj(r+1)(t+1)} \ge w_{kjrt}\,, j = 1, \dots, J_k\,,\ r = 1, \dots, \left(d_{kj} - 1\right), t = T_{1_{kj}}, \dots, \left(T_{2_{kj}} - 1\right) & (11) \\
\sum_{t=T_{1_{kj}}}^{T_{2_{kj}}} w_{kjrt} = 1\,,\ j = 1, \dots, J_k\,, r = 1, \dots, d_{kj} & (12) \\
\sum_{t=T_{1_{kj}}}^{T_{2_{kj}} - d_{kj} + 1} w_{kj1t} \ge 1\,,\ j = 1, \dots, J_k & (13) \\
w_{kjrt} \in \{0,1\}, j = 1, \dots, J_k\,, r = 1, \dots, d_{kj}\,,\ t = T_{1_{kj}}, \dots, T_{2_{kj}} & (14) \\
p_{kjt} \ge 0\,,\ j = 1, \dots, J_k\,, t = 1, \dots, T & (15)
\end{array}\right\} k = 1, \dots, N$$

In the lower level problem, consumers react to the electricity prices set by the retailer. For each consumer cluster $k \in \{1, \dots, N\}$, the objective function in the lower level problem (equation (6)) corresponds to the minimization of the electricity bill, i.e. the sum of the cost of the energy consumed by uncontrollable and shiftable loads in the planning period $\bar{T}$.

For controllable loads, each consumer cluster k should specify the preferences, i.e. the comfort time slots $\left[T_{1_{kj}}, T_{2_{kj}}\right] \subseteq \bar{T}$ in which each load j, $j \in \{1, \dots, J_k\}$, should operate and the power requested by load j at stage $r \in \left\{1, \dots, d_{kj}\right\}$ of its operation cycle, f_{kjr} (in kW), being d_{kj} the duration of the working cycle of load j. The lower level decision variables for each consumer cluster k are w_{kjrt} and p_{kjt}.

Variables w_{kjrt} specify whether appliance j is "on" or "off" at time $t \in \left[T_{1_{kj}}, T_{2_{kj}}\right]$ and stage r of its operation cycle; p_{kjt} is the power requested to the grid by appliance j at time $t \in \{1, \dots, T\}$.

Constraints (7) and (8) define the lower level variables p_{kjt}. Equation (7) defines p_{kjt} for $t \in \left[T_{1_{kj}}, T_{2_{kj}}\right]$, i.e., when load $j \in \{1, \dots, J_k\}$ is allowed to operate. Equation (8) does not allow appliance j to operate outside their comfort time slot.

Constraints (9) impose that the power contracted by consumer cluster $k \in \{1, \dots, N\}$, C_{kt}, is never exceed at any $t \in \{1, \dots, T\}$.

Constraints (10) ensure that at time t of the planning period, each load $j \in \{1, \dots, J_k\}$ of consumer cluster k is either "off" or "on" at only one stage r of its operation cycle. Constraints (11) ensure that if load j is "on" at time t and at stage $r < d_{kj}$ of its operation cycle, then it must also be "on" at time $t + 1$ and stage $r + 1$. Constraints (12) guarantee that each load $j \in \{1, \dots, J_k\}$ of consumer cluster k operates exactly once at stage r and this should occur within its comfort time slot. Constraints (13) guarantee that for each consumer cluster k, load j starts its working cycle within its allowed comfort time slot, i.e. at most at time $T_{2_{kj}} - d_{kj} + 1$, thus assuring that it never finishes later than $T_{2_{kj}}$.

Constraints (11-13) ensure that each controllable appliance j of cluster k operates precisely d_{kj} consecutive time intervals, thus forcing the lower level decision variables w_{kjrt} to be zero whenever appliance j is "off".

Constraints (14) define the lower level decision variables w_{kjrt} as binary variables, where 1 means that appliance j is "on" and 0 means that it is "off" at time t and stage r. Constraints (15) define decision variables p_{kjt} as non-negative for the whole planning period.

4. Hybrid population-based approaches with MILP solver

Since the BLMF problem is very difficult to solve, metaheuristics are used to explore the upper level search space combined with a MILP solver to find exact lower level solutions. Optimal solutions to the lower level problem should be obtained, since sub-optimal lower level solutions are infeasible to the BLMF problem. Therefore, the solutions yielded by the hybrid approaches may not be surely optimal solution to the BLMF problem, because metaheuristics are not exact methods, but they are certainly feasible solutions to the BLMF problem.

Even being able to compute the optimal solution of each lower level problem in a short time, many lower level problems need to be solved at each iteration of the algorithm (at least, one for each electricity price instantiation and each follower) and so, a high computational effort is always required at the lower level.

In order to assess the quality of the solution obtained, we have considered the so-called high point relaxation, which transforms the BL model into a single-level problem by optimizing the upper level objective function over the region defined by all (upper and lower level) constraints. This problem was linearized by transforming the product of binary and continuous variables that appear in the objective function (1) in linear terms, using additional binary variables and constraints. This led to a (high dimension) MILP model, which was solved by Cplex. However, this information may have low practical relevance, since it corresponds to one of the worst options the follower could make regarding his interests, which would result in a very high and unrealistic retailer's profit. These results will be presented in section 5.3.

Two hybrid population-based approaches are proposed to solve the BLMF model presented in section 3. These approaches use population-based metaheuristics (PSO and GA) to determine the optimal values of the upper level decision variables ((1) – (5)) and call an external MILP solver (*cplex*) to compute the optimal solutions to the lower level problems ((6) – (15)).

Population-based algorithms allow the simultaneous analysis of different electricity prices set by the retailer (upper level variables) for the model described in section 3. For each upper level variable setting, the solver obtains the optimal scheduling plans (lower level variables) for flexible loads.

The proposed hybrid algorithms present a similar structure. The algorithms start by creating an initial population of M individuals $x = (x_1, \ldots, x_I)$ representing the prices of electricity set by the retailer in the I sub-periods of the planning period, which should satisfy the upper level constrains (2)-(4). For this purpose, each $x_i, i \in \{1, \ldots, I\}$ is randomly generated in the range $\left[\underline{x_i}, \overline{x_i}\right]$, as imposed by the upper level constraints (2) and (3), which specify minimum and maximum prices in each sub-period. To ensure that each x also satisfies the upper level constraint (4), which imposes an average price in the planning period, the repair routine described in the sub-section 4.3 is applied. Then, for each individual x, the lower level problems (6)-(15) for $k = 1, \ldots, N$ are solved by *cplex*. The solutions should satisfy the upper level constraint (5) associated with the overall power requested. Therefore, after solving the lower level problem for all followers and aggregating the corresponding variable values to obtain the complete lower level solution, the algorithm checks the upper level constraint (5); only solutions satisfying all the upper level constraints are retained. The solutions at the lower level (w_{kjrt} and p_{kjt}, for consumer cluster k, shiftable load j, time t of the planning period and stage r of the working cycle of load j) indicate the time each shiftable load of each consumer cluster starts its operation cycle, $y_k = \left(y_{k_1}, \ldots, y_{k_{J_k}}\right)$. Each solution $(x, y_1, \ldots, y_N)$ to the BLMF model is then evaluated by the upper level objective function given by equation (1), $F(x, y_1, \ldots, y_N)$ (retailer's profit).

The algorithms run a pre-set number of iterations, Q; the best solution in the population in each iteration $q \in \{1, \ldots, Q\}$ of each algorithm represents the best combination of electricity prices (x^{best^q}) that results in the highest profit (F^{best^q}) for the retailer.

4.1. *Genetic Algorithm*

After randomly creating the initial population of M solutions $x = (x_1, \ldots, x_I)$ and evaluating their fitness $F(x, y_1, \ldots, y_N)$, the GA generates the offspring population. To generate each offspring, the algorithm randomly selects two individuals of the current population and the individual with the best F value is selected to be one parent. The other parent is randomly selected from the current population. The parent solutions are then subject to crossover to generate an offspring x^c, having both solutions the same probability of being the first or the second parent. If x' and x'' are the first and the second parent solutions, the one-point crossover operator produces the child $x^c = (x'_1, \ldots, x'_\varsigma, x''_{\varsigma+1}, \ldots, x''_I), \varsigma \in [2, I-1]$.

Then, a mutation operator with an adaptive probability is applied to x^c. For each position $x_i^c, i \in \{1, \ldots, I\}$ of x^c, a perturbation γ randomly generated in the range $\left[-\delta\left(\overline{x_i} - \underline{x_i}\right), \delta\left(\overline{x_i} - \underline{x_i}\right)\right]$ is added to x_i^c, i.e. $x_i^c \leftarrow x_i^c + \gamma$. Each run of the algorithm is initialized with a mutation probability p_m^0; if the value of F^{best} does not improve over a predefined number L of consecutive iterations $q \in \{1, \ldots, Q\}$, then the exploration capability is enhanced by increasing the mutation probability to $p_m^1 > p_m^0$. The value of the mutation probability decreases back to the previous value, p_m^0, just when F^{best} changes above a given threshold. For that purpose, it is considered that the change in F^{best} does not lead to a change in the mutation probability if inequality (16) is satisfied:

$$\frac{F^{best^q} - F^{best^{q-1}}}{F^{best^q}} < \tau \tag{16}$$

being τ the threshold value.

Several experiments were performed to assess the capability of this operator to enhance the upper level search. This adaptive mutation scheme led to improvements in the solutions obtained. In most runs (about 70%), this adaptive mutation led to better F^{best} values. The average improvement was around 0.5% after 100 generations. The F^{best} values started to be consistently better from around generation 65 onwards.

A repair routine is then applied to x^c (see subsection 4.3) to ensure that upper-level constraints (2)-(4) are satisfied. This process is repeated until M children are generated constituting the offspring population, which will compete with the current population to generate a new population for the next generation.

For each offspring solution x^c, the lower level problems (6)-(15) for $k = 1, \ldots, N$ are solved. The optimal solution obtained for the lower level problems is $y^c = (y^c{}_1, \ldots, y^c{}_N)$. The solutions (x^c, y^c) such that y^c does not satisfy the overall power requested from the grid (upper level constraint (5)) are not valid and, thus, they are discarded. Then, the upper level objective function is evaluated for each feasible (x^c, y^c) according to (1), $F(x^c, y^c) = F(x^c, {y^c}_1, \dots, {y^c}_N)$.

To select the population for the next generation, first the solution with the best F value, either in the current population or in the offspring set, is selected to the new population. This strategy guarantees that the best individual is preserved from one generation to the next. The remaining $M - 1$ individuals are selected by binary tournament without replacement between an individual from the current population and another one from the offspring, both randomly selected. The individual selected to integrate the new population is removed from its original pool (either from offspring or current population) preventing the repeated selection of the same individual.

The process is repeated until a given number of iterations is reached. The output of the algorithm is the solution with the highest retailer's profit.

4.2. Particle Swarm Optimization Algorithm

The PSO algorithm works with a population of candidate solutions, generally called swarm of particles. Each particle has a unique position in the search space. PSO solves an optimization problem by considering the movements of each particle in the search space. The velocity vector v of each particle x is influenced by the best position in the search space visited by this particle (individual best – x^{best}) and this is guided towards the best known position of the entire swarm (global best – g^{best}). In each iteration, the position of each particle is updated according to its own velocity vector. This procedure, guided by x^{best} and g^{best}, is expected to move the swarm towards the best solutions.

After randomly creating an initial population of M solutions $x = (x_1, \dots, x_I)$ and evaluating their fitness $F(x, y_1, \dots, y_N)$, solutions are changed from one iteration to the next by moving them into new positions. For each coordinate $i \in \{1, \dots, I\}$ of each solution x, the corresponding velocity component v_i for the next iteration q is given by the following equation:

$$ {v_i}^q = \eta {v_i}^{q-1} + \gamma_1 C_1 \left(x_i^{best} - {x_i}^{q-1}\right) + \gamma_2 C_2 \left(g_i^{best} - {x_i}^{q-1}\right) $$

where η is the inertia weight, C_1 and C_2 are the cognitive and social parameters, γ_1 and γ_2 are random numbers in the interval [0,1]. The new position of the particle (new solution x) is then given by:

$$ x^q = x^{q-1} + v^q $$

The individual best, x^{best}, and the global best, g^{best}, are updated whenever better positions are found by each individual solution x and the best particle of the swarm, respectively. Particles interact with each other, as they learn from their own experience, and gradually the members of the population move to better regions of the search space.

Similarly to GA, when the value of F^{best} does not improve over a predefined number L of consecutive iterations, i.e. when the previous inequality (16) is satisfied through L consecutive iterations, then some turbulence in the population is induced with a probability p_m. Therefore, for each solution x^q, after its movement has been performed as described above, each coordinate ${x_i}^q, i \in [1, I]$ is subject to a perturbation γ randomly generated in the range $\left[-\delta\left(\overline{x_i} - \underline{x_i}\right), \delta\left(\overline{x_i} - \underline{x_i}\right)\right]$, i.e., ${x_i}^q \leftarrow {x_i}^q + \gamma$.

The repair routine (see subsection 4.3.) is then applied to each solution *x* to ensure that upper-level constraints (2)-(4) are satisfied. For each updated and repaired solution, the lower level problems (6)-(15) are solved for the *N* followers and the optimal solution $y = (y_1, \dots, y_N)$ is obtained. Solutions that do not satisfy the upper level constraint (5) are not allowed to move, returning to their previous positions. Next, the upper level objective function (1) is evaluated for each (x, y), $F(x, y) = F(x, y_1, \dots, y_N)$.

This process is repeated for all solutions of the population until a given number of iterations is reached. The output of the algorithm is the solution $(x, y) = (x, y_1, \dots, y_N)$ such that $x = g^{best}$, which gives the highest retailer's profit $F(x, y) = F(x, y_1, \dots, y_N)$.

4.3. Repair routine for variables with fixed number of decimals

Each component x_i of *x* should satisfy constraints (2) – (3) and *x* must satisfy the average price equality constraint (4). At the beginning, each coordinate x_i is randomly generated in the range $\left[\underline{x_i}, \overline{x_i}\right]$ satisfying constraints (2) and (3). At each iteration, if x_i is out of the bounds imposed by such constraints then it is pushed to the closest bound. To ensure that *x* also satisfies constraint (4), the repair routine described below is applied.

Since each x_i defines an electricity price for a given period P_i, $i \in \{1, \dots, I\}$, i.e. they are components of a commercial offer of a retailer, a pre-defined number of decimal places must be considered (5 in our study). In this context, electricity prices can be seen as integer variables within a scale factor. The repair routine was designed taking into account this condition, which leads to additional computational difficulties, namely with respect to the repair routine proposed in (Alves et al. 2016) that did not consider this additional condition. For each solution of electricity prices *x*, the repair routine allows a tolerance for the upper level constraint (4). A deviation of ε in relation to the average of electricity prices is allowed - the average price of *x* can be lower but not higher than the one imposed in the model (i.e., the average price communicated to consumers). The aim is to stay as close as possible to the average price, but if it is not possible to meet it exactly, the optimal electricity prices *x* should not disappoint the consumer.

Repair Routine

1. $s = \sum_{i=1}^{I} \overline{P_i}\, x_i$
2. $A = \{1, \dots, I\}$ (A is the set of indices i of x_i that are allowed to be changed)
3. while $A \neq \emptyset$ then
4. if $s < (x^{AVG} - \varepsilon)T$ or $s > (x^{AVG})T$ then

5. $\nabla = T(x^{AVG} - \varepsilon/2) - s$
6. $P = \sum_{i \in A} \overline{P_i}$
7. for $x_i, i \in A$ do
8. $x_i \leftarrow x_i + \frac{\nabla}{P}$
9. end for
10. end if
11. for $i = 1, \dots, I$ do
12. if $x_i < \underline{x_i}$ then
13. $x_i \leftarrow \underline{x_i}$ and $A \leftarrow A \backslash \{i\}$
14. else if $x_i > \overline{x_i}$ then
15. $x_i \leftarrow \overline{x_i}$ and $A \leftarrow A \backslash \{i\}$
16. end if
17. end for
18. $s = \sum_{i=1}^{I} \overline{P_i}\, x_i$
19. if $s \in [(x^{AVG} - \varepsilon)T, (x^{AVG})T]$ then
20. Stop and return x
21. else if $A = \emptyset$ then
22. Stop and discard x
23. else
24. Go to 4.
25. end if
26. end while

5. Experimental results and discussion

Given the complexity of solving a BL optimization model with multiple followers, the main focus of the current work is the algorithmic comparison between the two population-based optimization algorithms, GA and PSO, coupled with a MILP solver presented in the previous section. In the following, the problem data and the model features are described, and the results obtained are presented and analyzed.

5.1. Problem data

A planning period of 24 hours was considered, which was split into intervals of 15 minutes, generating a planning period of 96 units of time, i.e., $\bar{T} = \{1, \dots, 96\}$ where each unit $t \in \bar{T}$ represents a quarter-hour. The original data for periods of 1 minute were thus aggregated for the quarter-hour time units by considering average values of intervals of 15 minutes. Analogously, the prices in €/kWh were converted into quarter-hour periods (i.e. converted into €/kW$\hbar$ with $\hbar = {}^1/_4$ h)

In order to perform a comparative analysis between the hybrid GA and PSO algorithms, a number of realistic problems were generated. First, four consumer profiles and four retailer instances were created. The electricity prices (table 1) are based on time-of-use tariffs presently used in Portugal, incorporating also information from ongoing pilot projects of dynamic tariff components (namely concerning network access terms). Comfort slots for appliance operation (table 2) are derived from energy audits in previous R&D projects and time use surveys. The power required by the appliance operation cycles (table 3) result from manufacturers data and energy audits carried out in previous projects.

Then, in order to assess different combinations of consumer profiles and retailers, ten problems were generated, each one consisting of a retailer and three consumer profiles randomly selected among those previously defined. Note that the retailers are not competitors, but distinct instances of leaders created for experimental purposes.

The electricity prices ($\pi_t,\ t \in \{1,\dots,96\}$, in €/kWh) that the retailers pay in the spot market can be seen in Table SM-1 (Supplementary Material). Regarding the electricity prices charged to the consumers by the retailer, tariffs with 7 sub-periods $P_i = \left[P_{1_i}, P_{2_i}\right] \subset \bar{T},\ i \in \{1,\dots,7\}$, were considered for all retailer instances. The lower limit of the sub-period $P_i,\ i \in \{2,\dots,7\}$, P_{1_i}, is always one unit of time above the upper limit of the previous sub-period, i.e. $P_{1_i} = P_{2_{i-1}} + 1$. For the first sub-period $P_{1_1} = 1$.

Table 1 displays the minimum, the maximum and the average electricity prices ($\underline{x_i}$, $\overline{x_i}$ and x^{AVG}, respectively) to be charged to consumers in each sub-period $P_i,\ i \in \{1,\dots,7\}$ for the four retailer instances. The maximum power requested to the grid C_t =2500W, $t \in \{1,\dots,T\}$, for each retailer instance.

Table 1. Minimum ($\underline{x_i}$), maximum ($\overline{x_i}$) and average (x^{AVG}) of the electricity prices (in €/kWh) that can be charged to the consumers by all retailer instances, in each sub-period $P_i,\ i \in \{1,\dots,7\}$.

Prices (€/kWh)		P_1 [1,28]	P_2 [29,38]	P_3 [39,44]	P_4 [45,60]	P_5 [61,76]	P_6 [77,84]	P_7 [85,96]	x^{AVG}
Retailer 1	$\underline{x_i}$	0.039	0.081	0.048	0.090	0.045	0.099	0.042	0.116
	$\overline{x_i}$	0.102	0.270	0.210	0.282	0.150	0.300	0.111	
Retailer 2	$\underline{x_i}$	0.030	0.075	0.040	0.088	0.041	0.092	0.038	0.098
	$\overline{x_i}$	0.092	0.200	0.170	0.240	0.135	0.270	0.108	
Retailer 3	$\underline{x_i}$	0.050	0.090	0.055	0.120	0.053	0.102	0.048	0.140
	$\overline{x_i}$	0.092	0.200	0.170	0.240	0.135	0.270	0.108	
Retailer 4	$\underline{x_i}$	0.050	0.090	0.055	0.120	0.053	0.102	0.048	0.158
	$\overline{x_i}$	0.110	0.300	0.240	0.320	0.178	0.342	0.118	

Regarding the consumer profiles, a total of five shiftable loads ($J = 5$) were considered: dishwasher (DW), laundry machine (LM), clothes dryer (CD), electric vehicle (EV) and electric water heater (EWH). The operation cycle of each controllable appliance (flexible load) can be scheduled within the planning period and the corresponding working slot can be shifted according to the consumers' preferences and the electricity prices. The comfort time slots $\left[T_{1_{kj}}, T_{2_{kj}}\right] \subseteq \bar{T}$ allowed for the operation of each controllable appliance $j \in \{1, \dots, J_k\}$ of the consumer cluster k are displayed in Table 2. The power requested to the grid (f_{kjr}) by each controllable appliance $j \in \{1, \dots, J_k\}$ at stage $r \in \{1, \dots, d_{kj}\}$ of its operation cycle is displayed in Table 3.

Table 2. Comfort time slots, $\left[T_{1_{kj}}, T_{2_{kj}}\right]$ for $j \in \{1, \dots, J_k\}$, allowed for the operation cycles of the flexible loads of each consumer profile $k \in \{1, \dots, 4\}$.

Consumer Profiles	Shiftable Appliances				
	DW	LM	CD	EV	EWH
Profile 1	[1,36]	[32,60]	[76,96]	[1,48]	[24,40]
Profile 2	[1,31]	[32,58]	[70,91]		
Profile 3		[35,55]	[75,93]		
Profile 4		[1,36]	[75,93]	[4,48]	[32,60]

Table 3. Operation cycles of the controllable loads for all consumer profiles.

Consumer Profiles	Appliance	Power required by the appliance at each stage of its operation cycle (W)								
		1	2	3	4	5	6	7	8-31	32-36
Profile 1	DW	1724	1272	104	1676	799				
	LM	2040	1028	88	180	228	215			
	CD	1808	1740	282						
	EV	1500	1500	1500	1500	1500	1500	1500	1500	1500
	EWH	1500	1500	1500	1500	1500				
Profile 2	DW	1634	805	1047	974					
	LM	1704	404	195	219					
	CD	1808	1740	282						
Profile 3	LM	2040	1028	88	180	228	215			
	CD	1808	1763	1250	282					
Profile 4	LM	2040	1028	88	180	228	215			
	CD	1500	1500	1500	1500	1500				
	EV	1500	1500	1500	1500	1500	1500	1500	1500	
	EWH	1808	1763	1250	282					

Fig. 1 and Table SM-2 display the base load for all consumer profiles ($b_{kt}, k \in \{1, \dots, 4\}$ and $t \in \{1, \dots, 96\}$), which corresponds to the power requested to the grid by appliances that are not deemed for control. Consumer clusters 1 and 4 demand the same base load and thus the corresponding lines in the diagram overlap.

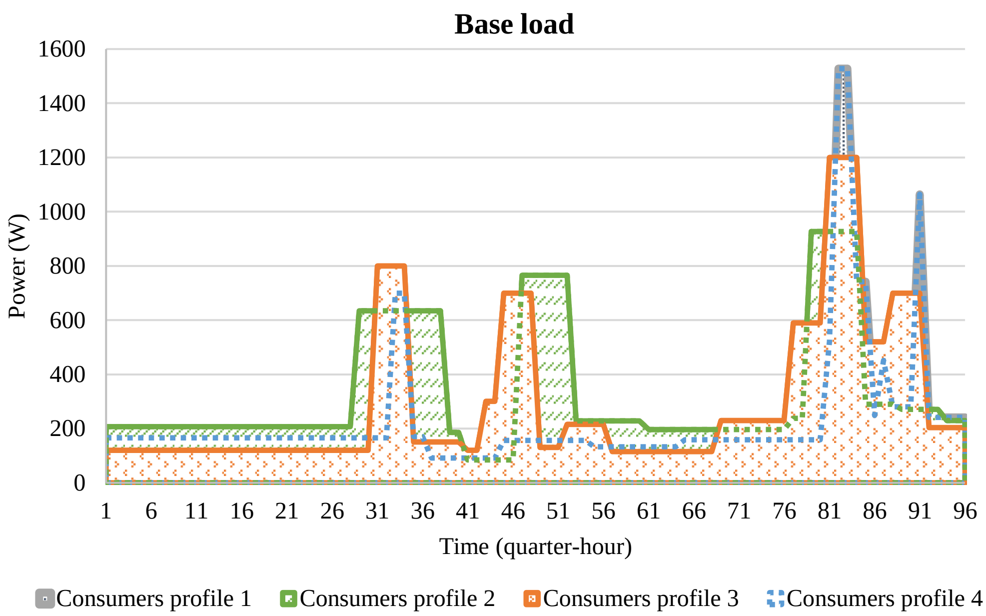

Fig. 1. Power requested to the grid by (non-controllable) base load for all consumer profiles.

Each consumer cluster has a given pattern of contracted power (C_{kt} in (9)) with two levels. For $t \in [28, 84]$, $C_{1t} = C_{3t} = 4600$ W and $C_{2t} = 4000$ W. For the other $t \in \bar{T}$, $C_{1t} = C_{3t} = 3000$ W and $C_{2t} = 2500$ W. For the consumer profile 4, $C_{4t} = 4800$ W, $t \in [29{,}88]$, and $C_{4t} = 2800$ W for the other $t \in \bar{T}$.

Table 4 summarizes the data of the ten problems randomly created with one leader and three followers, corresponding to distinct scenarios of retail pricing values and consumption patterns, for algorithm assessment purpose in a range of plausible scenarios with diversity of retailer prices and consumption patterns. The leader in each problem was randomly chosen among the four retailer instances defined. Similarly, three followers were randomly selected from the four possible consumer profiles.

Table 4. Retailer instances and consumer profiles of the ten problems (scenarios) randomly generated with one retailer and three consumer profiles.

Problems	Retailer instances				Consumer profiles			
	1	2	3	4	1	2	3	4
Problem 1	x				x	x		x
Problem 2			x			x	x	x
Problem 3		x			x	x		x
Problem 4		x			x	x	x	
Problem 5				x	x	x	x	
Problem 6	x					x	x	x
Problem 7		x				x	x	x
Problem 8	x				x		x	x
Problem 9				x	x	x		x
Problem 10			x		x		x	x

The number of lower level binary and continuous variables as well as the number of lower level constraints for each upper level variables configuration are displayed in Table 5. There are seven upper level variables (periods) for each resolution of the MILP lower level problems for the 3 followers.

Table 5. Dimension of the lower level problems

Consumer Profiles	Number of Variables		Number of Constraints	
	Binary (w_{kjrt})	Continuous (p_{kjt})	Inequality	Equality
Profile 1	2230	151	2281	206
Profile 2	298	80	389	91
Profile 3	202	40	292	50
Profile 4	1832	129	1890	175

5.2. *Parameters of the algorithms*

In the computational experiments, both hybrid algorithms GA and PSO were run 20 independent times for each problem. Each lower level MILP problem is solved to optimality by CPLEX for each price setting (and this resolution is very fast, about 1/9 sec). However, since stochastic optimizers are used to explore the upper level solution space, a sufficient number of independent runs of the hybrid algorithms should be performed to assess the statistical validity of conclusions.

Each run consisted of 100 iterations, with a population of 30 individuals (price settings). These values were tuned after experimentation regarding the algorithm performance vis-à-vis the number of iterations and population size. When the parameter Q increases above 100 iterations, slight or no improvements were observed. Nevertheless, decreasing the value of Q below 100 iterations could impair convergence. For the parameter M (population size in the GA and PSO algorithms), the values M=20 and M=30 have been tested and slightly better results were observed considering M=30. Time units of 5 minutes and 1 minute (respectively, $T = 288$ and $T = 1440$) were studied but the computational effort showed impracticable. Therefore, the planning period discretization in units of 15 minutes (which leads to $T = 96$) was considered in this study.

The two algorithms were implemented under similar configurations to ensure fair comparisons. The upper-level search parameters fixed for each algorithm are described below.

Both algorithms were written in R language and all the runs were carried out in a computer with an Intel Core i7-7700K CPU@3.6GHz and 64GB RAM. On average, the time spent in performing one iteration is similar in the two hybrid approaches – approximately 10 seconds. One iteration involves the upper level operations plus solving 90 MILP problems (30 individuals × 3 followers).

In both algorithms, $\tau = 0.001$ in (16) and the parameter δ = 0.2 in the mutation process. L=5 is the number of consecutive iterations without improvement of F^{best} that leads to the change of p_m in GA or to trigger mutation in PSO. The probability values adopted for the adaptive mutation in the GA were

$p_m^0 = 0.05$ and $p_m^1 = 0.1$, and the mutation probability considered in the PSO was p_m=0.1. In the PSO, the inertia parameter η was set to 0.4 and the learning parameters in both cognitive and social components were kept equal, $C_1 = C_2 = 1$.

In the repair routine, for each electricity prices solution, x, the parameter ε = 0.0001 in the two algorithms.

All parameters were set after experimentation, considering the quality of results obtained vs. the computational effort.

5.3. Results

The two population-based algorithms were tested on ten problems, which are summarized in Table 4. Information about the best solutions, the ones presenting the highest value for F, in both approaches is presented in Table 6. The maximum, minimum, median and the inter-quartile range (IQR) of F^{best} for the twenty runs in the ten problems are displayed for the GA and PSO algorithms.

Table 6. Statistics of F^{best} in 20 independent runs with 100 iterations each for the 10 problems.

Problems	Algorithm	F^{best}			
		Maximum	Minimum	Median	IQR
Problem 1	PSO	1294.579	1276.385	**1287.464**	7.123
	GA	1293.530	1259.623	1279.621	9.964
Problem 2	PSO	1015.265	1006.015	**1012.429**	0.534
	GA	1012.509	1005.650	1010.117	3.355
Problem 3	PSO	877.167	844.637	**873.188**	8.882
	GA	871.06	847.116	862.671	9.063
Problem 4	PSO	662.298	633.622	**660.114**	1.043
	GA	658.491	632.551	641.738	13.361
Problem 5	PSO	1352.069	1347.120	**1349.983**	1.983
	GA	1350.940	1331.664	1345.397	4.820
Problem 6	PSO	847.374	832.703	**846.503**	2.304
	GA	841.942	815.291	832.324	2.075
Problem 7	PSO	545.298	540.829	**542.684**	1.910
	GA	541.115	515.228	530.517	6.475
Problem 8	PSO	1251.595	1229.085	**1248.637**	7.129
	GA	1250.669	1223.703	1243.225	6.541
Problem 9	PSO	1897.617	1874.349	**1894.486**	4.710
	GA	1896.272	1867.233	1889.448	12.780
Problem 10	PSO	1336.982	1335.057	**1335.649**	1.071
	GA	1334.953	1327.800	1334.071	1.772

The Mann-Whitney test for the median at the 5% significance level was performed for each problem, indicating a statistically significant difference between PSO and GA for all problems, with PSO being better than GA in all problems. The better median values are displayed in bold in Table 6.

Table 6 shows that the PSO algorithm obtained the best solutions in all problems, regarding the maximum and the median values of F^{best} values over the 20 runs. The minimum value of F^{best} is also higher for PSO in almost all problems. Only Problem 3 displays a minimum value of F^{best} slightly smaller for PSO than for GA. The information in Table 6 also shows that the PSO algorithm has a smaller variability across runs, as measured by IQR. PSO displays a marginally higher variability only in problems 6 and 8. In general, the results obtained reveal the robustness of the PSO algorithm to obtain consistently better solutions than those reached by GA.

Fig. 2 shows the behavior of F^{best} over 100 iterations considering the average value of the different runs for the GA and PSO algorithms.

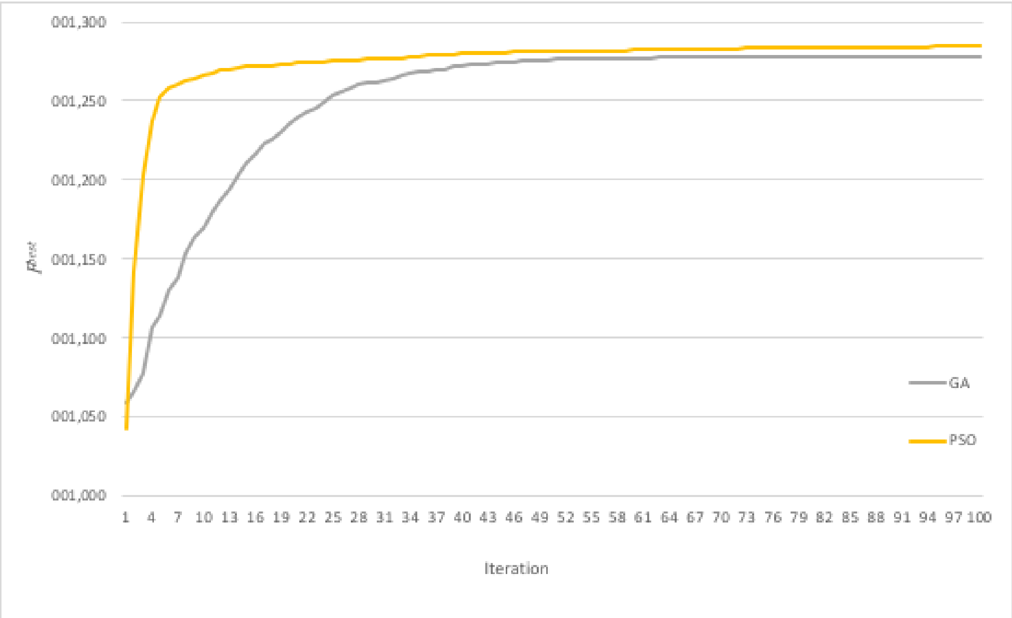

Fig. 2. Behavior of the average F^{best} over the iterations for the GA and PSO algorithms.

Since the pattern of results is similar on the ten problems, with PSO consistently better than GA, the description and analysis are also quite similar. Therefore, Problem 1 is explored with more detail. The results for the remaining problems are included as supplementary material.

Regarding the PSO algorithm, the solution with the maximum F^{best} value in the twenty runs of Problem 1 presents a retailer's profit $F = 1294.579$€ and consumers' costs $f_1 = 3433.438$€, $f_2 = 1445.757$€ and $f_4 = 2672.778$€ for the three consumer profiles involved. Concerning the GA, the solution of Problem 1 with maximum F^{best} value in the 20 runs results in a retailer's profit F of 1293.530€ and consumers' costs of $f_1 = 3431.87$, $f_2 = 1438.706$ and $f_4 = 2671.659$. The electricity prices charged by the retailer to the consumers in each of the seven sub-periods of the planning period time (in €/KWh) are displayed in Table 7 (upper-level solutions). Table 8 shows the appliance working intervals of the best solution of each algorithm for each consumer profile $k \in \{1,2,4\}$

considered in the Problem 1 (lower-level solutions). The corresponding schedule plans are illustrated in load diagrams of Fig. 3.

Table 7. Electricity prices (€/kWh) in the seven sub-periods of $\overline{T}$ that result in the maximum F^{best} value for the 20 runs of Problem 1.

Prices (€/kWh)	Algorithm	P_1 [1,28]	P_2 [29,38]	P_3 [39,44]	P_4 [45,60]	P_5 [61,76]	P_6 [77,84]	P_7 [85,96]
Retailer 1	PSO	0.10200	0.26972	0.10260	0.10296	0.04500	0.26164	0.04200
	GA	0.10200	0.27000	0.10296	0.10360	0.04688	0.24968	0.04640

Table 8. Working intervals, [initial time, final time], for the operation cycles of the controllable appliances for each consumer profile $k \in \{1,2,4\}$ of Problem 1.

Consumer Profiles	Algorithm	Shiftable Appliances				
		DW	LM	CD	EV	EWH
Profile 1	PSO	[1,5]	[41,46]	**[94,96]**	[5,40]	[36,40]
	GA	[1,5]	[41,46]	**[88,90]**	[5,40]	[36,40]
Profile 2	PSO	[1,4]	[39,42]	**[85,87]**		
	GA	[1,4]	[39,42]	**[88,90]**		
Profile 4	PSO		[1,6]	**[87,90]**	[4,35]	[39,44]
	GA		[1,6]	**[85,88]**	[4,35]	[39,44]

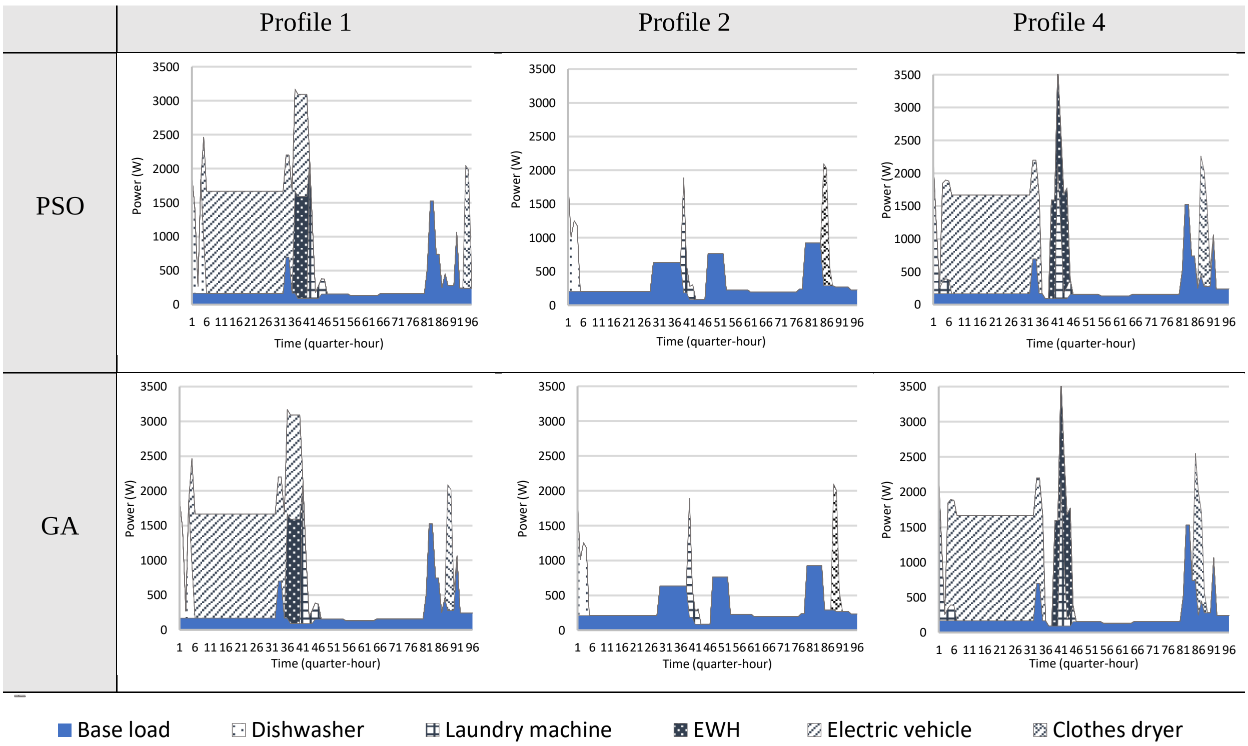

Fig. 3. Load diagrams corresponding to the best solution in both algorithms for the consumer profiles $k \in \{1,2,4\}$ of Problem 1.

The PSO algorithm outperforms GA since it consistently provided the highest profit to the retailer. In what concerns the load diagrams of the schedule plans of each consumer profile in both algorithms, the difference for the common controllable appliances resides in the starting time of the clothes dryer operation (working intervals marked in bold in Table 8). However, the times set by the two algorithms to begin the work cycle of this appliance are inside the same sub-period of electricity prices defined by the retailer instance of Problem 1.

Notice that the BL problem is the leader's problem and thus the optimal solution is the one resulting in the highest retailer's profit. The consumers react to the electricity prices defined by the retailer by scheduling the loads. The optimal solution for the retailer does not have to lead to global minimum costs to the consumers. On the contrary, since the retailer seeks to maximize its revenue, the consumers' cost is generally higher in the bilevel optimal solution than in other feasible solutions (i.e., solutions that optimize the lower level problems for different pricing settings).

Although the BL problem is seen from the retailer's perspective, a further analysis of the solutions from the consumers' point of view can be performed. Considering Problem 1, some experiments with a flat tariff were done. The idea is to mimic the retail electricity market in which consumers are allowed to choose the tariff that better suits their needs. Different commercial offers with a fixed tariff for whole planning period were assessed, namely adopting a flat tariff equal to the average electricity price defined by the retailer and considering flat tariffs giving the retailer the same revenue as the one obtained with the PSO and GA algorithms. Table 9 shows the consumers' costs for two different flat tariffs: $x^{FT\text{-}avg}$ =0.116 €/kWh, which is equal to the x^{AVG} value set by Retailer 1, the retailer considered in Problem 1; $x^{FT\text{-}TR}$ =0.13463 €/kWh, the price for a flat tariff that would provide the retailer a Total Revenue similar to the one obtained with the best solution given by PSO for the dynamic tariff (whose prices are shown in the first row of Table 7). We have considered herein the solution given by PSO to define the flat tariff but, if the solution given by GA was used instead this price would be 0.13445 €/kWh, which is very close to the one given by PSO. The consumers' costs for the best retailer's solution obtained with the two hybrid algorithms with dynamic tariffs (solutions in Table 8 for prices in Table 7) are also presented.

Table 9. Costs for each consumer profile $k \in \{1,2,4\}$ (€/kWh) in Problem 1.

Costs (€/kWh)	Flat tariff (€/kWh)		Dynamic tariff (€/kWh)	
			PSO	GA
	$x^{FT\text{-}avg}$ =0.116	$x^{FT\text{-}TR}$ =0.13463	x^{AVG} = 0.116	
Profile 1	2806.562	3257.191	3433.438	3431.871
Profile 2	1236.299	1434.803	1445.757	1438.706
Profile 4	2464.304	2859. 979	2672.778	2671.659

Table 9 shows that consumer profiles 1 and 2 are better off with a flat tariff while consumer profile 4 profits from a dynamic tariff in comparison with the flat tariff defined assuming the same retailer's revenue (i.e., 0.13463 €/kWh).

Further experiments were carried out considering different values of x^{AVG} higher than 0.116 €/kWh. Table 10 shows: the consumers' costs in the solutions obtained by PSO and GA for Problem 1 with x^{AVG} =0.165; the consumers' costs with a flat tariff of $x^{FT\text{-}avg}$ =0.165; and the consumers' costs with a flat tariff of $x^{FT\text{-}TR}$ =0.163, which has been defined on the basis that the retailer's total revenue is equal to the one obtained with the solution given by PSO for the dynamic tariff.

Table 10. Costs for each consumer profile $k \in \{1,2,4\}$ (€/kWh) in Problem 1 for x^{AVG} = 0.165.

Costs (€/kWh)	Flat tariff (€/kWh)		Dynamic tariff (€/kWh)	
			PSO	GA
	$x^{FT\text{-}avg}$ =0.165	$x^{FT\text{-}TR}$ =0.163	x^{AVG} = 0.165	
Profile 1	3992.093	3943.704	3968.282	3968.170
Profile 2	1758.529	1737.213	1956.422	1955.851
Profile 4	3505.260	3462.772	3218.726	3218.613

As can be seen in Table 10, when the average price increases, the comparison between dynamic tariffs and flat tariffs leads to different conclusions concerning consumers' outcomes. For x^{AVG} =0.165 vs. a flat tariff with equal price $x^{FT\text{-}avg}$, consumer profiles 1 and 4 gain with the dynamic tariff and only consumer profile 2 has a lower cost with the flat tariff. If the flat tariff is defined assuming the same retailer's revenue as in the dynamic tariff, the resulting price ($x^{FT\text{-}TR}$ =0.163) is lower than x^{AVG}. For this electricity price, profile 4 still profits from a dynamic tariff but consumer profiles 1 and 2 are better off with the flat tariff.

Note that, independently of the value of x^{AVG} considered, there are always some consumers that benefit from a flat tariff $x^{FT\text{-}TR}$ and others that get a higher cost in comparison with dynamic tariffs with an average price x^{AVG}, because the total revenue for the leader is kept constant.

It is noteworthy that for each combination of electricity prices decided by the retailer in the BLMF model, the upper level power constraints limit the power requested to the grid by all consumer clusters in each time unit. This can influence the corresponding appliances schedule, thus leading to higher costs for the consumers.

To further assess the quality of the solutions, the high point relaxation of the BL problem was solved for problem 1, giving the upper level objective function value of 2794.374. As expected, this information has low practical relevance (please compare with values in Table 6 for problem 1) since it corresponds to one of the worst options the follower would make regarding his interests. Therefore, this solution is not useful to assess the metaheuristic outcome because it is too far from the feasible solutions of the BL problem.

6. Conclusions

In this work, two BL hybrid population-based approaches were developed to deal with the interaction between a retailer (the leader) and multiple consumers (followers) in the retail electricity market. One approach is based on a GA and the other on PSO. The retailer controls the upper level variables of the BL model, the aim being to determine the optimal electricity prices that should be charged to the consumers in order to maximize his profit. Consumers react to these prices set by the retailer through the scheduling of their loads within time slots defined for appliance operation according to comfort requirements in order to minimize their electricity bills. The proposed algorithms use an exact MILP solver to obtain optimal solutions to the lower level problems for a given upper level solution. In spite of the lower level problems being high dimensional MILP problems, they were solved very fast by an external solver.

To assess the ability of the proposed algorithms to deal with this type of model, ten problems with distinct retailer instances and consumer profiles were generated. The results obtained consistently revealed that the PSO algorithm performed better than GA to compute the best solution. Also, PSO generally presented a smaller variability than GA. Only in two problems a higher variability of PSO was observed, but the corresponding IQR values are quite similar to those presented by GA. The results also enabled to assess the interest to consumers to engage in dynamic tariff schemes.

In the future, we intend to include other appliances into the model, namely interruptible and thermostatically controlled appliances in both single and multi-follower BL models. Further work will also contemplate expanding the model to consider other types of consumers beyond those of the residential sector. Algorithmic developments are underway to obtain better estimates for the leader's objective function upper bounds in order to evaluate more precisely the quality of solutions computed by metaheuristics and hybrid approaches.

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