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# An illumination-invariant phase-shifting algorithm for three-dimensional profilometry

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## ABSTRACT

Uneven illumination is a common problem in real optical systems for machine vision applications, and it contributes significant errors when using phase-shifting algorithms (PSA) to reconstruct the surface of a moving object. Here, we propose an illumination-reflectivity-focus (IRF) model to characterize this uneven illumination effect on phase-measuring profilometry. With this model, we separate the illumination factor effectively, and then formulate the phase reconstruction as an optimization problem. To simplify the optimization process, we calibrate the uneven illumination distribution beforehand, and then use the calibrated illumination information during surface profilometry. After calibration, the degrees of freedom are reduced. Accordingly, we develop a novel illumination-invariant phase-shifting algorithm (II-PSA) to reconstruct the surface of a moving object under an uneven illumination environment. Experimental results show that the proposed algorithm can improve the reconstruction quality both visually and numerically. Therefore, using this IRF model and the corresponding II-PSA, not only can we handle uneven illumination in a real optical system with a large field of view (FOV), but we also develop a robust and efficient method for reconstructing the surface of a moving object.

**Keywords:** Three-dimensional image acquisition, industrial inspection, surface measurements

## 1. INTRODUCTION

Over the past few years, advances in surface reconstruction have inspired many applications. However, surface profiling is particularly challenging when applied to the semiconductor industry, due to two conflicting requirements: a small feature size requiring a high precision, and the need for a high throughput.<sup>1</sup> For certain inspections, explicit three-dimensional reconstruction is not needed and various efficient methods have been proposed. For example, Dong et al. proposed a biplanar disparity matrix measure to measure the height of wafer bumps based on a specially designed lighting setup.<sup>2</sup> Also, Cheng et al. projected binary patterns on the surface of the integrated circuits (IC) samples, and then optimized the bit-pairing codification for obtaining more sampled profile data.<sup>3,4</sup> However, in other applications such as in volume measurement and surface inspection, a more complete profile information is necessary, then we need different ways to reconstruct the full surface profile.

Phase-measuring profilometry, especially the phase-shifting algorithm (PSA), is one of the popular methods for high precision dense surface reconstruction and attracts more and more attentions from both academia and industry in these years.<sup>5,6</sup> Combined with an encoded marker as the reference, high-resolution three-dimensional absolute coordinates of the surface can be measured.<sup>7</sup> Besides, PSA can be efficiently implemented so that the corresponding core calculation is parallelized inside Graphics Processing Unit (GPU) for high speed applications.<sup>8</sup> Most of these advances in PSA are based on a conventional phase-measuring profilometry model.<sup>9</sup> We can describe the relationship for a point  $x_0$ , with a total of  $n$  images, as

$$I_k(x_0) = B(x_0) + C(x_0) \cos \phi(x_0, h) + N_k(x_0), \quad (1)$$

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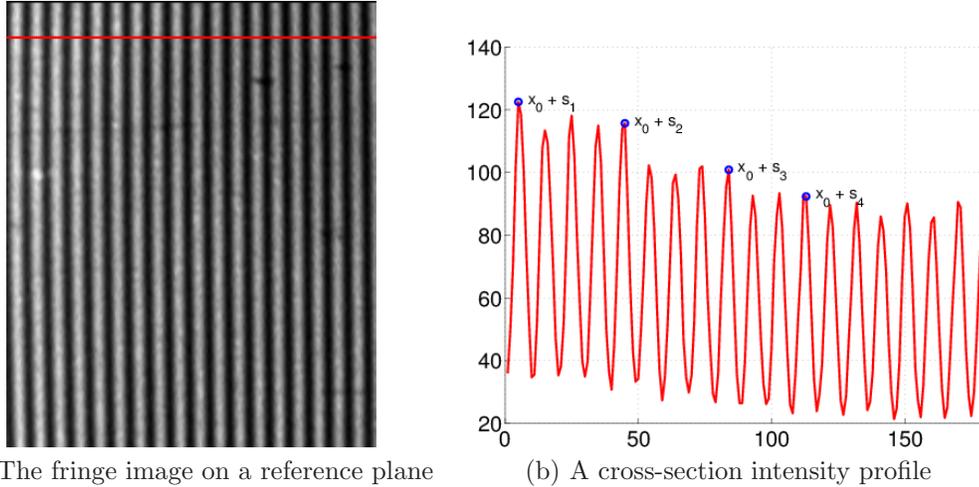


Figure 1. The fringe pattern on a homogenous reference plane from a real system under uneven illumination. (b) is one of the cross-section intensity profile in (a) (displayed in red). When a point  $x_0$  on the homogenous plane moves along this profile, its intensity is degenerated according to the position  $x_0 + s_k$ .

where  $k = 1, \dots, n$ ;  $B(x_0)$  is the background intensity;  $C(x_0)$  is the fringe contrast;  $\phi(x_0, h)$  is the phase offset as a function of height  $h$ ; and  $N_k(x_0)$  is additive noise at the point  $x_0$  for the  $k$ th image. It is common to use four images, with phase shifts at  $\theta(s_k) = (k - 1)\pi/2$  where  $k = 1, 2, 3, 4$  for reconstruction. In the absence of noise, the corresponding four equations imply that

$$\tan \phi(x_0, h) = \frac{I_4(x_0) - I_2(x_0)}{I_1(x_0) - I_3(x_0)}, \quad (2)$$

and then we can map the phase offset  $\phi(x_0, h)$  to height offset to a reference plane.<sup>10</sup> In an analogous manner, the whole surface of an object can be obtained point-by-point.

## 2. ILLUMINATION-REFLECTIVITY-FOCUS MODEL

The above analytical solution allows for an efficient reconstruction of stationary objects because the conventional model assumes the object for surface reconstruction is stationary during the measurements. Therefore, each point on the surface receives a consistent illumination, so that  $B(x_0)$  and  $C(x_0)$  are constants for the same reconstructed point  $x_0$ . However, this stationary requirement for objects limits its applications in reconstructing objects with size larger than the FOV or moving objects. In many practical applications, such as real-time inspection of semiconductors on the production line, the object to be reconstructed is moving along a conveyor belt. Most optical systems for these applications have large FOVs for a high throughput. Accordingly, the uneven illumination problem becomes more pronounced when shifting the object to different positions of this large FOV in a real system, and it causes the background intensity and fringe contrast to vary according to the shift  $s_k$  of the object when capturing the  $k$ th images  $I_k(x_0)$ .

Figure 1(a) shows a fringe image on a homogenous reference plane from a real system. Under even illumination, the cross-section intensity profile of the projected fringe pattern should be an ideal sinusoidal signal. Then, based on the intensity variations in these multiple fringe images at different shifts  $s_k$ , we can accurately estimate the phase  $\phi(x_0, h)$  from Equation 2. However, due to uneven illumination, this cross-section intensity profile is degenerated according to different positions in the FOV as shown in Figure 1(b). This cross-section profile also characterizes the intensity profile of a point  $x_0$  on the homogeneous plane when it moves horizontally to different positions in the FOV. For example, when the point  $x_0$  is moving to the peaks of the sinusoidal signal  $x_0 + s_k$  ( $k = 1, 2, 3, 4$ ) as shown in Figure 1(b), these intensities should be the same under an even illumination. However, these intensities are degenerated according to  $x_0 + s_k$ . This degeneration in intensities due to uneven illumination causes difficulties in distinguishing the intensity variations from the phase shifts of the projected

sinusoidal signal when using the conventional PSA to recover the phase  $\phi(x_0, h)$ . Furthermore, it introduces a major error source when reconstructing a moving object under an uneven illumination.

In literature, most of the existing techniques to address uneven illumination problem can be classified as active techniques and passive techniques. In active approaches, additional specific illumination sources or sensors are needed to obtain different modalities of images such as infrared image in order to obtain uniform intensity.<sup>11</sup> These devices increase the cost and occupy much space of an optical system. Other optical components such as optical films and diffusers can be used to reduce the uneven illumination effect.<sup>12</sup> However, they reduce the overall brightness at the same time. Also, lens arrays are used to improve the illumination evenness,<sup>13</sup> which complicate the design of an optical system. In passive approaches, the illumination variation model either requires many testing samples under different illumination conditions to characterize the radiance distribution or requires the exact parameters of the components such as the shapes and sizes of the emitting die inside LED sources. And some emission characteristics such as the surface of the object being Lambertian are assumed.<sup>14</sup> However, these assumptions may not be valid in practical applications.

Here, we propose a novel method to model and calibrate the illumination distribution by incorporating the uneven illumination factor explicitly into a more general phase-measuring model. For practical applications such as the real-time high-precision surface inspection for semiconductor components on a conveyor belt, the illumination can be well controlled so that the illumination intensity distribution within each image is a repeatable pattern. Therefore, we can use this calibrated illumination information for real-time reconstruction. To reconstruct the height of a point  $x_0$  under uneven illumination, we introduce variant  $B(x_0 + s_k)$  and  $C(x_0 + s_k)$  ( $k = 1, \dots, n$ ) in the phase-measuring model when the point is shifted to  $x_0 + s_k$  and then the degenerated intensity  $I_k(x_0)$  due to uneven illumination can be described as

$$I_k(x_0) = B(x_0 + s_k) + C(x_0 + s_k) \cos[\phi(x_0, h) + \theta(s_k)] + N_k(x_0). \quad (3)$$

Equation 1 is a special case of this when  $B(x_0 + s_k) = B(x_0)$  and  $C(x_0 + s_k) = C(x_0)$ . However, this generalization introduces two variables  $B(x_0 + s_k)$  and  $C(x_0 + s_k)$  for each measurement  $I_k(x_0)$ , which causes difficulties in recovering the phase  $\phi(x_0, h)$  without prior knowledge of the model. Therefore, we model the variant  $B(x_0 + s_k)$  and  $C(x_0 + s_k)$  in details based on the reflectivity factor  $R(x_0)$  at the point  $x_0$  on the surface, the illumination factor  $L(x_0 + s_k)$  and focus factor  $F(x_0 + s_k)$  at the position  $x_0 + s_k$  as

$$B(x_0 + s_k) = L(x_0 + s_k)R(x_0) \quad \text{and} \quad C(x_0 + s_k) = L(x_0 + s_k)R(x_0)F(x_0 + s_k). \quad (4)$$

This perspective shows how the uneven illumination factor  $L(x_0 + s_k)$  affects the background intensity  $B(x_0 + s_k)$  and fringe contrast  $C(x_0 + s_k)$  respectively. Then we propose the following illumination-reflectivity-focus (IRF) model:

$$I_k(x_0) = L(x_0 + s_k)R(x_0) \{1 + F(x_0 + s_k) \cos[\phi(x_0, h) + \theta(s_k)]\} + N_k(x_0). \quad (5)$$

Based on this IRF model, we formulate phase reconstruction as an optimization problem and propose a novel phase-shifting algorithm in the following section.

### 3. PROBLEM FORMULATION AND II-PSA

We use the four-frame algorithm in Section 1 as an example for illustration and consider the phase reconstruction at a specific point  $x_0$  during optimization formulation. Let  $E_k(x_0)$  be the residual error of the model in Equation 5 at phase shift  $(k - 1)\pi/2$ , i.e.,

$$E_k(x_0) = I_k(x_0) - L(x_0 + s_k)R(x_0) \{1 + F(x_0 + s_k) \cos[\phi(x_0, h) + (k - 1)\pi/2]\}. \quad (6)$$

In an optimization framework, we aim to find the optimal  $\phi(x_0, h)$  at  $x_0$  by minimizing

$$E(x_0) = \sum_{k=1}^4 E_k^2(x_0). \quad (7)$$

Using the IRF model to formulate the cost function above, the degrees of freedom are more than the number of the data points we collect, so it is difficult to solve for  $\phi(x_0, h)$  without prior knowledge. Although the illumination distribution  $L(x)$  and focus distribution  $F(x)$  vary according to the positions in this IRF model, they remain the same during the reconstruction provided that the projection and imaging systems are stationary. Therefore, we use a homogenous reference plane to calibrate the illumination and the focus factor first, so that  $L(x)$  and  $F(x)$  are known within the FOV during the reconstruction.

After optical calibration, the distributions  $L(x)$  and  $F(x)$  are known within the FOV. When a specific point  $x_0$  on the surface is shifted to  $x_0 + s_k$ , where we denote the corresponding illumination factor as  $L_k$  and the focus factor as  $F_k$  respectively, the phase reconstruction problem at point  $x_0$  is cast as minimizing

$$E(x_0) = \sum_{k=1}^4 \{I_k(x_0) - L_k R(x_0) - L_k R(x_0) F_k \cos[\phi(x_0, h) + (k-1)\pi/2]\}^2. \quad (8)$$

There are only two variables,  $R(x_0)$  and  $\phi(x_0, h)$ , in this optimization formulation, which model the magnitude modulation and the phase modulation process respectively when forming the fringe images. If more images are used in the reconstruction for better robustness and accuracy, no extra degree of freedom is introduced. Therefore, we name these two variables the fundamental variables in the phase-shifting algorithms. After calibrating the distributions  $L(x)$  and  $F(x)$ , we can correct the effect from uneven illumination on magnitude modulation when forming the fringe intensities at different positions  $x_0 + s_k$ . Therefore, to make the reconstruction result invariant to uneven illumination at different positions in an optical system, we normalize the intensity first and then recover the fundamental variables  $R(x_0)$  and  $\phi(x_0, h)$  by minimizing

$$\hat{E}(x_0) = \sum_{k=1}^4 \left\{ \frac{I_k(x_0)}{L_k F_k} - \frac{R(x_0)}{F_k} - R(x_0) \cos[\phi(x_0, h) + (k-1)\pi/2] \right\}^2. \quad (9)$$

From this formulation, we recover the reflectivity  $R(x_0)$  by

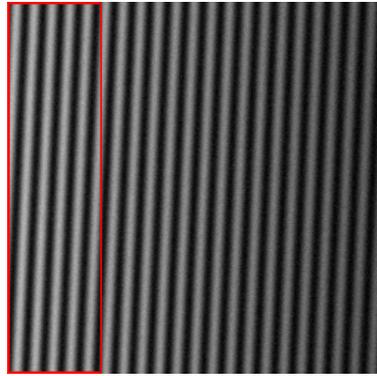
$$R(x_0) = \frac{\sum_{k=1}^4 I_k(x_0)/(L_k F_k)}{\sum_{k=1}^4 1/F_k}. \quad (10)$$

Substituting this  $R(x_0)$  into Equation 9, we can solve  $\cos \phi(x_0, h)$  and  $\sin \phi(x_0, h)$  by a standard least square optimization with a quadratic constraint  $\cos^2 \phi(x_0, h) + \sin^2 \phi(x_0, h) = 1$ .<sup>15</sup> Then we get phase  $\phi(x_0, h)$  at the point  $x_0$  and the final surface profile. We name this phase reconstruction method the illumination invariant phase-shifting algorithm (II-PSA) because the cost function in the above formulation is invariant to the illumination intensity. In the next section, we use II-PSA to reconstruct the surface of a moving object under an uneven illumination environment and compare with the results from PSA.

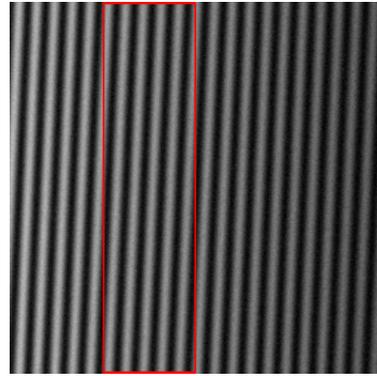
#### 4. NUMERICAL ANALYSIS AND EXPERIMENTS

In the experiments, we use every four consecutive images from image sequences and four-frame phase shift algorithm with phase shift at  $\pi/2$  apart for reconstruction a moving surface under uneven illumination. Since the reconstruction of the surface is at the focal plane of the optical system, the focus factor can be assumed a constant on the FOV during studying the uneven illumination effect. And in real systems, a projection optics design tilted projection of fringe pattern can ensure the measuring plane is uniformly focused.<sup>16</sup> Then we focus on the dominant uneven effect from illumination in experiments.

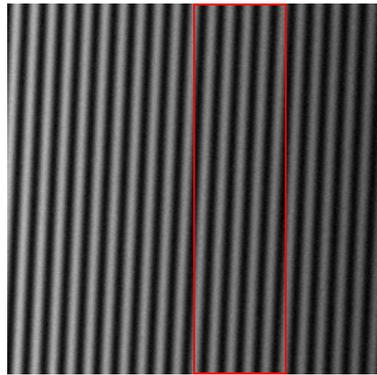
Figure 2 (a) to (d) capture one set of sample images of a homogenous plane when this plane is moving along the  $x$  direction across the FOV of the optical system. In this case, the period of the sinusoidal pattern is  $P = 12$  pixels, and the images are captured at four sequential positions  $\{s_1 = 0, s_2 = 63, s_3 = 126, s_4 = 189\}$  pixels relative to the initial position. The red boxes on these four images in the figure mark the region of interest (ROI) on the plane when this region is moving to different positions of the FOV during capture. We align these ROIs from the image sequences as shown in Figure 2 (e) to (h) before surface reconstruction.



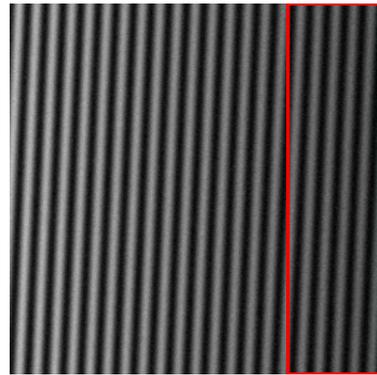
(a) The 1st fringe image



(b) The 2nd fringe image



(c) The 3rd fringe image



(d) The 4th fringe image



(e) The 1st ROI



(f) The 2nd ROI



(g) The 3rd ROI

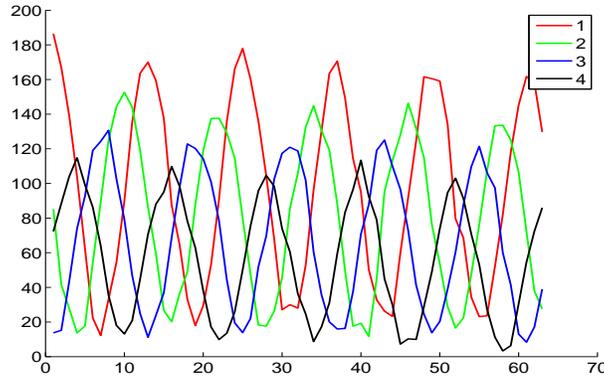


(h) The 4th ROI

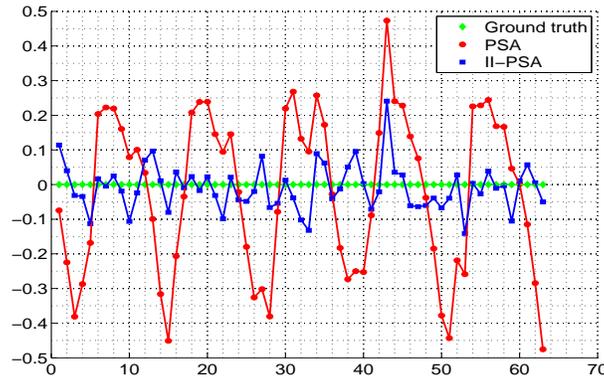
Figure 2. In the image sequences (a), (b), (c), and (d), the ROI corresponding to the same physical region of the object during motion movements are marked as red boxes and these regions are extracted, aligned and shown in (e), (f), (g) and (h).

Under an even illumination, the fringes in these aligned ROIs would have the same brightness. Based on uniformly illuminated fringe images, the conventional four-frame PSA can efficiently reconstruction the surface of the ROI with a high precision. However, due to uneven illumination across the FOV of the optical system, the aligned ROI in Figure 2 (e) is brighter than the aligned ROI in Figure 2 (h). Since these intensity variations from an uneven illumination introduce reconstruction error when calculating the phase, we propose II-PSA to improve the reconstruction performance.

In one of the experiments, the illumination distribution changes linearly from 100 gray levels on the left end of a  $256 \times 256$  image to 50 gray levels on the right end of the image as shown in Figure 2. The focus factor is



(a) Four synthetic intensity profiles at phase shifts  $\theta(s_k) = (k - 1)\pi/2$ .



(b) Reconstruction results from PSA and II-PSA.

Figure 3. Reconstruction results based on one-dimensional model (unit: radian).

assumed to be a constant 0.8 on a homogenous plane. We synthesize the images according to the IRF model in Equation 5 plus additive Gaussian noise with standard deviation  $\sigma = 5$ . After calibrating the illumination distribution and focus distribution, we align the ROI for a common physical region within the image sequences as shown in Figure 2 (e) to (h). One of the one-dimensional reconstruction results is shown in Figure 3. Figure 3 (a) shows the corresponding four cross-section intensity profiles extracted from the center row of the aligned ROI in Figure 2. From these intensity profiles, we see that the uneven illumination contributes a dominant effect on the uniformity of the sinusoidal signal, compared with camera noise. The resulting nonuniform intensities would be interpreted as phase offset from the height variation in PSA and would cause a large periodic phase error disturbance on the reconstructed phase. On the other hand, in our II-PSA, this periodic error pattern is removed by illumination normalization and only a random error pattern due to the camera noise remains in the result as shown in Figure 3 (b). The standard deviations of the error are 0.23 radian for PSA and 0.07 radian for II-PSA respectively.

## 5. CONCLUSION

In this paper, we propose a novel IRF model to characterize the uneven illumination factor for phase-measuring profilometry and then have developed a novel II-PSA to reconstruct the surface profile under uneven illumination. Experimental results show that our algorithm can significantly improve the reconstruction quality both visually and numerically.

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