# EXPLICIT BRACKET IN THE EXCEPTIONAL SIMPLE LIE SUPERALGEBRA $\mathfrak{c v e c t}(0 \mid 3)_{*}$ 

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#### Abstract

This note is devoted to a more detailed description of one of the five simple exceptional Lie superalgebras of vector fields, $\mathfrak{c v e c t}(0 \mid 3)_{*}$, a subalgebra of $\mathfrak{v e c t}(4 \mid 3)$. We derive differential equations for its elements, and solve these equations. Hence we get an exact form for the elements of $\mathfrak{c v e c t}(0 \mid 3)_{*}$. Moreover we realize $\mathfrak{c v e c t}(0 \mid 3)_{*}$ by "glued" pairs of generating functions on a (3|3)-dimensional periplectic (odd symplectic) supermanifold and describe the bracket explicitly. 1991 Mathematics Subject Classification: 17A70. Keywords: Lie superalgebra, Cartan prolongation.


March 3, 1997

## Introduction

V. Kac [3] classified simple finite-dimensional Lie superalgebras over $\mathbb{C}$. Kac further conjectured [3] that passing to infinite-dimensional simple Lie superalgebras of vector fields with polynomial coefficients we only acquire the straightforward analogues of the four well-known Cartan series: $\mathfrak{v e c t}(n)$, $\mathfrak{s v e c t}(n), \mathfrak{h}(2 n)$ and $\mathfrak{k}(2 n+1)$ (of all, divergence-free, Hamiltonian and contact vector fields, respectively, realized on the space of dimension indicated).

It soon became clear [4], [1], [5], [6] that the actual list of simple vectoral Lie superalgebras is much larger. Several new series were found.

Next, exceptional vectoral algebras were discovered [8], [9]; for their detailed description see [10], 2]. All of them are obtained with the help of a Cartan prolongation or a generalized prolongation, cf. [8]. This description is, however, not always satisfactory; a more succinct presentation (similar to the one via generating functions for the elements of $\mathfrak{h}$ and $\mathfrak{k}$ ) and a more explicit formula for their brackets is desirable.

The purpose of this note is to give a more lucid description of one of these exceptions, $\mathfrak{c v e c t}(0 \mid 3)_{*}$. In particular we offer a multiplication table for $\mathfrak{c v e c t}(0 \mid 3)_{*}$ that is simpler than previous descriptions, by use of "glued" pairs of generating functions for the elements of $\mathfrak{c v e c t}(0 \mid 3)_{*}$.

This note can be seen as a supplement to 10. To be self-contained and to fix notations we introduce some basic notions in section 0 .

Throughout, the ground field is $\mathbb{C}$.

[^0]
## §0. Background

0.1. We recall that a superspace $V$ is a $\mathbb{Z} / 2$-graded space; $V=V_{\overline{\overline{0}}} \oplus V_{\overline{1}}$. The elements of $V_{\overline{0}}$ are called even, those of $V_{\overline{1}}$ odd. When considering an element $x \in V$, we will always assume that $x$ is homogeneous, i.e. $x \in V_{\overline{0}}$ or $x \in V_{\overline{1}}$. We write $p(x)=\bar{i}$ if $x \in V_{\bar{i}}$. The superdimension of $V$ is $(n \mid m)$, where $n=\operatorname{dim}\left(V_{\overline{0}}\right)$ and $m=\operatorname{dim}\left(V_{\overline{1}}\right)$.

For a superspace $V$, we denote by $\Pi(V)$ the same superspace with the shifted parity, i.e., $\Pi\left(V_{\bar{i}}\right)=V_{\bar{i}+\overline{1}}$.
0.2. Let $x=\left(u_{1}, \ldots, u_{n}, \xi_{1}, \ldots, \xi_{m}\right)$, where $u_{1}, \ldots, u_{n}$ are even indeterminates and $\xi_{1}, \ldots, \xi_{m}$ odd indeterminates. In the associative algebra $\mathbb{C}[x]$ we have that $x \cdot y=(-1)^{p(x) p(y)} y \cdot x$ (by definition) and hence $\xi_{i}^{2}=0$ for all $i$. The derivations $\mathfrak{d e r}(\mathbb{C}[x])$ of $\mathbb{C}[x]$ form a Lie superalgebra; its elements are vector fields. These polynomial vector fields are denoted by $\mathfrak{v e c t}(n \mid m)$. Its elements are represented as

$$
D=\sum_{i} f_{i} \frac{\partial}{\partial u_{i}}+\sum_{j} g_{j} \frac{\partial}{\partial \xi_{j}}
$$

where $f_{i} \in \mathbb{C}[x]$ and $g_{j} \in \mathbb{C}[x]$ for all $i, j=1$..n. We have $p(D)=p\left(f_{i}\right)=$ $p\left(g_{j}\right)+\overline{1}$ and the Lie product is given by the commutator

$$
\left[D_{1}, D_{2}\right]=D_{1} D_{2}-(-1)^{p\left(D_{1}\right) p\left(D_{2}\right)} D_{2} D_{1}
$$

On the vector fields we have a map, div : $\mathfrak{v e c t}(n \mid m) \rightarrow \mathbb{C}[x]$, defined by

$$
\operatorname{div} D=\operatorname{div}\left(\sum_{i=1}^{n} f_{i} \frac{\partial}{\partial u_{i}}+\sum_{j=1}^{n} g \frac{\partial}{\partial \xi_{j}}\right)=\sum_{i=1}^{n} \frac{\partial f_{i}}{\partial u_{i}}-(-1)^{p(D)} \sum_{j=1}^{n} \frac{\partial g_{j}}{\partial \xi_{j}}
$$

A vector field $D$ that satisfies $\operatorname{div} D=0$ is called special. The linear space of special vector fields in $\mathfrak{v e c t}(n \mid m)$ forms a Lie superalgebra, denoted by $\mathfrak{s v e c t}(n \mid m)$.
0.3. Next we discuss the Lie superalgebra of Leitesian vector fields $\mathfrak{l e}(n)$. It consists of the elements $D \in \mathfrak{v e c t}(n \mid n)$ that annihilate the 2 -form $\omega=$ $\sum_{i} d u_{i} d \xi_{i}$. Hence $\mathfrak{l e}(n)$ is an odd superanalogon of the Hamiltonian vector fields (in which case $\omega=\sum_{i} d p_{i} d q_{i}$ ). Similar to the Hamiltonian case, there is a map Le : $\mathbb{C}[x] \rightarrow \mathfrak{l e}(n)$, with $x=\left(u_{1}, \ldots, u_{n}, \xi_{1}, \ldots, \xi_{n}\right)$ :

$$
\mathrm{Le}_{f}=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial u_{i}} \frac{\partial}{\partial \xi_{i}}+(-1)^{p(f)} \frac{\partial f}{\partial \xi_{i}} \frac{\partial}{\partial u_{i}}\right)
$$

Note that Le maps odd elements of $\mathbb{C}[x]$ to even elements of $\mathfrak{l e}(n)$ and vice versa. Moreover $\operatorname{Ker}(\mathrm{Le})=\mathbb{C}$. We turn $\mathbb{C}[x]$ (with shifted parity) into a Lie superalgebra with (Buttin) bracket $\{f, g\}$ defined by

$$
\mathrm{Le}_{\{f, g\}}=\left[\mathrm{Le}_{f}, \mathrm{Le}_{g}\right]
$$

A straightforward calculation shows that

$$
\{f, g\}=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial u_{i}} \frac{\partial g}{\partial \xi_{i}}+(-1)^{p(f)} \frac{\partial f}{\partial \xi_{i}} \frac{\partial g}{\partial u_{i}}\right)
$$

This way $\Pi \mathbb{C}[x] / \mathbb{C} \cdot 1$ is a Lie superalgebra isomorphic to $\mathfrak{l e}(n)$. We call $f$ the generating function of $\mathrm{Le}_{f}$. Here and throughout $p(f)$ will denote the
parity in $\mathbb{C}[x]$, not in $\Pi \mathbb{C}[x]$. So $p(f)$ is the parity of the number of $\xi$ in a term of $f$.
0.4. The algebra $\mathfrak{l e}(n)$ contains certain important subalgebras. First of all there is $\mathfrak{s l e}(n)$, the space of special Leitesian vector fields:

$$
\mathfrak{s l e}(n)=\mathfrak{l e}(n) \cap \mathfrak{s v e c t}(n \mid n)
$$

We have seen that if $D \in \mathfrak{l e}(n)$ then $D=\operatorname{Le}_{f}$ for some $f \in \mathbb{C}[x]$. Now $D \in \mathfrak{s l e}(n)$ iff $f$ is harmonic in the following sense

$$
\Delta(f):=\sum_{i=1}^{n} \frac{\partial^{2} f}{\partial u_{i} \partial \xi_{i}}=0
$$

Usually we simply say $f \in \mathfrak{s l e}(n)$, identifying $f$ and $\mathrm{Le}_{f}$. This $\Delta$ satisfies the condition $\Delta^{2}=0$ and hence $\Delta: \mathfrak{l e}(n) \rightarrow \mathfrak{s l e}(n)$. The image $\Delta(\mathfrak{l e}(n))=$ : $\mathfrak{s l e}^{\circ}(n)$ is an ideal of codimension 1 on $\mathfrak{s l e}(n)$. This ideal, $\mathfrak{s l e}^{\circ}(n)$, can also be defined by the exact sequence

$$
0 \longrightarrow \mathfrak{s l e}{ }^{\circ}(n) \longrightarrow \mathfrak{s l e}(n) \longrightarrow \mathbb{C} \cdot \operatorname{Le}_{\xi_{1} \ldots \xi_{n}} \longrightarrow 0
$$

Note that if $\Phi=\sum u_{i} \xi_{i}$ and $f \in \mathfrak{s l e}(n)$, then

$$
\Delta(\Phi f)=\left(n+\operatorname{deg}_{u} f-\operatorname{deg}_{\xi} f\right) \cdot f
$$

Let $\nu(f)=n+\operatorname{deg}_{u} f-\operatorname{deg}_{\xi} f$. Then $\nu(f) \neq 0$ iff $f \in \mathfrak{s l e}^{\circ}(n)$. So on $\mathfrak{s l e}^{\circ}(n)$ we can define the right inverse $\Delta^{-1}$ to $\Delta$ by the formula

$$
\Delta^{-1} f=\frac{1}{\nu(f)}(\Phi f)
$$

0.5. Cartan prolongs. We will repeatedly use Cartan prolongation. So let us recall the definition. Let $\mathfrak{g}$ be a Lie superalgebra and $V$ a $\mathfrak{g}$-module. Set $\mathfrak{g}_{-1}=V, \mathfrak{g}_{0}=\mathfrak{g}$ and for $i>0$ define the $i$-th Cartan prolong $\mathfrak{g}_{i}$ as the space of all $X \in \operatorname{Hom}\left(\mathfrak{g}_{-1}, \mathfrak{g}_{i-1}\right)$ such that

$$
X\left(w_{0}\right)\left(w_{1}, w_{2}, \ldots, w_{i}\right)=(-1)^{p\left(w_{0}\right) p\left(w_{1}\right)} X\left(w_{1}\right)\left(w_{0}, w_{2}, \ldots, w_{i}\right)
$$

for all $w_{0}, \ldots, w_{i} \in \mathfrak{g}_{-1}$.
The Cartan prolong (the result of Cartan's prolongation) of the pair $(V, \mathfrak{g})$ is $\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0}\right)_{*}=\oplus_{i \geq-1} \mathfrak{g}_{i}$.

Suppose that the $\mathfrak{g}_{0}$-module $\mathfrak{g}_{-1}$ is faithful. Then
$\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0}\right)_{*} \subset \mathfrak{v e c t}(n \mid m)=\mathfrak{d e r}(\mathbb{C}[x]), \quad$ where $n=\operatorname{dim}\left(V_{\overline{0}}\right)$ and $m=\operatorname{dim}\left(V_{\overline{1}}\right)$ and $x=\left(u_{1}, \ldots, u_{n}, \xi_{1}, \ldots, \xi_{m}\right)$. We have for $i \geq 1$

$$
\mathfrak{g}_{i}=\left\{D \in \mathfrak{v e c t}(n \mid m): \operatorname{deg} D=i,[D, X] \in \mathfrak{g}_{i-1} \text { for any } X \in \mathfrak{g}_{-1}\right\}
$$

The Lie superalgebra structure on $\mathfrak{v e c t}(n \mid m)$ induces one on $\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0}\right)_{*}$. This way the commutator of vector fields $[g, v]$, corresponds to the action $g \cdot v$, $g \in \mathfrak{g}$ and $v \in V$.

We give some examples of Cartan prolongations. Let $\mathfrak{g}_{-1}=V$ be an $(n \mid m)$-dimensional superspace and $\mathfrak{g}_{0}=\mathfrak{g l}(n \mid m)$ the space of all endomorphisms of $V$. Then $\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0}\right)_{*}=\mathfrak{v e c t}(n \mid m)$. If one takes for $\mathfrak{g}_{0}$ only the supertraceless elements $\mathfrak{s l}(n \mid m)$, then $\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0}\right)_{*}=\mathfrak{s v e c t}(n \mid m)$, the algebra of vector fields with divergence 0 .
§1. The structure of $\mathfrak{v e c t}(0 \mid 3)_{*}$
1.1. In this note our primary interest is in a certain Cartan prolongation (denoted by $\left.\mathfrak{v e c t}(0 \mid 3)_{*}\right)$ and the extension $\mathfrak{c v e c t}(0 \mid 3)_{*}$ thereof. Here we will discuss $\mathfrak{v e c t}(0 \mid 3)_{*}$. Now $\mathfrak{v e c t}(0 \mid 3)_{*}$ is a short-hand notation for the Cartan prolongation with

$$
V=\mathfrak{g}_{-1}=\Pi \Lambda\left(\eta_{1}, \eta_{2}, \eta_{3}\right) / \mathbb{C} \text { and } \mathfrak{g}_{0}=\mathfrak{d e r} V
$$

So $V$ is a superspace of dimension (4|3), with

$$
V_{\overline{0}}=\left\langle\eta_{1} \eta_{2} \eta_{3}, \eta_{1}, \eta_{2}, \eta_{3}\right\rangle ; \quad V_{\overline{1}}=\left\langle\eta_{2} \eta_{3}, \eta_{3} \eta_{1}, \eta_{1} \eta_{2}\right\rangle
$$

and $\operatorname{dim} \mathfrak{g}_{0}=(12 \mid 12)$.
The elements of $\mathfrak{g}_{-1}$ and $\mathfrak{g}_{0}$ can be expressed as vector fields in $\mathfrak{v e c t}(4 \mid 3)$. Choosing

$$
\eta_{1} \eta_{2} \eta_{3} \simeq-\partial_{y} ; \quad \eta_{i} \simeq-\partial_{u_{i}} ; \quad \frac{\partial \eta_{1} \eta_{2} \eta_{3}}{\partial \eta_{i}} \simeq-\partial_{\xi_{i}}
$$

it is subject to straightforward verification that the elements of $\mathfrak{g}_{0}$, expressed as elements of $\mathfrak{v e c t}(4 \mid 3)$ are of the form:

$$
\begin{aligned}
& \partial_{\eta_{1}} \simeq-y \partial_{\xi_{1}}-\xi_{2} \partial_{u_{3}}+\xi_{3} \partial_{u_{2}} \quad-\eta_{1} \partial_{\eta_{1}} \simeq u_{1} \partial_{u_{1}}+\xi_{2} \partial_{\xi_{2}}+\xi_{3} \partial_{\xi_{3}}+y \partial_{y} \\
& \partial_{\eta_{2}} \simeq-y \partial_{\xi_{2}}-\xi_{3} \partial_{u_{1}}+\xi_{1} \partial_{u_{3}} \quad-\eta_{2} \partial_{\eta_{2}} \simeq u_{2} \partial_{u_{2}}+\xi_{1} \partial_{\xi_{1}}+\xi_{3} \partial_{\xi_{3}}+y \partial_{y} \\
& \partial_{\eta_{3}} \simeq-y \partial_{\xi_{3}}-\xi_{1} \partial_{u_{2}}+\xi_{2} \partial_{u_{1}} \quad-\eta_{3} \partial_{\eta_{3}} \simeq u_{3} \partial_{u_{3}}+\xi_{1} \partial_{\xi_{1}}+\xi_{2} \partial_{\xi_{2}}+y \partial_{y} \\
& \eta_{1} \partial_{\eta_{2}} \simeq-u_{2} \partial_{u_{1}}+\xi_{1} \partial_{\xi_{2}} \quad \eta_{2} \partial_{\eta_{1}} \simeq-u_{1} \partial_{u_{2}}+\xi_{2} \partial_{\xi_{1}} \quad \eta_{1} \eta_{2} \eta_{3} \partial_{\eta_{1}} \simeq-u_{1} \partial_{y} \\
& \eta_{2} \partial_{\eta_{3}} \simeq-u_{3} \partial_{u_{2}}+\xi_{2} \partial_{\xi_{3}} \quad \eta_{3} \partial_{\eta_{2}} \simeq-u_{2} \partial_{u_{3}}+\xi_{3} \partial_{\xi_{2}} \quad \eta_{1} \eta_{2} \eta_{3} \partial_{\eta_{2}} \simeq-u_{2} \partial_{y} \\
& \eta_{3} \partial_{\eta_{1}} \simeq-u_{1} \partial_{u_{3}}+\xi_{3} \partial_{\xi_{1}} \quad \eta_{1} \partial_{\eta_{3}} \simeq-u_{3} \partial_{u_{1}}+\xi_{1} \partial_{\xi_{3}} \quad \eta_{1} \eta_{2} \eta_{3} \partial_{\eta_{3}} \simeq-u_{3} \partial_{y} \\
& \eta_{1} \eta_{2} \partial_{\eta_{3}} \simeq-u_{3} \partial_{\xi_{3}} \quad \eta_{1} \eta_{2} \partial_{\eta_{1}} \simeq-u_{1} \partial_{\xi_{3}}-\xi_{2} \partial_{y} \quad \eta_{1} \eta_{2} \partial_{\eta_{2}} \simeq-u_{2} \partial_{\xi_{3}}+\xi_{1} \partial_{y} \\
& \eta_{2} \eta_{3} \partial_{\eta_{1}} \simeq-u_{1} \partial_{\xi_{1}} \quad \eta_{2} \eta_{3} \partial_{\eta_{2}} \simeq-u_{2} \partial_{\xi_{1}}-\xi_{3} \partial_{y} \quad \eta_{2} \eta_{3} \partial_{\eta_{3}} \simeq-u_{3} \partial_{\xi_{1}}+\xi_{2} \partial_{y} \\
& \eta_{3} \eta_{1} \partial_{\eta_{2}} \simeq-u_{2} \partial_{\xi_{2}} \quad \eta_{3} \eta_{1} \partial_{\eta_{3}} \simeq-u_{3} \partial_{\xi_{2}}-\xi_{1} \partial_{y} \quad \eta_{3} \eta_{1} \partial_{\eta_{1}} \simeq-u_{1} \partial_{\xi_{2}}+\xi_{3} \partial_{y}
\end{aligned}
$$

1.2. Now we will give a more explicit description of $\mathfrak{v e c t}(0 \mid 3)_{*}$. It will turn out that $\mathfrak{v e c t}(0 \mid 3)_{*}$ is isomorphic to $\mathfrak{l e}(3)$ as Lie superalgebra; however considered as $\mathbb{Z}$-graded algebras we have to define a different grading. The $\mathbb{Z}$-graded Lie superalgebra $\mathfrak{l e}(3 ; 3)$ is $\mathfrak{l e}(3)$ as Lie superalgebra with $\mathbb{Z}$-degree of $D$

$$
D=\sum_{i} f_{i} \frac{\partial}{\partial u_{i}}+\sum_{j} g_{j} \frac{\partial}{\partial \xi_{j}}
$$

the $u$-degree of $f_{i}$ minus 1 (or the $u$-degree of $g_{j}$ ), i.e. $\operatorname{deg} \xi_{i}=0$.
Consider the map $i_{1}: \mathfrak{l e}(3 ; 3) \rightarrow \mathfrak{v e c t}(4 \mid 3)$ given by
a.) If $f=f(u)$ then

$$
i_{1}\left(\operatorname{Le}_{f}\right)=\operatorname{Le}_{\sum \frac{\partial f}{\partial u_{i}} \xi_{j} \xi_{k}-y f}
$$

where $y$ is treated as a parameter and $(i, j, k) \in A_{3}$ (even permutations of $\{1,2,3\}$ ).
b.) If $f=\sum f_{i}(u) \xi_{i}$ then

$$
i_{1}\left(\mathrm{Le}_{f}\right)=\mathrm{Le}_{f}-\varphi(u) \sum \xi_{i} \partial_{\xi_{i}}+\left(-\varphi(u) y+\Delta\left(\varphi(u) \xi_{1} \xi_{2} \xi_{3}\right)\right) \partial_{y}
$$

where $\varphi(u)=\Delta(f)$ and $\Delta$ as given in section 0.4.
c.) If $f=\psi_{1}(u) \xi_{2} \xi_{3}+\psi_{2}(u) \xi_{3} \xi_{1}+\psi_{3}(u) \xi_{1} \xi_{2}$ then

$$
i_{1}\left(\operatorname{Le}_{f}\right)=-\Delta(f) \partial_{y}-\sum_{i=1}^{3} \psi_{i}(u) \frac{\partial}{\partial \xi_{i}}
$$

d.) If $f=\psi(u) \xi_{1} \xi_{2} \xi_{3}$ then

$$
i_{1}\left(\mathrm{Le}_{f}\right)=-\psi(u) \partial_{y} .
$$

Note that $i_{1}$ preserves the $\mathbb{Z}$-degree. We have the following lemma.
1.3. Lemma. The map $i_{1}$ is an isomorphism of $\mathbb{Z}$-graded Lie superalgebras between $\mathfrak{l e}(3 ; 3)$ and $\mathfrak{v e c t}(0 \mid 3)_{*} \subset \mathfrak{v e c t}(4 \mid 3)$.

Proof. That $i_{1}$ is an embedding can be verified by direct computation. To prove that the image of $i_{1}$ is in $\mathfrak{v e c t}(0 \mid 3)_{*}$ it is enough to show that this is the case on the components $\mathfrak{l e}(3 ; 3)_{-1} \oplus \mathfrak{l e}(3 ; 3)_{0}$, i.e. on functions $f(u, \xi)$ of degree $\leq 1$ with respect to $u$, as the Cartan prolongation is the biggest subalgebra $\mathfrak{g}$ of $\mathfrak{v e c t}(4 \mid 3)$, with given $\mathfrak{g}_{-1}$ and $\mathfrak{g}_{0}$. The proof that $i_{1}$ is surjective onto $\mathfrak{v e c t}(0 \mid 3)_{*}$ is given in corollary 4.6.
A generalized version of Lemma 1.3 can be found in [10] and [7]. It states that $\mathfrak{l e}(n ; n)$ and $\mathfrak{v e c t}(0 \mid n)_{*}$ are isomorphic for all $n \geq 1$.

## §2. The construction of $\mathfrak{c v e c t}(0 \mid 3)_{*}$

2.1. Let us describe a general construction, which leads to several new simple Lie superalgebras. Let $\mathfrak{u}=\mathfrak{v e c t}(m \mid n)$, let $\mathfrak{g}=\left(\mathfrak{u}_{-1}, \mathfrak{g}_{0}\right)_{*}$ be a simple Lie subsuperalgebra of $\mathfrak{u}$. Moreover suppose there exists an element $d \in \mathfrak{u}_{0}$ that determines an exterior derivation of $\mathfrak{g}$ and has no kernel on $\mathfrak{u}_{+}$. Let us study the prolong $\tilde{\mathfrak{g}}=\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0} \oplus \mathbb{C} d\right)_{*}$.

Lemma. Either $\tilde{\mathfrak{g}}$ is simple or $\tilde{\mathfrak{g}}=\mathfrak{g} \oplus \mathbb{C} d$.
Proof. Let $I$ be a nonzero graded ideal of $\tilde{\mathfrak{g}}$. The subsuperspace $\left(\operatorname{ad} \mathfrak{u}_{-1}\right)^{k+1} a$ of $\mathfrak{u}_{-1}$ is nonzero for any nonzero homogeneous element $a \in \mathfrak{u}_{k}$ and $k \geq$ 0 . Since $\mathfrak{g}_{-1}=\mathfrak{u}_{-1}$, the ideal $I$ contains nonzero elements from $\mathfrak{g}_{-1}$; by simplicity of $\mathfrak{g}$ the ideal $I$ contains the whole $\mathfrak{g}$. If, moreover, $\left[\mathfrak{g}_{-1}, \tilde{\mathfrak{g}}_{1}\right]=\mathfrak{g}_{0}$, then by definition of the Cartan prolongation $\tilde{\mathfrak{g}}=\mathfrak{g} \oplus \mathbb{C} d$.

If, instead, $\left[\mathfrak{g}_{-1}, \tilde{\mathfrak{g}}_{1}\right]=\mathfrak{g}_{0} \oplus \mathbb{C} d$, then $d \in I$ and since $\left[d, \mathfrak{u}_{+}\right]=\mathfrak{u}_{+}$, we derive that $I=\tilde{\mathfrak{g}}$. In other words, $\tilde{\mathfrak{g}}$ is simple.

As an example, take $\mathfrak{g}=\mathfrak{s v e c t}(m \mid n) ; \mathfrak{g}_{0}=\mathfrak{s l}(m \mid n), d=1_{m \mid n}$. Then $\left(\mathfrak{g}_{-1}, \mathfrak{g}_{0} \oplus \mathbb{C} d\right)_{*}=\mathfrak{v e c t}(m \mid n)$.
2.2. Definition. The Lie superalgebra $\mathfrak{c v e c t}(0 \mid 3)_{*} \subset \mathfrak{v e c t}(4 \mid 3)$ is the Cartan prolongation with $\mathfrak{c v e c t}(0 \mid 3)_{-1}=\mathfrak{v e c t}(0 \mid 3)_{-1}$ and $\mathfrak{c v e c t}(0 \mid 3)_{0}=\mathfrak{v e c t}(0 \mid 3)_{0} \oplus$ $\mathbb{C} d$, with

$$
d=\sum u_{i} \partial_{u_{i}}+\sum \xi_{i} \partial_{\xi_{i}}+y \partial_{y} .
$$

If now

$$
f=\sum_{i=1}^{3} \xi_{i} \partial_{\xi_{i}}+2 y \partial_{y},
$$

then it is clear that $f \in \mathfrak{v e c t}(0 \mid 3) \oplus \mathbb{C} d$, but $f \notin \mathfrak{v e c t}(0 \mid 3)$.
2.3. Theorem. The Lie superalgebra $\mathfrak{c v e c t}(0 \mid 3)_{*}$ is simple.

Proof. We know that $\mathfrak{v e c t}(0 \mid 3)_{*} \cong \mathfrak{l e}(3 ; 3)$ is simple. According to Lemma 2.1 it is sufficient to find an element $F \in \mathfrak{c v e c t}(0 \mid 3)_{1}$, which is not in $\mathfrak{v e c t}(0 \mid 3)_{1}$. For $F$ one can take

$$
F=y \xi_{1} \partial_{\xi_{1}}+y \xi_{2} \partial_{\xi_{2}}+y \xi_{3} \partial_{\xi_{3}}+y^{2} \partial_{y}-\xi_{1} \xi_{2} \partial_{u_{3}}-\xi_{3} \xi_{1} \partial_{u_{2}}-\xi_{2} \xi_{3} \partial_{u_{1}}
$$

Indeed, one easily checks that $\partial_{y} F=f$, while

$$
\left[\partial_{\xi_{i}}, F\right]=-\partial_{\eta_{i}} \quad(i=1,2,3),
$$

and moreover $\left[\partial_{u_{i}}, F\right]=0$. This proves the claim.
Similar constructions are possible for general $n$. For $n=2$ we obtain $\mathfrak{c v e c t}(0 \mid 2)_{*} \cong \mathfrak{v e c t}(2 \mid 1)$, while for $n>3$ one can prove that $\mathfrak{c v e c t}(0 \mid n)_{*}$ is not simple. For details, we refer to [10].
2.4. Lemma. A vector field

$$
D=\sum_{i=1}^{3}\left(P_{i} \partial_{\xi_{i}}+Q_{i} \partial_{u_{i}}\right)+R \partial_{y}
$$

in $\mathfrak{v e c t}(4 \mid 3)$ belongs to $\mathfrak{c v e c t}(0 \mid 3)_{*}$ if and only if it satisfies the following system of equations:

$$
\begin{gather*}
\frac{\partial Q_{i}}{\partial u_{j}}+(-1)^{p(D)} \frac{\partial P_{j}}{\partial \xi_{i}}=0 \text { for any } i \neq j ;  \tag{2.1}\\
\frac{\partial Q_{i}}{\partial u_{i}}+(-1)^{p(D)} \frac{\partial P_{i}}{\partial \xi_{i}}=\frac{1}{2}\left(\sum_{1 \leq j \leq 3} \frac{\partial Q_{j}}{\partial u_{j}}+\frac{\partial R}{\partial y}\right) \text { for } i=1,2,3 ;  \tag{2.2}\\
\frac{\partial Q_{i}}{\partial \xi_{j}}+\frac{\partial Q_{j}}{\partial \xi_{i}}=0 \text { for any } i, j ; \text { in particular } \frac{\partial Q_{i}}{\partial \xi_{i}}=0 ;  \tag{2.3}\\
\frac{\partial P_{i}}{\partial u_{j}}-\frac{\partial P_{j}}{\partial u_{i}}=-(-1)^{p(D)} \frac{\partial R}{\partial \xi_{k}} \tag{2.4}
\end{gather*}
$$

for any $k$ and any even permutation $\left(\begin{array}{lll}1 & 2 & 3 \\ i & j & k\end{array}\right)$.

$$
\begin{gather*}
\frac{\partial Q_{i}}{\partial y}=0 \text { for } i=1,2,3  \tag{2.5}\\
\frac{\partial P_{k}}{\partial y}=(-1)^{p(D)} \frac{1}{2}\left(\frac{\partial Q_{i}}{\partial \xi_{j}}-\frac{\partial Q_{j}}{\partial \xi_{i}}\right) \tag{2.6}
\end{gather*}
$$

for any $k$ and for any even permutation $\left(\begin{array}{lll}1 & 2 & 3 \\ i & j & k\end{array}\right)$.
Proof. Denote by $\mathfrak{g}=\oplus_{i \geq-1} \mathfrak{g}_{i}$ the superspace of solutions of the system (2.1)-(2.6). Clearly, $\mathfrak{g}_{-1} \cong \mathfrak{v e c t}(4 \mid 3)_{-1}$. We directly verify that the images of the elements from $\mathfrak{v e c t}(0 \mid 3) \oplus \mathbb{C} d$ satisfy (2.1)-(2.6). Actually, we composed the system of equations (2.1)-(2.6) by looking at these images.

The isomorphism $\mathfrak{g}_{0}=\mathfrak{v e c t}(0 \mid 3) \oplus \mathbb{C} d$ follows from dimension considerations.

Set

$$
\begin{aligned}
D_{u_{j}}(D) & =\sum_{i \leq 3}\left(\frac{\partial P_{i}}{\partial u_{j}} \frac{\partial}{\partial \xi_{i}}+\frac{\partial Q_{i}}{\partial u_{j}} \frac{\partial}{\partial u_{i}}\right)+\frac{\partial R}{\partial u_{j}} \frac{\partial}{\partial y} \\
D_{y}(D) & =\sum_{i \leq 3}\left(\frac{\partial P_{i}}{\partial y} \frac{\partial}{\partial \xi_{i}}+\frac{\partial Q_{i}}{\partial y} \frac{\partial}{\partial u_{i}}\right)+\frac{\partial R}{\partial y} \frac{\partial}{\partial y} ; \\
\tilde{D}_{\xi_{j}}(D) & =(-1)^{p(D)} \sum_{i \leq 3}\left(\frac{\partial P_{i}}{\partial \xi_{j}} \frac{\partial}{\partial \xi_{i}}+\frac{\partial Q_{i}}{\partial \xi_{j}} \frac{\partial}{\partial u_{i}}\right)+(-1)^{p(D)} \frac{\partial R}{\partial \xi_{j}} \frac{\partial}{\partial y} .
\end{aligned}
$$

The operators $D_{u_{j}}, D_{y}$ and $\tilde{D}_{\xi_{j}}$, clearly, commute with the $\mathfrak{g}_{-1}$-action. Observe: the operators commute, not supercommute.

Since the operators in the equations (2.1)-(2.6) are linear combinations of only these operators $D_{u_{j}}, D_{y}$ and $\tilde{D}_{\xi_{j}}$, the definition of Cartan prolongation itself ensures isomorphism of $\mathfrak{g}$ with $\mathfrak{c v e c t}(0 \mid 3)_{*}$.
2.5. Remark. The left hand sides of eqs. (2.1)-(2.6) determine coefficients of the 2 -form $L_{D} \omega$, where $L_{D}$ is the Lie derivative and $\omega=\sum_{1 \leq i \leq 3} d u_{i} d \xi_{i}$. It would be interesting to interpret the right-hand side of these equations in geometrical terms as well.
2.6. Remark. Lemma 2.4 illustrates how $\mathfrak{c v e c t}(0 \mid 3)_{*}$ can be characterized by a set of first order, constant coefficient, differential operators. This is a general fact of Cartan prolongations; one just replaces the linear constraints on $\mathfrak{g}_{0}$ by such operators. For example, for $\mathfrak{v e c t}(0 \mid 3)_{*}$ we have the equations (2.1)-(2.6) and

$$
\begin{equation*}
\frac{\partial R}{\partial y}-\sum_{i=1}^{3} \frac{\partial Q_{i}}{\partial u_{i}}=0 \tag{2.7}
\end{equation*}
$$

Indeed, this equation is satisfied by all elements of $\mathfrak{v e c t}(0 \mid 3)_{0}$, see section 1.1, but not by $d$.

## §3. Solution of differential equations (2.1) - (2.6)

Set $D_{\xi}^{3}=\frac{\partial^{3}}{\partial \xi_{1} \partial \xi_{2} \partial \xi_{3}}$.
3.1. Theorem. Every solution of the system (2.1) - (2.6) is of the form:

$$
\begin{gather*}
D=\mathrm{Le}_{f}+y A_{f}-(-1)^{p(f)}\left(y \Delta(f)+y^{2} D_{\xi}^{3} f\right) \partial_{y}+  \tag{3.1}\\
A_{g}-(-1)^{p(g)}\left(\Delta(g)+2 y D_{\xi}^{3} g\right) \partial_{y},
\end{gather*}
$$

where $f, g \in \mathbb{C}[u, \xi]$ are arbitrary and the operator $A_{f}$ is given by the formula:

$$
\begin{equation*}
A_{f}=\frac{\partial^{2} f}{\partial \xi_{2} \partial \xi_{3}} \frac{\partial}{\partial \xi_{1}}+\frac{\partial^{2} f}{\partial \xi_{3} \partial \xi_{1}} \frac{\partial}{\partial \xi_{2}}+\frac{\partial^{2} f}{\partial \xi_{1} \partial \xi_{2}} \frac{\partial}{\partial \xi_{3}} . \tag{3.2}
\end{equation*}
$$

Proof. First, let us find all solutions of system (2.1)-(2.6) for which $Q_{1}=$ $Q_{2}=Q_{3}=0$. In this case the system takes the form

$$
\begin{align*}
\frac{\partial P_{j}}{\partial \xi_{i}} & =0 \text { for } i \neq j \\
(-1)^{p(D)} \frac{\partial P_{i}}{\partial \xi_{i}} & =\frac{1}{2} \frac{\partial R}{\partial y} \text { for } i=1,2,3
\end{align*}
$$

$$
\begin{gather*}
\frac{\partial P_{i}}{\partial u_{j}}-\frac{\partial P_{j}}{\partial u_{i}}=-(-1)^{p(D)} \frac{\partial R}{\partial \xi_{k}} \text { for }(i, j, k) \in A_{3} \\
\frac{\partial P_{k}}{\partial y}=0 \text { for } k=1,2,3
\end{gather*}
$$

From (2.1'), $\left(2.2^{\prime}\right)$ and $\left(2.6^{\prime}\right)$ it follows that

$$
P_{i}=\Psi_{i}\left(u_{1}, u_{2}, u_{3}\right)+\xi_{i} \varphi\left(u_{1}, u_{2}, u_{3}\right),
$$

where $\varphi=\frac{1}{2}(-1)^{p(D)} \frac{\partial R}{\partial y}$. For brevity we will write $\Psi_{i}(u)$ and $\varphi(u)$. Then $R=(-1)^{p(D)} \cdot 2 \varphi(u) y+R_{0}(u, \xi)$.

Let us expand the 3 equations of type (2.4'); their explicit form is:

$$
\begin{aligned}
& \frac{\partial R_{0}}{\partial \xi_{1}}=-(-1)^{p(D)}\left(\frac{\partial \Psi_{2}}{\partial u_{3}}-\frac{\partial \Psi_{3}}{\partial u_{2}}\right)+(-1)^{p(D)}\left(\frac{\partial \varphi}{\partial u_{2}} \xi_{3}-\frac{\partial \varphi}{\partial u_{3}} \xi_{2}\right), \\
& \frac{\partial R_{0}}{\partial \xi_{2}}=-(-1)^{p(D)}\left(\frac{\partial \Psi_{3}}{\partial u_{1}}-\frac{\partial \Psi_{1}}{\partial u_{3}}\right)+(-1)^{p(D)}\left(\frac{\partial \varphi}{\partial u_{3}} \xi_{1}-\frac{\partial \varphi}{\partial u_{1}} \xi_{3}\right), \\
& \frac{\partial R_{0}}{\partial \xi_{3}}=-(-1)^{p(D)}\left(\frac{\partial \Psi_{1}}{\partial u_{2}}-\frac{\partial \Psi_{2}}{\partial u_{1}}\right)+(-1)^{p(D)}\left(\frac{\partial \varphi}{\partial u_{1}} \xi_{2}-\frac{\partial \varphi}{\partial u_{2}} \xi_{1}\right) .
\end{aligned}
$$

The integration of these equations yields

$$
\begin{aligned}
& R_{0}=(-1)^{p(D)}\left(\Psi_{0}(u)-\left(\frac{\partial \Psi_{2}}{\partial u_{3}}-\frac{\partial \Psi_{3}}{\partial u_{2}}\right) \xi_{1}-\right. \\
& \left.\quad\left(\frac{\partial \Psi_{3}}{\partial u_{1}}-\frac{\partial \Psi_{1}}{\partial u_{3}}\right) \xi_{2}-\left(\frac{\partial \Psi_{1}}{\partial u_{2}}-\frac{\partial \Psi_{2}}{\partial u_{3}}\right) \xi_{3}-\left(\frac{\partial \varphi}{\partial u_{2}} \xi_{3} \xi_{1}+\frac{\partial \varphi}{\partial u_{1}} \xi_{2} \xi_{3}+\frac{\partial \varphi}{\partial u_{3}} \xi_{1} \xi_{2}\right)\right) \\
& \quad=(-1)^{p(D)}\left(\Psi_{0}(u)+\Delta\left(-\Psi_{1} \xi_{2} \xi_{3}-\Psi_{2} \xi_{3} \xi_{1}-\Psi_{3} \xi_{1} \xi_{2}-\varphi \xi_{1} \xi_{2} \xi_{3}\right)\right) .
\end{aligned}
$$

Therefore, any vector field $D$ with $Q_{1}=Q_{2}=Q_{3}=0$ satisfying (2.1) (2.6) is of the form

$$
\begin{aligned}
D & =\sum_{i=1}^{3} \Psi_{i}(u) \partial_{\xi_{i}}+\varphi(u) \sum_{i=1}^{3} \xi_{i} \partial_{\xi_{i}}+(-1)^{p(D)} \\
& \cdot\left(\Psi_{0}(u)+\Delta\left(-\Psi_{1} \xi_{2} \xi_{3}-\Psi_{2} \xi_{3} \xi_{1}-\Psi_{3} \xi_{1} \xi_{2}-\varphi \xi_{1} \xi_{2} \xi_{3}\right)+2 \varphi(u) y\right) \partial_{y}
\end{aligned}
$$

where, as before,

$$
\Delta=\sum_{i=1}^{3} \frac{\partial}{\partial u_{i}} \frac{\partial}{\partial \xi_{i}} .
$$

Set

$$
g(u, \xi)=g_{0}(u, \xi)-\Psi_{1} \xi_{2} \xi_{3}-\Psi_{2} \xi_{3} \xi_{1}-\Psi_{3} \xi_{1} \xi_{2}-\varphi \xi_{1} \xi_{2} \xi_{3},
$$

with $\Delta g_{0}=\Psi_{0}$ and $\operatorname{deg}_{\xi}\left(g_{0}\right) \leq 1$. Then

$$
A_{g}=\sum_{i=1}^{3} \Psi_{i} \partial_{\xi_{i}}+\varphi \sum_{i=1}^{3} \xi_{i} \partial_{\xi_{i}} ; \quad D_{\xi}^{3} g=\varphi \text { and }(-1)^{p(D)}=(-1)^{p(g)+1}
$$

for functions $g$ homogeneous with respect to parity. In the end we get:

$$
\begin{align*}
D & =A_{g}+(-1)^{p(D)}\left(\Delta(g)+2 y D_{\xi}^{3} g\right) \partial_{y} \\
& =A_{g}-(-1)^{p(g)}\left(\Delta(g)+2 y D_{\xi}^{3} g\right) \partial_{y} . \tag{3.3}
\end{align*}
$$

Let us return now to the system (2.1) - (2.6). Equations (2.3), (2.5), (2.6) imply that there exists a function $f(u, \xi)$ (independent of $y$ !) such that

$$
Q_{i}=-(-1)^{p(D)} \frac{\partial f}{\partial \xi_{i}} \text { for } i=1,2,3
$$

Then (2.1) implies that

$$
P_{i}=\frac{\partial f}{\partial u_{i}}+f_{i}\left(u, \xi_{i}, y\right) .
$$

From (2.6) it follows that

$$
\frac{\partial f_{i}}{\partial y}=\partial_{\xi_{j}} \partial_{\xi_{k}} f \text { for even permutations }(i, j, k)
$$

or

$$
f_{i}=y\left(\partial_{\xi_{j}} \partial_{\xi_{k}} f\right)+\tilde{P}_{i}\left(u, \xi_{i}\right)
$$

Observe that $\tilde{P}_{i}$ satisfy $\left(2.1^{\prime}\right)$ and $\left(2.6^{\prime}\right)$; hence, in view of $(2.2), \frac{\partial \tilde{P}_{i}}{\partial \xi_{i}}$ does not depend on $i$. Therefore, we can choose $\tilde{R}$ so that ( $\left.\tilde{P}_{i}, \tilde{R}\right)$ satisfy eqs. $\left(2.1^{\prime}\right),\left(2.2^{\prime}\right),\left(2.4^{\prime}\right),\left(2.6^{\prime}\right)$. Thanks to the linearity of system (2.1) - (2.6) the vector field $D$ is then of the form

$$
\begin{equation*}
D=D_{f}+\tilde{D} \tag{3.4}
\end{equation*}
$$

where $D_{f}$ and $\tilde{D}$ are solutions of (2.1) - (2.6) such that $\tilde{D}=\sum \tilde{P}_{i} \partial_{\xi_{i}}+\tilde{R} \partial_{y}$ (i.e., $\tilde{D}$ is of the form (3.3)) and

$$
\begin{aligned}
D_{f} & \left.=\sum\left(-(-1)^{p(D)} \frac{\partial f}{\partial \xi_{i}} \partial_{u_{i}}+\frac{\partial f}{\partial u_{i}} \partial_{\xi_{i}}\right)+\sum y\left(\partial_{\xi_{j}} \partial_{\xi_{k}} f\right) \partial_{\xi_{i}}\right)+R_{f} \cdot \partial_{y} \\
& =\mathrm{Le}_{f}+y A_{f}+R_{f} \partial_{y} .
\end{aligned}
$$

It remains to find $R_{f}$. Equation (2.2) takes the form

$$
(-1)^{p(D)} y D_{\xi}^{3} f=\frac{1}{2}\left(-(-1)^{p(D)}(\Delta f)+\frac{\partial R_{f}}{\partial y}\right)
$$

Hence,

$$
R_{f}=(-1)^{p(D)}\left(y^{2} D_{\xi}^{3} f+y \cdot(\Delta f)+R_{0}(u, \xi)\right) .
$$

Then, we can rewrite (2.4) as

$$
-y \frac{\partial \Delta f}{\partial \xi_{k}}+\frac{\partial R_{0}}{\partial \xi_{k}}=y \partial_{u_{j}} \partial_{\xi_{j}} \partial_{\xi_{k}} f-y \partial_{u_{i}} \partial_{\xi_{k}} \partial_{\xi_{i}} f
$$

Observe that the right hand side of the last equation is equal to $-y \frac{\partial \Delta f}{\partial \xi_{k}}$. This means that $\frac{\partial R_{0}}{\partial \xi_{k}}=0$ or $R_{0}=R_{0}(u)$. Therefore, replacing $\tilde{R}$ with $\tilde{R}+R_{0}$ we may assume that $R_{0}=0$. Then

$$
\begin{equation*}
D_{f}=\mathrm{Le}_{f}+y A_{f}+(-1)^{p(D)}\left(y(\Delta f)+y^{2} D_{\xi}^{3} f\right) \partial_{y} \tag{3.5}
\end{equation*}
$$

By uniting (3.3) - (3.5) we get (3.1).
$\S 4$ How to Generate $\mathfrak{c v e c t}(0 \mid 3)_{*}$ by pairs of functions
We constructed $\mathfrak{c v e c t}(0 \mid 3)_{*}$ as an extension of $\mathfrak{v e c t}(0 \mid 3)_{*} \cong \mathfrak{l e}(3 ; 3)$, see lemma 1.3. Using the results of section 3 , we obtain another embedding $i_{2}: \mathfrak{l e}(3) \rightarrow \mathfrak{v e c t}(0 \mid 3)_{*}$.
4.1. Lemma. The map

$$
\begin{equation*}
i_{2}: \mathrm{Le}_{f} \rightarrow \mathrm{Le}_{f}+y A_{f}-(-1)^{p(f)}\left(y \Delta(f)+y^{2} D_{\xi}^{3} f\right) \partial_{y} \tag{4.1}
\end{equation*}
$$

determines an embedding of $\mathfrak{l e}(3)$ into $\mathfrak{c v e c t}(0 \mid 3)_{*}$. This embedding preserves the standard grading of $\mathfrak{l e}(3)$.

Proof. We have to verify the equality

$$
i_{2}\left(\operatorname{Le}_{\{f, g\}}\right)=\left[i_{2}\left(\operatorname{Le}_{f}\right), i_{2}\left(\operatorname{Le}_{g}\right)\right] .
$$

Comparison of coefficients of different powers of $y$ shows that the above equation is equivalent to the following system:

$$
\begin{gather*}
\operatorname{Le}_{\{f, g\}}=\left[\mathrm{Le}_{f}, \mathrm{Le}_{g}\right] .  \tag{4.2}\\
A_{\{f, g\}}=\left[\mathrm{Le}_{f}, A_{g}\right]+\left[A_{f}, \mathrm{Le}_{g}\right]-(-1)^{p(f)}\left(\Delta(f) \cdot A_{g}+(-1)^{p(f) p(g)} \Delta(g) A_{f}\right) .  \tag{4.3}\\
{\left[A_{f}, A_{g}\right]=(-1)^{p(f)}\left(D_{\xi}^{3} f \cdot A_{g}+(-1)^{p(f) p(g)} D_{\xi}^{3} g A_{f}\right) .}  \tag{4.4}\\
\Delta(\{f, g\})=\{\Delta f, g\}-(-1)^{p(f)}\{f, \Delta g\} .  \tag{4.5}\\
D_{\xi}^{3}\{f, g\}=\left\{D_{\xi}^{3} f, g\right\}-(-1)^{p(f)}\left\{f, D_{\xi}^{3} g\right\}-(-1)^{p(f)}\left(A_{f}(\Delta g)\right.  \tag{4.6}\\
\left.+(-1)^{p(f) p(g)} A_{g}(\Delta f)\right)+\Delta f D_{\xi}^{3} g-D_{\xi}^{3} f \Delta g .
\end{gather*}
$$

Equation (4.2) is known, see section 0.3. The equalities (4.3)-(4.6) are subject to direct verification.

We found two embeddings $i_{1}: \mathfrak{l e}(3 ; 3) \rightarrow \mathfrak{v e c t}(0 \mid 3)_{*}$ and $i_{2}: \mathfrak{l e}(3) \rightarrow$ $\mathfrak{c v e c t}(0 \mid 3)_{*}$. Let us denote

$$
\alpha_{g}=A_{g}-(-1)^{p(g)}\left(\Delta g+2 y D_{\xi}^{3} g\right) \partial_{y} .
$$

We want to prove that the sum of the images of $i_{1}$ and $i_{2}$ cover the whole $\mathfrak{c v e c t}(0 \mid 3)_{*}$. According to Theorem 3.1, it is sufficient to represent $\alpha_{g}$ in the form $\alpha_{g}=i_{1} g_{1}+i_{2} g_{2}$. For convenience we simply write $f$ instead of $\mathrm{Le}_{f}$.
4.2. Lemma. For $\alpha_{g}$ we have:

$$
\alpha_{g}=\left\{\begin{array}{cc}
0 & \text { if } \operatorname{deg}_{\xi} g=0 \\
i_{1}\left(-(\Delta g) \xi_{1} \xi_{2} \xi_{3}\right) & \text { if } \operatorname{deg}_{\xi} g=1 \\
i_{1}(g) & \text { if } \operatorname{deg}_{\xi} g=2 \\
i_{1}\left(-\Delta^{-1}\left(D_{\xi}^{3} g\right)\right)+i_{2}\left(\Delta^{-1}\left(D_{\xi}^{3} g\right)\right) & \text { if } \operatorname{deg}_{\xi} g=3 .
\end{array}\right.
$$

The right inverse $\Delta^{-1}$ of $\Delta$ is given in section 0.4.
The proof of Lemma 4.2 is a direct calculation.
4.3. A wonderful property of $\mathfrak{s l e}{ }^{\circ}(3)$. In the standard grading of $\mathfrak{g}=$ $\mathfrak{s l e}{ }^{\circ}(3)$ we have: $\operatorname{dim} \mathfrak{g}_{-1}=(3 \mid 3), \mathfrak{g}_{0} \cong \mathfrak{s p e}(3)$. For the regraded superalgebra $R \mathfrak{g}=\mathfrak{s l e}{ }^{\circ}(3 ; 3) \subset \mathfrak{l e}(3 ; 3)$ we have: $\operatorname{dim} R \mathfrak{g}_{-1}=(3 \mid 3), R \mathfrak{g}_{0}=\mathfrak{s v e c t}(0 \mid 3) \cong$ $\mathfrak{s p e}(3)$. For the definition of $\mathfrak{s p e}(3)$ we refer to [3] or [10]. Therefore, for $\mathfrak{s l e}{ }^{\circ}(3)$ and only for it among the $\mathfrak{s l e}{ }^{\circ}(n)$, the regrading $R$ determines a nontrivial automorphism. In terms of generating functions the regrading is determined by the formulas:

1) $\operatorname{deg}_{\xi}(f)=0: R(f)=\Delta\left(f \xi_{1} \xi_{2} \xi_{3}\right)$;
2) $\operatorname{deg}_{\xi}(f)=1: R(f)=f$;
3) $\operatorname{deg}_{\xi}(f)=2: R(f)=D_{\xi}^{3}\left(\Delta^{-1} f\right)$.

Note that $R^{2}(f)=(-1)^{p(f)+1} f$. Now we can formulate the following proposition.
4.4. Proposition. The nondirect sum of the images of $i_{1}$ and $i_{2}$ covers the whole $\mathfrak{c v e c t}(0 \mid 3)_{*}$, i.e.,

$$
i_{1}(\mathfrak{l e}(3 ; 3))+i_{2}(\mathfrak{l e}(3))=(\mathfrak{c v e c t}(0 \mid 3))_{*} .
$$

We also have

$$
i_{1}(\mathfrak{l e}(3 ; 3)) \cap i_{2}(\mathfrak{l e}(3)) \cong \mathfrak{s l e}{ }^{\circ}(3 ; 3) \cong \mathfrak{s l e}^{\circ}(3) .
$$

Proof. The first part follows from Lemma 4.2. The second part follows by direct calculation from solving $i_{2}\left(\mathrm{Le}_{f}\right)=i_{1}\left(\mathrm{Le}_{g}\right)$. Note that $\mathrm{Le}_{f} \in \mathfrak{s l e}{ }^{\circ}(3)$ iff $\Delta(f)=0$ and $D_{\xi}^{3} f=0$, and similar for $\operatorname{Le}_{g} \in \mathfrak{s l e}^{\circ}(3 ; 3)$. The equation $i_{2}\left(\mathrm{Le}_{f}\right)=i_{1}\left(\mathrm{Le}_{g}\right)$ is only solvable if $f \in \mathfrak{s l e}^{\circ}(3)$ and $g \in \mathfrak{s l e}^{\circ}(3 ; 3)$, and in this case we obtain $g=(-1)^{p(f)+1} R f$.
Therefore, we can identify the space of the Lie superalgebra $\mathfrak{c v e c t}(0 \mid 3)_{*}$ with the quotient space of $\mathfrak{l e}(3 ; 3) \oplus \mathfrak{l e}(3)$ modulo

$$
\left\{(-1)^{p(g)+1} R g \oplus(-g), g \in \mathfrak{s l e}^{\circ}(3)\right\} .
$$

In other words, we can represent the elements of $\mathfrak{c v e c t}(0 \mid 3)_{*}$ in the form of the pairs of functions

$$
\begin{equation*}
(f, g), \quad \text { where } \quad f, g \in \Pi \mathbb{C}[u, \xi] / \mathbb{C} \cdot 1 \tag{4.7}
\end{equation*}
$$

subject to identifications

$$
(-1)^{p(g)+1}(R g, 0) \sim(0, g) \quad \text { for any } \quad g \in \mathfrak{s l e}^{\circ}(3) .
$$

### 4.5. Corollary. The map $\varphi$ defined by the formula

$$
\left.\varphi\right|_{i_{1}(\operatorname{le}(3 ; 3))}=\operatorname{sign} i_{2} i_{1}^{-1} ;\left.\quad \varphi\right|_{i_{2}(\operatorname{le}(3))}=i_{1} i_{2}^{-1}
$$

is an automorphism of $\mathfrak{c v e c t}(0 \mid 3)_{*}$. Here $\operatorname{sign}(D)=(-1)^{p(D)} D$.
The map $\varphi$ may be represented in inner coordinates of $\mathfrak{v e c t}(4 \mid 3)$ as a regrading by setting $\operatorname{deg} y=-1 ; \operatorname{deg} u_{i}=1 ; \operatorname{deg} \xi_{i}=0$.

In the representation (4.7) we have

$$
\varphi(f, g)=\left(g,(-1)^{p(f)+1} f\right)
$$

Now we can complete the proof of Lemma 1.3.
4.6. Corollary. The embedding $i_{1}: \mathfrak{l e}(3) \rightarrow \mathfrak{c v e c t}(0 \mid 3)_{*}$ is a surjection onto $\mathfrak{v e c t}(0 \mid 3)_{*}$.

Proof. By Proposition 4.4 we merely have to prove that $i_{2}\left(\operatorname{Le}_{f}\right) \in \mathfrak{v e c t}(0 \mid 3)_{*}$ iff $\Delta f=0$ and $D_{\xi}^{3} f=0$. Applying equation (2.7) to $i_{2}\left(\operatorname{Le}_{f}\right)$, this follows immediately.

## §5 The bracket in $\mathfrak{c v e c t}(0 \mid 3)_{*}$

Now we can determine the bracket in $\mathfrak{c v e c t}(0 \mid 3)_{*}$ in terms of representation $(f, g)$ as stated in formula (4.7).

We do this via $\alpha_{g}$. By Theorem 3.1 any $D \in \mathfrak{c v e c t}(0 \mid 3)_{*}$ is of the form $D=i_{2}(f)+\alpha_{g}$ for some generating functions $f$ and $g$. To determine the bracket $\left[i_{2}(f), i_{1}(h)\right]$, we

1. Compute the brackets $\left[i_{2} f, \alpha_{g}\right]$ for any $f, g \in \mathbb{C}[u, \xi] / \mathbb{C} \cdot 1$;
2. Represent $i_{1}(h)$ in the form

$$
\begin{equation*}
i_{1}(h)=i_{2} a(h)+\alpha_{b(h)} \text { for any } h \in \mathbb{C}[u, \xi] / \mathbb{C} \cdot 1 ; \tag{5.1}
\end{equation*}
$$

In Lemma 4.2 we expressed $\alpha_{g}$ in $i_{1}$ and $i_{2}$.
Remark. The functions $a(h)$ and $b(h)$ above are not uniquely defined. Any representation will do.
5.1. Lemma. For any functions $f, g \in \mathbb{C}[u, \xi] / \mathbb{C} \cdot 1$ the bracket $\left[i_{2} f, \alpha_{g}\right]$ is of the form

$$
\begin{equation*}
\left[i_{2} f, \alpha_{g}\right]=i_{2} F+\alpha_{G} \tag{5.2}
\end{equation*}
$$

where

$$
F=f \cdot D_{\xi}^{3} g-(-1)^{(p(f)+1)(p(g)+1)} A_{g} f \quad \text { and } \quad G=-f \Delta g
$$

Proof. Direct calculation gives that

$$
\begin{aligned}
{\left[i_{2} f, \alpha_{g}\right] } & =\left[\operatorname{Le}_{f}, A_{g}\right]+(-1)^{p(f) p(g)+p(f)+1} \Delta g \cdot A_{f} \\
\quad+ & y\left(\left[A_{f}, A_{g}\right]+(-1)^{p(f) p(g)+p(f)+1} \cdot 2 \cdot D_{\xi}^{3} g \cdot A_{f}\right) \\
+ & (-1)^{p(g)+1}\left(\{f, \Delta g\}+(-1)^{p(f)} \Delta f \cdot \Delta g\right) \partial_{y} \\
+ & \left((-1)^{p(g)+1} A_{f}(\Delta g)+(-1)^{p(f) p(g)+p(g)+1} A_{g}(\Delta f)\right. \\
& \left.+2 \cdot(-1)^{p(g)+1}\left\{f, D_{\xi}^{3} g\right\}+2 \cdot(-1)^{p(f)+p(g)+1} D_{\xi}^{3} f \cdot \Delta g\right) y \partial_{y} \\
& +(-1)^{p(f)+p(g)+1} 2 \cdot D_{\xi}^{3} f \cdot D_{\xi}^{3} g \cdot y^{2} \partial_{y} .
\end{aligned}
$$

In order to find the functions $F$ and $G$, it suffices to observe that the coefficient of $\partial_{y}$, non-divisible by $y$, should be equal to $(-1)^{p(G)+1} \Delta G$. This implies the equations:

$$
(-1)^{p(G)+1} \Delta G=(-1)^{p(g)+1}\left(\{f, \Delta g\}+(-1)^{p(f)} \Delta f \cdot \Delta g\right)
$$

or

$$
(-1)^{p(G)+1} \Delta G=(-1)^{p(f)+p(g)+1} \Delta(f \cdot \Delta g) .
$$

Here $p(G)=p(f \cdot \Delta g)=p(f)+p(g)+1$. Hence, $\Delta G=\Delta(-f \Delta g)$. Since $G$ is defined up to elements from $\mathfrak{s l e}{ }^{\circ}(3)$, we can take $G=-f \Delta g$.

The function $F$ to be found is determined from the equation

$$
\begin{equation*}
i_{2} F=\left[i_{2} f, \alpha_{g}\right]-\alpha_{G} . \tag{5.3}
\end{equation*}
$$

By comparing the coefficients of $y \partial_{y}$ in the left and right hand sides of (5.3) we get

$$
\begin{aligned}
(-1)^{p(F)+1} \Delta F & =(-1)^{p(g)+1} A_{f}(\Delta g)+(-1)^{p(f) p(g)+p(g)+1} A_{g}(\Delta f) \\
& +2(-1)^{p(g)+1}\left\{f, D_{\xi}^{3} g\right\}+(-1)^{p(f)+p(g)+1} 2 \cdot D_{\xi}^{3} f \cdot \Delta g \\
& -2 \cdot(-1)^{p(f)+p(g)} D_{\xi}^{3}(-f \Delta g)
\end{aligned}
$$

Observe that

$$
\begin{aligned}
D_{\xi}^{3}(f \Delta g) & =\left(D_{\xi}^{3} f\right) \Delta g+(-1)^{p(f)} A_{f}(\Delta g)+\sum_{i=1}^{3} \frac{\partial f}{\partial \xi_{i}} \frac{\partial}{\partial u_{i}}\left(D_{\xi}^{3} g\right) \\
& =\left(D_{\xi}^{3} f\right) \cdot \Delta g+(-1)^{p(f)} A_{f}(\Delta g)+(-1)^{p(f)}\left\{f, D_{\xi}^{3} g\right\}
\end{aligned}
$$

Then

$$
(-1)^{p(F)+1}(\Delta F)=(-1)^{p(g)} A_{f}(\Delta g)+(-1)^{p(f) p(g)+p(g)+1} A_{g}(\Delta f)
$$

By comparing parities we derive that

$$
p(F)+1=p\left(A_{f}(\Delta g)\right)=p(f)+1+p(g)+1=p(f)+p(g)
$$

It follows that

$$
\Delta F=(-1)^{p(f)} A_{f}(\Delta g)+(-1)^{p(f) p(g)+p(f)+1} A_{g}(\Delta f)
$$

Let us transform the right hand side of the equality obtained. The sums over $i, j, k$ are over $(i, j, k) \in A_{3}$ :

$$
\begin{aligned}
& (-1)^{p(f)} A_{f}(\Delta g)+(-1)^{p(f) p(g)+p(f)+1} A_{g}(\Delta f) \\
= & \sum(-1)^{p(f)} \partial_{\xi_{j}} \partial_{\xi_{k}} f \cdot \partial_{\xi_{i}}\left(\sum_{s=1}^{3} \partial_{u_{s}} \partial_{\xi_{s}} g\right) \\
& +(-1)^{p(f) p(g)+p(f)+1} \sum \partial_{\xi_{j}} \partial_{\xi_{k}} g \partial_{\xi_{i}}\left(\sum_{s=1}^{3} \partial_{u_{s}} \partial_{\xi_{s}} f\right) \\
= & (-1)^{p(f) p(g)+p(f)} \cdot \sum\left(\left(\partial_{u_{j}} \partial_{\xi_{i}} \partial_{\xi_{j}} g+\partial_{u_{k}} \partial_{\xi_{i}} \partial_{\xi_{k}} g\right) \cdot \partial_{\xi_{j}} \partial_{\xi_{k}} f\right) \\
& -(-1)^{p(f) p(g)+p(f)} \cdot \sum_{=}\left(\partial_{\xi_{j}} \partial_{\xi_{k}} g\left(\partial_{u_{j}} \partial_{\xi_{i}} \partial_{\xi_{j}} f+\partial_{u_{k}} \partial_{\xi_{i}} \partial_{\xi_{k}} f\right)\right) \\
= & (-1)^{p(f) p(g)+p(f)} \sum \partial_{u_{k}}\left(\partial_{\xi_{i}} \partial_{\xi_{k}} g \cdot \partial_{\xi_{j}} \partial_{\xi_{k}} f+\partial_{\xi_{j}} \partial_{\xi_{k}} g \cdot \partial_{\xi_{k}} \partial_{\xi_{i}} f\right) \\
= & \left.(-1)^{p(f) p(g)+p(f)+p(g)} \sum_{k=1}^{3}\left(\partial_{u_{k}} \partial_{\xi_{k}}\left(A_{g} f\right)-\partial_{u_{k}} D_{\xi}^{3} g \cdot \partial_{\xi_{k}} f\right)\right) \\
= & -(-1)^{(p(f)+1)(p(g)+1)} \Delta\left(A_{g} f\right)+(-1)^{p(f) p(g)+p(f)} \Delta\left(D_{\xi}^{3} g \cdot f\right) \\
= & \Delta\left(f \cdot D_{\xi}^{3} g\right)-(-1)^{(p(f)+1)(p(g)+1)} \Delta\left(A_{g} f\right) .
\end{aligned}
$$

Then

$$
F=f \cdot D_{\xi}^{3} g-(-1)^{(p(f)+1)(p(g)+1)} A_{g} f+F_{0}, \quad \text { where } \quad \Delta F_{0}=0
$$

We have shown how to find functions $F$ and $G$. To prove Lemma 5.1 it only remains to compare the elements of the same degree in $y$ in the righthand and the left-hand side, i.e., to verify the following three equalities:

$$
\begin{aligned}
(-1)^{p(F)+1} D_{\xi}^{3} F & =2(-1)^{p(f)+p(g)+1} D_{\xi}^{3} f \cdot D_{\xi}^{3} g \\
\mathrm{Le}_{F}+A_{G} & =\left[\operatorname{Le}_{f}, A_{g}\right]+(-1)^{p(f) p(g)+p(f)+1} \Delta g \cdot A_{f} \\
A_{F} & =\left[A_{f}, A_{g}\right]+2 \cdot(-1)^{p(f) p(g)+p(f)+1} D_{\xi}^{3} g \cdot A_{f}
\end{aligned}
$$

The verification is a direct one.
5.3. Lemma. The representation of $i_{1} h$ in the form (5.1) is as follows:

$$
i_{1} h=\left\{\begin{array}{cl}
i_{2}\left(\Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right) & \text { if } \operatorname{deg}_{\xi} h=0 \\
i_{2} h+\alpha_{(\Delta h) \xi_{1} \xi_{2} \xi_{3}} & \text { if } \operatorname{deg}_{\xi} h=1, \\
\alpha_{h} & \text { if } \operatorname{deg}_{\xi} h=2, \\
\alpha_{\Delta^{-1}\left(D_{\xi}^{3} h\right)} & \text { if } \operatorname{deg}_{\xi} h=3
\end{array}\right.
$$

Proof. It suffices to compare the definition of $\alpha_{g}$ with the definitions of $i_{1}$ and $i_{2}$. If $\operatorname{deg}_{\xi} h=0$ use the equalities $\sum \frac{\partial f}{\partial u_{i}} \xi_{j} \xi_{k}=\Delta\left(f \xi_{1} \xi_{2} \xi_{3}\right)$ and $A_{\Delta\left(f \xi_{1} \xi_{2} \xi_{3}\right)}=\mathrm{Le}_{f}$. In the remaining cases the verification is not difficult.

Making use of the Lemmas 5.1, Lemma 5.2 and Lemma 4.2 we can compute the whole multiplication table of $\left[i_{2} f, i_{1} h\right]$ :

- $\operatorname{deg}_{\xi} h=0$. Then

$$
i_{1} h=i_{2}\left(\Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right) \text { and }\left[i_{2} f, i_{1} h\right]=i_{2}\left\{f, \Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right\} .
$$

We also have

$$
\left\{f, \Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right\}=\left\{\begin{array}{cc}
0 & \text { if } \operatorname{deg}_{\xi} f=3 \\
-\{\Delta f, h\} \xi_{1} \xi_{2} \xi_{3} & \text { if } \operatorname{deg}_{\xi} f=2
\end{array}\right.
$$

- $\operatorname{deg}_{\xi} h=1$. Then

$$
\begin{gathered}
{\left[i_{2} f, i_{1} h\right]=\left[i_{2} f, i_{2} h+\alpha_{(\Delta h)} \xi_{1} \xi_{2} \xi_{3}\right]=} \\
i_{2}\{f, h\}-i_{2}(f \Delta h)+i_{2}\left(\Delta h \cdot \sum \xi_{i} \partial_{\xi_{i}} f\right)+\alpha_{-f \cdot \Delta\left((\Delta h) \xi_{1} \xi_{2} \xi_{3}\right)} .
\end{gathered}
$$

- $\operatorname{deg}_{\xi} h=2$. Then

$$
\begin{aligned}
& {\left[i_{2} f, i_{1} h\right]=\left[i_{2} f, \alpha_{h}\right]=(-1)^{p(f)} i_{2}\left(A_{h} f\right)-\alpha_{(f \Delta h)}=} \\
& \left\{\begin{array}{cc}
i_{1}\left(\{f, \Delta h\} \xi_{1} \xi_{2} \xi_{3}\right) & \text { if } \operatorname{deg}_{\xi} f=0 \\
i_{1}(\Delta(f h)-f \Delta h) & \text { if } \operatorname{deg}_{\xi} f=1 \\
i_{2}\left(A_{h} f\right)-i_{2}\left(\Delta^{-1} D_{\xi}^{3}(f \Delta h)\right)+i_{1}\left(\Delta^{-1} D_{\xi}^{3}(f \Delta h)\right) & \text { if } \operatorname{deg}_{\xi} f=2 \\
-i_{2}\left(h D_{\xi}^{3} f\right) & \text { if } \operatorname{deg}_{\xi} f=3 .
\end{array}\right.
\end{aligned}
$$

- $\operatorname{deg}_{\xi} h=3$. Then

$$
\begin{gathered}
{\left[i_{2} f, i_{1} h\right]=\left[i_{2} f, \alpha_{\Delta^{-1}\left(D_{\xi}^{3} h\right)}\right]=-\alpha_{f \cdot D_{\xi}^{3} h}=} \\
\left\{\begin{array}{cl}
0 & \text { if } \operatorname{deg}_{\xi} f=0 \\
i_{1}\left(-\Delta\left(f \cdot D_{\xi}^{3} h\right) \xi_{1} \xi_{2} \xi_{3}\right)=i_{1}(-f \Delta h-\Delta f \cdot h) & \text { if } \operatorname{deg}_{\xi} f=1 \\
i_{1}\left(-f \cdot D_{\xi}^{3} g\right) & \text { if } \operatorname{deg}_{\xi} f=2 \\
i_{1}\left(\Delta^{-1}\left(D_{\xi}^{3} f \cdot D_{\xi}^{3} g\right)\right)-i_{2}\left(\Delta^{-1}\left(D_{\xi}^{3} f \cdot D_{\xi}^{3} g\right)\right) & \text { if } \operatorname{deg}_{\xi} f=3
\end{array}\right.
\end{gathered}
$$

The final result is represented in the following tables.

The brackets $\left[i_{2} f, i_{1} h\right]$

| $\operatorname{deg}_{\xi}(f)$ | $\operatorname{deg}_{\xi}(h)=0$ | $\operatorname{deg}_{\xi}(h)=1$ |
| :---: | :---: | :---: |
| 0 | $i_{2}\left(\left\{f, \Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right\}\right)$ | $-i_{1}\left(\left\{\Delta\left(f \xi_{1} \xi_{2} \xi_{3}\right), h\right\}\right)$ |
| 1 | $i_{2}\left(\left\{f, \Delta\left(h \xi_{1} \xi_{2} \xi_{3}\right)\right\}\right) i_{2}$ | $\begin{gathered} i_{1}\left(\Delta^{-1}\{f, \Delta h\}\right)+ \\ i_{2}\left(\{f, h\}-\Delta^{-1}\{f, \Delta h\}\right) \\ \hline \end{gathered}$ |
| 2 | $-i_{2}\left(\{\Delta f, h\} \xi_{1} \xi_{2} \xi_{3}\right)$ | $i_{2}(\Delta(f h)-\Delta(f) h)$ |
| 3 | 0 | $i_{2}(f \Delta \Delta(h)+\Delta(f) h)$ |
| $\operatorname{deg}_{\xi}(f)$ | $\operatorname{deg}_{\xi}(h)=2$ | $\operatorname{deg}_{\xi}(h)=3$ |
| 0 | $i_{1}\left(\{f, \Delta h\} \xi_{1} \xi_{2} \xi_{3}\right)$ | 0 |
| 1 | $-i_{1}(\Delta(f h)+f \Delta h)$ | $i_{1}(-f \Delta(h)-\Delta(f) h)$ |
| 2 | $\begin{gathered} i_{1}\left(\Delta^{-1} D_{\xi}^{3}(f \Delta h)\right)+ \\ i_{2}\left(A_{h} f-\Delta^{-1} D_{\xi}^{3}(f \Delta h)\right) \end{gathered}$ | ) ${ }^{\text {a }}$ i $\left(-f D_{\xi}^{3} h\right)$ |
| 3 | $i_{2}\left(-h D_{\xi}^{3} f\right)$ | $\begin{gathered} \hline i_{1}\left(\Delta^{-1}\left(D_{\xi}^{3} f \cdot D_{\xi}^{3} h\right)\right)- \\ i_{2}\left(\Delta^{-1}\left(D_{\xi}^{3} f \cdot D_{\xi}^{3} h\right)\right) \\ \hline \end{gathered}$ |

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[^0]:    I.Shch. expresses her thanks: to D. Leites for rising the problem and help; to RFBR grant 95-01-01187 and NFR (Sweden) for part of financial support; University of Twente and Stockholm University for hospitality; to P. Grozman whose computer experiments encouraged her to carry on with unbearable calculations.

