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The school bus routing problem with student choice: a bilevel approach and a simple and effective metaheuristic

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Abstract

The school bus routing problem (SBRP) involves interrelated decisions such as selecting the bus stops, allocating the students to the selected bus stops, and designing the routes for transporting the students to the school taking into account the bus capacity constraint, with the objective of minimizing the cost of the routes. This paper addresses the SBRP when the reaction of students to the selection of bus stops is taken into account, that is, when students are allowed to choose the selected bus stop that best suits them. A bilevel optimization model with multiple followers is formulated, and its transformation into a single-level mixed integer linear programming (MILP) model is proposed. A simple and effective metaheuristic algorithm is also developed to solve the problem. This algorithm involves solving four MILP problems at the beginning, which can be used to obtain tight upper bounds of the optimal solution. Extensive computational experiments on SBRP benchmark instances from the literature show the effectiveness of the proposed algorithm in terms of both the quality of the solution found and the required computing time.

Keywords: school bus routing problem; preferences; bilevel optimization; metaheuristic

1. Introduction

The school bus routing problem (SBRP) aims at determining a set of routes for picking up a given set of students from a set of potential bus stops and transporting them to the school. Generally, the objective is to minimize the cost of the routes taking into account various constraints, for example, on the capacity of the buses or on the maximum walking distance of the students, etc. Park and Kim (2010) and Ellegood et al. (2019) provide comprehensive reviews on the subject. Several papers

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have considered simultaneously the problems of selecting the bus stops to be visited among a set of potential locations, allocating the students to them, and designing the bus routes. The papers differ in the order in which they deal with the three subproblems involved—location (L), allocation (A), and routing (R)—as well as in the method proposed to solve the SBRP. Dulac et al. (1980) propose an LAR approach, while Chapleau et al. (1985) and Bowerman et al. (1995) develop ARL approaches. Schittekat et al. (2013) propose an LRA strategy in which, after selecting the bus stops and computing the routes, the allocation of students is solved by an exact method. Calvete et al. (2020b) develop an L-partial-AR approach in which the allocation is done gradually with the aim of keeping the routing cost low. Previous papers use heuristic or metaheuristic algorithms to solve the SBRP, sometimes combined with exact methods to solve some of the subproblems involved. Regarding the use of exact methods, Riera-Ledesma and Salazar-González (2012, 2013) propose several formulations of the problem as well as a branch-and-cut-and-price algorithm, while Kinable et al. (2014) develop a branch-and-price algorithm. It is worth mentioning that in none of these papers can the students choose the selected bus stop where they are picked up.

In this paper, we focus on the SBRP with bus stop selection when each student has an order of preference for the potential bus stops and is free to choose his/her most preferred selected bus stop. For this purpose, we consider a two-level decision process in which, at the upper level of the hierarchy, a decision maker (representing a central authority, the school, or the company picking up the students) selects from a set of potential stops the bus stops that are visited, and the routes that ensure the service to all the students aiming to minimize the routing cost. Besides the bus capacity, the decision maker needs to take into account that, at the lower level of the hierarchy, the students choose their most convenient selected bus stop. To facilitate the reading of the paper, in the following, we will use the term "select" when referring to the selection of the bus stops that are made available to students, whereas we will use the term "choose" when referring to the choice of a selected bus stop by a student. The routes will visit selected bus stops that are chosen by at least one student.

Bilevel optimization has been proposed in the literature to model decision processes involving decisions ranked in accordance with a hierarchical structure. A bilevel model is formulated as an optimization problem, which involves another optimization problem in the constraint set. An updated review on this subject can be found in Dempe (2018). Concerning the application of bilevel optimization in SBRP, Parvasi et al. (2017, 2019) formulate a bilevel optimization model that considers the possibility of predicting the student's response and follows an LAR strategy to solve it. They focus on the design of the public transportation system in which the upper level decision maker attempts to locate bus stops and routes buses to these stops while, based on this information, at the lower level, the students decide whether to use this system or choose other services. Hence, in these papers, students may be reluctant to choose any of the bus stops that are visited by a route and decide to use an alternative transportation system.

In a more general routing context, Chen et al. (2020) aim to optimize the existing bus routes and enhance the accessibility of nearby bus stops for seniors living in age-restricted communities. Seniors are considered to be the upper level decision makers, and transit agencies are regarded as the lower level decision makers. Sadati et al. (2020) formulate a bilevel optimization problem for the determination of the most critical depots in a vehicle routing context. The attacker is the decision maker in the upper level problem who chooses a number of depots to interdict with certainty. The defender is the decision maker in the lower level problem who optimizes the vehicle routes in the wake of the attack. Also, bilevel optimization has been used to reformulate well-known problems such as the capacitated vehicle routing problem (CVRP) (Marinakis et al., 2007) or the ring-star problem (Calvete et al., 2013) with the purpose of developing efficient algorithms. On the other hand, preferences have been considered in facility location problems. Hanjoul and Peeters (1987) and Hansen et al. (2004) analyze the discrete facility location problem when customers are free to choose an open facility. The model assumes that the decision maker knows the preference orderings of customers and takes them into account when deciding which facilities should be opened to minimize the total cost of opening facilities and allocating customers. Calvete et al. (2020a) generalize this problem to include cardinality constraints on the facilities. They propose two different approaches to this problem and a matheuristic based on the structure of an evolutionary algorithm.

The first contribution of this paper is to address the reaction of students to the selection of bus stops. This allows for a more realistic and fair transportation system in which the preferences of the students are needed. This means that the transportation system must ensure that it selects bus stops and establishes routes so that all students can take the bus at their preferred selected bus stop. This problem is modeled as a bilevel mixed integer programming problem with multiple followers, from now on called the bilevel SBRP with bus stop selection (B-SBRP-SS). Unlike Parvasi et al. (2017, 2019), we assume that the transportation system must accommodate all the students and every student must be able to choose a bus stop visited by a route in accordance with his/her preferences. Note that the preference ordering interacts with the bus capacity constraint since, after the selection of the bus stops, the students freely choose their preferred bus stop, and then the routes, visiting these bus stops, must accommodate all the students. Therefore, no bus stop chosen by a number of students exceeding the capacity of the bus may be selected. Taking into account the special structure of the lower level problems, the B-SBRP-SS is then reformulated as a single-level mixed-integer linear programming (MILP) model. The second contribution of the paper is to develop a simple and effective metaheuristic called bilevel school bus metaheuristic (BSBM), which shows a very good performance in terms of the quality of the solution found and of the computing time involved.

The paper is organized as follows. Section 2 presents the problem and formulates the bilevel mixed integer programming model and its transformation as a single-level MILP model. In Section 3, the proposed metaheuristic is described. Section 4 assesses the computational performance of the algorithm. Moreover, some insight is given into the implications of the bilevel model, and the behavior of the model with respect to the number of bus stops accessible to students. Finally, Section 5 concludes the paper with some final remarks.

2. Formulation of the B-SBRP-SS

Let $G = (V, A)$ be a complete directed graph, where V is the node set and A is the arc set. The node set is defined as $V = \{0\} \cup W$, where node 0 represents the depot and W is the set of potential bus stops. The set of arcs is defined as $A = \{(i, j) : i, j \in V, i \neq j\}$, where the arcs refer to the links which are used to construct the routes.

There is a nonnegative routing cost c_{ij} associated with each arc $(i, j) \in A$, representing the cost of connecting node i with node j. The cost c_{0i}, $i \in W$, includes the routing cost from the depot to the bus stop i. Similarly, c_{i0}, $i \in W$, includes the routing cost from the bus stop i to the school and from there to the depot. Each route starts at the depot, visits a subset of bus stops, and finishes visiting the depot. The routes are node-disjoint, except for the depot, at which there is a fleet of identical buses, each with a fixed capacity Q (representing the maximum number of students that can be transported by a bus). Each bus may perform a single route. We assume that there is no limit on the maximum number of buses used.

Let U denote the set of students. We assume that each student $k \in U$ has a known set $W_k \subseteq W$ of bus stops that he/she can access. Moreover, the students have ranked their accessible bus stops from the best to the worst, that is, each student $k \in U$ has a set of predefined nonnegative distinct preferences $p_{ki} \in \{1, \dots, |W_k|\}$, $i \in W_k$, where $|W_k|$ stands for the cardinality of W_k. We assume that the smaller the value, the greater is the preference. Also, for each bus stop $i \in W$, let U_i denote the set of students who can access the bus stop i, that is, $U_i = \{k \in U : i \in W_k\}$.

The goal of the B-SBRP-SS is to select a subset of the set of potential bus stops and to build the routes that connect them in order to minimize the total routing cost, while the bus capacity constraint and the reaction of the students in terms of their preferences are taken into account.

In order to formulate the B-SBRP-SS as a bilevel mixed integer optimization model, we define the upper level decision variables:

$$\text{for } i \in W, \qquad z_i = \begin{cases} 1, & \text{if bus stop } i \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_i = \text{number of students in the bus after visiting the bus stop } i;$$

$$\text{for } (i,j) \in A, \quad x_{ij} = \begin{cases} 1, & \text{if arc } (i,j) \text{ is in a route} \\ 0, & \text{otherwise;} \end{cases}$$

and the lower level decision variables:

$$\text{for } k \in U, i \in W_k, \quad y_{ki} = \begin{cases} 1, & \text{if student } k \text{ chooses bus stop } i \\ 0, & \text{otherwise.} \end{cases}$$

To simplify the notation, we denote $\{z_i,\ i \in W\}$, $\{\delta_i,\ i \in W\}$, $\{x_{ij},\ (i,j) \in A\}$, and $\{y_{ki},\ k \in U, i \in W_k\}$, respectively, by z, δ, x, and y. Then, the B-SBRP-SS can be formulated as follows:

$$\min_{z,\delta,x} \quad \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1a}$$

s.t.

$$\sum_{i \in V:(i,j)\in A} x_{ij} = z_j, \quad j \in W \tag{1b}$$

$$\sum_{j \in V:(i,j)\in A} x_{ij} = z_i, \quad i \in W \tag{1c}$$

$$\sum_{i\in W} x_{0i} = \sum_{i \in W} x_{i0} \tag{1d}$$

$$\delta_i \leq Q z_i, \quad i \in W \tag{1e}$$

$$\sum_{k \in U_i} y_{ki} \leq \delta_i, \quad i \in W \tag{1f}$$

$$\delta_i - \delta_j + \sum_{k \in U_j} y_{kj} \leq Q(1 - x_{ij}), \quad (i, j) \in A \tag{1g}$$

$$z_i \in \{0, 1\}, \ \delta_i \geq 0 \quad i \in W \tag{1h}$$

$$x_{ij} \in \{0, 1\}, \quad (i, j) \in A, \tag{1i}$$

where, for each student $k \in U$, the variables y_{ki}, $i \in W_k$, solve:

$$\min_{y_{ki}, i \in W_k} \quad \sum_{i \in W_k} p_{ki} y_{ki} \tag{1j}$$

s.t.

$$\sum_{i \in W_k} y_{ki} = 1 \tag{1k}$$

$$y_{ki} \leq z_i, \quad i \in W_k \tag{1l}$$

$$y_{ki} \in \{0, \ 1\}, \quad i \in W_k. \tag{1m}$$

The objective function (1a) minimizes the routing cost. Constraints (1b) and (1c) enforce that exactly one arc enters and leaves each bus stop if and only if the bus stop is selected. Constraint (1d) ensures that as many buses leave the depot as enter the depot. Constraints (1e)–(1g) guarantee connectivity of routes as well as capacity requirements. It is worth pointing out that constraints (1f) and (1g) are coupling constraints, that is, they involve upper and lower level variables. Bilevel optimization problems are very sensitive to coupling constraints, as has been highlighted in Audet et al. (2006), Calvete and Galé (2007), and Mersha and Dempe (2006). Constraints (1h) and (1i) ensure the requirements of the decision variables (z, δ, x). Problem (1) involves $|U|$ optimization problems in the set of constraints, one for each student. The lower level problem corresponding to the student $k \in U$ is defined by (1j)–(1m). The objective function (1j) minimizes the preference of the bus stop chosen by the student k. Constraints (1k) and (1l) guarantee that the student k chooses a single bus stop, which has been selected by the upper level decision maker. Constraints (1m) ensure that the variables y_{ki}, $i \in W_k$ are binary. Note that the lower level problems are independent in the sense that each of them involves only the lower level decision variables of the corresponding student and the upper level variables. These bilevel problems with multiple followers have been analyzed in Calvete and Galé (2007), where it was shown that the $|U|$ lower level problems can be transformed into a single lower level problem whose objective function is the sum of the $|U|$ objective functions and whose set of constraints involves all the constraints together.

Problem (1) guarantees that every student takes the bus at his/her most preferred selected bus stop. Note that if a bus stop is selected, that is, it can be visited by a route, due to the lower level problems every student for which this bus stop is the most preferred among all selected bus stops is guaranteed to choose it. As a consequence of coupling constraints (1f) and (1g) in which the capacity of the bus is involved, only those values of the variables (z, δ, x) for which values of the variables y exist so that (z, δ, x, y) is a feasible solution are permissible. Therefore, those (z, δ, x) for which the optimal solutions of the lower level problems assign more than Q students to a selected bus stop are not permissible and should be rejected. Moreover, those (z, δ, x) for which there is not at least one accessible bus stop for every student must also be rejected. These properties will be efficiently used by the proposed algorithm.

In order to exactly solve problem (1), we reformulate it as a single-level optimization problem. For this purpose, we transform the lower level problem (1j)–(1m) corresponding to every student $k \in U$. For a permissible $(\tilde{z}, \tilde{\delta}, \tilde{x})$ and a student $k \in U$, let $I_k = \{i \in W_k : \tilde{z}_i = 1\}$. On the one hand, due to constraints (1l), $\tilde{y}_{ki} = 0$ for all $i \in W_k \setminus I_k$. On the other hand, the linear programming problem (2) defined in the following has a unique optimal solution, which is integer, due to constraint (2b) and the fact that the preferences are distinct numbers.

$$\min_{y_{ki}, i \in I_k} \quad \sum_{i \in I_k} p_{ki} y_{ki} \tag{2a}$$

s.t.

$$\sum_{i \in I_k} y_{ki} = 1 \tag{2b}$$

$$y_{ki} \geq 0, \quad i \in I_k. \tag{2c}$$

Therefore, problem (2) provides the optimal solution of the lower level problem (1j)–(1m) corresponding to the student $k \in U$. Taking into account either the duality theory or the special characteristics of problem (2), a feasible solution $\tilde{y}_{ki}$, $i \in I_k$ to this problem is an optimal solution if and only if it satisfies:

$$\sum_{i \in I_k} p_{ki} \tilde{y}_{ki} \leq p_{kj}, \quad j \in I_k. \tag{3}$$

Therefore, by substituting for each student $k \in U$ the lower level problem (1j)–(1m) in problem (1) with $\tilde{y}_{ki} = 0$ for all $i \in W_k \setminus I_k$ and constraints (2b), (2c), and (3) for all $i \in I_k$, the B-SBRP-SS can be reformulated as the following single-level MILP model:

$$\min_{z,\delta,x,y} \quad \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{4a}$$

s.t.

$$(1b) - (1i) \tag{4b}$$

$$\sum_{i \in W_k} y_{ki} = 1, \quad k \in U \tag{4c}$$

$$y_{ki} \leq z_i, \quad k \in U, i \in W_k \tag{4d}$$

$$\sum_{i \in W_k} p_{ki} y_{ki} \leq p_{kj} z_j + M(1 - z_j), \quad k \in U, j \in W_k \tag{4e}$$

$$y_{ki} \in \{0,\ 1\}, \quad k \in U,\ i \in W_k, \tag{4f}$$

where M is a large enough positive constant to ensure that constraints (4e) are imposed only when the bus stop j is selected. In this problem, the number of bus stops $|W|$ is a valid value for M. Note that, together with the feasibility of the lower level problem, which is enforced by constraints (4c), (4d), and (4f), constraints (4e) allow us to ensure that each student $k \in U$ chooses his/her preferred bus stop, as the lower level problem requires. Indeed, let $k \in U$ and $j \in W_k$ such that $z_j = 0$. Then, the bus stop j is not selected, so it has no students assigned. Since $\sum_{i \in W_k} y_{ki} = 1$ and y_{ki} are binary variables, it follows that

$$\sum_{i \in W_k} p_{ki} y_{ki} \leq |W_k| \leq |W|$$

and so (4e) is trivially held. Otherwise, if $z_j = 1$, j is a selected bus stop and the constraint

$$\sum_{i \in W_k} p_{ki} y_{ki} \leq p_{kj} \tag{5}$$

guarantees that the student k chooses either j or a selected bus stop with a better preference than that of j. Since constraint (5) needs to be satisfied for any $j \in W_k$ such that $z_j = 1$, the student k chooses his/her preferred bus stop.

3. BSBM: a metaheuristic algorithm for solving the B-SBRP-SS

BSBM constructs in each iteration a feasible solution of the B-SBRP-SS, usually called bilevel feasible solution. This consists of a selection of bus stops, the optimal allocation of students to these bus stops in accordance with their preferences, that is, the optimal solution of the lower level problems (1j)–(1m), and the construction of the routes taking into account the bus capacity constraint. Note that, because of the free choice of their selected bus stop by the students and the capacity constraint of the buses, not every selection of the bus stops z provides a permissible (z, δ, x). In fact, every bus stop which, if selected, would be chosen by more than Q students whatever the other selected stops were, should be removed. These bus stops cannot be selected in any bilevel feasible solution. After removing them, more bus stops with these characteristics could appear that would need to be removed. All these bus stops are banned stops. Moreover, if after removing all the banned stops there is some student $k \in U$ for which his/her accessible bus stops are all banned, we

Algorithm 1. Checking the feasibility of the B-SBRP-SS

$y_{ki} = 0, \quad k \in U, i \in W_k$;
$y_{kj} = 1, \quad k \in U, j = \text{argmin}\{p_{ki} : i \in W_k\}$;
$W_b = \emptyset$;
$\widetilde{W}_b = \{i \in W : \sum_{k \in U_i} y_{ki} > Q\}$;
while ($\widetilde{W}_b \neq \emptyset$) **do**
  $W_b = W_b \cup \widetilde{W}_b$;
  if ($W_k \setminus W_b = \emptyset$ for some $k \in U$) **then**
    return infeasible;
  else
    $y_{ki} = 0, \quad k \in U, i \in W_k \setminus W_b$;
    $y_{kj} = 1, \quad k \in U, j = \text{argmin}\{p_{ki} : i \in W_k \setminus W_b\}$;
    $\widetilde{W}_b = \{i \in W \setminus W_b : \sum_{k \in U_i} y_{ki} > Q\}$;
  end if
end while
return feasible;

can conclude that the B-SBRP-SS is infeasible. This fact is checked by the proposed metaheuristic in the initialization step described in Section 3.1.

If the B-SBRP-SS is feasible, each iteration of BSBM consists of two main steps. In the first step, described in Section 3.2, the algorithm selects a subset of the set of potential bus stops and allows the students to choose their preferred bus stop among the selected ones. Note that, since the lower level problems only depend on the values of variables z and y, the students can choose their preferred accessible bus stop after knowing the selected bus stops. In the second step, described in Section 3.3, after knowing the number of students who take the bus at each bus stop, the routes are computed by solving the corresponding CVRP and applying some local search procedures.

The algorithm proceeds by computing in each iteration a bilevel feasible solution until the termination condition is met. Next, we explain the previously mentioned steps.

3.1. Initialization step: checking feasibility of the B-SBRP-SS

The purpose of the initialization step is to identify the set of banned stops W_b in order to remove them and to check if the B-SBRP-SS is feasible. The procedure is described in Algorithm 1. Assuming that all bus stops are enabled at the beginning, the algorithm starts by letting the students choose their preferred one and computing the banned bus stops (those chosen by more than Q students). Then, in each iteration, if for some student $k \in U$ all his/her accessible bus stops W_k have been declared banned, the algorithm finishes by concluding that the B-SBRP is infeasible. Otherwise, the procedure looks for the banned stops among the current available stops and updates W_b. At termination, if the B-SBRP-SS is feasible, the set of banned stops W_b is removed from the original W and, accordingly, the graph G and each derived set is updated.

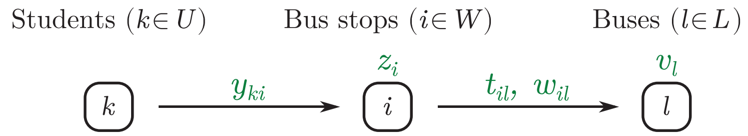


Fig. 1. A scheme of the variables in problem (6).

3.2. Selecting the set of active stops and allowing the students to choose their preferred bus stop

From now on, a selected bus stop will be called an active stop, and the set of active stops will be denoted by W_a. The algorithm involves two ways for selecting the set W_a. The first method, described in Section 3.2.1, involves computing the set W_a of active stops and assigning the students in accordance with their preferences by solving an MILP model. The second method, described in Section 3.2.2, involves the random selection of the bus stops together with a repairing process to ensure that a bilevel feasible solution can be obtained from them. In order to take advantage of the knowledge provided by the solutions that are being computed throughout the algorithm, a pool with the best bilevel feasible solutions found is maintained. This pool is initialized at the beginning with the empty set, and it is updated each time a better bilevel feasible solution is obtained.

3.2.1. Selecting W_a by solving an MILP

The idea is to solve an MILP problem to identify the set of active stops and the active stop preferred by each student without explicitly computing the routes. This method is applied in the first four iterations of BSBM, in which four MILP problems with the same set of constraints and different objective functions are solved. Each objective function aims to minimize an easily quantifiable cost related to the routing cost. We have found that the proposed problems lead, in general, to good bilevel feasible solutions (supported by the computational results reported in Section 4).

In addition to the variables y and z already introduced, we define the following variables (see Fig. 1):

$$v_l = \begin{cases} 1 & \text{if bus } l \text{ is used} \\ 0 & \text{otherwise} \end{cases} \qquad l \in L$$

$$w_{il} = \begin{cases} 1 & \text{if bus stop } i \text{ is visited by bus } l \\ 0 & \text{otherwise} \end{cases} \qquad i \in W,\ l \in L$$

$$t_{il} = \text{number of students which choose bus stop } i \text{ and are picked up by bus } l,\ i \in W,\ l \in L,$$

where L denotes the set of buses. Since as many buses as bus stops could be used, we assume $|L| = |W|$. As mentioned above, all the MILP models have the same feasible region thus we explain it first. This feasible region ensures that the values of z and y will be able to provide a bilevel feasible solution. Hence, the preferences of students and the bus capacity constraint play an important role in its definition:

$$\sum_{i \in W_k} y_{ki} = 1, \qquad k \in U \tag{6a}$$

$$\sum_{k\in U_i} y_{ki} \le Qz_i, \qquad i \in W \tag{6b}$$

$$\sum_{i\in W_k} p_{ki}y_{ki} \le p_{kj}z_j + M(1-z_j), \quad k\in U,\ \ j\in W_k \tag{6c}$$

$$\sum_{l\in L} w_{il} = z_i, \qquad i\in W \tag{6d}$$

$$z_i \le \sum_{k\in U_i} y_{ki}, \qquad i\in W \tag{6e}$$

$$t_{il} \le Qw_{il}, \qquad i\in W,\ \ l\in L \tag{6f}$$

$$\sum_{k\in U_i} y_{ki} = \sum_{l\in L} t_{il}, \qquad i\in W \tag{6g}$$

$$\sum_{i\in W} t_{il} \le Qv_l, \qquad l\in L \tag{6h}$$

$$w_{il} \le t_{il}, \qquad i\in W,\ \ l\in L \tag{6i}$$

$$y_{ki}\in\{0,1\},\ \ k\in U,\ \ i\in W_k \tag{6j}$$

$$w_{il}, z_i, v_l \in\{0,1\},\ \ t_{il}\ge 0,\ \ i\in W,\ \ l\in L. \tag{6k}$$

Constraints (6a)–(6c) guarantee that each student chooses his/her best accessible bus stop and that only active stops are selected. As in Section 2, we take $M = |W|$. Constraints (6d) guarantee that each active stop is visited by exactly one bus. Constraints (6e) ensure that those bus stops with no students are not selected. Constraints (6f) guarantee that bus stops that are not visited by a bus do not receive students for that bus. Constraints (6g) ensure that the students who choose a bus stop are picked up by a bus that visits the bus stop. Constraints (6h) ensure that only buses that follow a route can pick up students, and that the number of students picked up by the bus does not exceed the bus capacity. Constraints (6i) impose that bus stops at which no students are picked up by a particular bus are not assigned to that bus. Finally, constraints (6j) and (6k) guarantee the requirements of the variables.

The objective functions of the four problems are as follows:

$$\begin{array}{lll} \text{Problem 1} & \min & \sum\limits_{l\in L} \max\limits_{i\in W} \hat{c}_i w_{il} \\ \text{Problem 2} & \min & \sum\limits_{i\in W} \hat{c}_i z_i \\ \text{Problem 3} & \operatorname{lex\,min} & \left(\sum\limits_{l\in L} \max\limits_{i\in W} \hat{c}_i w_{il},\ \ \sum\limits_{i\in W} \hat{c}_i z_i\right) \\ \text{Problem 4} & \operatorname{lex\,min} & \left(\sum\limits_{i\in W} \hat{c}_i z_i,\ \ \sum\limits_{l\in L} \max\limits_{i\in W} \hat{c}_i w_{il}\right), \end{array} \tag{7}$$

Algorithm 2. General procedure to select the set of active stops W_a

Input: $g_i,\ i \in W$;
$W_a = \{i \in W : |W_k| = 1 \text{ for some } k \in U_i\}$;
while $(\bigcup_{i \in W_a} U_i \neq U)$ **do**
  Randomly select a bus stop $i_0 \in W \setminus W_a$, where the probability of each non-active stop i to be selected is $\dfrac{1 + g_i}{\sum_{j \in W \setminus W_a} (1 + g_j)}$
  $W_a = W_a \cup \{i_0\}$;
end while
$y_{ki} = 0,\quad k \in U,\ i \in W_k$;
$y_{kj} = 1,\quad k \in U,\ j = \text{argmin}\{p_{ki} : i \in W_k \cap W_a\}$;
while $(\sum_{k \in U_i} y_{ki} > Q$ for some $i \in W_a)$ **do**
  Select a bus stop $i_1 \in W_a$ such that $\sum_{k \in U_{i_1}} y_{ki_1} > Q$;
  Select a student $k_1 \in U_{i_1}$ such that $y_{k_1 i_1} = 1$ and $p_{k_1 i_1} > 1$;
  Randomly select a bus stop $i_2 \in W_{k_1}$ such that $p_{k_1 i_2} < p_{k_1 i_1}$;
  $W_a = W_a \cup \{i_2\}$;
  $y_{ki} = 0,\quad k \in U,\ i \in W_k$;
  $y_{kj} = 1,\quad k \in U,\ j = \text{argmin}\{p_{ki} : i \in W_k \cap W_a\}$;
end while
$W_a = W_a \setminus \{i \in W : \sum_{k \in U_i} y_{ki} = 0\}$;
Output: W_a;

where $\hat{c}_i = c_{0i} + c_{i0}$ denotes the cost of going from the depot to the bus stop i and back. Problem 1 considers for each bus the cost corresponding to the farthest bus stop on its route, and minimizes the sum for all buses of these costs. Note that the expression $\max_{i \in W} \hat{c}_i w_{il}$ can be linearized by introducing the variable $r_l,\ l \in L$, together with the constraints $\hat{c}_i w_{il} \leq r_l,\ i \in W,\ l \in L$, and replacing the expression by r_l in the objective function. Hence, Problem 1 minimizes $\sum_{l \in L} r_l$. Problem 2 minimizes the sum of the costs of all active stops. Problems 3 and 4 are biobjective problems that lexicographically optimize (Ehrgott, 2005) the above-mentioned objective functions.

Therefore, in the first iteration of BSBM, Problem 1 is solved. Its optimal solution provides the set W_a of active stops, which are those for which $z_i = 1,\ i \in W$, as well as the active stop chosen by each student $k \in U$, which is that $i \in W_k$ such that $y_{ki} = 1$. With this set of active stops, each with a demand equal to the number of students who have chosen it, BSBM computes the set of routes, as explained in Section 3.3, and the corresponding bilevel feasible solution is included in the pool. The subsequent three iterations do the same with Problems 2–4, respectively. The remaining iterations of BSBM compute the set W_a of active stops applying a general procedure explained in Section 3.2.2.

3.2.2. General procedure to select W_a

The general procedure to select the set W_a of active stops is described in Algorithm 2 and is applied in all the iterations other than the first four. It guarantees that there exists at least one active stop accessible for each student and that each student can choose an active stop in accordance with his/her preferences.

Note that, if the set of accessible bus stops W_k contains a single element for some $k \in U$, this is a fixed stop and must be included in W_a, since it must be included in any bilevel feasible solution. Hence, the iteration starts initializing the set W_a of active stops with the fixed stops (line 1 in Algorithm 2).

Then, one nonactive stop at a time is included in set W_a until it is guaranteed that $\bigcup_{i \in W_a} U_i = U$, that is, every student has an active stop accessible (lines 2–5). For selecting the new active stop, BSBM takes advantage of the pool of the best bilevel feasible solutions found in the previous iterations. Then, a probability is assigned to each current nonactive stop, which is proportional to the number of times it appears in the solutions of the pool plus one, and the new active stop is randomly selected based on these probabilities (line 3, where g_i denotes the number of times that bus stop i is an active stop in the pool's solutions).

At this time (line 5), set W_a contains at least one accessible bus stop for each student. However, it is not guaranteed that a bilevel feasible solution can be built from this set of active stops due to the free choice of the bus stops by the students and the bus capacity constraint. Therefore, the algorithm checks if it is possible to compute a bilevel feasible solution, and repairs W_a otherwise.

As previously mentioned, knowing the set W_a of active stops allows us to solve the lower level problems (1j)–(1m) for every $k \in U$. This means that every student k chooses the most preferred active stop, that is, the value of y_{ki}, $i \in W_k$ is determined (lines 6 and 7). If for some $i \in W_a$, $\sum_{k \in U_i} y_{ki} > Q$, the algorithm "repairs" the set W_a by activating additional bus stops (lines 8–15). For this purpose, the algorithm looks for a student $k_1 \in U$ who has chosen a bus stop i_1 preferred by more than Q students and has at least one accessible nonactive stop that he/she would prefer to the current active stops (lines 9 and 10). Then, one of these bus stops is randomly selected (line 11) and activated, that is, it is added to W_a (line 12). After updating W_a, the students update their choice of the bus stop (lines 13 and 14). This procedure is repeated until every active stop is chosen by at most Q students. We can ensure that the procedure finishes with a selection of active stops able to provide a bilevel feasible solution because the initialization step 3.1 has concluded that problem (1) is feasible. Finally, BSBM removes from W_a the active stops that have not been chosen by any student (line 16).

With the set W_a of active stops and the choice made by the students, the algorithm computes the set of routes, as explained in Section 3.3, and the corresponding bilevel feasible solution is added to the pool if appropriate.

3.3. Solving the capacitated vehicle routing problem

Whatever the procedure for computing the set W_a of active stops is applied, at this time for every $i \in W_a$, we have that $\sum_{k \in U_i} y_{ki} \leq Q$. Hence, BSBM proceeds by solving the CVRP corresponding to the nodes in $\{0\} \cup W_a$, with the demand of a node equal to the number of students who have chosen the active stop corresponding to the node. After having solved the CVRP, three local search procedures are applied, which seek to improve the objective function (4a):

- *Removing active stops*
  In random order, it is attempted to select every active stop that is not fixed and to remove it from its route, linking the two adjacent active stops. If it is possible to reassign its students to the

remaining active stops, taking into account the preferences and bus capacity constraints of the current routes, and the objective function (4a) decreases (which occurs if the triangle inequality is satisfied by costs c_{ij}), then the bus stop is actually removed.

- *Replacing active stops by nonactive ones*
  In random order, for every active stop in each route that is not fixed, we search for a nonactive stop that can replace the incumbent one (i.e., if replaced, every student must have at least one active stop accessible) and would reduce the objective function (4a). If such a bus stop is found, after updating W_a, the lower level problems (1j)–(1m), $k \in U$, are solved. If after reassigning the students, the number of students in each route is less than or equal to Q, the bus stops are actually interchanged.
- *Exchanging active stops in different bus routes*
  For every pair of active stops in distinct routes, we check if the routing cost is reduced when exchanging their position in their respective routes. If this is the case and the number of students in each route remains less than or equal to Q, they are actually exchanged. Note that in this case, the set of active bus stops does not change and therefore neither does the choice of the students.

After having applied the three local search procedures (in the order in which they have been described) and removed the active stops with no students, if there has been any change we solve the CVRP again with the current allocation of students, using the current routes as initial routes. If this improves the objective function value (4a), the local search procedures are applied again only once. To finish the iteration, those active stops that finally have not been chosen by any student are removed from the routes and the objective function (4a) is computed.

4. Computational experiments

The numerical experiments have been performed on a PC Intel Core i7-6700 with 3.4×8 GHz, 32.0 GB RAM, and Windows 10 64-bit as the operating system. The metaheuristic has been coded in C++, TDM-GCC 4.9.2. In the computational experiment we have selected the algorithm VRP_RTR developed by C. Goer, which is an implementation of the Record-To-Record (RTR) metaheuristic to generate good solutions to a CVRP instance. This algorithm is available at the VRPH library: https://sites.google.com/site/vrphlibrary/home. Since the B-SBRP-SS has not been previously studied, no benchmark instances are available. Therefore, we decided to adapt the set of benchmark instances of the SBRP with bus stop selection and a maximum walking distance constraint proposed in Schittekat et al. (2013). This set consists of 112 instances, each with a different geographical distribution of bus stops and students, which we have classified in five groups according to the number of potential bus stops. Table 1 displays the characteristics of each group. The bus capacity Q is 25 or 50. The school and the depot are located at the same place. Besides the costs c_{ij}, $(i, j) \in A$, these instances include a distance d_{ki}, $k \in U$, $i \in W$ and a parameter called maximum walking distance wd, which takes the values 5, 10, 20, and 40. The complete instance characteristics can be seen in Table A1.

From this set of instances, we have generated a set of instances of the B-SBRP-SS by defining, for each student $k \in U$, the set of accessible bus stops as $W_k = \{i \in W : d_{ki} \leq wd\}$. This set is called B-$wd$. We have also generated an additional set of instances, called B-nwd, in which all bus stops are

Table 1
Characteristics of the instances proposed in Schittekat et al. (2013)

Group	Instances	No. of instances	$\lvert W\rvert$	$\lvert U\rvert$
S_1	1–24	24	5	25,50,100
S_2	25–48	24	10	50,100,200
S_3	49–72	24	20	100, 200, 400
S_4	73–96	24	40	200, 400, 800
S_5	97–112	16	80	400, 800

accessible for every student, that is, $W_k = W, \forall k \in U$, aiming to show how the number of accessible bus stops can affect the value of the objective function. In both sets, the preferences of the bus stops are assigned according to their distance from the student location. The closest is the most preferred (i.e., for each student $k \in U$, according to nondecreasing values of the distances d_{ki}, $i \in W$). In case of a tie, the bus stop with the lowest index is preferred.

In the following subsections, we present the results of the computational experiments. In order to analyze the performance of the proposed metaheuristic, in the first part of the study, we compare the solutions provided by the proposed metaheuristic with the optimal solution (or the best known bilevel feasible solution) of the B-SBRP-SS obtained by solving model (4) through an MILP solver. In the second part, we compare the objective function values of each instance in set B-*wd* and the same instance in B-*nwd*. Moreover, we illustrate with some instances the differences in the optimal solution when the students' choice is taken into account and when it is not.

4.1. Evaluating the performance of BSBM

To solve exactly the B-SBRP-SS, defined by model (4), we have applied IBM ILOG CPLEX 12.9.0 with the default settings. The termination criterion has been set at 3600 seconds. When the CPLEX run is interrupted before providing the optimal solution, the best solution at this time is saved.

Bearing in mind the stochastic nature of BSBM, each instance has been solved 10 times. The size of the pool mentioned in Section 3.2.2 has been set at 20. The stopping criterion of the metaheuristic has been established in terms of the number of iterations without objective function (4a) improvement (20 iterations) or computing time (10 minutes), whichever is earlier. Problems (7) have also been solved using CPLEX 12.9.0 with the default settings, and the stopping criterion set at a time limit of 30 seconds for each problem and instance. If the run is interrupted before providing the optimal solution, the best solution at this time is saved.

The first thing to point out is that, although all instances are feasible when the preferences are not taken into account (see Schittekat et al., 2013, where the original 112 instances are proposed), the B-SBRP-SS may not be feasible if it is impossible to select bus stops guaranteeing each student's choice. This happens in 18 of the 112 instances in set B-*wd* and in 17 instances in set B-*nwd*. The infeasible instances coincide except for instance 39, which is feasible in the second set. The first two columns of Table 2 display the group and the number of those instances that are not feasible. In most of them, the bus capacity is $Q = 25$ (15 instances in B-*wd* and 14 instances in B-*nwd*).

Table 2
Infeasible benchmark instances and CPU time in seconds required for detecting infeasibility

Set B-*wd*				Set B-*nwd*			
Group	Instance	CPLEX	BSBM	Group	Instance	CPLEX	BSBM
S_1	21	0.01	0.00	S_1	21	0.01	0.00
	23	0.01	0.00		23	0.01	0.00
S_2	39	0.03	0.00	S_2			
	45	0.02	0.00		45	0.07	0.00
	47	0.04	0.00		47	0.06	0.00
S_3	63	0.03	0.00	S_3	63	0.80	0.00
	67	0.02	0.00		67	2.84	0.00
	69	0.03	0.00		69	4.23	0.00
	71	0.04	0.00		71	2.35	0.00
	72	1.01	0.00		72	2.94	0.00
S_4	87	14.11	0.00	S_4	87	15.77	0.00
	91	0.14	0.00		91	54.32	0.00
	93	0.18	0.00		93	55.37	0.00
	95	0.30	0.00		95	55.30	0.00
	96	0.31	0.00		96	57.02	0.00
S_5	103	26.28	0.00	S_5	103	105.92	0.00
	111	188.08	0.00		111	287.09	0.00
	112	376.95	0.00		112	3402.14	0.00

As could be expected, Q is an influential parameter on the B-SBRP-SS feasibility. The remaining columns in this table provide the CPU time in seconds required for both algorithms to detect infeasibility. It is worth pointing out that BSBM requires in all instances a negligible amount of time, while CPLEX requires less than 1.01 seconds in 14 of 18 instances of set B-*wd*, but requires more than three minutes and more than six minutes for detecting infeasibility in the two largest instances. This trend is especially evident in set B-*nwd* in which the largest instance requires almost one hour to detect infeasibility. Finally, Algorithm 1 finishes with a number of banned stops which is lower than $|W|$ only in four infeasible instances of set B-*wd*. These are the instances 39 ($|W_b| = 2$), 45 ($|W_b| = 9$), 67 ($|W_b| = 3$), and 91 ($|W_b| = 21$). In the remaining instances of both sets, $W_b = W$.

Next, we pay attention to the feasible instances. In this case, instance 39 ($|W_b| = 2$) for set B-*nwd* and instances 88 ($|W_b| = 2$) and 79, 104, and 109 ($|W_b| = 3$)) for both sets have banned stops. To assess the quality of BSBM, we compare the results provided by this algorithm with the optimal solution (or the best known feasible solution) values provided by CPLEX. To make the paper easier to be read, in this section, we summarize the results aiming to have a quick and precise insight into the performance of the algorithm. The detailed results of the experiments are presented in the Appendix (see Tables A2 and A3).

Table 3 outlines the results on the performance of BSBM. The first and second columns display the name of the set and the group. The third and fourth columns show the number of feasible instances and the number of them that are solved to optimality. The fifth to seventh columns display the number of instances in each group for which each procedure provides the best solution.

Table 3
The number of feasible instances solved to optimality and procedure that provides the best solution by group of instances

				Best solution				
Set	Group	Feasible instances	Solved to optimality	Both	CPLEX	BSBM	$\text{Gap}_{\text{CPLEX}}$	Gap_{BSBM}
B-*wd*	S_1	22	22	22	–	–	–	–
	S_2	21	21	20	1	–	–	0.01
	S_3	19	16	11	8	–	–	0.01
	S_4	19	1	4	5	10	0.04	0.01
	S_5	13	–	–	–	13	0.08	–
B-*nwd*	S_1	22	22	22	–	–	–	–
	S_2	22	22	21	1	–	–	0.02
	S_3	19	19	18	1	–	–	0.01
	S_4	19	1	3	4	12	0.13	0.01
	S_5	13	–	–	–	13	0.29	–

The eighth column shows, for the instances for which BSBM is better than CPLEX, the average gap between the solution value provided by CPLEX and the best solution value provided by BSBM, defined as $(f^e - f^a_{\min})/f^a_{\min}$, where f^e refers to the best objective function value provided by CPLEX, and $f^a_{\min}$ is the minimum of the objective function values obtained in the 10 runs executed for the considered instance. The ninth column is similar, but exchanging the role of CPLEX and BSBM, that is, in this column the gap is defined as $(f^a_{\min} - f^e)/f^e$. With respect to set B-*wd*, CPLEX provides the optimal solution within the prescribed computing time (3600 seconds) in 60 of 94 feasible instances. Moreover, in 57 instances both procedures provide the same solution, in 14 instances CPLEX provides a better solution and in 23 instances the best solution is provided by BSBM. With respect to set B-*nwd*, these results are somewhat better. CPLEX provides the optimal solution in 64 of 95 feasible instances. Moreover, in 64 instances both procedures provide the same solution, in 6 instances CPLEX provides a better solution. and in 25 instances the best solution is provided by BSBM. For both sets, BSBM provides better results than those found by CPLEX when the instance size increases. Moreover, according to $\text{Gap}_{\text{CPLEX}}$ and Gap_{BSBM}, when a procedure does not provide the best solution, the gap is notably narrower for BSBM.

It is also worth pointing out that, in 69 of the 94 feasible instances (73%) of set B-*wd*, BSBM provides the same objective function value in all the 10 runs. For the remaining instances, the percentage gap defined as $100 \times (f^a_{\max} - f^a_{\min})/f^a_{\min}$, where $f^a_{\max}$ is the maximum of the objective function values obtained in the 10 runs, is less than 6.29. For set B-*nwd*, the values are 75 of 95 (79%), and the percentage gap is less than 14.69. These results show the accuracy of the proposed algorithm.

Table 4 summarizes the information in Tables A2 and A3 concerning the CPU times (in seconds) by computing the average by group. The first column displays the name of the group. The second and the third columns show the average of the time required by CPLEX and the average of the time at which CPLEX finds the reported solution for set B-*wd*. The fourth and the fifth columns display the average of the computing time in the 10 runs of BSBM and the average of the standard deviation for set B-*wd*. For set B-*nwd*, the corresponding values are reported in the last four columns. We can see the excellent performance of BSBM in terms of computing time. On average, BSBM requires less than two minutes to solve the B-SBRP-SS. The time required by BSBM is slightly longer for the instances of the set B-*nwd* than for those of the set B-*wd*.

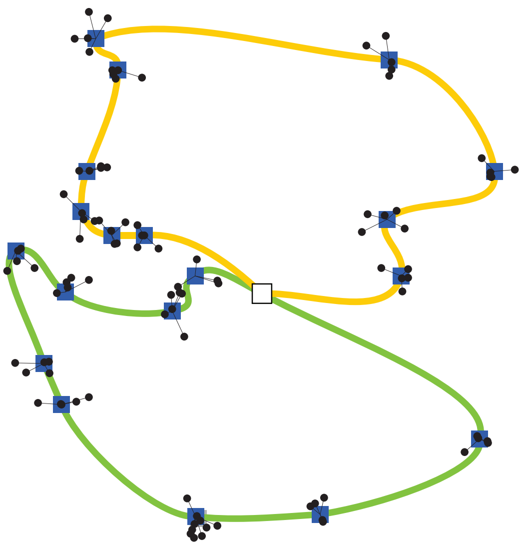
Table 4
Average computing times in seconds by group of instances

	Set B-*wd*				Set B-*nwd*			
	CPLEX		BSBM		CPLEX		BSBM	
Group	$\overline{t^e}$	$\overline{t^e_0}$	$\overline{t^a_{\text{mean}}}$	$\overline{Stdev^a}$	$\overline{t^e}$	$\overline{t^e_0}$	$\overline{t^a_{\text{mean}}}$	$\overline{Stdev^a}$
S_1	0.03	0.02	0.11	0.00	0.06	0.04	0.26	0.02
S_2	0.15	0.10	0.56	0.03	0.44	0.34	1.22	0.04
S_3	1011.02	53.71	18.47	0.14	244.05	84.93	26.76	0.12
S_4	3414.04	2399.24	76.75	1.12	3414.85	2242.54	100.86	0.35
S_5	3602.99	3271.07	124.65	6.25	3600.51	2919.69	126.01	5.27

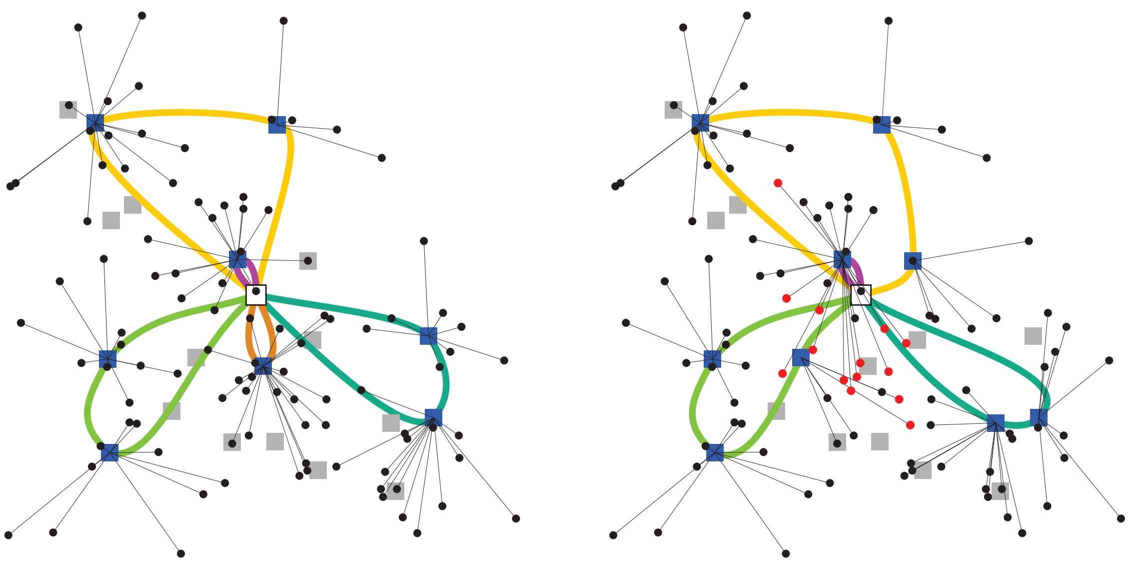
Table 5
The number of runs in each instance for which the best solution corresponding to Problems 1–4 coincides with the best solution provided by BSBM

	Set B-*wd*			Set B-*nwd*		
	No. of	Runs		No. of	Runs	
Group	instances	All	None	instances	All	None
S_1	22	22	–	22	22	–
S_2	21	20	–	22	21	–
S_3	19	12	4	19	18	–
S_4	19	7	6	19	13	–
S_5	13	3	7	13	–	13

Finally, we justify a previous assertion. In Section 3.2.1, it was pointed out that the solution of Problems 1–4 leads, in general, to good bilevel feasible solutions of problem (4). To support this assertion, we now ask ourselves how good the bilevel feasible solutions obtained from the solutions of Problems 1–4 are, compared with the best solution provided by BSBM. For this purpose, we count in how many of the 10 runs the best solution obtained after applying just the way of selecting the set of active stops described in Section 3.2.1 already yields the best solution provided by BSBM. Table 5 highlights the outstanding performance of this way of selecting the set of active stops. The first column displays the name of the group. The second to fourth columns refer to set B-*wd*. The second column displays the number of feasible instances in each group. The third column shows the number of instances for which in all the runs the best solution corresponding to Problems 1–4 coincides with the best solution provided by BSBM. The fourth column displays the number of instances in which in none of the runs both solutions coincide. Obviously, for the remaining instances they coincide from 1 to 9 runs. The fifth to seventh columns provide the corresponding information concerning set B-*nwd*. For set B-*wd*, in 64 of the 94 feasible instances (68.09%) the objective function value provided by BSBM coincides with the best value at the end of these four iterations in the 10 runs. Only for 17 instances did none of the runs deliver the best objective function provided by BSBM. For set B-*nwd*, the corresponding numbers are 74 of the 95 feasible instances (77.89%) and 13 instances. Consequently, especially for the smaller instances, we can think of this step as a good method to obtain tight upper bounds of the optimal solution.



(a) Instance 50: $|W| = 20$, $|U| = 100$, $wd = 5$, $Q = 50$. Routing costs are 408.36 (left) and 406.65 (right)



(b) Instance 53: $|W| = 20$, $|U| = 100$, $wd = 20$, $Q = 25$. Routing costs are 267.01 (left) and 245.17 (right)

Fig. 2. Geographical distribution of two feasible instances. On the left is the optimal solution of problem B-SBRP-SS. On the right is the optimal solution when the students' choice is not taken into account. The white square represents the depot. The remaining squares represent the bus stops (the selected ones in blue). The balls correspond to students. Red balls represent students whose preference is not satisfied.

4.2. *Remarks on the modeling approach of the B-SBRP-SS*

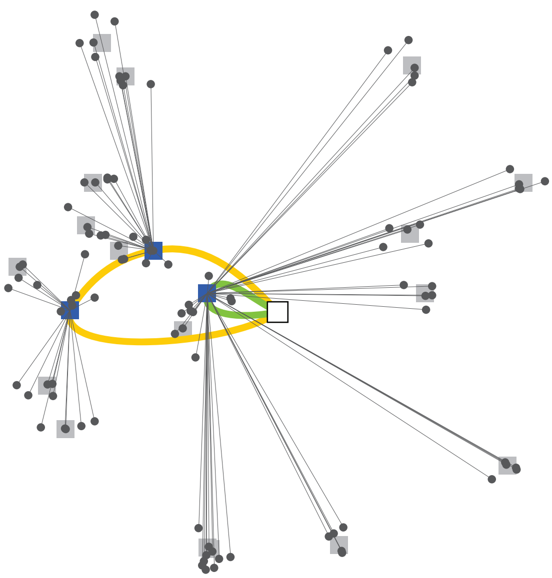
In this subsection, we illustrate some issues involved in the B-SBRP-SS using instances of B-*wd* and B-*nwd*. First, we present Fig. 2 to visualize the consequences of including the choices of the students in the formulation of the problem, that is, to show the differences between the B-SBRP-SS, in which the students choose their preferred bus stop, and the traditional SBRP with bus stop selection in which the students are allocated to a bus stop within their prespecified walking distance (Schittekat et al., 2013; Kinable et al., 2014; Calvete et al., 2020b). Figure 2 displays the optimal solutions of two feasible instances, 50 and 53, when the student's choice is considered (instance in set B-*wd* on the left-hand side) and not considered (classical instance on the right-hand side). The white square represents the depot. The remaining squares represent the bus stops (the selected ones in blue). The balls correspond to students. Red balls represent students whose preference is not satisfied.

According to these solutions, for instance 50 only one student is not allocated to his/her preferred bus stop when the reaction of students is not considered, the selected bus stops are same in both cases and the routes change only slightly. To take into account, students' choice increases the routing cost from 406.65 to 408.36. However, for instance 53, if the students' choice is not considered the optimal solution allocates 14 students to a bus stop that is not their preferred stop. Comparing this solution to the solution of the B-SBRPP-SS, the selected stops are quite different and one more route is needed to satisfy students' choice. For this instance, the objective function value increases from 245.17 to 267.01.

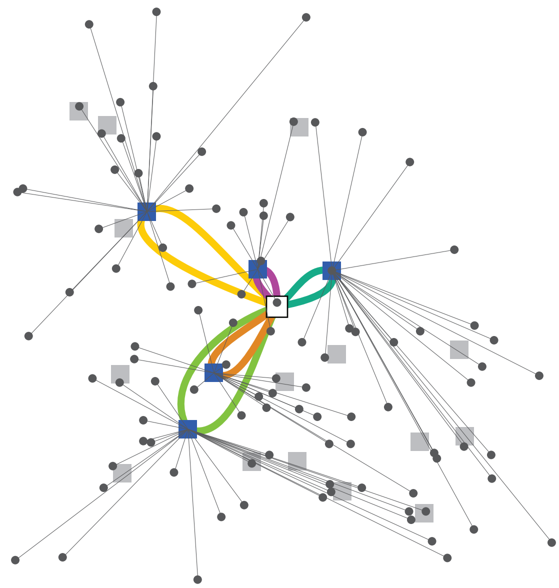
Next, we assess how the number of available bus stops, that is, the cardinality of W_k, $k \in U$, could influence the objective function value of problem (4) by comparing the results obtained for sets B-*wd* and B-*nwd*. We can consider set B-*nwd* as representing a borderline situation where it does not matter how far students have to walk. For this comparison to be fair, only those instances for which the optimal solution in both sets has been reached are analyzed. The ratio between the objective function values of the same instance in B-*wd* and B-*nwd* is observed to vary between 1 and 10.40, with a mean of 2.69 and a standard deviation of 2.11. Obviously, the reduction in the objective function value of the instances in set B-*nwd* is obtained at the price of not restricting in any way the distance walked by students. As an illustration, Fig. 3 displays the optimal solutions of instances 50 and 53 of set B-*nwd*. As above, the white square represents the depot. The remaining squares represent the bus stops (the selected ones in blue). The balls correspond to students. This figure should be compared to the left part in Fig. 2. Note that for instance 50, the optimal solution still requires two routes, but the bus stops are selected to considerably reduce the cost (which decreases from 408.36 to 98.67) and so students are forced to walk long distances (these increase from less than 5 to less than 60). Something similar happens for instance 53. In this case, the optimal solution requires one more route, the routing cost decreases from 267.01 to 147.36, and the walking distance increases from less than 20 to less than 51.

5. Conclusions

The present paper addresses for the first time an SBRP with bus stop selection in which the students are allowed to choose their most preferred bus stop among those selected by the company responsible for picking up the students and taking them to school. In this problem, the selection of the bus stops among the set of potential bus stops and the construction of the bus routes take into account both bus capacity and student preferences. In such a hierarchical decision process, bilevel programming provides a suitable theoretical framework to formulate the problem. Thus, a bilevel mixed integer problem with multiple followers is proposed. According to this model, every



(a) Instance 50: $|W| = 20$, $|U| = 100$, $Q = 50$. Routing cost is 98.67



(b) Instance 53: $|W| = 20$, $|U| = 100$, $Q = 25$. Routing cost is 147.36

Fig. 3. For the set B-*nwd*, on the left is the optimal solution of instance 50 and on the right is the optimal solution of instance 53. The white square represents the depot. The remaining squares represent the bus stops (the selected ones in blue). The balls correspond to students.

student must be able to take the bus at his/her most preferred available bus stop. That is to say, if a bus stop is available every student for whom this is the most preferred among all the available bus stops should be able to access the bus at this bus stop. It is worth pointing out that this assumption can seriously conflict with the bus capacity constraint, even making the problem infeasible. Therefore, as future lines of research, other ways of handling the reaction of students could be proposed. Moreover, the possibility of relaxing the constraint imposing that each bus stop is visited only once could also be analyzed to provide the possibility of those bus stops highly preferred to be visited several times by different buses. Finally, additional constraints could also be included to establish a maximum traveled time constraint in order to guarantee that students do not spend too long time in the bus.

The bilevel model is transformed into a single-level MILP model by taking advantage of the characteristics of the lower level problems. The free choice of the students narrows the feasibility region especially when the bus capacity is small, and thus allows this problem to be efficiently solved through an MILP solver such as CPLEX in small computing times when the size is small. For solving larger instances, a metaheuristic is also proposed, whose efficiency has been shown in terms of the quality of the solution found and of the CPU time required.

To assess the performance of the metaheuristic, a well-known set of benchmark instances of the SBRP with bus stop selection and a maximum walking distance constraint has been adapted. We have defined two sets of instances depending on the number of bus stops that each student can access. The first set considers that all the bus stops are accessible for every student. In the second set, each student can access only those bus stops located at a distance smaller than the maximum walking distance. For the small- to medium-sized instances, which have 5, 10, and 20 potential bus stops and less than 400 students, the exact solution of the MILP model requires reasonable computational times, while for the larger instances, the computational times exceed in most cases the prescribed total time. In contrast, the metaheuristic requires very small computational times in all cases, and it is particularly competitive when all the bus stops are accessible.

Acknowledgments

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Appendix

Table A1 displays the characteristics of the instances.

Table A1
Characteristics of the instances proposed in Schittekat et al. (2013)

Group	Instance	$\|W\|$	$\|U\|$	*wd*	*Q*	Group	Instance	$\|W\|$	$\|U\|$	*wd*	*Q*	Group	Instance	$\|W\|$	$\|U\|$	*wd*	*Q*
S_1	1	5	25	5	25	S_3	49	20	100	5	25	S_5	97	80	400	5	25
	2				50		50				50		98				50
	3			10	25		51			10	25		99			10	25
	4				50		52				50		100				50
	5			20	25		53			20	25		101			20	25
	6				50		54				50		102				50
	7			40	25		55			40	25		103			40	25
	8				50		56				50		104				50
	9	5	50	5	25		57	20	200	5	25		105	80	800	5	25
	10				50		58				50		106				50
	11			10	25		59			10	25		107			10	25
	12				50		60				50		108				50
	13			20	25		61			20	25		109			20	25
	14				50		62				50		110				50
	15			40	25		63			40	25		111			40	25
	16				50		64				50		112				50
	17	5	100	5	25		65	20	400	5	25						
	18				50		66				50						
	19			10	25		67			10	25						
	20				50		68				50						
	21			20	25		69			20	25						
	22				50		70				50						

Continued

Table A1
(Continued)

Group	Instance	$\lvert W\rvert$	$\lvert U\rvert$	*wd*	*Q*	Group	Instance	$\lvert W\rvert$	$\lvert U\rvert$	*wd*	*Q*	Group	Instance	$\lvert W\rvert$	$\lvert U\rvert$	*wd*	*Q*
	23			40	25		71			40	25						
	24				50		72				50						
S_2	25	10	50	5	25	S_4	73	40	200	5	25						
	26				50		74				50						
	27			10	25		75			10	25						
	28				50		76				50						
	29			20	25		77			20	25						
	30				50		78				50						
	31			40	25		79			40	25						
	32				50		80				50						
	33	10	100	5	25		81	40	400	5	25						
	34				50		82				50						
	35			10	25		83			10	25						
	36				50		84				50						
	37			20	25		85			20	25						
	38				50		86				50						
	39			40	25		87			40	25						
	40				50		88				50						
	41	10	200	5	25		89	40	800	5	25						
	42				50		90				50						
	43			10	25		91			10	25						
	44				50		92				50						
	45			20	25		93			20	25						
	46				50		94				50						
	47			40	25		95			40	25						
	48				50		96				50						

Tables A2 and A3 display the results corresponding to the feasible instances of sets B-*wd* and B-*nwd*, respectively. The first and second columns display the number of the group and the instance. The third column displays the "status" of the solution provided by CPLEX. If CPLEX finishes with the optimal solution an "O" is written; otherwise, if the run is interrupted, an "F" is written. The fourth and fifth columns show, for each instance, the information regarding the objective function provided by CPLEX: "f^e" is the optimal objective function value (best objective function value if the run is interrupted), and "Gap^e" is the relative gap provided by CPLEX (this value is not shown if CPLEX reaches the optimal solution). The sixth and seventh columns display the information provided by BSBM: "$f^a_{\min}$" and "$f^a_{\max}$" are, respectively, the minimum and the maximum of the objective function values obtained in the 10 runs executed for the considered instance. If $f^a_{\min}$ equals $f^a_{\max}$, a symbol "=" is written in the column of $f^a_{\max}$. The eighth column compares the objective function values f^e and $f^a_{\min}$. A symbol "=" indicates that both are equal, "Exact" refers to the case $f^e < f^a_{\min}$, while "BSBM" means that $f^e > f^a_{\min}$. The 9th–12th columns display the information relative to CPU times: "t^e" is the CPU time required by CPLEX, "t^e_0" is the time at which CPLEX finds the reported solution, "t^a_{mean}" is the average computing time in seconds in the 10 runs of BSBM, and "$Stdev^a$" is the standard deviation.

Table A2
Results of the feasible benchmark instances of set B-*wd* (CPU times in seconds)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Instance	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
S_1	1	O	141.01		141.01	=	=	0.02	0.00	0.05	0.00
	2	O	161.62		161.62	=	=	0.03	0.02	0.05	0.00
	3	O	182.14		182.14	=	=	0.02	0.02	0.08	0.00
	4	O	195.80		195.80	=	=	0.02	0.02	0.05	0.00
	5	O	111.65		111.65	=	=	0.02	0.02	0.08	0.00
	6	O	103.18		103.18	=	=	0.03	0.03	0.07	0.00
	7	O	7.63		7.63	=	=	0.04	0.03	0.21	0.01
	8	O	25.64		25.64	=	=	0.03	0.02	0.14	0.01
	9	O	286.68		286.68	=	=	0.02	0.02	0.05	0.00
	10	O	197.20		197.20	=	=	0.03	0.03	0.12	0.00
	11	O	193.55		193.55	=	=	0.05	0.00	0.11	0.00
	12	O	215.86		215.86	=	=	0.02	0.02	0.15	0.00
	13	O	130.53		130.53	=	=	0.02	0.02	0.10	0.01
	14	O	96.26		96.26	=	=	0.03	0.02	0.12	0.00
	15	O	19.60		19.60	=	=	0.05	0.03	0.22	0.00
	16	O	30.24		30.24	=	=	0.02	0.02	0.12	0.00
	17	O	360.35		360.35	=	=	0.00	0.00	0.05	0.01
	18	O	304.23		304.23	=	=	0.02	0.02	0.05	0.00
	19	O	294.21		294.21	=	=	0.01	0.01	0.12	0.00
	20	O	229.41		229.41	=	=	0.02	0.00	0.13	0.00
	22	O	144.41		144.41	=	=	0.04	0.00	0.16	0.00
	24	O	52.70		52.70	=	=	0.08	0.02	0.22	0.01
S_2	25	O	242.85		242.85	=	=	0.06	0.06	0.15	0.00
	26	O	282.12		282.12	=	=	0.03	0.02	0.67	0.02
	27	O	244.54		244.54	=	=	0.06	0.03	0.21	0.01
	28	O	288.33		288.33	=	=	0.06	0.03	0.13	0.01
	29	O	108.98		108.98	=	=	0.10	0.09	0.74	0.07
	30	O	157.48		157.48	=	=	0.05	0.05	0.35	0.02
	31	O	35.89		35.89	=	=	0.26	0.25	0.95	0.05
	32	O	36.66		36.66	=	=	0.13	0.13	0.60	0.05
	33	O	403.18		403.18	=	=	0.15	0.08	0.24	0.01
	34	O	296.53		296.53	=	=	0.44	0.06	0.22	0.00
	35	O	404.10		404.10	=	=	0.12	0.02	0.48	0.01
	36	O	294.80		294.80	=	=	0.13	0.05	0.21	0.00
	37	O	217.64		217.64	=	=	0.31	0.30	1.29	0.13
	38	O	184.67		184.67	=	=	0.24	0.09	0.82	0.10
	40	O	38.05		38.05	=	=	0.24	0.23	0.95	0.05
	41	O	735.27		735.27	=	=	0.01	0.01	0.05	0.00
	42	O	506.06		506.06	=	=	0.17	0.16	0.08	0.00
	43	O	541.03		541.03	=	=	0.02	0.02	0.86	0.00
	44	O	490.29		493.83	=	Exact	0.17	0.03	0.49	0.00
	46	O	254.94		254.94	270.19	=	0.16	0.09	0.97	0.07
	48	O	81.37		81.37	=	=	0.29	0.28	1.37	0.11

Continued

Table A2
(Continued)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Instance	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
S_3	49	O	507.81		507.81	=	=	3369.98	0.38	0.76	0.01
	50	O	408.36		408.36	=	=	59.56	0.56	0.57	0.01
	51	F	425.84	0.16	425.84	445.50	=	3604.98	42.72	2.76	0.18
	52	O	366.40		366.53	=	Exact	20.08	15.97	0.55	0.02
	53	O	267.01		267.01	=	=	1317.74	292.63	7.79	0.33
	54	O	185.22		185.22	194.25	=	100.65	78.08	2.18	0.32
	55	O	74.37		74.37	76.19	=	244.26	84.63	5.62	0.15
	56	O	20.98		20.98	=	=	3.66	0.86	2.81	0.11
	57	O	928.55		928.55	=	=	38.48	22.42	12.54	0.00
	58	O	476.05		479.19	=	Exact	2008.17	23.42	0.86	0.01
	59	O	681.19		681.25	=	Exact	70.34	0.14	24.60	0.01
	60	F	464.58	0.08	476.68	=	Exact	3603.28	4.94	1.46	0.01
	61	O	478.58		484.44	488.32	Exact	305.86	31.95	80.94	0.71
	62	F	276.24	0.18	276.24	=	=	3600.07	73.13	34.35	0.32
	64	O	66.02		66.02	=	=	238.70	238.67	36.06	0.06
	65	O	1355.48		1355.48	=	=	0.02	0.02	19.32	0.00
	66	O	768.51		770.48	=	Exact	57.97	2.95	16.65	0.01
	68	O	645.80		647.39	=	Exact	36.42	16.88	35.89	0.01
	70	O	398.75		403.03	412.69	Exact	529.16	90.06	65.29	0.47
S_4	73	F	808.70	0.42	808.70	809.83	=	3608.63	3417.61	87.17	0.60
	74	F	573.81	0.23	559.57	563.84	BSBM	3602.35	3289.94	11.30	0.39
	75	F	773.09	0.45	760.97	765.98	BSBM	3610.49	2207.11	102.92	3.17
	76	F	508.64	0.34	485.87	=	BSBM	3603.38	235.56	24.89	0.32
	77	F	460.34	0.67	441.26	441.66	BSBM	3602.80	3561.64	98.76	0.53
	78	F	311.68	0.47	309.19	309.45	BSBM	3603.16	2465.73	67.71	0.74
	79	F	136.91	0.49	137.76	=	Exact	3606.31	218.28	72.91	0.12
	80	F	90.18	0.64	83.74	86.80	BSBM	3605.20	3593.16	58.85	1.64
	81	F	1440.06	0.32	1440.06	=	=	3602.37	1925.53	92.46	0.21
	82	F	872.61	0.39	834.81	=	BSBM	3602.96	3271.55	61.43	0.01
	83	F	1054.61	0.19	1054.85	1060.60	Exact	3602.03	3466.30	75.85	3.91
	84	F	802.64	0.46	799.38	801.97	BSBM	3602.30	3582.33	103.84	3.27
	85	F	845.42	0.38	847.39	860.42	Exact	3601.44	3523.25	100.45	0.75
	86	F	501.09	0.65	466.33	=	BSBM	3600.06	3585.38	76.10	0.40
	88	F	140.82	0.49	146.99	=	Exact	3605.71	312.89	91.84	0.47
	89	O	3085.11		3085.11	=	=	0.17	0.17	60.74	0.01
	90	F	1452.52	0.27	1452.52	=	=	3602.60	65.03	91.82	0.01
	92	F	1135.36	0.30	1143.25	1181.72	Exact	3601.35	3360.75	84.86	3.37
	94	F	906.37	0.46	876.72	901.42	BSBM	3603.46	3503.34	94.42	1.41
S_5	97	F	1561.78	0.58	1478.89	1482.46	BSBM	3605.73	456.16	99.08	4.10
	98	F	1077.07	0.41	1002.53	1015.87	BSBM	3602.72	3583.36	95.07	6.78
	99	F	1419.13	0.72	1323.40	1349.13	BSBM	3604.34	3527.91	129.69	10.21
	100	F	837.92	0.56	782.82	792.40	BSBM	3605.95	3587.14	124.58	8.43
	101	F	909.20	0.86	823.13	=	BSBM	3600.11	3600.03	137.30	1.92

Continued

Table A2
(Continued)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Instance	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
	102	F	529.41	0.75	482.83	513.19	BSBM	3600.30	3537.48	132.21	9.96
	104	F	181.46	0.96	148.31	=	BSBM	3600.13	3569.61	126.79	0.26
	105	F	2725.28	0.70	2709.34	=	BSBM	3605.11	3555.06	104.73	1.52
	106	F	1602.50	0.61	1483.59	1486.11	BSBM	3606.23	3465.75	97.35	3.60
	107	F	2234.48	0.74	2209.79	2237.32	BSBM	3603.92	3402.05	138.61	8.83
	108	F	1322.73	0.69	1294.64	1312.86	BSBM	3600.50	3587.19	148.59	21.60
	109	F	1832.02	0.62	1761.02	=	BSBM	3603.67	3538.09	145.53	1.92
	110	F	1037.91	0.91	882.08	=	BSBM	3600.17	3114.06	140.99	2.13

Table A3
Results of the feasible benchmark instances of set B-*nwd* (CPU times in seconds)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Inst.	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
S_1	1	O	34.17		34.17	=	=	0.10	0.03	0.21	0.01
	2	O	48.89		48.89	=	=	0.04	0.03	0.18	0.01
	3	O	41.72		41.72	=	=	0.04	0.03	0.17	0.01
	4	O	51.45		51.45	=	=	0.04	0.03	0.17	0.01
	5	O	45.80		45.80	=	=	0.03	0.03	0.16	0.00
	6	O	36.88		36.88	=	=	0.05	0.05	0.18	0.01
	7	O	7.63		7.63	=	=	0.04	0.03	0.18	0.01
	8	O	10.24		10.24	=	=	0.03	0.03	0.20	0.01
	9	O	214.21		214.21	=	=	0.04	0.03	0.64	0.03
	10	O	18.97		18.97	=	=	0.05	0.05	0.19	0.01
	11	O	118.12		118.12	=	=	0.03	0.03	0.28	0.03
	12	O	65.97		65.97	=	=	0.07	0.06	0.20	0.01
	13	O	78.87		78.87	=	=	0.10	0.03	0.42	0.07
	14	O	23.98		23.98	=	=	0.08	0.06	0.21	0.01
	15	O	17.13		17.13	=	=	0.09	0.08	0.33	0.04
	16	O	6.34		6.34	=	=	0.07	0.03	0.20	0.01
	17	O	360.35		360.35	=	=	0.03	0.02	0.20	0.00
	18	O	225.98		225.98	=	=	0.07	0.03	0.33	0.02
	19	O	294.21		294.21	=	=	0.02	0.02	0.20	0.00
	20	O	147.66		147.66	=	=	0.08	0.08	0.36	0.03
	22	O	89.33		89.33	=	=	0.05	0.05	0.37	0.02
	24	O	31.70		31.70	=	=	0.10	0.08	0.41	0.02
S_2	25	O	83.21		83.21	=	=	0.28	0.27	1.21	0.04
	26	O	39.58		39.58	=	=	0.20	0.14	0.36	0.01
	27	O	61.27		61.27	=	=	0.23	0.14	0.72	0.04
	28	O	42.62		42.62	=	=	0.25	0.23	0.40	0.02
	29	O	34.43		34.43	=	=	0.28	0.27	0.80	0.04

Continued

Table A3
(Continued)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Inst.	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
	30	O	15.73		15.73	=	=	0.15	0.14	0.42	0.02
	31	O	19.33		19.33	=	=	0.25	0.25	0.90	0.04
	32	O	8.33		8.33	=	=	0.20	0.14	0.47	0.02
	33	O	318.76		318.76	=	=	0.33	0.31	1.34	0.08
	34	O	123.60		123.60	=	=	1.23	0.70	1.25	0.04
	35	O	253.12		253.12	=	=	0.29	0.28	1.35	0.05
	36	O	158.54		161.52	=	Exact	1.20	0.84	1.32	0.02
	37	O	169.83		169.83	=	=	0.27	0.16	1.22	0.05
	38	O	74.16		74.16	=	=	0.61	0.48	1.33	0.03
	39	O	78.33		78.33	=	=	0.10	0.09	0.82	0.04
	40	O	18.60		18.60	=	=	0.48	0.36	1.58	0.07
	41	O	735.27		735.27	=	=	0.32	0.13	1.27	0.00
	42	O	318.73		318.73	=	=	0.48	0.47	2.41	0.05
	43	O	541.03		541.03	=	=	0.32	0.09	1.32	0.01
	44	O	357.22		357.22	=	=	0.52	0.34	2.01	0.04
	46	O	215.85		215.85	=	=	1.10	1.09	2.03	0.06
	48	O	72.65		72.65	=	=	0.60	0.47	2.25	0.06
S_3	49	O	192.01		192.01	=	=	50.24	17.42	20.68	0.03
	50	O	98.67		98.67	=	=	60.19	5.20	4.59	0.04
	51	O	261.27		261.27	=	=	2155.10	81.56	27.03	0.03
	52	O	50.51		50.51	=	=	4.78	4.63	2.66	0.15
	53	O	147.36		147.36	=	=	176.66	3.91	20.29	0.04
	54	O	28.02		28.02	=	=	3.34	3.33	1.91	0.05
	55	O	42.12		42.12	=	=	40.55	34.86	13.16	0.04
	56	O	12.79		12.79	=	=	4.08	2.75	2.85	0.03
	57	O	648.49		648.49	=	=	10.82	10.73	35.81	0.36
	58	O	159.40		159.40	=	=	255.49	205.89	36.38	0.04
	59	O	428.51		428.51	=	=	25.70	12.58	37.41	0.05
	60	O	182.31		182.31	=	=	527.28	526.14	36.64	0.23
	61	O	362.51		362.51	=	=	154.28	2.16	36.23	0.06
	62	O	120.05		120.05	=	=	126.43	27.63	35.56	0.32
	64	O	59.88		59.88	=	=	435.63	351.77	43.19	0.05
	65	O	1355.48		1355.48	=	=	3.00	1.28	20.92	0.00
	66	O	561.90		568.99	=	Exact	16.03	15.73	38.19	0.08
	68	O	457.26		457.26	=	=	104.12	84.80	42.48	0.10
	70	O	324.11		324.11	324.20	=	483.14	221.22	52.49	0.56
S_4	73	F	465.18	0.71	469.54	476.44	Exact	3600.05	3516.08	113.99	0.55
	74	F	168.79	0.55	130.52	=	BSBM	3600.08	3302.61	56.97	0.04
	75	F	413.83	0.74	404.36	=	BSBM	3604.91	3209.38	122.48	0.06
	76	F	177.03	0.69	130.30	=	BSBM	3600.07	3583.05	55.31	0.03
	77	F	337.42	0.75	342.39	353.15	Exact	3600.13	1158.33	123.41	1.00
	78	F	132.87	0.68	103.49	=	BSBM	3600.09	2486.02	59.37	0.04
	79	F	136.91	0.49	137.76	=	Exact	3601.94	803.19	79.42	0.06

Continued

Table A3
(Continued)

			Objective function value					CPU times			
			CPLEX		BSBM			CPLEX		BSBM	
Group	Inst.	Status	f^e	Gap^e	$f^a_{\min}$	$f^a_{\max}$	Best	t^e	t^e_0	t^a_{mean}	$Stdev^a$
	80	F	35.58	0.57	35.47	=	BSBM	3600.07	3461.41	73.73	0.03
	81	F	1111.65	0.07	1130.62	=	Exact	3601.76	499.02	86.39	0.15
	82	F	528.99	0.78	455.26	456.51	BSBM	3600.07	3572.94	123.00	0.53
	83	F	812.41	0.19	800.47	=	BSBM	3602.80	892.66	88.41	0.11
	84	F	506.71	0.76	458.67	479.28	BSBM	3600.80	3397.83	123.53	0.62
	85	F	801.40	0.50	748.23	=	BSBM	3604.77	3432.89	107.47	0.14
	86	F	383.59	0.81	325.59	341.30	BSBM	3600.07	106.72	123.89	1.14
	88	F	109.54	0.44	109.54	=	=	3603.08	1796.99	104.20	0.10
	89	O	3085.11		3085.11	=	=	56.91	10.17	103.90	0.01
	90	F	1132.46	0.14	1132.46	=	=	3602.35	1915.97	115.52	0.13
	92	F	1029.19	0.34	1021.11	1029.19	BSBM	3601.54	1869.17	128.33	1.73
	94	F	822.76	0.53	809.02	=	BSBM	3600.66	3593.84	126.98	0.11
S_5	97	F	1201.49	0.90	1041.40	1106.14	BSBM	3600.20	3426.53	138.91	6.56
	98	F	731.53	0.95	452.47	505.06	BSBM	3600.33	3349.05	128.01	1.72
	99	F	1254.84	0.93	1013.39	1092.92	BSBM	3600.20	1178.76	138.68	4.96
	100	F	642.72	0.95	400.37	459.17	BSBM	3600.33	1703.11	127.44	1.59
	101	F	849.57	0.92	738.70	766.68	BSBM	3600.21	2295.34	132.52	3.45
	102	F	473.71	0.95	267.97	301.10	BSBM	3600.41	3586.16	128.00	1.85
	104	F	210.00	0.98	159.82	172.45	BSBM	3600.26	2453.91	68.46	3.77
	105	F	2691.78	0.92	2501.10	2548.80	BSBM	3600.60	2767.30	130.72	7.44
	106	F	1281.91	0.94	1057.66	1137.45	BSBM	3600.51	3559.33	141.22	5.93
	107	F	2281.74	0.91	2066.71	2134.33	BSBM	3600.93	3398.50	130.72	10.66
	108	F	1229.02	0.94	972.89	1037.52	BSBM	3601.03	3486.97	140.51	6.23
	109	F	1801.68	0.67	1731.48	1772.81	BSBM	3600.54	3585.25	91.66	6.33
	110	F	1017.44	0.95	799.34	835.39	BSBM	3601.09	3165.80	141.24	7.99