

# ACTIVE VS. PASSIVE DEFENSE AGAINST A STRATEGIC ATTACKER

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The article analyzes how a defender determines a balance between protecting an object (passive defense) and striking preventively against an attacker seeking to destroy the object (active defense). The attacker analogously determines a balance between attacking and protecting against the preventive strike. The defender makes its decision about striking preventively based on its estimate of the probability of being attacked. In both cases of preventive strike and no preventive strike, the defender anticipates the most harmful attacker's strategy. The influence of the ratio between the player's resources and the contest intensities on the solution of the game is analyzed.

*Keywords:* Survivability; active defense; passive defense; attack; protection; contest intensity.

## 1. Introduction

The distinction between active and passive defense is important. Some defensive measures, such as protective shields, are by their nature defensive. Other measures, and especially those equipped with manpower, can generate active defense which means exerting effort when certain conditions are met. Various kinds of sensors can also cause conditional active defense.

Earlier research has considered passive defense in the sense of defending against incoming attacks. Azaiez and Bier [2007] consider the optimal resource allocation for security in reliability systems. They determine closed-form results for moderately general systems, assuming that the cost of an attack against any given component increases linearly in the amount of defensive investment in that component.

Bier *et al.* [2005] and Bier and Abhichandani [2002] assume that the defender minimizes the success probability and expected damage of an attack. Bier *et al.* [2005] analyze the protection of series and parallel systems with components of different values. They specify optimal defenses against intentional threats to system reliability, focusing on the tradeoff between investment cost and security. The optimal defense allocation depends on the structure of the system, the cost-effectiveness of infrastructure protection investments, and the adversary's goals and constraints. Levitin [2007] considers the optimal element separation and protection in complex multi-state series-parallel system and suggests an algorithm for determining the expected damage caused by a strategic attacker. Patterson and Apostolakis [2007] introduced importance measures for ranking the system elements in complex systems exposed to terrorist actions. Michaud and Apostolakis [2006] analyzed such measures of damage caused by the terror as impact on people, impact on environment, impact on public image etc.

Bier *et al.* [2006] assume that a defender allocates defense to a collection of locations while an attacker chooses a location to attack. They show that the defender allocates resources in a centralized, rather than decentralized, manner, that the optimal allocation of resources can be non-monotonic in the value of the attacker's outside option. Furthermore, the defender prefers its defense to be public rather than secret. Also, the defender sometimes leaves a location undefended and sometimes prefers a higher vulnerability at a particular location even if a lower risk could be achieved at zero cost. Dighe *et al.* [2009] consider secrecy in defensive allocations as a strategy for achieving more cost-effective attacker deterrence. Zhuang and Bier [2007] consider defender resource allocation for countering terrorism and natural disasters.

It is sometimes suggested that attack is the best defense, but not always. This paper seeks to determine when it is optimal to stay on the defensive and await the blow, and when it is optimal to go on the offensive and strike preventively. Our focus is on the quite challenging defense optimization where the defender has a fixed resource which can be used passively or actively. The attacker has one resource which is used for attack, and one resource which is used to protect against a preventive strike by the defender. The attacker distributes its fixed resource between attacking the defender and protecting itself against a preventive strike by the defender.

Consider as a motivating example the battle situation when the attacker is determined to attack the defender's facility that is camouflaged. The attacker attacks the facility if it succeeds to locate it (with an exogenously given probability  $z$ ). The defender can passively wait for the attack hoping that the attacker either will not locate the facility or the facility protection will withstand the attack. Alternatively, the defender can try to destroy the attacker's facilities. In this case the defender reveals its location and will be certainly attacked if it fails to destroy the attack facilities.

Section 2 presents the model. Section 3 solves the model and analyzes the solutions. Section 4 concludes.

## 2. The Model

### *Nomenclature*

$r$	Total defender's resource
$R$	Total attacker's resource
$\rho$	Ratio $r/R$ between defender's and attacker's resources
$X$	Fraction of attacker's resource allocated to attack
$x$	Fraction of defender's resource allocated to preventive strike
$V(x, X)$	Vulnerability of attacker's facility
$v(0, 1)$	Vulnerability of defended object in the case of no preventive strike
$v(x, X)$	Vulnerability of defended object in the case of preventive strike
$m$	Contest intensity in strike against the attacker
$\mu$	Contest intensity in attack against the defender
$z$	Estimated probability of attack against the defender if there is no preventive strike
$z_{\min}$	Threshold value of attack probability to justify preventive strike
$P(0, 1)$	Probability of destruction of the defended object in the case of no preventive strike
$P(x, X)$	Probability of destruction of the defended object in the case of preventive strike
$W$	Probability of destruction of the defended object in the minmax game

The defender anticipates an attack from the attacker and estimates the probability of attack to be  $0 \leq z \leq 1$ . The attack can be directed against an object owned or controlled by the defender, or against the defender itself. The defender can defend its object in two ways: implementing the preventive strike against the potential attacker (active defense) and protecting its object against the impact (passive defense). In the case of the preventive strike, the defender distributes its resource  $r$  between strike effort  $xr$  and protection effort  $(1 - x)r$  (where  $x > 0$ ). If the attacker survives the preventive strike, it attacks the defender with probability 1 (revenge attack). In the case of no preventive strike, the defender allocates its entire resource  $r$  into protection ( $x = 0$ ). We exclude the case of preventive strike without the use of resources ( $x = 0$ ) from consideration because this strategy just provokes a revenge attack without having chance to destroy the attacker's facilities. This strategy is *a priori* worse than the no preventive strike strategy because it increases the attack probability from  $z$  to 1 without affecting the players' resource distribution (the players' effort distribution is the same for no preventive strike and preventive strike with  $x = 0$ ).

The probability of the object destruction is obtained as a product of object vulnerability (conditional probability of the object destruction given it is attacked)

and the probability that the object is attacked. In the case of no preventive strike the attack probability is  $z$ . In the case of the preventive strike the attack probability is equal to the probability of the attacker's survival in the preventive strike. The object vulnerability depends on the protection and attack efforts and contest intensity as it will be shown later.

The attacker distributes its resource between the attack effort  $XR$  and protection effort  $(1 - X)R$ , where  $0 < X \leq 1$ . We exclude  $X = 0$  since that guarantees zero destruction probability which the attacker maximizes. The defender has one free choice variable  $x$  and minimizes the probability of the object destruction. The attacker has one free choice variable  $X$  and maximizes the probability of the object destruction. The defender builds the system over time. The attacker takes it as given when it chooses its attack strategy. Consequently, we consider a two period zero sum game where the defender moves first choosing  $x$  and anticipating the attacker's best response. The attacker can move with choosing  $X$  or changing  $X$  after the defender chooses  $x$ , but before the preventive strike. (The attacker can or cannot observe  $x$ . In the former case it optimizes  $X$ , in the latter case it can just guess). We require that the preventive strike, if it occurs, occurs before the revenge attack.

The defender's and attacker's resources can be monetary budgets, but can also be actual resources which may be combinations of manpower, ammunition, etc. In order to divide the resources into  $x$  versus  $1 - x$  and  $X$  versus  $1 - X$  we require some degree of quantitative divisibility for the various defense and attack resources.

The vulnerability of the attacked object is determined by the common ratio form of the attacker-defender contest success function (Tullock, 1980; Skaperdas, 1996)

$$v = \frac{T^\mu}{T^\mu + t^\mu}, \quad (1)$$

where  $T$  is the attacker's effort,  $t$  is the defender's effort,  $\partial v / \partial T > 0$ ,  $\partial v / \partial t < 0$ , and  $\mu \geq 0$  is a parameter for the contest intensity. When  $\mu = 0$ ,  $t$  and  $T$  have equal no impact on  $v$  regardless of their size which gives 50% vulnerability. When  $0 < \mu < 1$ , there is a disproportional advantage of exerting less effort than one's opponent. When  $\mu = 1$ , the efforts have proportional impact on the  $v$ . When  $\mu > 1$ , exerting more effort than one's opponent gives a disproportional advantage. Finally,  $\mu = \infty$  gives a step function where "winner-takes-all". The parameter  $\mu$  can be illustrated by the history of warfare. Low intensity occurs in situations where neither the defender nor the attacker can get a significant upper hand. Examples are the time prior to cannons and modern fortifications in the fifteenth century, and entrenchment used with the machine gun in World War I (Hirshleifer 1995:32–33). High  $\mu$  occurs when one or the other opponent more easily can get the upper hand. Airplanes, tanks, and mechanized infantry in World War II allowed both the offense and defense to concentrate firepower more rapidly, which intensified the effect of force superiority.

In the case of no preventive strike,  $x = 0$ , the greatest destruction probability is achieved when the attacker chooses  $X = 1$ . Inserting  $t = r$  and  $T = R$  into (1), the probability of destruction  $P(0, 1)$  of the defended object is

$$P(0, 1) = zv(0, 1) = z \frac{R^\mu}{R^\mu + r^\mu} = \frac{z}{1 + \rho^\mu}. \quad (2)$$

where  $v(0, 1)$  is the vulnerability of the defended object in the case of no preventive strike,  $T = R$  is the attacker's attack effort,  $t = r$  is the defender's resource, and  $\mu$  is the contest intensity in the attack against the defender.

In the case of preventive strike the defender exerts effort  $T = xr$  to strike and the attacker exerts effort  $t = (1 - X)R$  to defend its facility. The vulnerability of the attacker's facility is

$$V(x, X) = \frac{(xr)^m}{(xr)^m + [(1 - X)R]^m} = \frac{1}{1 + [(1 - X)/(\rho x)]^m}, \quad 0 < x, \quad X \leq 1. \quad (3)$$

where  $m$  has the same interpretation as  $\mu$ . In the revenge attack the defender exerts the remaining effort  $t = (1 - x)r$  to protect and the attacker exerts the effort  $T = XR$  to strike the object. The vulnerability of the defended object in the revenge attack given the attacker survives the preventive strike is

$$v(x, X) = \frac{(XR)^\mu}{(XR)^\mu + [(1 - x)r]^\mu} = \frac{1}{1 + [\rho(1 - x)/X]^\mu}, \quad 0 < x, \quad X \leq 1. \quad (4)$$

In the case of preventive strike the probability of destruction of the defended object is

$$\begin{aligned} P(x, X) &= [1 - V(x, X)]v(x, X) \\ &= \frac{1}{1 + [\rho x/(1 - X)]^m} \cdot \frac{1}{1 + [\rho(1 - x)/X]^\mu}, \quad 0 < x, \quad X \leq 1. \end{aligned} \quad (5)$$

The probability of the object destruction given the optimal defender's decision about the preventive strike is  $W = \min\{P(x^*, X^*), P(0, 1)\}$ , where  $(x^*, X^*)$  is the solution of the zero sum game with attacker utility (5), which is a negative utility for the defender.

Summing up, the formal definition of the zero-sum game in extensive form is that the defender moves first choosing the strategy  $x$  from the strategy set  $0 < x \leq 1$ , and deciding whether it is worth to attack preventively, i.e.  $0 < x \leq 1$  versus  $x = 0$ , while the attacker moves second choosing the strategy  $X$  from the strategy set  $0 < X \leq 1$ . The defender minimizes the probability  $W$  of object destruction, which can be interpreted as a negative utility which the defender maximizes. The attacker maximizes  $W$  which can be interpreted as the attacker's utility. The interpretation of  $z$  is the estimation by the defender of the probability of an attack.

### 3. Model Solution and Analysis

The local minmax solution of the game can belong either to the point  $x = 0, X = 1$  or to the point  $(x^*, X^*)$  where

$$\begin{cases} \frac{\partial P}{\partial X} = 0, \\ \frac{\partial P}{\partial x} = 0. \end{cases} \quad (6)$$

Solving the problem (6) and obtaining the optimal  $x^*$  and  $X^*$  the defender can decide whether the preventive strike is beneficial by comparing  $P(x^*, X^*)$  with  $P(0, 1)$  obtained in (2). The preventive strike is justified if  $P(x^*, X^*) < P(0, 1)$ , i.e.

$$\frac{1}{1 + [\rho x^*/(1 - X^*)]^m} \frac{1}{1 + [\rho(1 - x^*)/X^*]^\mu} < \frac{z}{1 + \rho^\mu}, \quad 0 < x^*, \quad X^* \leq 1. \quad (7)$$

The attacker exerts maximum attack effort  $R$  if the defender chooses passive defense, and exerts attack effort  $X^* R$  if it survives the defender's preventive strike.

It follows from (7) that the defender should strike preventively if according to its estimates the probability of the attack against the defended object  $z$  exceeds the threshold value  $z_{\min}$ , where

$$z_{\min} = \frac{1 + \rho^\mu}{\left[1 + \left(\frac{\rho x^*}{1 - X^*}\right)^m\right] \left[1 + \left(\frac{\rho(1 - x^*)}{X^*}\right)^\mu\right]}, \quad 0 < x^*, \quad X^* \leq 1. \quad (8)$$

Differentiating (5) gives

$$\begin{aligned} \frac{\partial P}{\partial X} &= \frac{\rho^m \left(\frac{x}{1-X}\right)^m \frac{m}{(1-X)} [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu] - \rho^\mu \left(\frac{1-x}{X}\right)^\mu \frac{\mu}{X} [1 + \rho^m \left(\frac{x}{1-X}\right)^m]}{\left[1 + \rho^m \left(\frac{x}{1-X}\right)^m\right]^2 [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu]^2}, \\ \frac{\partial P}{\partial x} &= \frac{\rho^m \left(\frac{x}{1-X}\right)^m \frac{m}{x} [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu] - \rho^\mu \left(\frac{1-x}{X}\right)^\mu \frac{\mu}{1-x} [1 + \rho^m \left(\frac{x}{1-X}\right)^m]}{\left[1 + \rho^m \left(\frac{x}{1-X}\right)^m\right]^2 [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu]^2}. \end{aligned} \quad (9)$$

System (6) can be rewritten in the form

$$\begin{cases} \rho^m \left(\frac{x}{1-X}\right)^m \frac{m}{(1-X)} [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu] \\ \quad = \rho^\mu \left(\frac{1-x}{X}\right)^\mu \frac{\mu}{X} [1 + \rho^m \left(\frac{x}{1-X}\right)^m] \\ \rho^m \left(\frac{x}{1-X}\right)^m \frac{m}{x} [1 + \rho^\mu \left(\frac{1-x}{X}\right)^\mu] \\ \quad = \rho^\mu \left(\frac{1-x}{X}\right)^\mu \frac{\mu}{1-x} [1 + \rho^m \left(\frac{x}{1-X}\right)^m] \end{cases} \quad (10)$$

Dividing both sides of the first equation by the corresponding sides of the second one we obtain

$$\frac{x}{1-X} = \frac{1-x}{X} \quad (11)$$

from which follows that  $x^* = 1 - X^*$ . Hence the defender's resource ratio allocated to preventive strike plus the attacker's resource ratio allocated to attack equals one. If the defender eagerly strikes preventively, the attacker must protect itself and decreases its attack.

Substituting  $1 - X$  with  $x$  in the first equation of (10) yields

$$x^* = \frac{m\rho^m(1+\rho^\mu)}{m\rho^m(1+\rho^\mu) + \mu\rho^\mu(1+\rho^m)}, \quad X^* = \frac{\mu\rho^\mu(1+\rho^m)}{m\rho^m(1+\rho^\mu) + \mu\rho^\mu(1+\rho^m)}, \quad (12)$$

$$\frac{x^*}{X^*} = \frac{m\rho^m(1+\rho^\mu)}{\mu\rho^\mu(1+\rho^m)}$$

Inserting (12) into (5) and (7) gives

$$P(x^*, X^*) = [(1+\rho^m)(1+\rho^\mu)]^{-1}, \quad z_{\min} = (1+\rho^m)^{-1}, \quad (13)$$

It is always possible for  $P(0, 1) = z(1+\rho^\mu)^{-1}$  to be lower than  $P(x^*, X^*)$  if  $z$  is low, but the preventive strike is preferable if  $z = 1$ . Equation (13) means that the threshold value of the attack probability to justify the preventive strike does not depend on the contest intensity in the attack against the defender. Comparing (13) and (2), the attacker strikes preventively if  $z > z_{\min}$ , and otherwise chooses passive defense.

It can be seen from (12) that in the case of identical contest intensities ( $m = \mu$ ) the resource distribution parameters for both players are the same:  $x = X = 1/2$  and  $P(x^*, X^*) = (1+\rho^m)^{-2}$ . This means that in the case of identical conditions of both attacks, both players allocate their resources equally among the protection and attack efforts independently of the value of the resource ratio  $\rho$  (see the discussion in the Appendix).

In the case of equal resources  $\rho = 1$  we get  $P(x^*, X^*) = 0.25$  and  $x^* = m/(m + \mu)$ ,  $X^* = \mu/(m + \mu)$ . Hence the defender allocates more resources, and the attacker less resources, into the preventive strike contest if that contest has the highest intensity.

The object destruction probabilities corresponding to the extreme values of contest intensities are presented in Table 1.

Table 1. Object destruction probabilities corresponding to the extreme values of contest intensities.

	$m = 0, \mu = 0$	$m = \infty, \mu = 0$	$m = 0, \mu = \infty$	$m = \infty, \mu = \infty$
$\rho < 1$	$W = \min\{0.25, z/2\}$	$W = z/2$	$W = \min\{0.5, z\}$	$W = z$
$\rho > 1$	$W = \min\{0.25, z/2\}$	$W = 0$	$W = 0$	$W = 0$
$\rho = 1$	$W = \min\{0.25, z/2\}$			

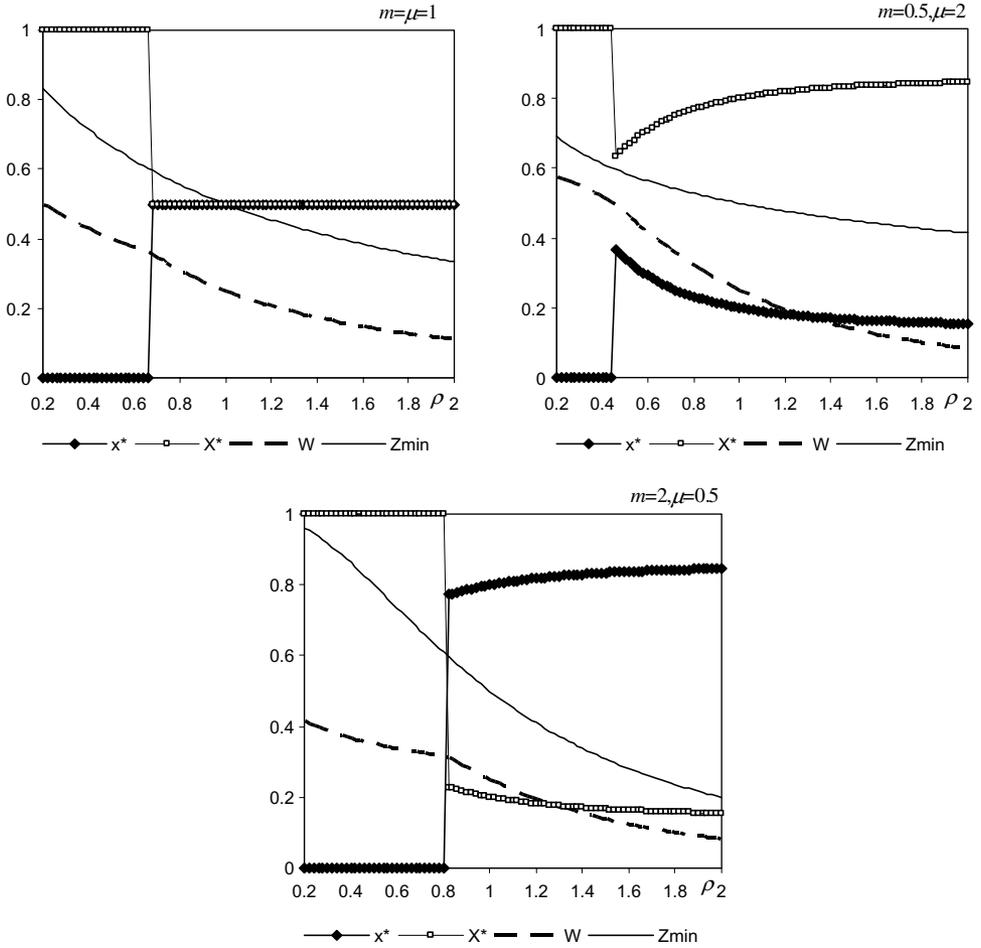


Fig. 1.  $x^*$ ,  $X^*$ ,  $W$  and  $z_{\min}$  as functions of  $\rho$  for  $z = 0.6$  and different combinations of  $m$  and  $\mu$ .

Figure 1 presents  $x^*$ ,  $X^*$ ,  $W$  and  $z_{\min}$  as functions of  $\rho$  for  $z = 0.6$  and different combinations of  $m$  and  $\mu$ .

It can be seen that  $z_{\min}$  decreases with  $\rho$ , which means that the preventive attack is not beneficial for the defender when it has much less resources than the attacker. When the preventive strike is beneficial both players allocate more resources into the preventive strike contest as  $\rho$  increases when  $m > 1$ , and allocate less resources into this contest as  $\rho$  increases when  $m < 1$ . When both contests have the same intensity both players distribute their resources evenly between the contests. It can be seen from (12) that with the growth of  $\rho$  the optimal defender's resource distribution parameter asymptotically approaches the value  $\lim_{\rho \rightarrow \infty} x^*(\rho) = (1 + \mu/m)^{-1}$ , which yields  $W = [(1 + (1 + \mu/m)^{-m})(1 + (1 + \mu/m)^{-\mu})]^{-1}$ .

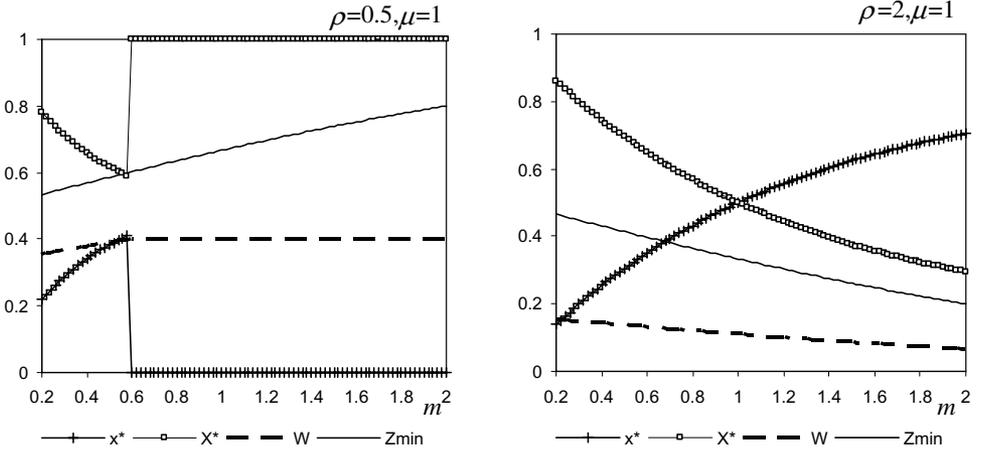


Fig. 2.  $x^*, X^*, W$  and  $z_{\min}$  as functions of  $m$  for  $z = 0.6$  and  $\mu = 1$ .

Figure 2 presents  $x^*, X^*, W$  and  $z_{\min}$  as functions of  $m$  for  $z = 0.6$  and  $\mu = 1$  for two values of  $\rho$ . When the attacker is superior ( $\rho = 0.5$ ), the defender does not strike preventively when  $m > 0.6$ , since it then succeeds poorly in the contest with the attacker which chooses  $X^* = 1$ . In this case the object destruction probability  $W = z/1.5$  does not depend on  $m$  according to (2). As  $m$  approaches infinity the object destruction probability in the case of the preventive strike approaches  $P = 1/1.5$  which is always greater than in the case of no preventive strike. When the defender is superior ( $\rho = 2$ ), the defender always strikes preventively, and increasingly so as  $m$  increases since a high  $m$  requires more resources to succeed with the preventive strike. Indeed, for  $\rho = 2, \mu = 1$  in the case of preventive strike  $P = \frac{1}{1+2} \cdot \frac{1}{1+2^m} = \frac{1}{3(1+2^m)}$  which is always lower than in the case of no preventive strike:  $\frac{z}{1+2} = \frac{0.6}{3} = \frac{1}{3 \cdot 1.6667}$ . As  $m$  approaches infinity the object destruction probability approaches 0.

Figure 3 presents  $x^*, X^*, W$  and  $z_{\min}$  as functions of  $\mu$  for  $z = 0.6$  and  $m = 1$  for two values of  $\rho$ . When the attacker is superior ( $\rho = 0.5$ ), the defender never strikes preventively since  $m = 1$  is too high to succeed against the attacker, and since  $z = 0.6$  is low and the defender prefers passive defense. As  $\mu$  approaches infinity the object destruction probability  $W$  approaches the value of  $z = 0.6$  (in the case of attack the defender cannot survive), whereas the preventive strike for  $\rho = 0.5, m = 1$  and  $\mu = \infty$  according to (13) yields  $P = 0.667$ . When the defender is superior ( $\rho = 2$ ), the defender always strikes preventively, and decreasingly so as  $\mu$  increases since a high  $\mu$  requires more resources to succeed with the passive defense. As  $\mu$  approaches infinity the object destruction probability approaches 0 in both cases with and without the preventive strike. However, for  $\rho = 0.5$  and  $m = 1$  this probability in the case of preventive strike equals  $\frac{1}{1+2} \cdot \frac{1}{1+2^\mu} \approx \frac{0.333}{1+2^\mu}$ , which is always lower than in the case of no preventive strike:  $\frac{z}{1+2^\mu} = \frac{0.6}{1+2^\mu}$ .

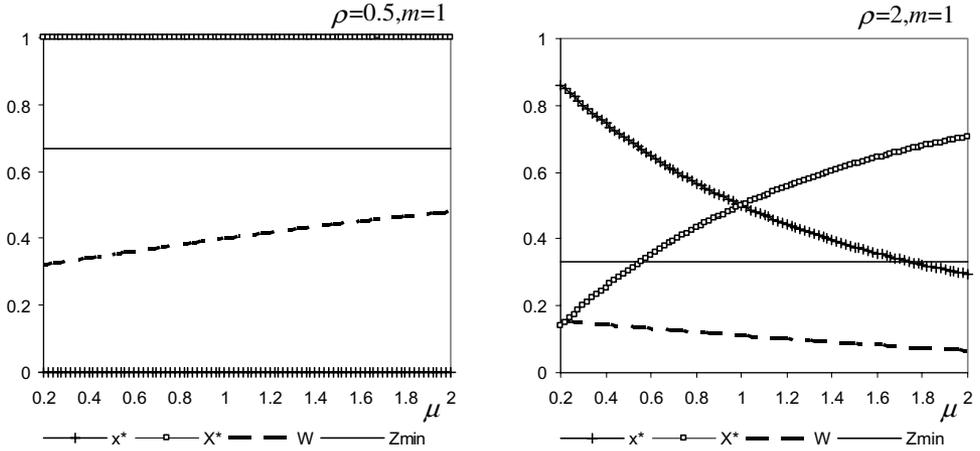


Fig. 3.  $x^*$ ,  $X^*$ ,  $W$  and  $z_{\min}$  as functions of  $\mu$  for  $z = 0.6$  and  $m = 1$ .

#### 4. Conclusion

We show how a defender allocates resources between passively protecting an object and actively striking preventively against an attacker which seeks to destroy the object. Analogously, the attacker allocates resources between attacking and protecting itself against the preventive strike. The preventive strike, if it occurs, occurs before the revenge attack.

In the case of the preventive strike, the probability of destruction of the defended object equals the attacker's survivability in the preventive strike multiplied with the object's vulnerability. The preventive strike is justified if the destruction probability is lower than when choosing passive defense which causes the attacker to allocate its entire resource to the attack. With passive defense the defender estimates the probability of being attacked. Passive defense is preferable if the defender estimates a low probability of attack, but the preventive strike is always preferable if the defender estimates a certainty of being attacked.

We define the game between the attacker and the defender and show that in the solution of this game the fraction of the attacker's resource allocated to attack plus the fraction of the defender's resource allocated to preventive strike always equals one. If the defender chooses a large preventive strike, the attacker protects itself and decreases its attack effort. If the contest intensities in the active preventive strike contest and the passive defense contest are equal, both players allocate their resources equally among the protection and attack efforts independently of how resourceful they are. If the players are equally resourceful, the defender allocates more resources, and the attacker less resources, into the preventive strike contest if that contest has the highest intensity. Analytical expression for the optimal resource distributions and the resulting probability of system destruction are obtained. The

condition of the preventive strike efficiency is formulated. Examples are provided for illustration.

## Appendix

The following is the proof that  $x = X = 1/2$  is the minmax solution of the game when  $m = \mu$  and  $x > 0$ :

$$\begin{aligned}
 g(x) &= P(x, 1/2) = \frac{1}{1 + (2\rho x)^m} \cdot \frac{1}{1 + (2\rho(1-x))^m}, \\
 g'(x) &= -\frac{(2\rho)^m x^{m-1} - (2\rho)^m (1-x)^{m-1} + (4\rho^2)^m x(1-x)^{m-1}(1-2x)}{(1 + (2\rho x)^m + (2\rho(1-x))^m + (4\rho^2 x(1-x))^m)^2} \\
 g'(1/2) &= 0; \\
 \text{when } 0 < x < 1/2, \quad g'(x) &< 0; \\
 \text{when } 1/2 < x \leq 1, \quad g'(x) &> 0
 \end{aligned} \tag{A.1}$$

This means that  $g(X)$  has local minimum at  $X = 1/2$  and global minimum at  $x = 0$  or  $x = 1/2$

$$\begin{aligned}
 h(X) &= P(1/2, X) = \frac{1}{1 + \left[\frac{\rho}{2(1-X)}\right]^m} \cdot \frac{1}{1 + \left[\frac{\rho}{2X}\right]^m} \\
 &= \frac{1}{1 + \left[\frac{\rho}{2(1-X)}\right]^m + \left[\frac{\rho}{2X}\right]^m + \left[\frac{\rho^2}{4X(1-X)}\right]^m} \\
 h'(X) &= -\frac{m(\rho/2)^m}{(1-X)^{m+1} X^{m+1}} \cdot \frac{X^{m+1} - (1-X)^{m+1} - \left[\frac{\rho}{2}\right]^m (1-2X)}{\left(1 + \left[\frac{\rho}{2(1-X)}\right]^m + \left[\frac{\rho}{2X}\right]^m + \left[\frac{\rho^2}{4X(1-X)}\right]^m\right)^2}. \\
 h'(1/2) &= 0; \\
 \text{when } 0 < X < 1/2, \quad h'(X) &> 0; \\
 \text{when } 1/2 < X \leq 1, \quad h'(X) &< 0
 \end{aligned} \tag{A.2}$$

This means that  $h(X)$  has local maximum at  $X = 1/2$  and global maximum at  $X = 1$  or  $X = 1/2$ . Thus, one of the points  $x = X = 1/2$  or  $x = 0, X = 1$  are the minmax solutions of the game for  $m = \mu$ .

Though we cannot prove that the points  $(x^*, X^*)$  determined by (12) or (0, 1) are the minmax solutions of the game for  $m \neq \mu$ , the numerical simulations show that they always are when  $x > 0$ .

## References

- Azaiez, N. and Bier, V. M. [2007] Optimal resource allocation for security in reliability systems, *European Journal of Operational Research* **181**, 773–786.
- Bier, V. M. and Abhichandani, V. [2002] Optimal allocation of resources for defense of simple series and parallel systems from determined adversaries, *Proceedings of the Engineering Foundation Conference on Risk-Based Decision Making in Water Resources X* (Santa Barbara, CA) American Society of Civil Engineers.
- Bier, V. M., Nagaraj, A. and Abhichandani, V. [2005] Protection of simple series and parallel systems with components of different values, *Reliability Engineering and System Safety* **87**, 315–323.
- Bier, V. M., Oliveros, S. and Samuelson, L. [2006] Choosing what to protect: strategic defense allocation against an unknown attacker, *Journal of Public Economic Theory* **9**(4), 563–587.
- Dighe, N., Zhuang J. and Bier, V. M. [2009] Secrecy in defensive allocations as a strategy for achieving more cost-effective attacker deterrence, *International Journal of Performance Engineering*, Special Issue on System Survivability and Defense against External Impacts **5**(2), 31–43.
- Hirshleifer, J. [1995] Anarchy and its breakdown, *Journal of Political Economy* **103**(1), 26–52.
- Levitin, G. [2007] Optimal defense strategy against intentional attacks, *IEEE Transactions on Reliability* **56**(1), 148–156.
- Michaud, D. and Apostolakis, G. [2006] Methodology for ranking the elements of water-supply networks, *Journal of Infrastructure Systems* **12**(3), 230–242.
- Patterson, S. and Apostolakis, G. [2007] Identification of critical locations across multiple infrastructures for terrorist actions, *Reliability Engineering & System Safety* **92**(9), 1183–1203.
- Skaperdas, S. [1996] Contest success functions, *Economic Theory* **7**, 283–290.
- Tullock, G. [1980] Efficient rent-seeking, in Buchanan, J. M., Tollison, R. D. and Tullock, G., *Toward a Theory of the Rent-Seeking Society*, Texas A. & M. University Press, College Station, 97–112.
- Zhuang, J. and Bier, V. M. [2007] Balancing terrorism and natural disasters — defensive strategy with endogenous attacker effort, *Operations Research* **55**(5), 976–991.