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A Framework for Interneural Dynamics in a Cerebral Cortex Center

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We model the innervation dynamics of interneurons in a cerebral cortex center A between the time of initial sensory input and acquisition of a sustained steady state. The model assumes that interneurons in A are heavily interconnected allowing synchronization. This invites modeling the dynamics by means of a discrete time map. The model takes into account the influence of excitatory and inhibitory cells and reflects the architecture of synapses along the axons. The acquisition of a sustained chaotic state is characterized by means of a natural invariant probability measure. The time to attain this probability measure can be estimated.

Keywords: Discrete time interneural maps, natural invariant probability measure, Frobenius-Perron operator.

1. Introduction

The nervous system is organized into many local regions called centers [Shepherd, 2004, Chapter 1] (dynamic cell assemblies). At each center input fibres activate interneurons which carry on local processing. Principal neurons then carry signals from one center to another [Rakic, 1976]. In this note we present a mathematical framework for studying the dynamical behavior of interneural activity in a center A from the time of initial sensory input until a steady state of activation is achieved. This activity occurs at close range in a small region and lasts typically around half a second. Our main objective is to describe the sustained activated state by means of an invariant probability measure, that describes the chaotic steady state behaviour.

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Interneurons of the same type are heavily interconnected by electrical gap junctions [Gibson et al., 1999; Galarreta and Hestrin, 1999], allowing them to readily synchronize. For example, activation of hippocampal interneuron networks generates coherent activity in the gamma-frequency range (30-80 Hz) [Bragin et al., 1995]. In the sequel we make the assumption that interneurons in a cortical center A re-act coherently to spike trains, allowing the use of a discrete time model to describe the activation dynamics. The model takes into account the activity of inhibitory as well as excitatory cells by synaptic integration [Kandel and Seigelbau, 2000]. We prove that there exists well defined sustained chaotic behavior of interneural activity in A in the sense that there exists a natural probability measure on A that describes the steady state activity.

In Section 2 we present the model for interneural activation in a center. In Section 3 we prove that, under certain conditions, a steady state probability measure is achieved. In Section 4 we present an example for which we computationally estimate the time to attain a steady state of activation.

2. Nonlinear Dynamical Model

The activation process in a center begins when a sensory neuron or a cluster of sensory neurons initiate activity in A with a coherent spike train. We assume there are X interneurons in a center A that can be innervated. Typically, X is of the order of millions of interneurons. Because there are thousands of synapses on a single axon, one interneuron activates many other interneurons. Furthermore, the architecture of the interneuron is such that its presynapses spread out seemingly randomly in groups along its axon. Thus, there are spatial gaps between activated interneurons.

Clusters of neurons in cortical centers have been studied in [Friston , 1997; Lehman et al. , 1987]. In [Schmidt et al. , 2010; Chavez et al. , 2010], there are studies of how individual neuronal dynamics determines large scale results.

We assume that, on average, each of N clusters in a cortical center has approximately the same number of interneurons. A common theory suggests that individual cells do not exchange signals among each other, but rather that exchange takes place between groups of cells [Shimazaki et al., 2012], where a mathematical model is developed to clarify the way neurons collaborate. Furthermore, it appears that these clusters of neurons can organize themselves within milliseconds into different clusters.

Key to our model are:

1. Discrete time [Cessac, 2008, 2011] dynamical model is a map which describes how clusters of interneurons in A innervate other interneural clusters in A.

2. We represent each of the N interneural clusters in A by a subinterval of length $1/N$ in the interval $I = [0, 1]$. In Figure 1 we show an example for $N = 20$ interneuron clusters and a gap parameter, α, $\alpha = 0.5$. This means any interneuron cluster can fail to activate at most $N\alpha = 10$ other interneuron clusters. The actual number of non-activated interneuron clusters is a random number between 0 and 10. This number and the location of the gaps are chosen randomly. For example, the first interneuron cluster in the connection pattern for clusters shown in Figure 1 fails to innervate the 2nd, 3rd, 7th, 11th and 19th clusters. Figure 2 shows another possible connectivity pattern for $N = 20$ clusters and $\alpha = 0.5$.

The connectivity pattern gives rise to a map T from $[0, 1]$ to $[0, 1]$. T is a semi-Markov map ([Góra and Boyarsky, 1993; Jabłoński et al., 1995]). A Markov map ([Boyarsky and Góra, 1997]) maps each subinterval (cluster of interneurons) onto one contiguous group of subintervals (interneural clusters). Although the deterministic dynamics of individual orbits of an activated interneuron cluster is unpredictable because of the nonlinear nature of T, the steady state behavior of this chaotic system can be studied statistically. This behavior is described by an invariant probability measure which characterizes the onset of the steady state chaotic dynamics. Such measures have been studied in the context of brain dynamics in [Froyland and Aihara, 2001; Boyarsky and Góra, 2006, 2009].

In order to analyze the steady state dynamics of T, we define a matrix M (Frobenius-Perron Operator [Boyarsky and Góra, 1997]), which displays the information in the map T, that is, which interneuron cluster innervates which, and how effectively this is accomplished as reflected by the slopes of the line segments in the graph of T. The slope depends on how (on average) an interneuron cluster integrates

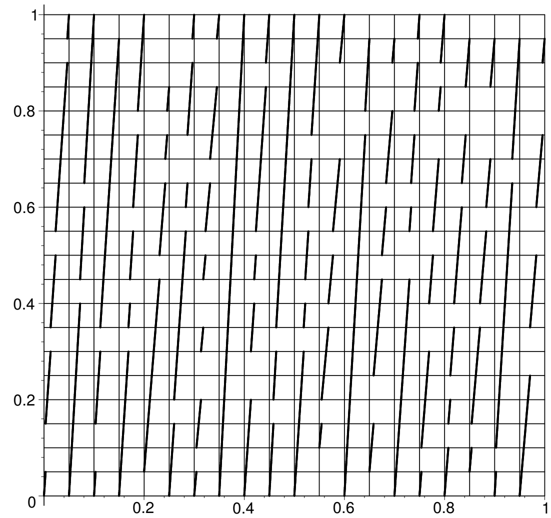

Fig. 1. Connectivity map for $N = 20$ clusters and $\alpha = 0.5$. Each subinterval of length 0.05 represents a cluster.

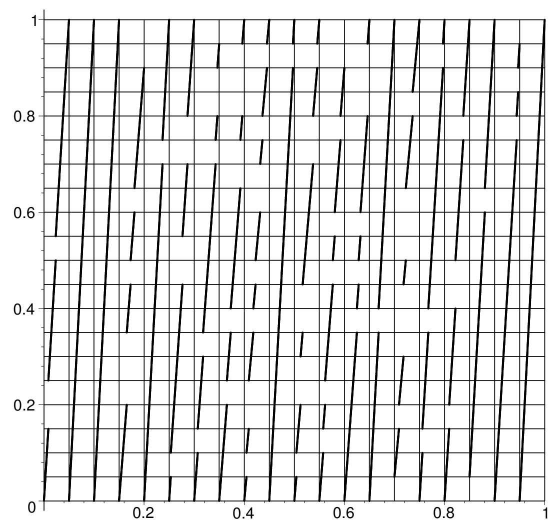

Fig. 2. Another possible connectivity map for $N = 20$ clusters and $\alpha = 0.5$.

(synaptic integration [Kandel and Seigelbau, 2000]) all the excitatory and inhibitory signals and responds by activation or passivity at the interneuron level.

The final piece in the model relates to time evolution: since the input to A "is composed of a whole ensemble of spike trains arriving through many parallel inputs" [Abeles, 1991], the map T is iterated at the rate of the spike train, denoted by τ. Multiplying the matrix by a normalized (total area = 1) left vector describes the action of interneuron clusters in A on an ensemble of parallel spike trains. The normalized left eigenvector of M, viewed as a function on I is the steady state of the dynamical system [Boyarsky and Góra, 1997, Chapter 9].

The 20×20 matrix M associated with the map T in Figure 3 is shown below. The gaps in the images over each subinterval in T appear as gaps in the rows of M.

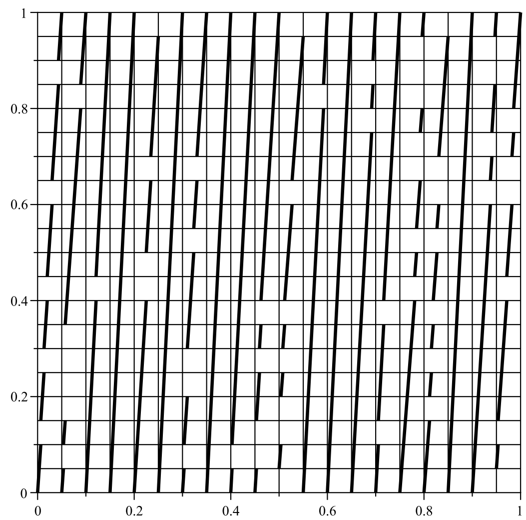

Fig. 3. Example of a semi-Markov transformation for $N = 20$, $\alpha = 0.4$.

$$
M = \begin{bmatrix}
1/15 & 1/15 & 0 & 1/15 & 1/15 & 0 & 1/15 & 1/15 & 0 & 1/15 & 1/15 & 1/15 & 0 & 1/15 & 1/15 & 1/15 & 1/15 & 0 & 1/15 & 1/15 \\
1/14 & 0 & 1/14 & 0 & 0 & 0 & 0 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 0 & 1/14 & 1/14 & 1/14 \\
1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 0 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 \\
1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 \\
1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 0 & 0 & 1/16 & 1/16 & 1/16 & 0 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 0 \\
1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 \\
1/15 & 0 & 1/15 & 1/15 & 0 & 0 & 1/15 & 1/15 & 1/15 & 1/15 & 0 & 1/15 & 1/15 & 0 & 1/15 & 1/15 & 1/15 & 1/15 & 1/15 & 1/15 \\
1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 \\
1/19 & 0 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 \\
1/16 & 0 & 0 & 1/16 & 1/16 & 0 & 1/16 & 0 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 \\
0 & 1/13 & 0 & 0 & 1/13 & 0 & 1/13 & 0 & 1/13 & 1/13 & 1/13 & 1/13 & 0 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 0 \\
1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 1/19 & 0 & 1/19 & 1/19 & 1/19 \\
1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 \\
1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 0 & 1/18 & 0 & 1/18 & 1/18 \\
1/18 & 1/18 & 0 & 1/18 & 1/18 & 1/18 & 1/18 & 0 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 & 1/18 \\
1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 0 & 1/13 & 0 & 1/13 & 1/13 & 0 & 0 & 1/13 & 0 & 0 & 0 & 1/13 \\
1/13 & 1/13 & 1/13 & 0 & 1/13 & 0 & 1/13 & 0 & 1/13 & 1/13 & 0 & 1/13 & 0 & 0 & 1/13 & 1/13 & 1/13 & 1/13 & 1/13 & 0 \\
1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 & 1/20 \\
1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 0 & 1/16 & 0 & 1/16 & 1/16 & 0 & 0 & 1/16 \\
0 & 1/14 & 0 & 1/14 & 1/14 & 1/14 & 1/14 & 1/14 & 0 & 1/14 & 1/14 & 1/14 & 0 & 0 & 1/14 & 0 & 1/14 & 1/14 & 1/14 & 1/14
\end{bmatrix}.
$$

The normalized left invariant eigenvector of M yields the invariant probability measure on the 20 interneuron clusters:

$$
[0.05374\ 0.04791\ 0.04358\ 0.05003\ 0.05383\ 0.03900\ 0.05768\ 0.04687\ 0.04404\ 0.05774\ 0.04915\ 0.06110\ 0.04295\ 0.04220\ 0.05351\ 0.05518\ 0.05022\ 0.04758\ 0.05351\ 0.05009]
$$

Figure 4 shows this invariant measure in grey. Note that it is almost uniform on the interval I which is interpreted as almost uniform activation of the center A.

In a related setting [Cessac, 2008, 2011], the existence and uniqueness of an invariant measure is established.

3. Main Result

If the matrix M is irreducible, that is, if any of the N clusters can eventually communicate (activate) any of the other clusters, then it follows from general Perron-Frobenius Theory [2] that M has a unique normalized left eigenvector, that is, a unique invariant probability measure. However, we do not know this measure unless we solve the matrix equation, which may be a formidable task for large matrices.

We now present the main result of this note. Let a center A have N equal-sized clusters that can be identified and assume there exists a number α which characterizes the random interaction between clusters: if α is small, then there is lots of interaction between the interneuron clusters in A. In the main theorem we shall show that, under general conditions, there exists an invariant probability measure on A that describes the steady state chaotic activity of interneurons as $N \to \infty$ and $\alpha \to 0$. Moreover, we present a condition under which the limit invariant measure is uniform on A.

Let $P(A)$ denote the probability of an event A and let $P(A|B)$ denote the probability of an event A given B. In order to generate random integers to construct the map T and the associated matrix M, we need to use the ceiling function $\lceil \cdot \rceil$ and floor function $\lfloor \cdot \rfloor$ as described in the Appendix.

Theorem 1. *Assume that α satisfies $-2\alpha^2 + 3\alpha \sim \frac{3}{N}$. Then, N approaching ∞ implies that α approaches 0, and the invariant measure of T approaches uniform distribution almost surely.*

Proof. For convenience, we drop the notations "$\lfloor\rfloor$" and "$\lceil\rceil$" from $\lfloor N(1-\alpha)\rfloor$ and $\lceil N\alpha \rceil$, respectively. First, we have

$$\begin{aligned}
P(M_{ij} = \frac{1}{N(1-\alpha)+n}) &= P(M_{ij} = \frac{1}{N(1-\alpha)+n}|X=n)P(X=n) \\
&\quad + P(M_{ij} = \frac{1}{N(1-\alpha)+n}|X \neq n)P(X \neq n) \\
&= \frac{N(1-\alpha)+n}{N}\frac{1}{N\alpha} + 0 \\
&= \frac{N(1-\alpha)+n}{N^2\alpha}.
\end{aligned}$$

Now, for two integers $s \neq t$, $1 \leq s, t \leq N$, we have

$$\begin{aligned}
P(M_{jt} = M_{js} = \frac{1}{N(1-\alpha)+n}) &= P(M_{jt} = M_{js} = \frac{1}{N(1-\alpha)+n}|X=n)P(X=n) \\
&\quad + P(M_{jt} = M_{js} = \frac{1}{N(1-\alpha)+n}|X \neq n)P(X \neq n) \\
&= \frac{\binom{N-2}{N(1-\alpha)+n-2}}{\binom{N}{N(1-\alpha)+n}}\frac{1}{N\alpha} + 0 \\
&= \frac{(N(1-\alpha)+n)(N(1-\alpha)+n-1)}{N(N-1)}\frac{1}{N\alpha}.
\end{aligned}$$

Thus,

$$P(M_{jt} = M_{js} \neq 0) = \sum_{n=1}^{N\alpha} \frac{(N(1-\alpha)+n)(N(1-\alpha)+n-1)}{N(N-1)}\frac{1}{N\alpha}.$$

Similarly, we have

$$\begin{aligned}
P(M_{jt} = M_{js} = 0) &= \sum_{n=1}^{N\alpha} P(M_{jt} = M_{js} = 0|X=n)P(X=n) \\
&= \sum_{n=1}^{N\alpha} \frac{\binom{N-2}{N\alpha-n-2}}{\binom{N}{N\alpha-n}}\frac{1}{N\alpha} \\
&= \sum_{n=1}^{N\alpha} \frac{(N\alpha-n)(N\alpha-n-1)}{N(N-1)}\frac{1}{N\alpha}.
\end{aligned}$$

Therefore, we have

$$
\begin{aligned}
P(M_{jt}=M_{js}) &= \sum_{n=1}^{N\alpha} \frac{(N(1-\alpha)+n)(N(1-\alpha)+n-1)+(N\alpha-n)(N\alpha-n-1)}{N^2\alpha(N-1)} \\
&= \sum_{n=1}^{N\alpha} \Bigg[\frac{N^2(1-\alpha)^2+N\alpha(N\alpha-1)-N(1-\alpha)}{N^2\alpha(N-1)} + \frac{2n^2}{N^2\alpha(N-1)} \\
&+ \frac{(2N-4N\alpha)n}{N^2\alpha(N-1)} \Bigg] \\
&= \frac{N^2(1-\alpha)^2+N\alpha(N\alpha-1)-N(1-\alpha)}{N(N-1)} + \frac{(N\alpha+1)(2N\alpha+1)}{3N(N-1)} \\
&+ \frac{(2N-4N\alpha)(N\alpha+1)}{2N(N-1)} \\
&= 1 + \frac{(2\alpha^2-3\alpha)N^2-3\alpha N+3N-1}{3N(N-1)}.
\end{aligned}
$$

On one hand,

$$
\lim_{N\to\infty} P(M_{jt}=M_{js}) = (1-\alpha)^2+\alpha^2+\frac{2\alpha^2}{3}+\frac{(2-4\alpha)\alpha}{2} = 1-\alpha+\frac{2}{3}\alpha^2.
$$

This shows that $P(M_{jt}=M_{js})$ is very close to 1 if N is very big. Since event "$M_{1t}=M_{1s}, M_{2t}=M_{2s},\ldots,M_{Nt}=M_{Ns}$" implies event "$M_t=M_s$", we have

$$
\begin{aligned}
P(M_t=M_s) &\geq P(M_{1t}=M_{1s}, M_{2t}=M_{2s},\ldots,M_{Nt}=M_{Ns}) \\
&= \left(1+\frac{(2\alpha^2-3\alpha)N^2-3\alpha N+3N-1}{3N(N-1)}\right)^N.
\end{aligned}
$$

Thus, for small α and big N, we obtain

$$
P(M_{1t}=M_{1s}, M_{2t}=M_{2s},\ldots,M_{Nt}=M_{Ns}) \sim \exp\left(N\frac{(2\alpha^2-3\alpha)N^2-3\alpha N+3N-1}{3N(N-1)}\right),
$$

while,

$$
N\frac{(2\alpha^2-3\alpha)N^2-3\alpha N+3N-1}{3N(N-1)} = -(-2\alpha^2+3\alpha)\frac{N}{3}\frac{N}{N-1}+\frac{N(1-\alpha)}{N-1}+\frac{1}{3(N-1)},
$$

which converges to 0 given $N\to\infty$ and the condition of the theorem is satisfied.

Therefore, we conclude that the sum of any two columns of M is equal with probability almost 1, which implies vector $(\frac{1}{N},\frac{1}{N},\ldots,\frac{1}{N})$ is very close to the left invariant eigenvector for big N (and small α) given the condition in this theorem. ■

4. Experimental estimation of the time of acquisition of the invariant probability measure

For a randomly chosen initial probability vector v we calculated vM^n for a few first n's. In Figure 4 we show the vector v (black) and the M-invariant vector (grey) representing invariant probability measure, both as piecewise constant functions on $[0,1]$.

In Figure 5 we present the third iteration vM^3 with the M-invariant vector. The difference between them is less than 10^{-3} so the graphs practically coincide. We see that experiment shows that the time of acquisition of the invariant probability measure is almost instantaneous.

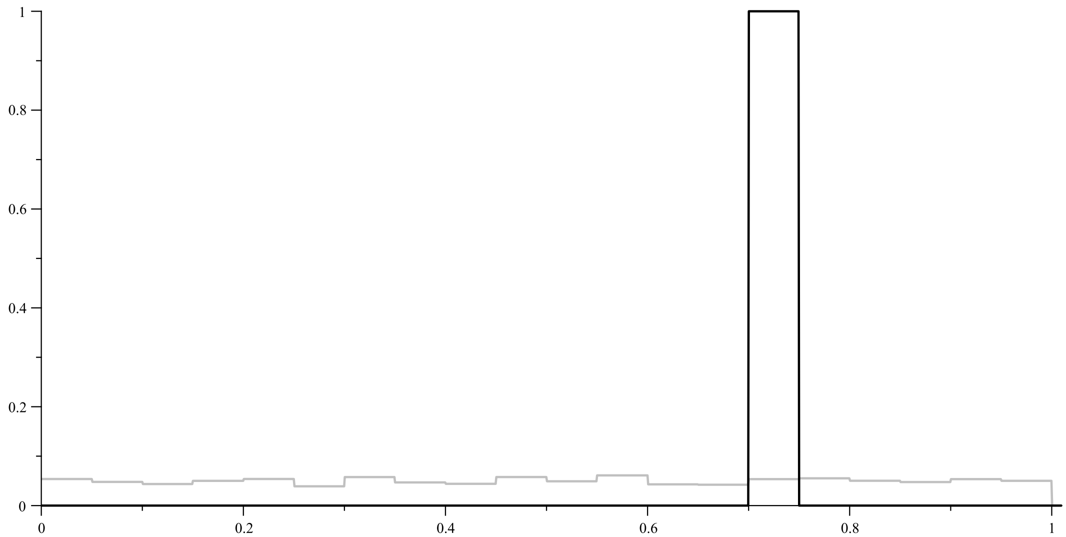

Fig. 4. Initial vector v (black) and the M-invariant vector (grey).

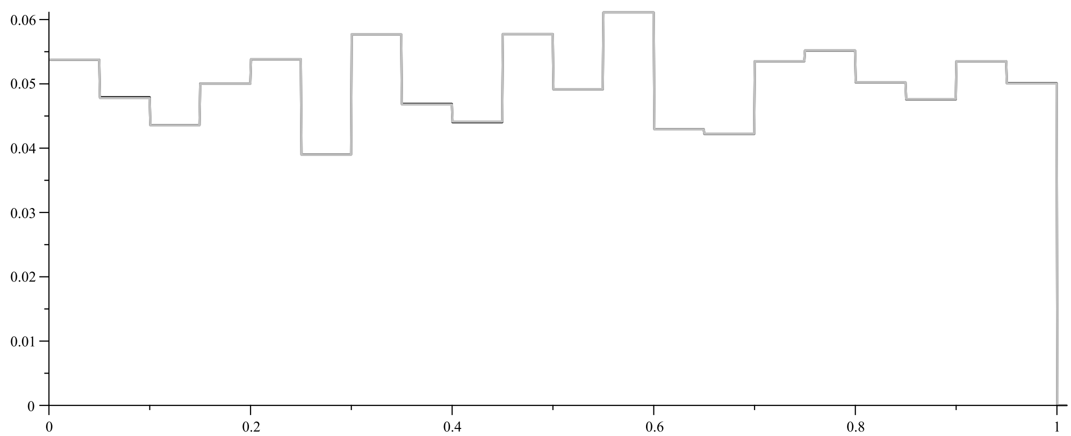

Fig. 5. Vector vM^3 (black) and the M-invariant vector (grey). Almost equal.

5. Appendix: Algorithm for construction of matrix M

Now we construct our model for the signal transmission among the neurons by stating a semi-Markov linear transformation (see [Góra and Boyarsky, 1993; Jabłoński et al., 1995]) $T : I \to I$, where $I = [0, 1]$, I is partitioned into N subintervals of equal length such that T is piecewise linear and monotone on each subinterval.

Let α be a number satisfying $0 < \alpha < 1$, we give the Perron-Frobenius matrix $M = (a_{i,j})_{1 \le i,j \le N}$ associated with T as follows, we start from $i = 1$:

(1) For row i, we generate one random integer k_i from $\{1, 2, \ldots, \lceil N\alpha \rceil\}$, where $\lceil x \rceil$ is the ceiling function, and k_i takes each integer with equal probability.

(2) Randomly choose $\lfloor N(1-\alpha) \rfloor + k_i$ integers from $\{1, 2, \ldots, N\}$, where $\lfloor x \rfloor$ is the floor function, storing these integers in set A we assign values with $a_{i,j} = \frac{1}{\lfloor N(1-\alpha) \rfloor + k_i}$ whenever $j \in A$, the rest of the elements in row i are 0.

(3) If $i < N$, set $i = i + 1$, and return to step (1).

6. Conclusion

We use rudimentary chaos theory to model the activation dynamics in a cerebral cortex center. The existence of an invariant probability measure characterizes the acquisition of the steady state of activity. In our model, the acquisition time of the invariant probability measure is almost instantaneous, which accords with what we know from experience, that interneurons transmit local signals very quickly throughout cortical centers.

References

M. Abeles, [1991], *Neural Circuits of the Cerebral Cortex*, Cambridge University Press.

A. Boyarsky and P. Góra, [1997], *Laws of Chaos. Invariant Measures and Dynamical Systems in One Dimension*, Probability and its Applications, Birkhaüser, Boston, MA.

A. Boyarsky and P. Góra, [2006], *Invariant measures in brain dynamics*, Physics Letters A **358**,27–30.

A. Boyarsky and P. Góra, [2009], *An ergodic theory of consciousness*, International Jour. of Bifurcations and Chaos **19**, No 4, 1397–1400.

A. Bragin, G. Jandó, Z. Nádasdy, J. Hetke, K. Wise, G. Buzsáki, [1995], *Gamma (40-100 Hz) oscillation in the hippocampus of the behaving rat*, J. Neurosci. 1995 Jan;15(1 Pt 1):47–60.

B. Cessac, [2008], *A discrete time neural network model with spiking*, Jour Math Biol. **56**, 311–345.

B. Cessac, [2011], *A discrete time neural network model with spiking II: Dynamics with noise*, Jour Math Biol. **62**, 863–900.

M. Chavez, M. Valencia, V. Latora, and J. Martinerie, [2010], Complex networks: new trends for the analysis of brain connectivity, Internat. J. Bifur. Chaos Appl. Sci. Engrg. **20**, no. 6, 1677–1686, MR2673671 (2011h:92013).

G. Froyland, K. Aihara, [2001], *Estimating statistic of neuronal dynamics via Markov chains,* Biol. Cybern, **84**, 31–40.

K.J. Friston, [1997], Transients, metastability and neuronal dynamics, NeuroImage **5**, 164–171.

M. Galarreta and S. Hestrin, [1999], *A network of fast-spiking cells ion the neocortex connected by electrical synapses*, Nature **402** (Nov. 1999), 72–75.

J. B. Gibson, M. Belerlein and B. W. Connors, [1999], *Two networks of electrically coupled inhibitory neurons in neocortex*, Nature **402**, 75–79, (Nov. 1999).

P. Góra and A. Boyarsky, [1993], *A matrix solution to the inverse Perron-Frobenius problem*, Proc. Amer. Math. Soc., **118**, 409–414.

M. Jabłoński, P. Góra, A. Boyarsky, [1995], *Invariant measures generated by sequences of approximating transformations*, Computers & Mathematics with Applications, **30**, 75–91.

E. Kandel and S. Seigelbaum, [2000], *Synaptic integration,* Principles of Neural Science, Fourth Edition, McGraw-Hill.

D. Lehman, A. Ozaki and I. Pal, [1987], EEG alpha map series: brain microstates by space-oriented adaptive segmentation, Electroencephalography and Clinical Neurophysiology **67**, 271–288.

P. Rakic, [1976], *Local Circuit Neurons*, Cambridge, Mass, MIT Press.

G. Schmidt, G. Zamora-Lpez and J. Kurths, [2010], *Simulation of large scale cortical networks by individual neuron dynamics*, Internat. J. Bifur. Chaos Appl. Sci. Engrg. **20**, no. 3, 859–867, MR2667691.

G. M. Shepherd, [2004], *The Synaptic Organization of the Brain*, Fifth Edition, Oxford University Press.

Hideaki Shimazaki, Shun-ichi Amari, Emery N. Brown and Sonja Grun, [2012], *State-Space Analysis of Time-Varying Higher-Order Spike Correlation for Multiple Neural Spike Train Data*, PLoS Computational Biology, **8**, issue 3, 1–27.