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# CLARIFICATION OF "TURN PERFORMANCE OF AIRCRAFT"*

WILLIAM F. FORD†

**Abstract.** A recent note analyzed the minimum turning radius of an airplane in terms of its airspeed and angle of bank. Unfortunately, some misconceptions concerning the underlying physics were introduced. This note is intended to clarify those areas.

**Key words.** aircraft performance, load factor, lift

**AMS subject classifications.** 76G25, 93A30

**Introduction.** Wolper's recent note [1] on an airplane's turn performance, although well motivated and mathematically correct, contains some statements about the aerodynamics that are either incorrect or misleading. Here are the major difficulties, in the order in which they are encountered.

1. The first paragraph of §1 states that an airplane turns because of its horizontal component of lift. This common belief is only half true.
2. The second paragraph declares that in analyzing the turn performance of an airplane we may essentially ignore variation of the lift coefficient $C_L$. This is false.
3. The fourth paragraph speaks of an airplane in level flight without enough lift to support its weight and states that the airplane is then "called stalled." This is confusing and incorrect.
4. In §2, the second paragraph states that if the airspeed in the turn is too low for the angle of bank, the airplane stalls and the turn radius becomes infinite. This contains a (potentially dangerous) hidden assumption.

**1. Why an airplane turns.** To explain this one must recognize that an airplane is an extended, nonsymmetric solid body, which therefore has five degrees of freedom. It follows that analysis of the turn must address both center-of-mass motion and motion relative to the center-of-mass.

An airplane does not turn "because" of lift. However, a pilot may choose to *employ* lift as one of the two ingredients necessary for the turn. Those ingredients are

1. a horizontal force at right angles to the flight path used to change the airplane's direction of flight and
2. a mechanism for rotating the airplane's longitudinal axis, and thereby the horizontal force, as the turn progresses.

Note that a horizontal force alone may not produce a turn. For instance, if the pilot rolls into a moderate bank, but then applies opposite rudder to prevent the fuselage from rotating about the vertical axis, a simple translation will result (albeit in a new direction). This is called a slip.

Also note that the pilot is not restricted to using lift as the horizontal force. For instance, if the pilot keeps the wings level but yaws the airplane with the rudder, the thrust now has a component lateral to the flight path that will supply the necessary horizontal force, and a turn will result. (This turn will be less efficient, of course, since at cruise speeds the thrust produced by a light airplane is 5 to 6 times smaller than its lift.)

In a normal turn that uses a bank, the necessary mechanism for keeping the horizontal force at right angles to the flight path is the relative wind, as it strikes the horizontal stabilizer at the tail of the airplane. In effect, the airplane acts like a weather vane.

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†NASA Lewis Research Center, Cleveland, OH 44135.

In the yaw-induced turn of the second example, the pilot keeps the rudder deflected to produce a moment that counteracts the weathervaning tendency. Interestingly enough, in this situation the stabilizer-rudder combination is acting like a small vertical wing, producing a lift at the tail that is basically horizontal and opposite to the direction of turn.

In casual conversation, a phrase like "lift makes the airplane turn" is perfectly acceptable, but in the classroom or the pages of a scientific journal, a higher standard of accuracy should be our goal.

**2. The coefficient of lift.** As the oncoming wind sweeps over the cambered, slightly up-tilted wings of an airplane, an aerodynamic force is produced. That force may be resolved into two components, one parallel to the oncoming wind or the flight path, the other perpendicular. The perpendicular component is by far the larger and has been historically referred to by the name *lift*.

The lift $L$ produced by the wings of an airplane is given by

$$L = \frac{1}{2}\rho V^2 C_L S, \tag{1}$$

where $\rho$ is the air density, $V$ the airspeed, $C_L$ the coefficient of lift, and $S$ the wing area. In the usual level turn made with bank angle $\theta$, the lift $L$ has a vertical component $L\cos\theta$ equal to the airplane's weight $W$ and a horizontal component equal to the required centripetal force,

$$L\sin\theta = m\frac{V^2}{R} = \frac{WV^2}{gR}, \tag{2}$$

where $g$ is the acceleration of gravity and $R$ the radius of the turn.

If we combine these results one way we find that $V^2 = gR\tan\theta$, but if we combine them another way we get

$$C_L = \frac{2W}{\rho V^2 S\cos\theta}, \tag{3}$$

which shows that, for fixed airspeed in a level turn, $C_L$ is inversely proportional to $\cos\theta$. It is quite clear, therefore, that the coefficient of lift, which at a given airspeed controls the load on the wing and must be increased to increase bank angle, cannot be dismissed as "essentially constant" for purposes of analysis.

**3. Lift and stall speed.** The term "stall speed" is something of a misnomer. It suggests that an airplane somehow measures the speed of the oncoming air, and upon finding it below an acceptable value, promptly refuses to fly. In this view all airplanes parked on the ramp are stalled.

In fact an airplane wing can be stalled at *any* airspeed, as the FAA has been emphasizing for decades without much success. On the other hand, there is a certain limiting airspeed below which an airplane cannot be maintained in steady level flight. This limiting speed is often referred to by the FAA as "minimum controllable airspeed," but it has the symbol $V_S$ and is popularly known as the "stall speed."

The reference, of course, is to the fact that an attempt to maintain altitude, yet fly even slower, will produce a stall. But the knowledgeable pilot is expected to understand that a stall — the sudden breakup of smooth, laminar flow over the wing resulting in a sharp reduction of lift — is caused not by any change in the speed of the airflow, but by the increase in the wing's angle of attack beyond a certain critical value.

From a philosophical perspective, decreased airspeed (and anticipated loss of lift) is the "formal" cause, namely, the motivation for the pilot's decision to increase the angle of attack. But the direct physical cause of a stall is the increased angle of attack.

Even without this knowledge, it is apparent that the phrase "level flight without enough lift to support the weight" is an oxymoron. But steady flight with $L < W$ is not only possible, it is common and safe, and happens at least twice during every flight. *An airplane in steady, unaccelerated flight, whose lift is less than its weight, is either ascending or descending*. (See the Appendix.)

**4. Stalls in the turn.** Stalls during a turn occur for the same reason that they do in level flight: the angle of attack is increased beyond its critical value. They do *not* occur simply because the airspeed becomes too low or the bank angle too large.

So what does happen when the airspeed becomes too low or the bank angle too large? Since the vertical component of lift is now less than the weight, the airplane will begin to *descend while it turns*. As long as the pilot does not increase the angle of attack, the airplane will not stall, nor will it fall out of the sky, nor will its turn radius become infinite. It will simply descend while it turns.

Prospective commercial pilots practice just such a maneuver while preparing for their check ride: the maneuver is called a gliding spiral. It is taught primarily as an emergency procedure, designed to keep the airplane directly over the selected landing area during the descent, and is typically performed at idle power, low airspeed, and moderately steep bank.

Despite the existence of the gliding spiral in the flight training syllabus, the myth persists that an airplane will automatically stall if the airspeed is low and/or the bank is steep. Why? It is because of the *hidden* assumption that the altitude must remain constant. I call it hidden because in so many texts and airplane manuals the typical charts that depict the "increase of stall speed" with bank angle do not include a warning or legend or note with the crucial proviso "at constant altitude." My experience as a flight instructor has convinced me that the failure to stress this assumption (let alone ignore its existence) is responsible for more accidents in general aviation than any other cause.

It is true that in Wolper's case the condition "level" is stated, not assumed. Nevertheless, a sentence that begins "If the airspeed is too low for the angle of bank then the airplane stalls. . ." misleadingly implies a causal relation between airspeed and stall that does not exist if the pilot simply abandons the attempt to maintain altitude.

**5. The maximum performance turn.** A maximum performance turn may be defined as one having minimum turn radius. Since both lift and centripetal acceleration are proportional to $V^2$, that quantity can be eliminated from (1) and (2) and the resulting equation solved for $R$, yielding

$$R = \frac{W}{\frac{1}{2}\rho g S C_L \sin\theta}. \tag{4}$$

Observe that the minimum turn radius is achieved by using maximum coefficient of lift and maximum bank angle, *independent of airspeed.*

Assuming from now on that $C_L$ has its maximum value $(C_L)_{\max}$, we can replace $(C_L)_{\max}$ by its expression in terms of weight and minimum controllable airspeed $V_S$, obtaining

$$R_{\min}(\theta) = \frac{R_0}{\sin\theta}, \qquad R_0 = \frac{V_S^2}{g}. \tag{5}$$

Of course $R_0$ can be achieved only with a 90° bank and consequent altitude loss which, for a single course reversal, can be estimated as

$$\Delta h = \frac{1}{2} g t^2 = \frac{1}{2} g \left(\frac{\pi R_0}{V}\right)^2 = \frac{1}{2}\left(\frac{\pi V_S}{V}\right)^2 R_0. \tag{6}$$

Evidently, to minimize altitude loss, $V$ should be as large as possible without risking undue stress on the wings. Every airplane has a particular airspeed, known as the "maneuvering speed" or "turbulent air penetration airspeed" (denoted by $V_a$), whose usage guarantees that the wings will not be accidentally overstressed. For a "Normal Category" airplane $V_a = \sqrt{3.8}V_S$, and so if this airspeed is used, $\Delta h \cong 1.3R_0$.

If a level turn is required then (3) comes into play, and from the form it takes when $C_L = (C_L)_{\max}$,

$$\cos\theta = \frac{V_S^2}{V^2},$$

we see that $V$ must be as large as possible to achieve maximum bank angle. Again, $V_a$ should be used for safety, with the result

$$\theta = \cos^{-1}\left(\frac{V_S}{V_a}\right)^2 \cong 75^\circ, \qquad R_{\text{level}} = \frac{R_0}{\sqrt{1-(V_S/V_a)^4}} \cong 1.03R_0. \tag{7}$$

There are other ways of defining a maximum performance turn that include climbing during the turn; many of these can produce course reversals requiring less than $2R_0$ of horizontal airspace.

**6. Appendix: Lift and weight.** Of the four major forces acting on the airplane (lift, drag, thrust, and weight) two have, by definition, directions that are *fixed* relative to the flight path: lift is always perpendicular and drag is always parallel. Except in rare instances,[1] the third force, thrust, also lies along the flight path. The weight, however, is independent of flight path and is always directed toward the earth.

Now consider an airplane whose flight path is inclined at an angle $\beta$ to the horizontal and is therefore climbing or descending. The weight of the airplane can be resolved into two components, one of them being a force $W\cos\beta$ perpendicular to the flight path. If the airplane is in steady state motion, i.e., in equilibrium, then there must be another force perpendicular to the flight path having exactly the same magnitude. We conclude that $L = W\cos\beta$, and therefore $L < W$ in a steady climb or descent.[2]

The limiting case, a vertical climb and descent with zero lift, may be observed at any airshow in the maneuver known as the "hammerhead."[3] To perform it, the airplane enters a totally vertical climb, slows down while rising because of its weight, and then, just as its airspeed reaches zero, gracefully arcs over to a totally vertical descent. Although the climb and descent portions are not steady, they are *straight*, and so the single perpendicular force, lift, must therefore vanish.

Once these facts are fully appreciated by the student a number of misconceptions disappear, and the quantity known as lift can be regarded simply as a force

1. conveniently perpendicular to the flight path,
2. used in part to counteract gravity and in part for maneuvering purposes,
3. whose production requires an angle of attack that may become close to a critical value, and thus must be treated with care.

## REFERENCE

[1] J. S. Wolper, *Turn performance of aircraft*, SIAM Rev., 36 (1994), pp. 470–473.

[1] One example is the yaw-induced turn described above.

[2] It is my experience that, despite all their training, 99 out of 100 pilots are unaware that lift is actually smaller than weight during a steady climb.

[3] Actually it is called a "hammerhead stall," but this too is a misnomer since no stall occurs.