# A Boosting Framework of Factorization Machine

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## Abstract

Recently, Factorization Machines (FM) has become more and more popular for recommendation systems, due to its effectiveness in finding informative interactions between features. Usually, the weights for the interactions is learnt as a low rank weight matrix, which is formulated as an inner product of two low rank matrices. This low rank can help improve the generalization ability of Factorization Machines. However, to choose the rank properly, it usually needs to run the algorithm for many times using different ranks, which clearly is inefficient for some large-scale datasets. To alleviate this issue, we propose an Adaptive Boosting framework of Factorization Machines (AdaFM), which can adaptively search for proper ranks for different datasets without re-training. Instead of using a fixed rank for FM, the proposed algorithm will adaptively gradually increases its rank according to its performance until the performance does not grow, using boosting strategy. To verify the performance of our proposed framework, we conduct an extensive set of experiments on many real-world datasets. Encouraging empirical results shows that the proposed algorithms are generally more effective than state-of-the-art other Factorization Machines.

### 1 Introduction

Originally introduced in [16], the Factorization Machines (FM) is proposed as a new model class that combines the advantages of linear models, such as Support Vector Machines (SVM) [5], with factorization models. Like linear model, FM is a general model which will learn a weight vector for any real valued feature vector. However, FM also learn a pairwise feature interaction matrix for all interactions between variables, thus it can estimate interactions for highly sparse data(like recommender systems) where linear models fail. The interaction matrix is learnt using factorized parameters with much smaller latent factor compared with the original dimension of the instances. This introduced several ben-

efits. Firstly, this acts as a kind of regularization, since the rank of the interaction matrix is no more than the latent factors and the number of parameters is much lower than that of the full matrix. Secondly, this makes the computation of the prediction score of FM can be calculated in linear time and thus FMs can be optimized directly. Because of these advantages, FM can be used for any supervised learning tasks, including classification, regression, and recommendation systems. On the other hand, FM can mimic most factorization models [19, 17], including standard matrix factorization [22], SVD++ [10], timeSVD++ [11], and PITF (Pairwise Interaction Tensor Factorization) [20], just by feature engineering. This property makes FM suitable to many application domains, where factorization models are appropriate. Practically, FM can achieve as good accuracy performance as the best specialized models on the Netflix and KDDcup 2012 challenges [18].

Although the original Factorization machine is successfully applied to optimize the accuracy of the model [16]. However, it is not guaranteed to optimize ranking performance for recommendation system [3, 4]. Recently the Pair-wised Ranking based Factorization Machines (PRFM) algorithm [15] is proposed to directly optimize the Area Under the ROC Curve (AUC) performance. However, AUC measure is not suitable for top-N recommendation tasks [14], where the higher accuracy at the top of the list is more important than that at the low-position (such as Normalized Discounted Cumulative Gain (NDCG) and Mean Reciprocal Rank (MRR) [12]). So, LambdaFM [25] is proposed to directly optimize the rank biased metrics, using the core ideas of LambdaRank [1] where top pairs are assigned with higher importance. Empirical results show that LambdaFM generally outperforms PRFM in terms of different ranking metrics. Although FM and their variants are successfully applied to many problems, it usually needs to run the algorithm for many times to choose the rank properly. This clearly is inefficient for some large-scale datasets.

Motivated by the above observations, we would like to design an algorithm that can adaptively search for a proper latent number for different datasets without retraining. To achieve this goal, we adopt boosting technique, which was proposed to improve the performance

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(such as, AUC, NDCG, MRR) of models by combining multiple weak models [8, 24], to propose an Adaptive boosting framework of Factorization Machine (AdaFM). Specifically, AdaFM works in rounds to build multiple component FMs bases on dynamically weighted training datasets, which are linearly combined to construct a strong FM. In this way, AdaFM will adaptively gradually increases its latent number according to its performance until the performance becomes saturated. As for component FM, we can either choose the original FM, PRMF or LambdaFM, according the performance that we would like to optimize. To verify the performance of our proposed framework, we conduct an extensive set of experiments on many large-scale real-world datasets. Encouraging empirical results shows that the proposed algorithms are more effective than state-of-the-art other Factorization Machines.

The rest of the paper is organized as follows. Section 2 presents the proposed framework and algorithms. Section 3 discusses our experimental results and Section 4 concludes our work.

# 2 Adaptive Boosting Factorization Machine

In this section, we will firstly introduce the problem setting and Factorization Machine. Then, we will present our Adaptive Boosting Factorization Machine framework, following which we will give several specific algorithms.

**2.1 Problem Settings** Our goal is to learn a function  $f: \mathbb{R}^d \to \mathcal{Y}$ , based on a dataset  $\{(\mathbf{x}_i, y_i) | i \in [n] := \{1, \ldots, n\}\}$ , where  $\mathbf{x}_i \in \mathbb{R}^d$  is the feature vector of the i-th instance,  $y_i \in \mathcal{Y}$  is the label of  $\mathbf{x}_i$ . There are many different choices of  $\mathcal{Y}$ , which corresponds to different problems. For example, when  $\mathcal{Y} = \{-1, +1\}$ , we can treat this problem as a classification problem.

**2.1.1 Factorization Machine** To learn a reasonable f, Factorization Machines (FM) can be adopted. Specifically, *second order* FM model predict the output for an instance  $\mathbf{x}$  using the following simple equation as:

$$(2.1) f_{\Theta}(\mathbf{x}) = \mathbf{w}^{\top} \mathbf{x} + \sum_{l=1}^{d} \sum_{m=l+1}^{d} (\mathbf{V} \mathbf{V}^{\top})_{lm} x_{l} x_{m}$$

where  $x_l$  is the l-th element of  $\mathbf{x}$ , and the model parameters  $\Theta$  to be learnt consists

$$\mathbf{w} \in \mathbb{R}^d$$
,  $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_k] \in \mathbb{R}^{d \times k}$ 

where  $k \ll d$  is a usually prefixed parameter which defines the rank of the factorization.

Intuitively, the vector  $\mathbf{w}$ , the linear part of the model, contains the weights of individual features for predicting y; while the positive semidefinite matrix  $\mathbf{V}\mathbf{V}^{\top}$ , the factorization part, captures all the pairwise interactions between all the variables. Using the factorized parametrization  $\mathbf{V}\mathbf{V}^{\top}$  instead of a full matrix is based on the assumption that the effect of pairwise interactions has a low rank. This explicit low rank assumption helps reduce the overfitting problem, and allows FM to estimate reliable parameters even in highly sparse data. In addition, this reduces the number of parameters to be learnt from  $d^2$  to kd, and allows to compute prediction efficiently by using

(2.2) 
$$f_{\Theta}(\mathbf{x}) = \mathbf{w}^{\top} \mathbf{x} + \frac{1}{2} (\|\mathbf{V}^{\top} \mathbf{x}\|_{2}^{2} - \sum_{s=1}^{k} \|\mathbf{v}_{s} \circ \mathbf{x}\|_{2}^{2})$$

where  $\circ$  is the element-wise product. So, FM can be computed efficiently with the computation cost O(kd) instead of  $O(d^2)$  when implemented naively.

Given the above parametric FM function, now we take  $\mathcal{Y} = \{-1, +1\}$  as a concrete example, which can be treated as a classification problem. In order to learn the optimal parameters for FM, we need introduce some loss function  $\ell(f_{\Theta}(\mathbf{x}_i), y_i)$  to measure the performance of  $f_{\Theta}$  on  $(\mathbf{x}_i, y_i)$ . One popular loss function is the well-known logistic regression loss,

$$\ell(f_{\Theta}(\mathbf{x}_i), y_i) = \ln(1 + \exp(-y_i f_{\Theta}(\mathbf{x}_i))),$$

which measures how much is violation of the desired constraint  $y_i f_{\Theta}(\mathbf{x}_i) \geq 0$  by the function f. Under these settings, FM is formulated as

(2.3) 
$$\min_{\Theta} \frac{1}{n} \sum_{i=1}^{n} \ell(f_{\Theta}(\mathbf{x}_{i}), y_{i}) + \frac{\gamma}{2} \|\Theta\|^{2}$$

where  $\|\Theta\|^2 = \|\mathbf{w}\|_2^2 + \|\mathbf{V}\|_F^2$ . The parameter  $\gamma > 0$  is a trade-off parameter for the regularization and empirical loss.

### 2.1.2 Pairwise Ranking Factorization Machine

Although traditional FM can be applied to many different problems with interactions hard to be estimated, it is usually designed to approximately minimize the classification error, or regression loss, which is apparently not appropriate for ranking tasks where the prediction score does not matters while the ranks matter.

To solve this task, Pairwise Ranking Factorization Machines (PRFM) is proposed. In PRFM, the dataset is firstly transformed to a new one which is

$$\{(\mathbf{x}_i, \mathbf{x}_j, y_{ij}) | i, j \in [n]\},$$

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where  $y_{ij} = 1$  if  $y_i > y_j$  and  $y_{ij} = -1$  otherwise. Then the objective function of PRFM is defined as

(2.4) 
$$\min_{\Theta} \frac{1}{n^2} \sum_{i,j=1}^{n} \ell(f_{\Theta}(\mathbf{x}_i) - f_{\Theta}(\mathbf{x}_j), y_{ij}) + \frac{\gamma}{2} \|\Theta\|^2$$

where  $\ell$  is the logistic regression loss, and  $\gamma > 0$  is a regularization parameter. Intuitively, PRFM model would assign higher sores for positive instances compared with negative instances, which is equivalent to approximately maximize a concave lower bound of AUC performance measure. In practice, PRFM dose work much better than FM in the setting of recommendation task measured by AUC.

2.1.3 Lambda Factorization Machine Although PRFM can achieve significant higher AUC performance compared with traditional FM. However, in PRMF, an incorrect pairwise ordering at the bottom of list impacts the score just as much as that at the top of the list, this makes it not suitable to top-N recommendation tasks, where the higher accuracy at the top of the list is more important to the recommendation quality than that at the low-position. This can be further explained using rank biased metrics, such as NDCG and MRR [12], for which higher weights are assigned to the top accurate instances.

To address this issue, LambdaFM is proposed to directly optimize the rank biased metrics, using the core ideas of LambdaRank where different pairs are assigned with different importance according to their positions in the list. Specifically, three strategies are proposed in LambdaFM. The first one is Static Sampler, in which the item  $\mathbf{x}_i$  is assigned to a sampling probability

(2.5) 
$$\exp[-(r(\mathbf{x}_i) + 1)/(|I| \times \rho)], \quad \rho \in (0, 1],$$

where  $r(\mathbf{x}_j)$  represents the rank of item  $\mathbf{x}_j$  among all items I according to its overall popularity,  $\rho > 0$  is a parameter. The second one is Dynamic Sampler. Dynamic sampler will first draw samples  $\mathbf{x}_{j_1}, \ldots, \mathbf{x}_{j_m}$  uniformly from unobserved item set  $I \setminus I_u$ , where  $I_u$  is the item set clicked by u, then sample one item according to the distribution

(2.6) 
$$p_j \propto \exp\left[-(r(\mathbf{x}_j) + 1)/(m\rho)\right],$$
where  $r(\mathbf{x}_{j_m}) \propto 1/\hat{y}(\mathbf{x}_{j_m}),$ 

where  $\hat{y}(\mathbf{x}) = f_{\Theta}(\mathbf{x})$ . Different from the first two samplers which would like to push non-positive items with higher ranks down from top positions, the third one is to pull positive items with lower ranks up from

the bottom positions. Specifically, for a pair of positive and non-positive items  $(\mathbf{x}_i, \mathbf{x}_j)$ , a rank-aware weight will be assigned to it, where the weight is

(2.7) 
$$\Gamma(r(\mathbf{x}_i)) = (\sum_{r=0}^{r(\mathbf{x}_i)} 1/(r+1))/\Gamma(I),$$
 
$$\Gamma(I) = \sum_{r=0}^{|I|} 1/(r+1).$$

However, it is impractical to compute  $r(\mathbf{x}_i)$  for large scale datasets. To remedy this issue, an approximate method is to repeatedly draw an item from I until we obtain  $\mathbf{x}_j$ , s.t.,  $\hat{y}(\mathbf{x}_i) - \hat{y}(\mathbf{x}_j) \leq \epsilon$  and  $y_i \succeq y_j$ , where  $\epsilon$  is a positive margin value. Let T denote the size of sampling trials before obtaining such an item, then  $\Gamma(r(\mathbf{x}_i)) \approx \lceil \frac{|I|-1}{T} \rceil$ . Empirical results show that the three variant of LambdaFM generally outperforms PRFM in terms of different ranking metrics, such as NDCG.

**2.2** Algorithm The proposed Adaptive Boosting Factorization Machine (AdaFM) framework aims to provide a general framework to optimize the loss function defined based on various ranking metrics.

To introduce the proposed algorithm, we briefly describe the problem details with some notations. Specifically, let U be the whole set of useres and I the whole set of items, then our goal is to utilize the interactions between U and I to recommend a target user u a list of items that he may prefer. In training, a set of user  $U = \{u_a | a = 1, \dots, n\}$  is given. Each user  $u_a$  is associated with a list of retrieved items  $I_a = \{i_{ab}|b =$  $1, \ldots, n_a$  and a list of labels  $Y_a = \{y_{ab} | b = 1, \ldots, n_a\},\$ where  $y_{ab}$  denotes the rank of item  $i_{ab}$  for user  $u_a$ . A feature vector  $\mathbf{x}_{ab}$  is created from each user-item pair  $(u_a, i_{ab})$ . The interaction  $y_{ab}$  belongs to the set of  $\mathcal{Y} =$  $\{r_1,\ldots,r_q\}$ . Thus the training set can be represented as  $S = \{(u_a, I_a, Y_a)\}$ . For a user  $u_a$  and item  $i_{ab}$ , we denote his historical items by  $I_{ab} = \{i_{ac} \in I_a | y_{ac} = y_{ab}\}$ and define  $I_{ab}^- = \{i_{ac} \in I_a | y_{ac} \prec y_{ab}\}.$ 

**2.2.1** AdaFM Our objective is to learn a Factorization Machine f, such that for each user  $u_a$  the function f can assign its item list  $I_a$  with prediction scores that generate a rank list as close as possible with  $Y_a$ . To achieve this goal, we introduce function  $\pi(u_a, I_a, f)$  to denote the rank list of items  $I_a$  for  $u_a$ , resulted by the learnt model f. Specifically, for  $I_a = \{i_{a1}, \ldots, i_{an_a}\}$ ,  $\pi(u_a, I_a, f)$  is defined as a bijection from  $\{1, \ldots, n_a\}$  to itself, where the b-th element of  $\pi(u_a, I_a, f)$  denotes the rank of item  $i_{ab} \in I_a$ .

Then the learning process is to maximize some

## **Algorithm 1** The Component Algorithm

**Input**: The observed dataset  $S = \{(u_a, I_a, Y_a)\}$ , the weight distribution  $\{p_a^t\}$ , the learning rate  $\eta$ , and the regularization parameter  $\gamma$ 

**Initialize:**  $w_l = 0$ , and  $v_{l,m}$  using  $\mathcal{N}(0,0.1)$ 

for e = 2, ..., MaxIter do

Uniformly draw  $u_a$  from UUniformly draw  $i_{ab}$  from  $I_{ab}$ Several methods to draw  $i_{ac}$  from  $I_{ab}^-$ :

- Uniformly draw  $i_{ac}$  [PRFM]
- Randomly draw  $i_{ac}$  by Eq (2.5) [LFM-S]
- Randomly draw  $i_{ac}$  by Eq (??) [LFM-D]
- Randomly draw  $i_{ac}$  by Eq (??) [LFM-W]

Update the model based on

- $\theta \leftarrow \theta \eta \frac{\partial [(p_a^t/n)\ell(\Delta_{abc}^t, 1) + \gamma/2 \|\Theta\|^2]}{\partial \theta},$  for PRFM, LFM-S, LFM-D
- $$\begin{split} \theta \leftarrow \theta \eta \frac{\partial [\Gamma(r(\mathbf{x}_{ab}))(p_a^t/n)\ell(\Delta_{abc}^t, 1) + \gamma/2\|\Theta\|^2]}{\partial \theta}, \\ \text{for LFM-W} \end{split}$$

end for

**Output**: the model  $h^t$ , or  $\Theta^t = \{\mathbf{w}^t, V^t\}$ 

performance which measures the match between  $\pi(u_a, I_a, f)$  and  $Y_u$ , for all users  $u_a, a = 1, \ldots, n$ . Specifically, we can use a general function  $E[\pi(u_a, I_a, f), Y_a]$ to denote the ranking accuracy associated with each user and its item list  $(u_a, I_a)$ . Then, the ranking accuracy in terms of a ranking metric, e.g., MAP, on the training data is re-written as below

$$\frac{1}{n} \sum_{a=1}^{n} E[\pi(u_a, I_a, f), Y_a] \propto \sum E[\pi(u_a, I_a, f), Y_a].$$

To maximize the ranking accuracy, we propose to minimize the following loss function:

$$\arg\min_{f\in\mathcal{F}}\sum_{a=1}^{n} \{1 - E[\pi(u_a, I_a, f), Y_a]\},\,$$

where  $\mathcal{F}$  is the set of all possible FM. Observation that this minimization is equivalent to maximizing the performance measures. However E is a non-continuous function, it is difficult to optimize the loss function defined above. To solve this issue, we propose to minimize its upper bound as follows:

$$\arg\min_{f\in\mathcal{F}}\sum_{a=1}^{n}\exp\{-E[\pi(u_a,I_a,f),Y_a]\}.$$

The primary idea of applying boosting for Factorization Machine is to learn a set of component FMs and then create an ensemble of the components to predict the users' preferences on items. Specifically, we can use a linear combination of component FM as the final AdaFM model:

$$f(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h^t(\mathbf{x})$$

where

$$h^t = \mathbf{w}_t^{\top} \mathbf{x} + \sum_{l=1}^d \sum_{m=l+1}^d (V_t V_t^{\top})_{lm} x_l x_m$$

is the t-th component FM with small rank k and  $\alpha_t$ is a positive weight assigned to  $h^t$  to determine its contribution in the final model. Therefor, for f we can get an equivalent formulation as:

$$f(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t \left[ \mathbf{w}_t^{\top} \mathbf{x} + \sum_{l=1}^{d} \sum_{m=l+1}^{d} (V_t V_t^{\top})_{lm} x_l x_m \right]$$
$$= \bar{\mathbf{w}}_T^{\top} \mathbf{x} + \sum_{l=1}^{d} \sum_{m=l+1}^{d} (\bar{V}_T \bar{V}_T^{\top})_{lm} x_l x_m$$

where

$$\bar{\mathbf{w}}_T = \sum_{t=1}^T \alpha_t \mathbf{w}_t \in \mathbb{R}^d, \quad \bar{V}_T = [\sqrt{\alpha_1} V_1, \dots, \sqrt{\alpha_T} V_T] \in \mathbb{R}^{d \times kT}.$$

This implies that the learnt f is still a Factorization Machine, which rank kT.

In the training process, AdaFM runs for T rounds, and one component FM is created at each round. At the t-th round, given the former t-1 components, the optimization problem is converted to

$$(\alpha_t, h^t) = \arg\min_{(\alpha, h)} \sum_{a=1}^n \exp\{-E[\pi(u_a, I_a, f^{t-1} + \alpha h), Y_a]\}$$

where 
$$f^{t-1} = \sum_{s=1}^{t-1} \alpha_s h^s$$
.

where  $f^{t-1} = \sum_{s=1}^{t-1} \alpha_s h^s$ . To solve the above optimization, we first create an optimal component  $h^t$  by using a re-weighting strategy, which assigns a dynamic weight  $\beta_a^t$  for each user  $u_a$ . At each round, AdaFM increase the weights of the observed users for which their item lists are not ranked well by the ensemble components created so far. The learning process of the next component will then pay more attention to those "hard" users. Once,  $h^t$  is given, the optimal  $\alpha_t$  can be solved. Finally, the details of the AdaFM is summarized in Algorithm 1.

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## Algorithm 2 The AdaFM Algorithm

Input: Dataset  $S = \{(u_a, I_a, Y_a)\}$ , and performance measure E and round T, and latent factor kInitialize:  $p_a^1 = 1/n, \forall a$ for  $t = 1, 2, \dots, T$  do
Solve  $h^t = CA(S, \{p_a^t\}, k)$ Compute  $E[\pi(u_a, I_a, h^t), Y_a], \forall a$ Compute  $\alpha_t = \frac{1}{2} \ln \frac{\sum_a p_a^t \{1 + E[\pi(u_a, I_a, h^t), Y_a]\}}{\sum_a p_a^t \{1 - E[\pi(u_a, I_a, h^t), Y_a]\}}$ Update  $f^t = \sum_{s=1}^t \alpha_s h^s$ Compute  $p_a^t = \frac{\exp\{-E[\pi(u_a, I_a, f^t), Y_a]\}}{\sum_a \exp\{-E[\pi(u_a, I_a, f^t), Y_a]\}} \ \forall a$ 

end for

**Output**: the model  $f = f^T$ 

In algorithm 1, there is a key step using Component Algorithm (CA)

$$h^t = CA(S, \{p_a^t\}, k, E),$$

for which the inputs are the data set S, the weights  $\{p_a^t\}$ , the latent factor k and the performance measure E; and the output is a FMs model with latent factor k, which is obtained through maximizing

$$\max_{h} \sum_{a=1}^{n} p_{a}^{t} E[\pi(u_{a}, I_{a}, h), Y_{a}].$$

The specific algorithm to solve the above problem will be presented in the next subsection.

**2.2.2 Component Algorithm** To construct component FMs, we can adopt the original FM, PRFM, or LambdaFM model. Specifically, for each user  $u_a$  and an item  $i_{ab} \in I_a$ , we can use the score of FM on  $\mathbf{x}_{ab}$  to model the the relation between the user  $u_a$  and item  $i_{ab}$ , as follows:

$$(2.8)h_{\Theta}(\mathbf{x}) = \mathbf{w}^{\top}\mathbf{x} + \sum_{l=1}^{d} \sum_{m=l+1}^{d} (\mathbf{V}\mathbf{V}^{\top})_{lm} x_{l} x_{m},$$

where  $\mathbf{w} \in \mathbb{R}^d$  and  $\mathbf{V} \in \mathbb{R}^{d \times k}$ . At each round, the accuracy of the component  $h^t$  can be evaluated by the ranking performance measure E weighted by  $p_a^t$ . The optimal  $h^t$  is then obtained by consistently optimizing the weighted ranking measure.

**PRFM** is selected as the component algorithm to optimize AUC, which is chosen as the ranking metric. Given the weight distribution  $p_a^t$ , the accuracy of the component  $h^t$  measured by weighted AUC, is defined as

follows:

$$\begin{aligned} wAUC &=& \sum_{a} \frac{p_a^t}{|H_a|} \sum_{H_a} \mathbb{I}(\pi_{ab}^t < \pi_{ac}^t) \\ &=& \sum_{a} \frac{p_a^t}{|H_a|} \sum_{H_a} \mathbb{I}(h_{ab}^t > h_{ac}^t) \end{aligned}$$

where  $H_a = \{(b,c)|Y_{ab} \succ Y_{ac}\}$ ,  $\pi^t_{ab}$  denotes the rank position of the item  $i_{ab}$  in the list ranked by  $h^t$  for  $u_a$ , and  $h^t_{ab} = h^t(\mathbf{x}_{ab})$ . Maximizing the weighted AUC is equivalent to minimizing the following loss function:

$$\min_{h} \sum_{a} \frac{p_a^t}{|H_a|} \sum_{H_a} \mathbb{I}(h_{ab}^t \leq h_{ac}^t)$$

To solve this problem, we replace the indicator function with a convex surrogate, i.e., the logistic regression loss function, as follows:

$$\ell(\Delta_{abc}^t, 1) = \ln\left(1 + \exp(-\Delta_{abc}^t, 1)\right)$$

where  $\Delta^t_{abc} = h^t_{ab} - h^t_{ac}$ . The optimal component  $h^t$  can be found by optimizing the following objective function:

$$\min_{h} \frac{1}{n} \sum_{a} p_{a}^{t} \sum_{H_{a}} \frac{1}{|H_{a}|} \ell(\Delta_{abc}^{t}, 1) + \frac{\gamma}{2} ||\Theta||^{2}$$

where  $\gamma > 0$  is a regularization parameter. The problem above can be solved by stochastic gradient descent, which firstly uniformly sample one user  $u_a$  from all the users, then sample a pair (b,c) from  $H_a$ , and finally update the model based on the following method:

$$\theta \leftarrow \theta - \eta \frac{\partial [(p_a^t/n)\ell(\Delta_{abc}^t, 1) + \gamma/2 \|\Theta\|^2]}{\partial \theta}$$

where  $\theta \in \{w_l, v_{l,m}\}$ , and  $\eta > 0$  is the learning rate. To calculate the gradient of the objective with respect to  $\theta$ , we can firstly derive the gradient using the property of Multi-linearity:

$$\frac{\partial h_{ab}^t}{\partial \theta} = \begin{cases} x_{ab}^l & \text{if } \theta \text{ is } w_l \\ x_{ab}^l \sum_{r=1}^d v_{r,m} x_{ab}^r - v_{l,m} (x_{ab}^l)^2 & \text{if } \theta \text{ is } v_{l,m} \end{cases}$$

Then, if we denote

$$\lambda_{abc} = \frac{\partial \ell(\Delta_{abc}^t, 1)}{\Delta_{abc}^t} = \frac{-\exp(-\Delta_{abc}^t)}{1 + \exp(-\Delta_{abc}^t)}$$

the stochastic gradient for  $w_l$  can be computed as

$$\frac{\partial [(p_a^t/n)\ell(\Delta_{abc}^t, 1) + \gamma/2||\Theta||^2]}{\partial w_l} = \frac{p_a^t}{n} \frac{\partial \ell(\Delta_{abc}^t, 1)}{\Delta_{abc}^t} \frac{\partial \Delta_{abc}^t}{\partial w_l} + \gamma w_l$$
$$= (p_a^t/n)\lambda_{abc}(x_{ab}^l - x_{ac}^l) + \gamma w_l$$

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and the stochastic gradient for  $v_{l,m}$  can be computed as

$$\frac{\partial [(p_a^t/n)\ell(\Delta_{abc}^t, 1) + \gamma/2||\Theta||^2]}{\partial v_{l,m}} \qquad \text{in Table 1.}$$

$$= \frac{p_a^t}{n} \frac{\partial \ell(\Delta_{abc}^t, 1)}{\Delta_{abc}^t} \frac{\partial \Delta_{abc}^t}{\partial v_{l,m}} + \gamma v_{l,m} \qquad \text{framework und adopt two stan}$$

$$= \frac{p_a^t}{n} \lambda_{abc} \left\{ \sum_{r=1}^d v_{r,m} (x_{ab}^r x_{ab}^l - x_{ac}^r x_{ac}^l) - v_{l,m} [(x_{ab}^l)^2 - (x_{ac}^l G_{ab}^{lim})] \right\} \qquad \text{NDCG}.$$

$$+ \gamma v_{l,m}. \qquad \qquad 3.2 \quad \text{Compar}$$

LambdaFM is selected to optimize NDCG, which is chosen as the performance metric. For this case, we can adopt the lambda sampling strategies [25] instead of the uniform sampling one, i.e, the popularity based Static Sampler (2.5), Rank-Aware Dynamic Sampler (??), and Rank-aware Weighted Approximation (??).

Finally, the algorithm for building the component is summarized in the following Algorithm 2.

#### 3 Experiments

In this section, we report a comprehensive suite of experimental results that help evaluate the performance of our proposed AdaFM algorithm on several recommendation tasks. The experiments are designed to answer the following open questions: (1) Whether the proposed boosting approach is effective to improve the ranking performances significantly? (2) Whether the weak learner's latent dimension has a great effect on ranking performances.

Table 1: Basic statistics of datasets. Each entry indicates whether a user has interacted with an item.

Datasets	# Users	# Items	#Entries
Yelp	17,526	85,539	875,955
Lastfm	992	60,000	$759,\!391$
Yahoo	$2,\!450$	$6,\!518$	$107,\!334$

**3.1 Experimental Testbed** We evaluate our proposed algorithm against several baselines on three publicly available Collaborative Filtering (CF) datasets, i.e., Yelp¹ (user-venue pairs), Lastfm² (user-music pairs), and Yahoo music³ (user-music pairs). To speed up the experiments, we perform the following sampling strategies on these datasets. For Yelp, we filter out the users with less than 20 interactions. For Yahoo, we derive a smaller dataset by randomly sampling a subset of

users and items from the original dataset. The statistics of the datasets after preprocessing are summarized in Table 1.

To test the performances of our proposed AdaFM framework under different optimization targets, we adopt two standard ranking metrics: Area Under ROC Curve (AUC) and Normalized Discounted Cumulative (AUC).

- **3.2 Comparison** Algorithms Our proposed AdaFM is a general framework for improving the performances of FM derived algorithms. Thus, we compare the performances of the following FM derived algorithms and their corresponding enhanced models using our proposed AdaFM framework.
- The Original FM that is designed for the rating prediction task, and its enhanced model using AdaFM and we name it AdaFM-O for short;
- Pariwise Ranking FM (PRFM), which aims to maximize the AUC metric, and its adaptive version (AdaFM-P);
- LambdaFM, which is designed to maximize the NDCG metric. We use three different sampling strategies to form the list pairs, i.e., Static sampler,
   Dynamic sampler, and rank-aware sampler, as described in Section 2.1.3, and we name them as LFM-S, LFM-D, and LFM-W, respectively. We also name their adaptive versions as AdaFM-S, AdaFM-D, and AdaFM-W, respectively.

# **3.3** Hyper-parameter Settings The main parameters to be tuned in our experiments are as follows:

**Learning rate**  $\eta$ : For base learners, we first apply the 5-fold cross validation to find the best  $\eta$  for FM when k=2, and then use the same  $\eta$  for the PRFM, LambdaFM, AdaFM.

**Latent dimension** k: In order to compare the performance of AdaFM and the base learners, we simply choose the latent dimension of AdaFM from  $k \in \{2, 3\}$ , and range the latent dimension of the FM derived algorithms in  $k \in [1, 20]$ .

**Regularization**  $\gamma$ : FM derived algorithms have several regularization parameters, including  $\gamma_{w_l}$  and  $\gamma_{v_{l,m}}$ , which represent the regularization parameters of  $w_l$  and  $v_{l,m}$ , respectively. During the experiments, we select the best values of  $\gamma$  in  $\{0.5, 0.1, 0.05, 0.01, 0.005\}$  for each FM derived algorithm. For simplicity, in our experiments, we restrict  $\gamma_{w_l}$  and  $\gamma_{v_{l,m}}$  to have the same value of  $\gamma$ .

**Distribution coefficient**  $\rho$ :  $\rho$  controls the sampling probability of Lambda FM, and is usually affected by data distribution. Thus, we select the best values of  $\rho$ 

https://www.yelp.com/dataset\\_challenge

<sup>2</sup>http://www.dtic.upf.edu/~ocelma/

MusicRecommendationDataset/lastfm-1K.html

 $<sup>{\</sup>it ^3} https://webscope.sandbox.yahoo.com/catalog.php? \\ {\it datatype=r}$ 

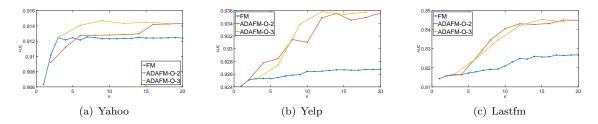


Figure 1: FM based learner's results on different datasets. For FM, the horizontal axis denotes the latent dimension. For AdaFM-O, the horizontal axis denotes the weak learner's latent dimension multiplied by the number of weak learners.

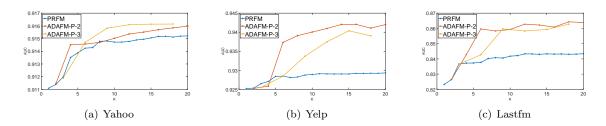


Figure 2: PRFM based learner's results on different datasets. The horizontal axis has the same meaning as Figure 1.

Table 2: Performance on NDCG. The best result is indicated in bold.								
Datasets	$\mathbf{FM}$	PRFM	LFM-S	AdaFM-S	LFM-D	AdaFM-D	LFM-W	AdaFM-W
Yelp	0.204	0.205	0.217	0.225	0.215	0.228	0.221	0.227
Yahoo	0.382	0.383	0.386	0.407	0.392	0.408	0.395	0.410

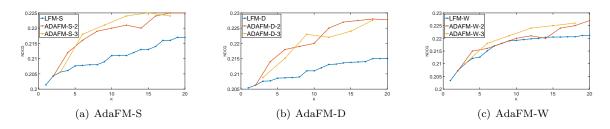


Figure 3: NDCG results on Yahoo dataset. The horizontal axis has the same meaning as Figure 1.

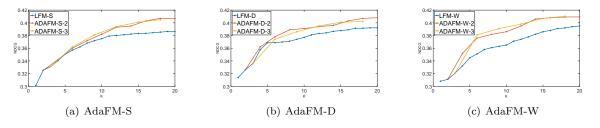


Figure 4: NDCG results on Yelp dataset. The horizontal axis has the same meaning as Figure 1.

Table 3: Performance comparision on AUC. The best result is indicated in bold.

Datasets	$\mathbf{FM}$	AdaFM-O	PRFM	AdaFM-P
Yelp	0.911	0.914	0.915	0.916
Lastfm	0.826	0.845	0.843	0.864
Yahoo	0.925	0.936	0.929	0.942

for LFM-S, LFM-D, and LFM-W in (0,1].

### 3.4 Performance Evaluation

**3.4.1 AUC Optimization** We start by evaluating the effectiveness of our proposed AdaFM framework on AUC maximization task. The detailed results are presented in Figure 1 and 2, and Table 3. Several insightful observations can be made.

First, after combine the Adaptive Boosting and FM, the final results are increased. As shown in Figure 1, when use FM as weak learner, compare with the FM, on Lastfm dataset, we get an 2.32% improvement. And as shown in Figure 2, when use PRFM as weak learner, compare with the PRFM, on Lastfm, we get an 2.49% improvement.

Second, AdaFM shows better results by using less parameters. This is clearly evident in Figure 1 and 2. For example, in all datasets, AdaFM with four weak learners (which latent dimension is 2) achieves a comparable or even better results than the base FM and PRFM with k=20. The results are encouraging as it shows in the cases when the base FM and PRFM stuck in a certain local optimum, our proposed boosting framework can help to achieve better results.

Last but not least, as shown in Table 3, the AdaFM-P has the best results on all the datasets. This shows when using a better weak learner, i.e., PRFM in our case, the AdaFM method achieves better results. This further demonstrates the effectiveness of our boosting framework.

3.4.2 NDCG Optimization We proceed to evaluate the effectiveness of our AdaFM framework on NDCG maximization task. We use LambdaFM with different samplers as our baselines, which are designed to optimize the NDCG metric. More specifically, we consider three variants of LambdaFM, i.e., LFM-S, LFM-D, and LFM-W, with their corresponding boosted versions, i.e., AdaFM-S, AdaFM-D and AdaFM-W. As shown in the Table 2, LambdaFM is better than FM and PRFM, as LambdaFM is designed to optimize the NDCG metric. But our AdaFM methods outperform all the three variants of LambdaFM: LFM-S, LFM-D,

and LFM-W. Specifically, on Yelp dataset, comparing with the original algorithm, AdaFM-S, AdaFM-D, and AdaFM-W get 3.6%, 6.04% and 2.7% improvement, respectively. On Yahoo dataset, AdaFM-S, AdaFM-D, and AdaFM-W get 4.8%, 2.19% and 3.8% improvement, respectively.

And as shown in Figure 3 and 4, the findings are similar to Figure 1 and 2 where our proposed AdaFM achieves better results.

**3.5** Effect of Latent Dimension In this section, we study whether weak learner's latent dimension affect the final results of our proposed AdaFM.

From the experiments in Figure 1, 2, 3, and 4, we find that: (1) with the increase of weak learner numbers, the performances of our proposed AdaFM first increase and then become stable, no matter the latent dimension of weak learners; (2) AdaFM tends to have similar performance even when the latent dimension of the weak learners are different. For example, the AUC performances of AdaFM-O-2 and AdaFM-O-3 both increase with the weak learner nubmers on Lastfm (i.e., Figure 1(c)), however, they achieve quite similar AUC performance after a certain weak learner numbers (i.e., 0.845 vs. 0.844). This finding indicates that it is easy for our proposed AdaFM to tune model parameters in practice.

#### 4 Conclusions

In this paper, we first proposed a novel Adaptive Boosting framework of factorization machine(AdaFM), which combines the advantages of adaptive boosting and FM. Our proposed AdaFM is a general framework that can be used to improve the performance of all the existing FM derived algorithms, e.g., FM, PRFM, and LambdaFM. We then presented the details of how to combine adaptive boosting technique and FM derived models. We finally performed thorough experiments to evaluate our model performance on three real public datasets. The results demonstrated that AdaFM is able to improve the prediction performances in both AUC and NDCG maximization tasks.

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