This is the Pre-Published Version.

Electronic version of an article published as Asia-Pacific Journal of Operational Research, 37(2), 2020, 2050005, Article DOI: 10.1142/S0217595920500050,

Quantity leadership for a dual-channel supply chain with retail service

Ciwei Dong

School of Business Administration, Zhongnan University of Economics and Law Wuhan 430073, China, dongciwei@zuel.edu.cn

Liu Yang

Business School, University of International Business and Economics, Chaoyang District, Beijing, 100029, China, yangliu@uibe.edu.cn

Chi To Ng*

Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, daniel.ng@polyu.edu.hk

Abstract

In a dual-channel supply, the manufacturer sells the products by both a traditional channel via the retailer and an online channel directly. Comparing with the direct channel, the retailer may provide additional services to the traditional channel. This paper studies the quantity leadership for a dual-channel supply chain with retail service. The manufacturer decides the wholesale price of the products and its selling quantity via the online channel, and the retailer decides the service level and its selling quantity via the traditional channel. We consider three Cournot competition games: Manufacturer-as-leader game, Retailer-as-leader game, and Simultaneous game. Optimal solutions are derived for these games. Based on the optimal solutions, we investigate the quantity leadership/followership decisions for the manufacturer and retailer, associated with the changes of some parameters. We observe that when the service sensitivity parameters are low, being a follower is a dominant strategy for the retailer; otherwise, both strategies of manufacturer-as-leader (retailer as the follower) and retailer-as-leader (manufacturer as the follower) are Nash equilibriums. We further conduct the numerical studies to investigate the impacts of parameters related to the retail service, and discuss the insights of the findings.

Key words: quantity leadership; dual-channel; retail service; channel competition

*Corresponding author.

1 Introduction

With the rapid development of the e-commerce and the third-party logistics, manufacturers have been selling products through the online channel directly (Chen et al., 2008; Chiang, 2010; Huang et al., 2012; Choi et al., 2013). For example, many brand name companies in a variety of industries, such as Hewlett & Packard, IBM, Lenovo, Nike, Estee Lauder and Walmart, have begun to sell their products to the customers directly by an online channel (Huang et al., 2012; Chiang et al., 2003; Tsay and Agrawal, 2004b; Dumrongsiri et al., 2008; Hua et al., 2010; Xu et al., 2014). More recently, the Chinese e-commerce giant, Alibaba, launched it's initial public offering (IPO) and became the largest IPO in the U.S. history, which shows in a way that online buying has become an important channel for the customers (Forbes.com, 2014). Through online buying, the customers can easily compare the selling prices of products on different web sites, and it may save costs for the customers. On the other hand, manufacturers may still sell their products via the retailers by the traditional channel, in addition to the online channel. Such a supply chain, co-existing the traditional channel and online channel, is called a dual-channel supply chain.

Facing the threat of the development of the online channel, the retailers need to put efforts, such as improving the service quality, to attract the customers buying products by the traditional channel. Because in a dual-channel supply chain, the manufacturer and its retailer sell the same products to the customers, there exists competition between the traditional channel and the online channel. Retail service plays an important role on the channel choice for the customers, and improving the retail service has significant positive effects on customers' preference for the traditional channel (Yan and Pei, 2009; Dan et al., 2012).

Many existing researches on dual-channel supply chain focus on pricing strategies (e.g., Chiang et al., 2003; Cattani et al., 2006; Cai et al., 2009). However, the quantity strategies of the manufacturer and the retailer for dual-channel supply chain, especially with the consideration of retail service, remain unknown. Besides, either the manufacturer or the retailer could act as a Stackelberg leader and first decide the production quantity. The party that first decides the production quantity may obtain a higher profit and exhibits first-mover advantage (Vives, 2001; Wang et al., 2013). But incentives of both parties in dual-channel supply chain in choosing the quantity leadership/followership are still unclear. These observations motivate this study.

In this paper we consider a dual-channel supply chain, in which a manufacturer sells its products to the customers by a traditional channel and an online channel. In the traditional channel, the products are sold to the customers via a retailer, and in the online channel, the products are sold to the customers directly. Comparing with the online channel, the retailer offers an additional retail service to the customer. Retail service can be characterized as the social interactions gained from shopping, or fitting, etc. The manufacturer decides its selling quantity via the online channel and the wholesale price of the products. The retailer decides its selling quantity and the service level. Here, the service level could be measured in terms of the service quality (e.g., the shopping environment of the retail store and the availability of the products in the retail store). There is Cournot competition between the manufacturer and the retailer. We consider three Cournot competition games: *Manufacturer-as-leader game*, *Retailer-as-leader game*, and *Simultaneous game*. In order to investigate the quantity leadership/followership, we further consider an extended two-stage game, namely the *endogenous timing game* (see, e.g., Hamilton and Slutsky, 1990; Amir and Stepanova, 2006). In the first stage of the extended game, the two parties simultaneously choose to be a leader or a follower. In the second stage of the extended game, the two parties play one of the three games (i.e., the Manufacturer-as-leader game, Retailer-as-leader game, and Simultaneous game).

We derive the optimal solutions of the three games for the manufacturer and the retailer. In any game, there is a positive correlation between the retailer's service level and selling quantity. That is, optimal response of the service level increases in the selling quantity, or the optimal response of the selling quantity increases in the service level. Besides, if the wholesale price increases, then the retailer's service level will be decreased. The relationships of other decision variables, such as the relationship of the service level and the manufacturer's selling quantity via the online channel, vary in different games. Our numerical results show that the quantity leadership depends on some factors, such as the service sensitivity parameters for the retail prices. For example, when the service sensitivity parameters for the retail prices in both channels are low, being a follower is a dominant strategy for the retailer; otherwise, both strategies: manufacturer-as-leader (retailer as the follower) and retailer-as-leader (manufacturer as the follower), are Nash equilibriums.

The remainder of this paper is organized as follows: In Section 2 we review the related literature. In Section 3 we present the general model setting of this paper. In Section 4 we analyze the three games. In Section 5 we study the preference of the leadership for the three games. In Section 6 we investigate the impacts of the parameters related to the retail service on the optimal solutions. In Section 7 we conclude the paper and suggest some topics for future research. We provide all the proofs in the Appendix.

2 Literature Review

There are many papers that study issues related to the dual-channel supply chain in the Operations Management / Operations Research (OM/OR) literature (as reviewed by Cattani et al., 2004; Tsay and Agrawal, 2004a). Chiang et al. (2003), Cattani et al. (2006), Cai et al. (2009), He et al. (2013), and Ding et al. (2016) investigate the pricing issues of the dual-channel supply chain. Chiang et al. (2003) and Cattani et al. (2006) focus on the channel conflict, which is incurred when the manufacturer opens up a direct channel in competition with the traditional channel. Tsay and Agrawal (2004b) investigate the channel conflict for the dual-channel supply chain from different perspectives. Some other papers study the dual-channel supply chain with the consideration of other issues, such as production, ordering decision or inventory strategies (e.g., Chiang, 2010; Huang et al., 2012; Chiang and Monahan, 2005; Liu et al., 2010; Takahashi et al., 2011; Niu et al., 2012; Hsieh et al., 2014), coordination of the supply chain (e.g., Xu et al., 2014; Hsieh et al., 2014; Yan, 2008; Cai, 2010; Chen et al., 2012; Cao, 2014), and information sharing (e.g., Yao et al., 2005; Yue and Liu, 2006; Cao et al., 2013).

There are some papers that consider the retail service for the dual-channel supply in the literature. Dumrongsiri et al. (2008) consider a dual-channel supply chain, where the manufacturer decides the price of the direct channel and the retailer decides its retail price and order quantity, and then the customers choose the purchase channel for given prices and service qualities. Chen et al. (2008) study the dual sales channel management with service competition, where traditional channel's service is measured by production availability, and the direct channel's service is measured by the delivery lead time for the product. Yan and Pei (2009) focus on the decision of the retail services in a dual-channel market. They show that improving the retail services can alleviate the channel competition and channel conflict, and improve the supply chain performance. Hua et al. (2010) study the price and lead time decisions in dual-channel supply chains, where the lead time could be regarded as a kind of services. Dan et al. (2012) investigate the pricing policies in a dual-channel supply chain with the consideration of retail services, for both the centralized and decentralized chains. Xu et al. (2013) study the impact of price comparison service on pricing strategy in a dual-channel supply chain. Yet unlike our paper, most of these papers focus on the pricing strategies rather than the quantity decisions. Besides, they do not consider the three game models and the endogenous timing game model as our paper does.

Endogenous timing game is widely examined in the Economics literature (see e.g., Amir and Grilo, 1999; van Damme and Hurkens, 2004; Bárcena-Ruiz, 2007). Only few papers have considered the issues of the endogenous timing game in the OM/OR literature. Wang et al. (2013) consider a supply chain with an original equipment manufacturer (OEM) and a contract manufacturer (CM). OEM and CM can play three games: a sequential game with OEM as the leader, a sequential game with the CM as leader, and a simultaneous game. Based on these three games, their paper investigates the quantity leadership for OEM and CM by using the endogenous timing game. They find that either OEM or CM may prefer to be a leader or a follower. Wang et al. (2014) adopt the endogenous timing game to study the effect of competition on the choice between efficient production (produce before new information of the demand becomes available) and responsive production (produce after new information of the demand becomes available). By considering the scenario where firms are competing in a market with uncertain demand and varying intensity of substitutability for the competitor's production, they characterize the efficient or responsive choice in equilibrium.

3 Modelling

In this paper, we consider a dual-channel supply chain with one manufacturer and one retailer. The manufacturer sells a single type of product to the customers not only through the retailer by a traditional channel, but also by an online channel directly. The customers can buy the product by any channel. For the online channel, the retail price is p_m. The quantity of the products sold via this channel is q_m. For the traditional channel, the retailer orders the products from the manufacturer with a wholesale price w. The order quantity is denoted by q_r. The retailer then sells the products to the customers with a retail price p_r. We consider that both the manufacturer and retailer are operating on the make-to-order basis. Under this setting, the manufacturer's production quantity is equal to the total demand of online and traditional channels, and the retailers order quantity is equal to the demand of traditional channel. In other words, manufacturer's production quantity is equal to $q_m + q_r$, and the quantity of products sold via the traditional channel is equal to q_r. Setting of the make-to-order system is wildly used in the dual-channel literature, e.g., Chiang et al. (2003) and Ding et al. (2016).

The customers buying the products via the traditional channel can get an additional service s, provided by the retailer. The channel structure is pictorially shown in Figure 1.

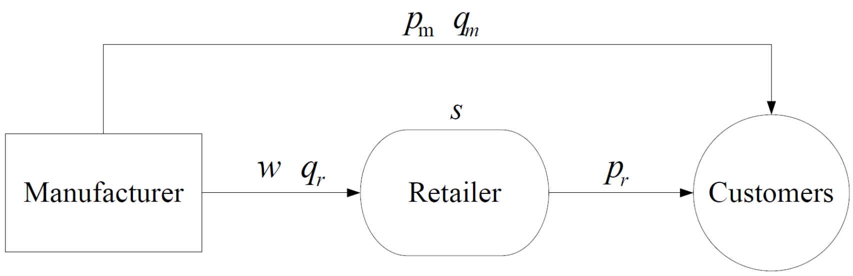


Figure 1: Channel structure

There exist competition issues between the online channel and the traditional channel in the consumer market. We consider that the two channels engage in the quantity-setting Cournot competition. Then the market prices are jointly determined by their selling quantities. That is, we model the market prices as inverse demand functions:

$$\begin{aligned} p_m &= a_m - q_m - b_m q_r + \beta_m s; \\ p_r &= a_r - q_r - b_r q_m + \beta_r s. \end{aligned}$$

Inverse demand functions are wildly used in the OM/OR literature (see, e.g., Wang et al., 2013; Farahat and Perakis, 2011). Here, $a_m > 0$ and $a_r > 0$ are potential market sizes of the online channel and traditional channel, respectively. $b_m > 0$ and $b_r > 0$ are *cross-quantity sensitivity* parameters that measure the cross effects of the selling quantities on the retail price of another channel. Without loss of generality, we set the *quantity sensitivity* parameters that measure the effects of the selling quantities on the retail price of its own channel equal to one. Then we have $b_m < 1$ and $b_r < 1$, which indicate the relationships among the quantity and cross-quantity sensitivity parameters. These relationships state that the retail price in each channel is more sensitive to a change in its own quantity than that of a simultaneous change in the quantity of another channel. $\beta_m > 0$ and $\beta_r > 0$ are service sensitivity parameters. We assume that in the inverse demand functions, the retail service has increase effects on the retail prices in both channels. It is because that if the retail service increases, then the retail price in the traditional channel will be increased, consequently, the manufacturer could increase the retail price in the online channel as well. It is reasonable to assume that increase of the retail price in the traditional channel due to the increase of the service level is larger than the increased price in the online channel. That is, $\beta_r > \beta_m$.

Let c denote the unit production cost. The retailer is subject to the investment cost for the retail service. Similar to Tsay and Agrawal (2004b), Yan and Pei (2009) and Dan et al. (2012), we consider that the investment cost for the retail service is a quadratic function of s, i.e., $c_s s^2/2$, where c_s is the service investment coefficient. The higher the value of c_s, the less efficient the investment is. In practice, it is true that improving the service level is not trivial. So we assume that c_s is high enough, i.e., $c_s > \beta_r^2$.

Let Π_m and Π_r denote the manufacturer's and retailer's profits, respectively. The profit functions can be presented as follows:

$$\begin{aligned}\Pi_m &= (p_m - c)q_m + (w - c)q_r \\ &= (a_m - q_m - b_m q_r + \beta_m s - c)q_m + (w - c)q_r.\end{aligned}$$

$$\begin{aligned}\Pi_r &= (p_r - w)q_r - \frac{c_s}{2}s^2 \\ &= (a_r - q_r - b_r q_m + \beta_r s - w)q_r - \frac{c_s}{2}s^2.\end{aligned}$$

In the manufacturer's profit function, the first term is the profit obtained from the online channel, and the second term is the profit obtained from the traditional channel. In the retailer's profit function, the first term is the profit from selling the products in the consumer market, and the second term is the investment cost for the retail service.

To avoid some trivial outcomes, it is necessary to introduce some additional notations and two relevant assumptions. Let $X = [4(2 - b_r b_m)c_s - (2\beta_r - b_r\beta_m)^2]/(4c_s)$, $Y = (2b_m c_s - 2\beta_m\beta_r + b_r\beta_m^2)/(2c_s)$, and $4X - Y^2 > 0$; and let $N = -4(2\beta_r - b_r\beta_m)/[(4 - b_r b_m)^2 c_s - 2(2\beta_r - b_r\beta_m)^2]$, $L = b_m/2 + (2\beta_m - b_m\beta_r)N/2$, $H = 1 - (2\beta_r - b_r\beta_m)N/2$, and $H(4 - b_r b_m) - 2L^2 > 0$. Otherwise, the manufacturer's profit function could be convex in the wholesale price if we release these two assumptions.

4 Analysis

In this paper, we investigate the quantity leadership for the dual-channel supply chain. The manufacturer and the retailer consider to play three games: Manufacturer-as-leader game, Retailer-as-leader game, and Simultaneous game. Next we discuss these three games in subsections 4.1, 4.2, and 4.3, respectively.

4.1 Manufacturer-as-leader game

For the Manufacturer-as-leader game, we consider a Stackelberg game. Figure 2 illustrates the game sequence. In the first step, the manufacturer, acting as the Stackelberg leader, decides the selling quantity via the online channel q_m and the wholesale price w. In the

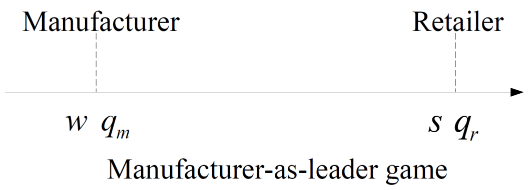


Figure 2: Decision sequence for the Manufacturer-as-leader game

second step, the retailer, acting as the Stackelberg follower, decides its selling quantity q_r and service level s.

The optimization problems for the manufacturer and the retailer in this game are as follows:

$$\max_{q_m, w} \Pi_m; \tag{1}$$

$$\max_{q_r, s} \Pi_r. \tag{2}$$

We use the backward induction approach to solve these optimization problems (1) and (2). That is, we first solve the retailer's problem (2) and obtain the optimal responses of its selling quantity and service level, for given q_m and w. Then we substitute the optimal responses into the manufacturer's profit function and solve the manufacturer's problem (1).

By solving the retailer's problem, we obtain the following results.

Proposition 1. *Given q_m and w, the optimal responses of retailer's selling quantity and service level are given as follows: If $a_r - b_r q_m - w \leq 0$, then $q_r = s = 0$; otherwise,*

$$q_r = \frac{(a_r - b_r q_m - w)c_s}{2c_s - \beta_r^2}; \tag{3}$$

$$s = \frac{\beta_r}{c_s} q_r. \tag{4}$$

Proposition 1 shows that if the quantity of the products sold via the online channel is large enough or the wholesale price is sufficiently high, i.e., $a_r - b_r q_m - w \leq 0$, then both the order quantity and service level equal to zero. In other words, only online channel exists in the market. Otherwise, i.e., $a_r - b_r q_m - w > 0$, the optimal responses of retailer's selling quantity and service level are determined by Equations (3) and (4). Then the retailer should increase its service level when the selling quantity increases. Both the retailer's optimal selling quantity and service level decrease in the manufacturer's selling quantity via the online channel and the wholesale price.

Note that when $a_r - b_r q_m - w \leq 0$, there only exists an online channel in the market. As we focus on the dual-channel supply chain, in the following, we only consider the case when $a_r - b_r q_m - w > 0$. Without further explanation, similar consideration will be applied for the Retailer-as-leader and Simultaneous games. Next we substitute the optimal responses of retailer's selling quantity and service level into the manufacturer's profit function. By solving the manufacturer's problem, we obtain the following results.

Proposition 2. *Manufacturer's optimal selling quantity via the online channel and the wholesale price are given as follows:*

$$\begin{aligned} q_m &= \frac{2c_s(a_m - c)(2c_s - \beta_r^2) - c_s(a_r - c)[(b_r + b_m)c_s - \beta_r\beta_m]}{4c_s(2c_s - \beta_r^2) - [(b_m + b_r)c_s - \beta_r\beta_m]^2}; \\ w &= \frac{(a_r + c)c_s + [(b_m - b_r)c_s - \beta_r\beta_m]q_m}{2c_s}. \end{aligned}$$

Proposition 2 shows the optimal quantity of the productions sold via the online channel and the optimal wholesale price. Note that in the proof of Proposition 2, we show that $4c_s(2c_s - \beta_r^2) - [(b_m + b_r)c_s - \beta_r\beta_m]^2 > 0$. Therefore, in order to guarantee $q_m > 0$, we need the condition $2(a_m - c)(2c_s - \beta_r^2) > (a_r - c)[(b_r + b_m)c_s - \beta_r\beta_m]$. From this condition, we can observe that when the potential market size of the online channel a_m is large, the optimal quantity of the products sold via the online channel is larger than zero; when the potential market size of the traditional channel a_r is large, the optimal quantity of the products sold via the online channel may be less than zero. That is intuitive. Besides, we need the condition $(a_r + c)c_s + [(b_m - b_r)c_s - \beta_r\beta_m]q_m > 0$ to guarantee that the wholesale price is positive.

4.2 Retailer-as-leader game

Figure 3 illustrates the game sequence for the Retailer-as-leader game. First, the manufacturer decides the wholesale price. Second, given the wholesale price w, the retailer decides the selling quantity q_r and the retail service level s. Finally, the manufacturer decides the selling quantity via the online channel q_m.

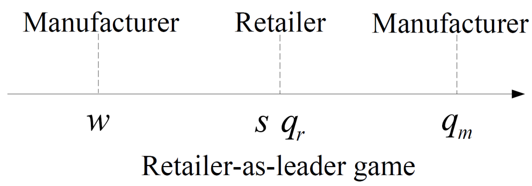


Figure 3: Decision sequence for the Retailer-as-leader game

Note that in this paper we focus on the quantity leadership, so the leadership is embodied in terms of the decision sequence for the selling quantities, for all games. For example, regarding the decision sequence for the Retailer-as-leader game, the retailer decides its selling quantity q_r first, followed by the manufacturer's decision for the selling quantity via the online channel q_m. Besides, for the Retailer-as-leader game, it is possible that the retailer decides the selling quantity and the retail service first, followed by the manufacturer's wholesale price decision. However, in this paper we focus on the effects of the retail service and only consider the decision sequence that the manufacturer decides the wholesale price first.

Similarly, we use the backward induction approach to solve the problems for the Retailer-as-leader game. First, given w, q_r and s, we solve the following optimization problem:

$$\max_{q_m} \Pi_m. \tag{5}$$

It is easy to verify that Π_m is concave in q_m. So we obtain the following result immediately.

Proposition 3. *Given w, q_r and s, the optimal response of manufacturer's selling quantity* *via the online channel is given by*

$$q_m = \frac{a_m - b_m q_r + \beta_m s - c}{2}.$$

Proposition 3 shows the optimal solution for the optimization problem (5). If the quantity of products sold via the traditional channel or the unit production cost are large enough, i.e., $a_m - b_m q_r + \beta_m s - c \le 0$, then the optimal response of manufacturer's selling quantity via the online channel equals to zero. Besides, we observe that the optimal response of manufacturer's selling quantity via the online channel decreases in the retailer's selling quantity and increases in the retailer's service level. It is straightforward to see that the optimal response of manufacturer's selling quantity via the online channel is independent of the wholesale price, for given retailer's decisions. By substituting the optimal response of q_m back into the manufacturer's profit function, we obtain that

$$\Pi_m = (\frac{a_m - b_m q_r + \beta_m s - c}{2})^2 + (w - c)q_r. \tag{6}$$

Next, we substitute the optimal response of q_m into the retailer's objective function Π_r, and solve the following optimization problem.

$$\max_{q_r, s} \Pi_r. \tag{7}$$

By solving the optimization problem (7), we obtain the following results.

Proposition 4. *Given w, the optimal responses of retailer's order quantity and the service level are given by*

$$q_r = \frac{[2a_r - (a_m - c)b_r - 2w]2c_s}{4(2 - b_r b_m)c_s - (2\beta_r - b_r\beta_m)^2};$$
$$s = \frac{(2\beta_r - b_r\beta_m)q_r}{2c_s}.$$

Proposition 4 shows the optimal responses of retailer's order quantity and the service level. Note that in the proof of Proposition 4, we show that $4(2 - b_r b_m)c_s - (2\beta_r - b_r\beta_m)^2 >$

0. Therefore, in order to guarantee $q_r > 0$, we need the condition $2a_r > (a_m - c)b_r + 2w$. From this condition, we can observe that when the potential market size of the traditional channel a_r is large, then the optimal order quantity is larger than zero; when the potential market size of the online channel a_r or the wholesale price is large, the optimal order quantity equals to zero. For the optimal response of the service level, we have $s \geq 0$ as $2\beta_r > b_r\beta_m$. We observe that the retailer's service level increases in its selling quantity. Combining this result with that the retailer's selling quantity decreases in the wholesale price, we find that the retailer's service level decreases in the wholesale price.

Finally, we substitute the optimal responses of q_r and s into the manufacturer's profit function Π_m in Equation (6) and solve the following optimization problem.

$$\max_{w} \Pi_m; \tag{8}$$

We obtain the following result for the optimization problem (8).

Proposition 5. *The optimal wholesale price set by the manufacturer is given by*

$$w = \frac{[2(a_m - c)X - (2a_r - (a_m - c)b_r)Y]Y + (2a_r - (a_m - c)b_r + 2c)2X}{2(4X - Y^2)}.$$

Proposition 5 shows the optimal wholesale price set by the manufacturer for the retailer. Similarly, we need the condition $[2(a_m - c)X - (2a_r - (a_m - c)b_r)Y]Y + (2a_r - (a_m - c)b_r + 2c)2X > 0$ to guarantee that the wholesale price is positive.

4.3 Simultaneous game

Figure 4 illustrates the game sequence for the Simultaneous game. First, the manufacturer decides the wholesale price w. Second, given the wholesale price, the retailer decides the service level s. Finally, the manufacturer and retailer decide their selling quantities simultaneously. Similar to the Retailer-as-leader game, in this simultaneous game, it is possible that the retailer decides the service level first, followed by the manufacturer's wholesale price decision. However, in this paper we focus on the effects of the retail service

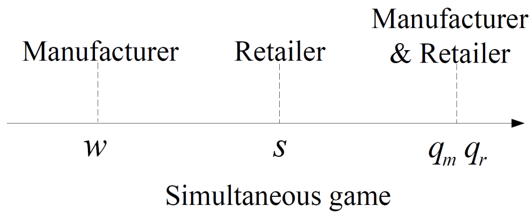


Figure 4: Decision sequence for the Simultaneous game

and only consider the decision sequence that the manufacturer decides the wholesale price first.

Regarding the decision sequences in this game, simultaneous move focuses on the decision sequences of the selling quantities. Similarly, we use the backward induction approach to solve the optimization problems in this game. First, given w and s, we solve the following optimization problems simultaneously.

$$\max_{q_m} \Pi_m; \tag{9}$$

$$\max_{q_r} \Pi_r. \tag{10}$$

Proposition 6. *Given w and s, optimal responses of the manufacturer's selling quantity via the online channel and optimal responses of the retailer's order quantity are given by*

$$\begin{aligned} q_m &= \frac{2(a_m - c) + (2\beta_m - b_m\beta_r)s - (a_r - w)b_m}{4 - b_r b_m} \\ q_r &= \frac{2(a_r - w) + (2\beta_r - b_r\beta_m)s - (a_m - c)b_r}{4 - b_r b_m}. \end{aligned}$$

Proposition 6 shows the optimal responses of the manufacturer's selling quantity via the online channel and the retailer's order quantity. In order to guarantee $q_m > 0$ and $q_r > 0$, we need the conditions $2(a_m - c) + (2\beta_m - b_m\beta_r)s > (a_r - w)b_m$ and $2(a_r - w) + (2\beta_r - b_r\beta_m)s > (a_m - c)b_r$. We find that manufacturer's optimal response of the selling quantity via the online channel may increase or decrease in the retail service, since $dq_m/ds = (2\beta_m - b_m\beta_r)/(4 - b_r b_m)$ which may be positive or negative. However, we find that retailer's selling quantity increases in the retail service, since $dq_r/ds = (2\beta_r - b_r\beta_m)/(4 - b_r b_m) > 0$. Another observation is that for a given s, retailer's selling quantity decreases in the wholesale price, whereas manufacturer's selling quantity via the online channel increases in the wholesale price.

By substituting the optimal responses of q_m and q_r into the manufacturer's and retailer's profit functions, we obtain that

$$\begin{aligned}\Pi_m &= [\frac{2(a_m-c)+(2\beta_m-b_m\beta_r)s-(a_r-w)b_m}{4-b_rb_m}]^2 \\ &\quad +(w-c)\frac{2(a_r-w)+(2\beta_r-b_r\beta_m)s-(a_m-c)b_r}{4-b_rb_m} \qquad (11)\\ \Pi_r &= [\frac{2(a_r-w)+(2\beta_r-b_r\beta_m)s-(a_m-c)b_r}{4-b_rb_m}]^2-\frac{c_s}{2}s^2. \qquad (12)\end{aligned}$$

Next, we solve the following optimization problem and obtain the optimal service level.

$$\max_s \Pi_r. \qquad (13)$$

Proposition 7. *Given w, the optimal response of service level is given by*

$$s = \frac{2(2\beta_r-b_r\beta_m)[2a_r-(a_m-c)b_r-2w]}{(4-b_rb_m)^2c_s-2(2\beta_r-b_r\beta_m)^2}$$

Proposition 7 shows the optimal solution for the optimization (13). Note that in the proof of Proposition 7, we show that $(4-b_rb_m)^2c_s-2(2\beta_r-b_r\beta_m)^2>0$. Therefore, in order to guarantee $s>0$, we need the condition $2a_r>(a_m-c)b_r+2w$. The explanation of this condition is similar to that in Proposition 4. We observe that retailer's service level decreases in the wholesale price.

Finally, we substitute the optimal response of s into the manufacturer's profit function (11), and solve the following optimization problem.

$$\max_w \Pi_m. \qquad (14)$$

The following proposition shows the optimal solution for the optimization problem (14).

Proposition 8. *The optimal wholesale price is given by*

$$\begin{aligned}w &= \frac{1}{8[H(4-b_rb_m)-2L^2]}\Big\{4L[4(a_m-c)-2b_ma_r-(2\beta_m-b_m\beta_r)(2a_r-(a_m-c)b_r)N] \\ &\quad +(4-b_rb_m)[4(cH+a_r)-2(a_m-c)b_r-(2\beta_r-b_r\beta_m)(2a_r-(a_m-c)b_r)N]\Big\} \qquad (15)\end{aligned}$$

Proposition 8 shows the optimal wholesale price set by the manufacturer for the retailer. Similarly, in order to guarantee that the wholesale price is positive, we need the condition that value of items in the brace in Equation (15) is larger than zero.

In the above analyses, we solve the optimization problems for all three games. We summarize some results from preceding analyses to the following proposition.

Proposition 9. *(1) In any game, the optimal response of s decreases in w, and there are positive correlations between q_r and s.*
(2) In the Manufacturer-as-leader game, the optimal responses of s and q_r decrease in q_m; in the Retailer-as-leader game, the optimal response of q_m decreases in q_r and increases in s; and in the Simultaneous game, the optimal response of q_m may increase or decrease in s but it increases in w for a given s.

As part (1) of Proposition 9 shows, there are positive correlations between the service level and the order quantity. If the retailer increases the service level, then the demand of the traditional channel will be increased, such that the retailer will increase the order quantity; or if the retailer increases its order quantity, then in order to sell the increased quantity of products, the retailer will increase its service level. Besides, we find that service level decreases in the wholesale price. One explanation is that when the wholesale price increases, the retailer will decrease the order quantity. Consequently, the service level will be decreased, due to the positive correlations between the retailer's order quantity and service level. Part (2) of Proposition 9 indicates the relationships among the retailer's order quantity, service level, and the manufacturer's selling quantity via the online channel. Recalling that in the Manufacturer-as-leader game, the manufacturer first decides its selling quantity via the online channel, which is followed by the retailer's decisions. We find that in the Manufacturer-as-leader game, the retailer should increase its order quantity and service level, when the manufacturer decreases its selling quantity via the online channel. In the Retailer-as-leader game, the retailer first decides its selling quantity and service level, which is followed by the manufacturer's selling quantity decision. We find that the manufacturer should increase its selling quantity via the online channel if the retailer decreases its selling quantity or increases the service level. In the Simultaneous game, the manufacturer and the retailers decide their selling quantities simultaneously. We find that the manufacturer may increase or decrease its selling quantity via the online channel, when the retailer increases the service level. But for a given service level, the manufacturer will increase its selling quantity via the online channel when it increases the wholesale price. One possible explanation is that when the wholesale price increases, the retailer will decrease its order quantity, and then the manufacturer can increase its selling quantity via the online channel to get more market share.

5 Preference of the Leadership for Three Games

In order to investigate the preference of the leadership for the manufacturer and the retailer, we consider a two-stage extended game, namely the endogenous timing game (see, e.g., Wang et al., 2013; Amir and Stepanova, 2006). In the first stage, the manufacturer and the retailer simultaneously choose either to move first and be the Stackelberg leader, or to move second and be the Stackelberg follower. Let L and F stand for the "Leader" and "Follower", respectively. The joint action of the manufacturer and the retailer for the Manufacturer-as-leader game, the Retailer-as-leader game, and the Simultaneous game are defined as (L, F), (F, L), and (S, S), respectively. The corresponding profits for the manufacturer and the retailer can be presented as (Π_m^L, Π_r^F), (Π_m^F, Π_r^L) and (Π_m^S, Π_r^S), respectively. After announcing the timing sequence of game, in the second stage the manufacturer and the retailer choose to play the game accordingly.

Table 1: Payoffs of each player for the games

		Retailer	
		Leader	Follower
Manufacturer	Leader	$(\Pi_m^S,\ \Pi_r^S)$	$(\Pi_m^L,\ \Pi_r^F)$
	Follower	$(\Pi_m^F,\ \Pi_r^L)$	$(\Pi_m^S,\ \Pi_r^S)$

Table 1 illustrates the payoffs of each player for different games. In the table, "Leader" and "Follower" indicate the roles for manufacturer and the retailer. (Π_m^i, Π_r^j) $(i, j \in \{L, F, S\})$ represents the profits of the manufacturer and the retailer. For example, (Π_m^L, Π_r^F) indicates that the profit of the manufacturer is Π_m^L and the profit of the retailer is Π_r^F, when the manufacturer is the leader and the retailer is the follower.

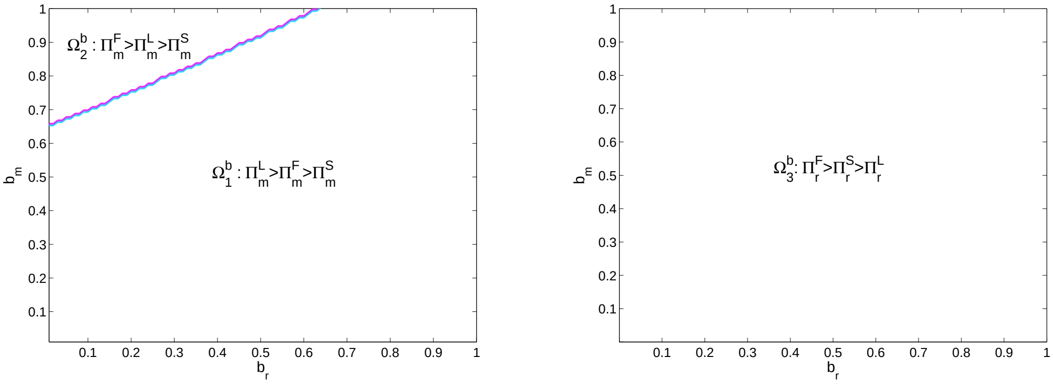
Recalling that in Section 4, we have solved the problems for the first stage of the endogenous timing game and obtained the optimal payoffs of each player for the three games. It is difficult to compare the payoffs of each player directly. In this section, we numerically investigate the preference of the leadership for the manufacturer and the retailer for three games, associated with the changes of the cross-quantity sensitivity parameters (b_r and b_m), service sensitivity parameters for the retail prices (β_r and β_m), and the potential market sizes of two channels (a_r and a_m).

5.1 Preference of the leadership for the changes of b_r and b_m

For numerical studies, we first set $a_m = a_r = 20$, $c = c_s = 5$, $\beta_m = 1$ and $\beta_r = 2$. Figure 5 shows the relationships of the manufacturer's payoffs for the three games, associated with the changes of b_r and b_m.

Figure 5 shows the relationship of the payoffs of the manufacturer and retailer for three games when b_r and b_m changes. X-axis represents the values of b_r and Y-axis represents the values of b_m. Figures 5(a) and 5(b) show the relationships of the manufacturer's profits and the retailer's profits, respectively, for the three games.

From Figure 5, we observe that cross-quantity sensitivity parameters will affect the relationship of the manufacturer's profit for the three games, whereas they will not affect the relationship of the retailer's profit for the three games. Specifically, from Figure 5(a), we find that the manufacturer's profits have the following relationships: in the region Ω_1^b, we have $\Pi_m^L > \Pi_m^F > \Pi_m^S$; and in the region Ω_2^b, we have $\Pi_m^F > \Pi_m^L > \Pi_m^S$. It indicates that when b_r is small and b_m is large, i.e., Ω_2^b, the manufacturer prefers to be a follower rather than be a leader. Otherwise, i.e., Ω_1^b, the manufacturer prefers to be a leader rather than be a follower. From Figure 5(b), we find that the retailer's profit have the following



(a) The relationship of the manufacturer's profits (b) The relationship of the retailer's profits

Figure 5: The relationship of the payoffs of each player for the changes of b_r and b_m

relationship: $\Pi_r^F > \Pi_r^S > \Pi_r^L$. It indicates that the retailer prefers being a follower to being a leader.

Besides, given that the manufacturer chooses to be a leader, the retailer will choose to be a follower, because $\Pi_r^F > \Pi_r^S$; and given that the manufacturer chooses to be a follower, the retailer will choose to be a follower as well, because $\Pi_r^S > \Pi_r^L$. Thus, we have the following observation for this numerical setting:

Observation 1. *F is a dominant strategy for the retailer for the region Ω_3^b.*

By combining the results from Figures 5(a) and 5(b), we obtain the results for the decisions of the first stage, i.e., quantity timing decision stage, of the extended timing game. Because for the region Ω_1^b, we have $\Pi_m^L > \Pi_m^S$ and $\Pi_r^F > \Pi_r^S$. Then based on Table 1, we find that if the retailer chooses to be a follower, then the manufacturer will choose to be a leader; if the manufacturer chooses to be a leader, the retailer will choose to be a follower. Thus, (L, F) is a pure Nash equilibrium for the region Ω_1^b. Similarly, we can find that (L, F) is also a pure Nash equilibrium for the region Ω_2^b. Therefore, by summarizing the above analyses, we have the following observation, which is pictorially shown in Figure 6, for this numerical setting:

Observation 2. *(L, F) is a pure Nash equilibrium for both the regions Ω_1^b and Ω_2^b.*

To this end, we summarize the preference of the leadership for the changes of the cross-

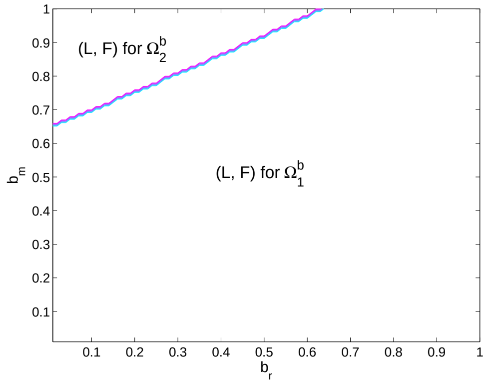


Figure 6: Impacts of b_r and b_m on quantity timing decision

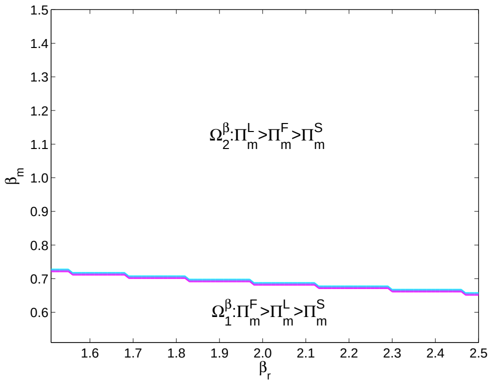
quantity sensitivity parameters. We observe that being a follower is a dominant strategy for the retailer and the strategy of manufacturer-as-leader (retailer as the follower) is a Nash equilibrium, even when cross-quantity sensitivity parameters change.

5.2 Preference of the leadership for the changes of β_r and β_m

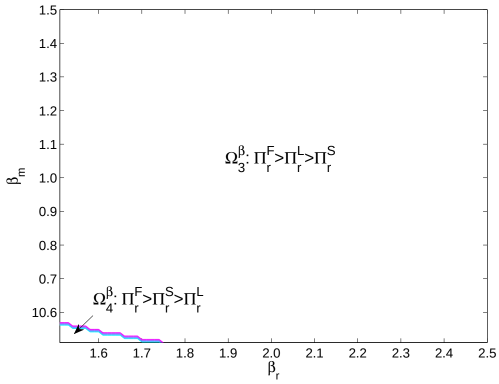
In this subsection, we study the preference of the leadership for three games, associated with the changes of β_r and β_m. We set $a_m = a_r = 20$, $c = 5$, $c_s = 9$, and $b_m = b_r = 0.5$.

Figure 7 shows the relationship of the payoffs of the manufacturer and retailer for three games when β_r and β_m changes. X-axis represents the values of β_r and Y-axis represents the values of β_m. Figures 7(a) and 7(b) show the relationships of the manufacturer's profits and the retailer's profits, respectively, for the three games.

From Figure 7, we find that the service sensitivity parameters will affect both the relationships of the manufacturer's profit and retailer's profit for the three games. Specifically, from Figure 7(a), we find that when β_m is small, i.e., Ω_1^β, the manufacturer prefers being a follower to being a leader; whereas when β_m is large, i.e., Ω_2^β, the manufacturer prefers being a leader to being a follower. From Figure 7(b), we find that in both the regions Ω_3^β and Ω_4^β, the retailer prefers being a follower to being a leader. But when β_r and β_m are large, i.e., Ω_3^β, the retailer's profit in the Retailer-as-leader game is larger than that in the Simultaneous game; and when β_r and β_m are small, i.e., Ω_4^β, the retailer's profit in the Retailer-as-leader game is smaller than that in the Simultaneous game.



(a) The relationship of the manufacturer's profits



(b) The relationship of the retailer's profits

Figure 7: The relationship of the payoffs of each player for the changes of β_r and β_m

Besides, for the region Ω_4^β, we have the following analyses. Given that the manufacturer chooses to be a leader, the retailer will choose to be a follower, because $\Pi_r^F > \Pi_r^S$; and given that the manufacturer chooses to be a follower, the retailer will choose to be a follower as well, because $\Pi_r^S > \Pi_r^L$. Thus, we have the following observation for this numerical setting:

Observation 3. *F is a dominant strategy for the retailer for the region Ω_4^β.*

By combining the results from Figures 7(a) and 7(b), we obtain the results for the quantity timing decision stage. Similar to the analyses for the changes of b_r and b_m, here, we summarize the results for the quantity timing decision stage, which are pictorially shown in Figure 8, in the following observation:

Observation 4. *(L, F) and (F, L) are two pure Nash equilibriums for both the regions Ω_2^β and $\Omega_1^\beta - \Omega_4^\beta$, and (L, F) is the unique pure Nash equilibrium for the region Ω_4^β. Here, $\Omega_1^\beta - \Omega_4^\beta$ is the region that is included in Ω_1^β but excludes Ω_4^β.*

To this end, we summarize the preference of the leadership for the changes of the service sensitivity parameters. We observe that if both the service sensitivity parameters β_m and β_r are small (e.g., Ω_4^β in Figure 8), being a follower is a dominant strategy for the retailer, and the strategy of manufacturer-as-leader (retailer as the follower) is a Nash equilibrium; otherwise, both strategies of manufacturer-as-leader (retailer as the follower)

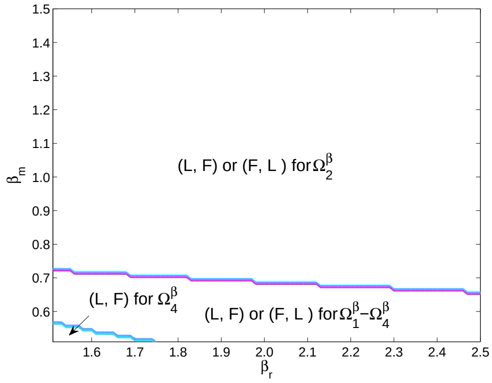


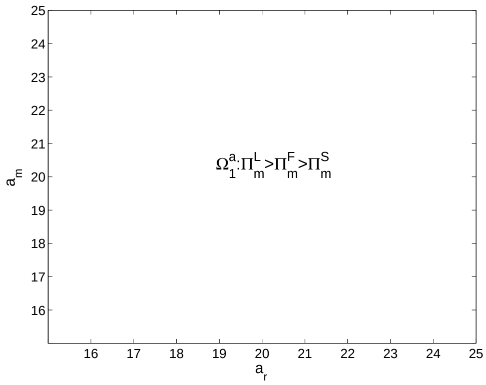
Figure 8: Impacts of β_r and β_m on quantity timing decision

and retailer-as-leader (manufacturer as the follower) are Nash equilibriums.

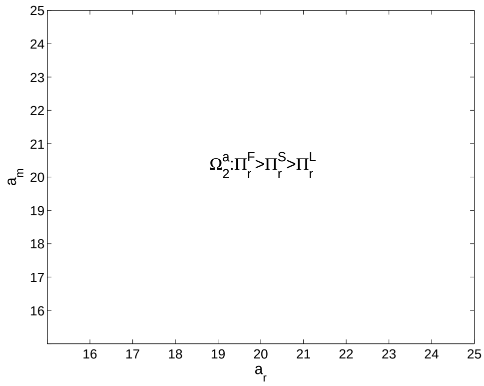
5.3 Preference of the leadership for the changes of a_r and a_m

In this subsection, we study the preference of the leadership for the three games, associated with the changes of a_r and a_m. We set $b_m = b_r = 0.5$, $\beta_m = 1$, $\beta_r = 2$, and $c = c_s = 5$.

Figure 9 shows the relationship of the payoffs of the manufacturer and retailer for three games when a_r and a_m changes. X-axis represents the values of a_r and Y-axis represents the values of a_m. Figures 9(a) and 9(b) show the relationships of the manufacturer's profits and the retailer's profits, respectively, for the three games.



(a) The relationship of the manufacturer's profits



(b) The relationship of the retailer's profits

Figure 9: The relationship of the payoffs of each player for the changes of a_r and a_m

From Figure 9, we observe that the potential market size will not affect both the relationships of the manufacturer's profit and retailer's profit for the three games. For all values of a_m and a_r, the manufacturer prefers being a leader to being a follower, and the retailer prefers being a follower to being a leader. Similar to the changes of b_r and b_m, here we have the following observation for this numerical setting:

Observation 5. *F is a dominant strategy for the retailer for the region Ω_2^a.*

By combining the results from Figures 9(a) and 9(b), we obtain the results for the quantity timing decision stage. Similarly, here, we summarize the results for the quantity timing decision stage, which are pictorially shown in Figure 10, in the following observation:

Observation 6. *(L, F) is a pure Nash equilibrium for the regions Ω_1^a (Ω_2^a).*

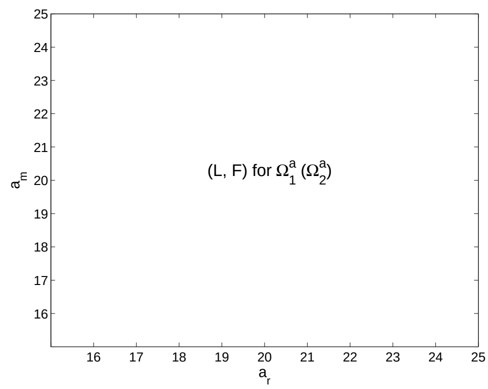
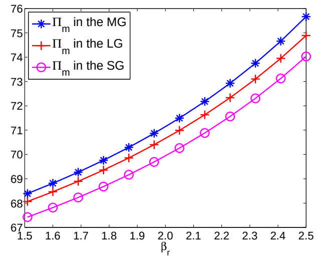


Figure 10: Impacts of a_r and a_m on quantity timing decision

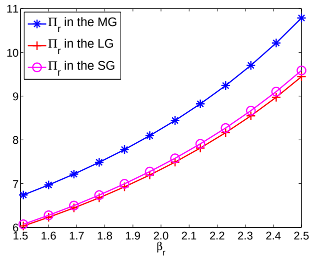
To this end, we summarize the preference of the leadership for the changes of potential market sizes. We observe that being a follower is a dominate strategy for the retailer and the strategy of manufacturer-as-leader (retailer as the follower) is a Nash equilibrium, even when the potential market sizes change.

6 Impacts of Parameters Related to the Retail Service

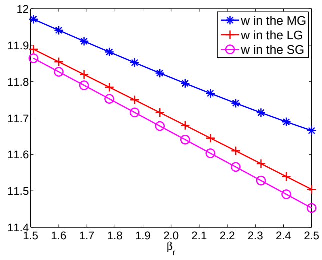
In this section, we numerically study the impacts of parameters related to the retail service, i.e., service sensitivity parameter for the retail price of the traditional channel (β_r), service sensitivity parameter for the retail price of the online channel (β_m) and service investment coefficient (c_s), on the optimal solutions. We first set $a_m = a_r = 20$, $b_m = b_r = 0.5$, $\beta_m = 1$, $\beta_r = 2$, $c = 5$, and $c_s = 9$. For the impacts of service sensitivity parameters, we change β_r from 1.5 to 2.5 and change β_m from 0.5 to 1.5, respectively; and for the impacts of service investment coefficient, we change c_s from 5 to 15.



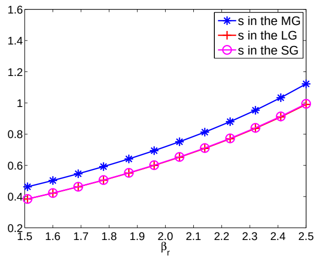
(a) Manufacturer's profits



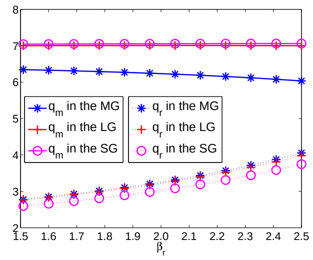
(b) Retailer's profits



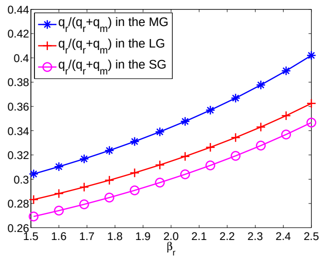
(c) Wholesale prices



(d) Service levels



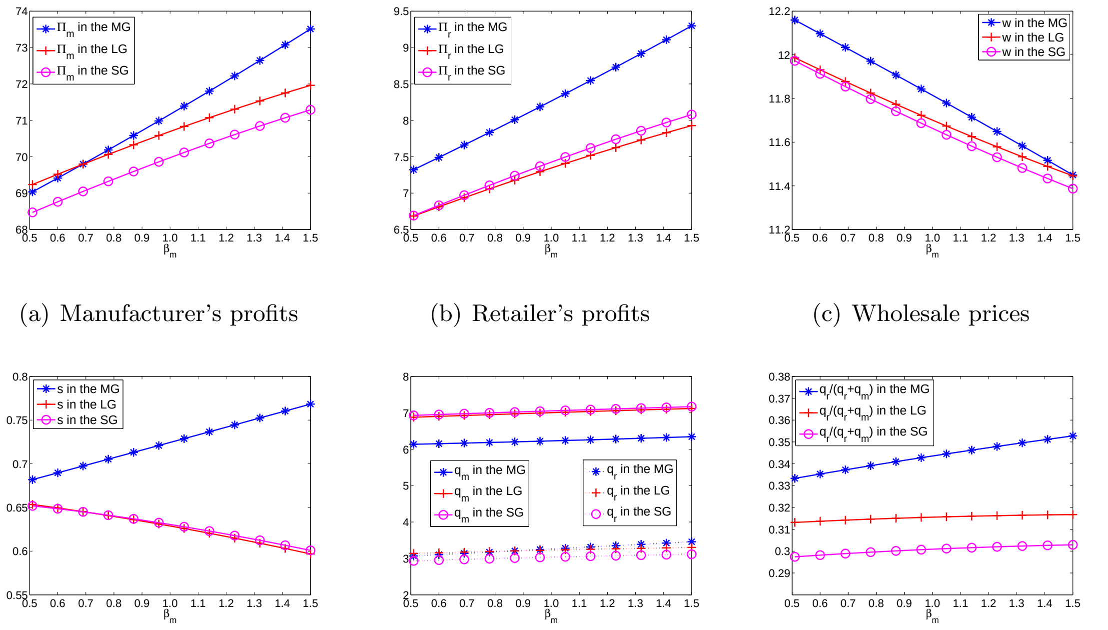
(e) Selling quantities



(f) Ratio of the retail's selling quantities

Figure 11: Impacts of the service sensitivity parameter for the retail price of the traditional channel (β_r)

Figure 11 shows the impacts of the service sensitive parameter β_r on the optimal solutions. Here, "MG" stands for the Manufacturer-as-leader game, "LG" stands for the Retailer-as-leader game, and "SG" stands for the Simultaneous game. We find that not only the retailer's profits but also the manufacturer's profits increase in β_r. It may be because that the service sensitivity parameter β_r has a positive effect on the retail price of the traditional channel. If the customers in the traditional channel are more sensitive to the service, then the retailer will increase its service level. As shown in Proposition 9, there are negative correlations between the service level and the wholesale price, then manufacturer will decrease the wholesale prices consequently. For the selling quantities, the impacts are not uniform. We put the impacts on q_m and q_r together in Figure 11(e), which does not show many fluctuations for the impacts on q_m in this figure. However, in fact, we find that manufacturer's selling quantity via the online channel is decreasing in β_r for MG, concave in β_r for LG, and increasing in β_r for SG. For the impacts on q_r, Figure 11(e) shows that the retailer will increase its order quantities when the customers for the traditional channel are more sensitive to the retail service. It is because high service level can stimulate the demand in the traditional channel. Although manufacturer's selling quantity via the online channel may increase in β_r, as Figure 11(f) shows, in any game, the ratios of the retailer's selling quantities over the total selling quantities increase in β_r. It indicates that when the customers in the traditional channel are more sensitive to the retailer service, the market share of the traditional channel will be increased.

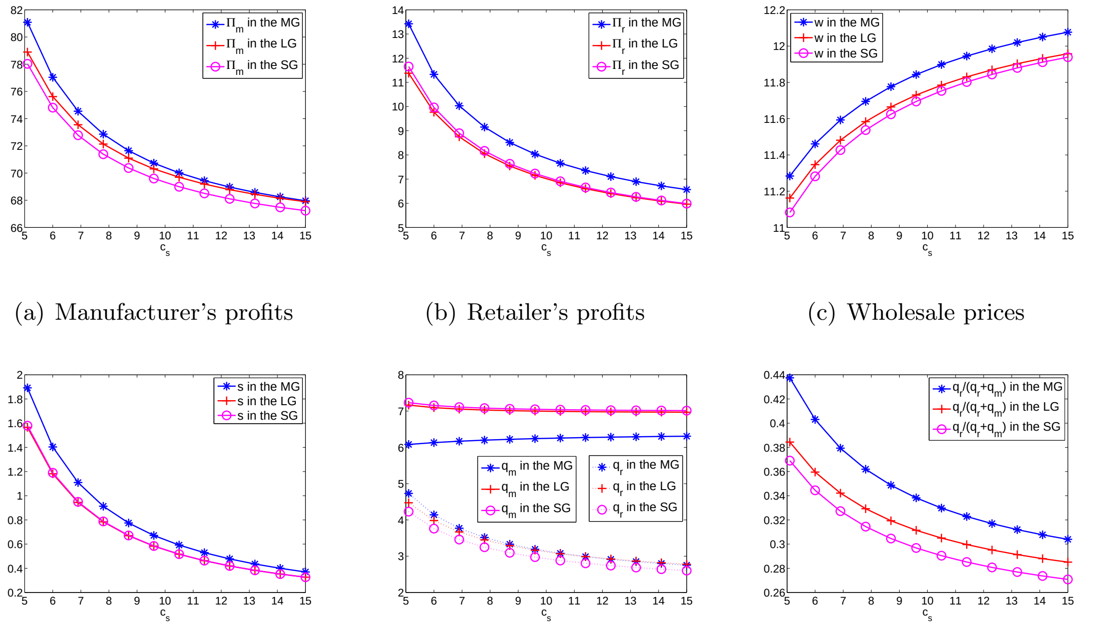


(a) Manufacturer's profits (b) Retailer's profits (c) Wholesale prices

(d) Service levels (e) Selling quantities (f) Ratio of the retail's selling quantities

Figure 12: Impacts of the service sensitivity parameter for the retail price of the online channel (β_m)

Figure 12 shows the impacts of the service sensitive parameter β_m on the optimal solutions. Similar to the impacts of β_r, in any game, the manufacturer's and the retailer's profits increase in β_m, the wholesale prices decrease in β_m, and ratios of the retailer's selling quantities over the total selling quantities increase in β_m. However, for the impacts on the service levels, s increases in β_m for the MG, and decreases in β_m for the LG and SG. Besides, differs from the impacts of β_r, here, in any game, manufacturer's selling quantities via the online channel and retailer's selling quantities increase in β_m.



(a) Manufacturer's profits (b) Retailer's profits (c) Wholesale prices

(d) Service levels (e) Selling quantities (f) Ratio of the retail's selling quantities

Figure 13: Impacts of the service investment coefficient (c_s)

Figure 13 shows the impacts of the service investment coefficient (c_s) on the optimal solutions. It is straightforward to see that manufacturer's profits, retailer's profits, and the service levels decrease in the service investment coefficient. The manufacturer will increase the wholesale prices when the service investment coefficient increases, because the wholesale price has a negative correlation with the service level which is decreasing in the service investment coefficient. For the impacts on the manufacturer's selling quantities via the online channel, we observe that q_m may increase or decrease in c_s for different games. Specifically, q_m increases in c_s for the MG, and decreases in c_s for the LG and SG. For the impacts on the retailer's selling quantities, we observe that in any game, q_r decreases in c_s. In any game, the ratios of the retailer's selling quantities over the total selling quantities decrease in c_s (the higher the value of c_s, the less the efficiency of the investment). It indicates that when the service investment becomes less efficient, the market share of the traditional channel will be decreased.

7 Conclusions

This paper studies the quantity leadership for a dual-channel supply chain with one manufacturer and one retailer, with the consideration of retail service. There exists the Cournot competition between the manufacturer and the retailer. We consider three games, i.e., Manufacturer-as-leader game, Retailer-as-leader game, and Simultaneous game, for the manufacturer and the retailer. We find the optimal selling quantity via the online channel and wholesale price of the products for the manufacturer, and the optimal selling quantity via the traditional channel and the service level for the retailer, for the three games.

To investigate the preference of the leadership for the manufacturer and the retailer, we adopt the endogenous timing game. Our numerical results show that the quantity leadership depends on some factors, such as the cross-quantity sensitivity parameters, service sensitivity parameters for the retail prices, and potential market sizes of the two channels. Regarding the changes of the cross-quantity sensitivity parameters and potential market sizes of the two channels, being a follower is a dominant strategy for the retailer and the strategy of manufacturer as the leader (retailer as the follower) is a Nash equilibrium for all changes in our setting. Regarding the changes of the service sensitivity parameters for the retail prices, being a follower is a dominant strategy for the retailer if the service sensitivity parameters for the retail prices in both channels are low; otherwise, both the strategies of manufacturer as the leader (retailer as the follower) and retailer as the leader (manufacturer as the follower) are Nash equilibriums in our setting.

We further study the impacts of parameters related to the retail service on the optimal solutions. We find that both the manufacturer's and the retailer's profits will be increased if the customers are more sensitive to the retail service. Although the impacts on the manufacturer's selling quantity via the online channel vary in different games, the market shares of the traditional channel increase in the service sensitivity parameters for the retail prices, for all games. For the impacts of the service investment coefficient, we find that the manufacturer's and the retailer's profits, the service levels, and the market shares of the traditional channel will be decreased when the service investment becomes less efficient.

There are some interesting future research directions. First, it is worth considering the risk issues for the dual-channel supply chain with the retail service. In this paper we consider the retail service in a dual-channel supply chain with deterministic demand. So second future research direction could be to consider the stochastic demand for the dual-channel supply chain. Besides, we assume that all information is known to both parties. An extension of the model to consider the asymmetric information may provide additional insights. It is also interesting to consider a scenario that the manufacturer and the retailer negotiate the wholesale price via a generalized Nash bargaining scheme in future research.

Acknowledgments

The authors thank the area editor and anonymous reviewers for their constructive comments. This research was supported in part by the National Natural Science Foundation of China (Nos. 71571043 and 71601187). It was also supported in part by the Zhongnan University of Economics and Law (No. 21141610815), and Fundamental Research Funds for the Central Universities in Zhongnan University of Economics and Law (No. 2016GS07).

Appendix

Proof of Proposition 1

Proof. In this proof we show that Π_r is jointly concave in q_r and s. Taking the first and second derivatives of Π_r with respect to q_r obtains that

$$\begin{aligned}\frac{\partial \Pi_r}{\partial q_r} &= -2q_r + a_r - b_r q_m + \beta_r s - w;\\ \frac{\partial^2 \Pi_r}{\partial q_r^2} &= -2 < 0.\end{aligned}$$

Taking the first and second derivatives of Π_r with respect to s obtains that

$$\begin{aligned}\frac{\partial \Pi_r}{\partial s} &= \beta_r q_r - c_s s;\\ \frac{\partial^2 \Pi_r}{\partial s^2} &= -c_s < 0.\end{aligned}$$

Taking the cross derivatives of Π_r with respect to q_r and s obtains that

$$\frac{\partial^2 \Pi_r}{\partial q_r \partial s} = \beta_r.$$

Then we show that

$$\frac{\partial^2 \Pi_r}{\partial q_r^2}\frac{\partial^2 \Pi_r}{\partial s^2} - (\frac{\partial^2 \Pi_r}{\partial q_r \partial s})^2 = 2c_s - \beta_r^2 > c_s - \beta_r^2 > 0.$$

Thus, we obtain that Π_r is jointly concave in q_r and s. Therefore, we can obtain the optimal responses of q_r and s by solving the equations of the first-order conditions: $\frac{\partial \Pi_r}{\partial q_r} = 0$ and $\frac{\partial \Pi_r}{\partial s} = 0$. □

Proof of Proposition 2

Proof. In this proof we show that Π_m is jointly concave in q_m and w. Taking the first and second derivatives of Π_r with respect to q_m obtains that

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial q_m} &= a_m - \frac{a_r - w}{2 - \frac{\beta_r^2}{c_s}}(b_m - \frac{\beta_r \beta_m}{c_s}) - c - 2q_m(1 - \frac{b_r b_m}{2 - \frac{\beta_r^2}{c_s}} + \frac{b_r}{2 - \frac{\beta_r^2}{c_s}}\frac{\beta_r \beta_m}{c_s}) - \frac{b_r}{2 - \frac{\beta_r^2}{c_s}}(w - c); \\
\frac{\partial^2 \Pi_m}{\partial q_m^2} &= -\frac{2}{2 - \frac{\beta_r^2}{c_s}}[2 - \frac{\beta_r^2}{c_s} + b_r(\frac{\beta_r \beta_m}{c_s} - b_m)] \\
&\leq -\frac{2}{2 - \frac{\beta_r^2}{c_s}}[1 + b_r(\frac{\beta_r \beta_m}{c_s} - b_m)] \\
&\leq -\frac{2}{2 - \frac{\beta_r^2}{c_s}} b_r \frac{\beta_r \beta_m}{c_s} \\
&< 0,
\end{aligned}$$

where the first inequality holds because $c_s \geq \beta_r^2$ and the second inequality holds because $b_r < 1$ and $b_m < 1$.

Taking the first and second derivatives of Π_m with respect to w obtains that

$$\begin{aligned}
\frac{\partial \Pi_m}{\partial w} &= \frac{1}{2 - \frac{\beta_r^2}{c_s}}[q_m(b_m - b_r - \frac{\beta_r \beta_m}{c_s}) + a_r + c - 2w]; \\
\frac{\partial^2 \Pi_m}{\partial w^2} &= -\frac{2}{2 - \frac{\beta_r^2}{c_s}} < 0.
\end{aligned}$$

Taking the cross derivatives of Π_m with respect to q_m and w obtains that

$$\frac{\partial^2 \Pi_m}{\partial q_m \partial w} = \frac{1}{2 - \frac{\beta_r^2}{c_s}}(b_m - b_r - \frac{\beta_r \beta_m}{c_s}).$$

Then we show that

$$\begin{aligned}\frac{\partial^2\Pi_m}{\partial q_m^2}\frac{\partial^2\Pi_m}{\partial w^2}-(\frac{\partial^2\Pi_m}{\partial q_m\partial w})^2 &= \frac{1}{(2-\frac{\beta_r^2}{c_s})^2}[4(2-\frac{\beta_r^2}{c_s})-(b_m+b_r-\frac{\beta_r\beta_m}{c_s})^2] \\ &= \frac{1}{(2-\frac{\beta_r^2}{c_s})^2}\{2[2-\frac{\beta_r^2}{c_s}+b_r(\frac{\beta_r\beta_m}{c_s}-b_m)] \\ &\quad +2(2-\frac{\beta_r^2}{c_s})-b_r^2-(b_m-\frac{\beta_r\beta_m}{c_s})^2\}.\end{aligned}$$

We have proved that the first part of the above equation is positive, i.e.,$2-\frac{\beta_r^2}{c_s}+b_r(\frac{\beta_r\beta_m}{c_s}-b_m)>0$, upon showing $\frac{\partial^2\Pi_m}{\partial q_m^2}<0$. Next we show that $2(2-\frac{\beta_r^2}{c_s})-b_r^2-(b_m-\frac{\beta_r\beta_m}{c_s})^2$ is non-negative:

$$\begin{aligned}2(2-\frac{\beta_r^2}{c_s})-b_r^2-(b_m-\frac{\beta_r\beta_m}{c_s})^2 &\geq 2-b_r^2-(b_m-\frac{\beta_r\beta_m}{c_s})^2 \\ &\geq 1-(b_m-\frac{\beta_r\beta_m}{c_s})^2 \\ &\geq 0,\end{aligned}$$

where the first inequality holds because $c_s\geq\beta_r^2$ and the second inequality holds because $b_r<1$. From $c_s\geq\beta_r^2$ and $\beta_r>\beta_m$, we can obtain that $-1\leq-\frac{\beta_r\beta_m}{c_s}\leq 0$. Combining it with $0\leq b_m\leq 1$, we obtain that $(b_m-\frac{\beta_r\beta_m}{c_s})^2\leq 1$. Thus the last inequality holds. Therefore, we obtain that $4(2-\frac{\beta_r^2}{c_s})-(b_m+b_r-\frac{\beta_r\beta_m}{c_s})^2>0$, which indicates $\frac{\partial^2\Pi_m}{\partial q_m^2}\frac{\partial^2\Pi_m}{\partial w^2}-(\frac{\partial^2\Pi_m}{\partial q_m\partial w})^2>0$.

Then we obtain the result that Π_m is jointly concave in q_m and w. We can obtain the optimal q_m and w by solving the equations of the first-order conditions: $\frac{\partial\Pi_m}{\partial q_m}=0$ and $\frac{\partial\Pi_m}{\partial w}=0$. □

Proof of Proposition 4

Proof. In this proof we show that Π_r is jointly concave in q_r and s. Taking the first and second derivatives of Π_r with respect to q_r obtains that

$$\begin{aligned}\frac{\partial\Pi_r}{\partial q_r} &= a_r-\frac{a_m-c}{2}b_r-w+(\beta_r-\frac{b_r}{2}\beta_m)s-(2-b_rb_m)q_r; \\ \frac{\partial^2\Pi_r}{\partial q_r^2} &= -(2-b_rb_m)<0.\end{aligned}$$

Taking the first and second derivatives of Π_r with respect to s obtains that

$$\begin{aligned}\frac{\partial \Pi_r}{\partial s} &= (\beta_r - \frac{b_r}{2}\beta_m)q_r - c_s s;\\ \frac{\partial^2 \Pi_r}{\partial s^2} &= -c_s < 0.\end{aligned}$$

Taking the cross derivatives of Π_r with respect to q_r and s obtains that

$$\frac{\partial^2 \Pi_r}{\partial q_r \partial s} = \beta_r - \frac{b_r}{2}\beta_m.$$

Then we show that

$$\begin{aligned}\frac{\partial^2 \Pi_r}{\partial q_r^2}\frac{\partial^2 \Pi_r}{\partial s^2} - (\frac{\partial^2 \Pi_r}{\partial q_r \partial s})^2 &= \frac{1}{4}[4(2 - b_r b_m)c_s - (2\beta_r - b_r\beta_m)^2]\\ &\geq c_s - b_r b_m c_s - \frac{b_r^2}{4}\beta_m^2 + b_r\beta_r\beta_m\\ &\geq b_r\beta_r\beta_m - \frac{b_r^2}{4}\beta_m^2\\ &\geq 0,\end{aligned}$$

where the first inequality holds because $c_s \geq \beta_r^2$, the second inequality holds because $b_r < 1$ and $b_m < 1$, and the last inequality holds because $\beta_r > \beta_m$ and $b_r < 1$.

Thus, we obtain that Π_r is jointly concave in q_r and s. Therefore, we can obtain the optimal responses of q_r and s by solving the equations of the first-order conditions: $\frac{\partial \Pi_r}{\partial q_r} = 0$ and $\frac{\partial \Pi_r}{\partial s} = 0$. □

Proof of Proposition 5

Proof. In this proof we show that Π_m is concave in w. Taking the first and second derivatives of Π_m with respect to w obtains that

$$\begin{aligned}\frac{d\Pi_m}{dw} &= (a_m - c - b_m q_r + \beta_m s)(\frac{b_m}{2}\frac{1}{X} - \frac{\beta_m}{2}\frac{\beta_r - \frac{b_r}{2}\beta_m}{c_s}\frac{1}{X}) + q_r - (w - c)\frac{1}{X}\\ \frac{d^2\Pi_m}{dw^2} &= -\frac{1}{2X^2}(4X - Y^2).\end{aligned}$$

Note that in the above equations q_r and s are functions of w, which are given in Proposition 4. Thus, we obtain that Π_m is concave in w when $4X - Y^2 \geq 0$. Therefore, we can obtain w by solving the equation of the first-order condition: $\frac{d\Pi_m}{dw} = 0$. □

Proof of Proposition 6

Proof. It is easy to verify that Π_m in Equation (9) is concave in q_m, and Π_r in Equation (10) is concave in q_r. Thus, by solving equations of $\frac{\partial \Pi_m}{\partial q_m} = 0$ and $\frac{\partial \Pi_r}{\partial q_r} = 0$, respectively, we obtain that

$$\begin{aligned} q_m &= \frac{a_m - b_m q_r + \beta_m s - c}{2} \\ q_r &= \frac{a_r - b_r q_m + \beta_r s - w}{2}. \end{aligned}$$

Then, by jointly solving the above two equations, we have

$$\begin{aligned} q_m &= \frac{2(a_m - c) - (a_r - w)b_m + (2\beta_m - b_m\beta_r)s}{4 - b_r b_m} \\ q_r &= \frac{2(a_r - w) - (a_m - c)b_r + (2\beta_r - b_r\beta_m)s}{4 - b_r b_m}. \end{aligned}$$

□

Proof of Proposition 7

Proof. In this proof we show that Π_r presented in Equation (12) is concave in s. Taking the first and second derivatives of Π_r with respect to s obtains that

$$\begin{aligned} \frac{d\Pi_r}{ds} &= \frac{2(2\beta_r - b_r\beta_m)(2a_r - (a_m - c)b_r - 2w)}{(4 - b_r b_m)^2} + (\frac{2(2\beta_r - b_r\beta_m)^2}{(4 - b_r b_m)^2} - c_s)s \\ \frac{d^2\Pi_r}{ds^2} &= -\frac{1}{(4 - b_r b_m)^2}[(4 - b_r b_m)^2 c_s - 2(2\beta_r - b_r\beta_m)^2] \\ &\leq -\frac{1}{(4 - b_r b_m)^2}[(4 - b_r b_m)^2 \beta_r^2 - 2\beta_r^2(2 - b_r\frac{\beta_m}{\beta_r})^2] \\ &= -\frac{\beta_r^2}{(4 - b_r b_m)^2}(4 - b_r b_m + \sqrt{2}(2 - b_r\frac{\beta_m}{\beta_r}))(1 - b_r b_m + 3 - 2\sqrt{2} + \sqrt{2}b_r\frac{\beta_m}{\beta_r}) \\ &\leq 0, \end{aligned}$$

where the first inequality holds because $c_s \geq \beta_r^2$ and the second inequality holds because $b_r < 1$, $b_m < 1$, and $\beta_r > \beta_m$. Thus, Π_r is concave in s and we can obtain the optimal response of s by solving the equation of the first-order condition: $\frac{d\Pi_r}{ds} = 0$. □

Proof of Proposition 8

Proof. In this proof we show that Π_m is concave in w. Taking the first and second derivatives of Π_m with respect to w obtains that

$$\begin{aligned}\frac{d\Pi_m}{dw} &= 4q_m\frac{L}{4-b_rb_m} - (w-c)\frac{2H}{4-b_rb_m} + q_r\\ \frac{d^2\Pi_m}{dw^2} &= -\frac{4}{(4-b_rb_m)^2}[H(4-b_rb_m)-2L^2].\end{aligned}$$

Note that in the above equations q_r and q_m are functions of w, which are given in Proposition 6. Thus, we obtain that Π_m is concave in w when $H(4-b_rb_m)-2L^2 \geq 0$. Therefore, we can obtain w by solving the equation of the first-order condition: $\frac{d\Pi_m}{dw} = 0$. □

References

Amir, R and I Grilo (1999). Stackelberg versus Cournot equilibrium. *Games and Economic Behavior*, **26**, 1–21.

Amir, R and A Stepanova (2006). Second-mover advantage and price leadership in Bertrand duopoly. *Games and Economic Behavior*, **55**, 1–20.

Bárcena-Ruiz, JC (2007). Endogenous timing in a mixed duopoly: Price competition. *Journal of Economics*, **91**(3), 263–272.

Cai, G (2010). Channel Selection and Coordination in Dual-Channel Supply Chains. *Journal of Retailing*, **86**, 22–36.

Cai, G, ZG Zhang and M Zhang (2009). Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics*, **117**, 80–96.

Cao, E (2014). Coordination of dual-channel supply chains under demand disruptions management decisions. *International Journal of Production Research*, DOI: 10.1080/00207543.2014.938835.

Cao, E, Y Ma, C Wan and M Lai (2013). Contracting with asymmetric cost information in a dual-channel supply chain. *Operations Research Letters*, **41**, 410–414.

Cattani, KD, WG Gilland and JM Swaminathan (2004). Coordinating traditional and internet supply chains. In *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D Simchi- Levi, SD Wu and ZJ Shen (Eds). Springer US.

Cattani, K, W Gilland, HS Heese and J Swaminathan (2006). Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Producation and Operations Management*, **15**(1), 40–56.

Chen, KY, M Kaya and Ö Özer (2008). Dual sales channel management with service competition. *Manufacturing & Service Operations Management*, **10**(4), 654–675.

Chen, J, H zhang and Y Sun (2012). Implementing coordination contracts in a manufacturer Stackelberg dual-channel supply chain. *Omega*, **40**, 571–583.

Chiang, WK (2010). Product availability in competitive and cooperative dual-channel distribution with stock-out based substitution. *European Journal of Operational Research*, **200**, 111–126.

Chiang, WK, D Chhajed and JD Hess (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science*, **49**(1), 1–20.

Chiang, WK and GE Monahan (2005). Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, **162**, 325–341.

Choi, TM, PS Chow, B Kwok, SC Liu and B Shen (2013). Service quality of online shopping platforms: A case-based empirical and analytical study. *Mathematical Problems in Engineering*, http://doi:10.1155/2013/128678.

Dan, B, G Xu and C Liu (2012). Pricing policies in a dual-channel supply chain with retail services. *International Journal of Production Economics*, **139**, 312–320.

Ding, Q, C Dong and Z Pan (2016). A hierarchical pricing decision process on a dual-channel problem with one manufacturer and one retailer. *International Journal of Production and Economics*, **175**, 197–212.

Dumrongsiri, A, M Fan, A Jain and K Moinzadeh. (2008). A supply chain model with direct and retail channels. *European Journal of Operational Research*, **187**, 691–718.

Farahat, A and G Perakis. 2011. A comparison of bertrand and cournot profits in oligopolies with differentiated products. *Operations Research*, **59**(2), 507–513.

Forbes.com. 2014. Alibaba's IPO by the numbers: Bigger than Google, Facebook, and Twitter combined. http://www.forbes.com/sites/liyanchen/2014/09/19/alibabas-ipo-by-the-numbers-bigger-than-google-facebook-and-twitter-combined/ [Accessed August 23, 2016].

Hamilton, JH and SM Slutsky (1990). Endogenous timing in duopoly games: Stackelberg or cournot equilibria. *Games and Economic Behavior*, **2**(1), 29–46.

He, Y, H Song and P Zhang (2013). Optimal selling strategy in dual-channel supply chains. *Journal of System and Management Sciences*, **3**(2), 10–19.

Hsieh, CC, YL Chang and CH Wu (2014). Competitive pricing and ordering decisions in a multiple-channel supply chain. *International Journal of Production Economics*, **154**, 156–165.

Hua, G, S Wang and TCE Cheng (2010). Price and lead time decisions in dual-channel supply chains. *European Journal of Operational Research*, **205**, 113–126.

Huang, S, C Yang and X Zhang (2012). Pricing and production decisions in dual-channel supply chains with demand disruptions. *Computers & Industrial Engineering*, **62**, 70–83.

Liu, B, R Zhang and M Xiao (2010). Joint decision on production and pricing for online dual channel supply chain system. *Applied Mathematical Modelling*, **34**, 4208–4218.

Niu, RH, X Zhao, I Castillo and T Joro (2012). Pricing and inventory strategies for a two-stage dual-channel supply chain. *Asia-Pacific Journal of Operational Research*, **29**(1), 1240004-1–1240004-38.

Takahashi, K, T Aoi, D Hirotani and K Morikawa (2011). Inventory control in a two-echelon dual-channel supply chain with setup of production and delivery. *International Journal of Production Economics*, **133**, 403–415.

Tsay, AA and N Agrawal (2004a). Modeling conflict and coordination in multi-channel distribution systems: A review. In *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D Simchi- Levi, SD Wu and ZJ Shen (Eds). Springer US.

Tsay, AA and N Agrawal (2004b). Channel conflict and coordination in the e-commerce age. *Production and Operations Management*, **13**(1), 93–110.

van Damme, E and S Hurkens (2004). Endogenous price leadership. *Games and Economic Behavior*, **47**, 404–420.

Vives, X (2001). *Oligopoly pricing: Old edeas and new tools.* MIT Press, Cambridge, MA.

Wang, Y, B Niu and P Guo (2013). On the advantage of quantity leadship when outsourcing production to a competitive contract manufacturer. *Production and Operations Management*, **22**(1), 104–119.

Wang, T, DJ Thomas and N Rudi (2014). The effect of competition on the efficient–responsive choice. *Production and Operations Management*, **23**(5), 829–846.

Xu, G, B Dan, X Zhang and C Liu (2014). Coordinating a dual-channel supply chain with risk-averse under a two-way revenue sharing contract. *International Journal of Production Economics*, **147**, 171–179.

Xu, Q, Z Liu and B Shen (2013). The impact of price comparison service on pricing strategy in a dual-channel supply chain. *Mathematical Problems in Engineering*, http://dx.doi.org/10.1155/2013/613528.

Yan, R (2008). Profit sharing and firm performance in the manufacturer-retailer dual-channel supply chain. *Electronic Commerce Research*, **8**, 155–172.

Yan, R and Z Pei (2009). Retail services and firm profit in a dual-channel market. *Journal of Retailing and Consumer Services*, **16**, 306–314.

Yao, DQ, X Yue, X Wang and JJ Liu (2005). The impact of information sharingon a returns policy with the addition of a direct channel. *International Journal of Production Economics*, **97**, 196–209.

Yue, X and J Liu (2006). Demand forecast sharing in a dual-channel supply chain. *European Journal of Operational Research*, **174**, 646–667.