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# Faking Patience with Tacit Collusion\*

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## Abstract

This paper analyses coordination in tacit collusion when firms' discount factor is private information. We consider an infinitely repeated duopoly where two states of the world randomly occur, with different incentives for collusion. Depending on its own discount factor, a firm chooses cooperative behaviour in both states (patient), in none of the states (impatient) or in one state (mildly patient). The presence of different states affects the strategic role of beliefs. A mildly patient firm has an incentive in “faking patience” to get the deviation profit. Interestingly, this effect prevents or delays collusion when the belief in patience is strong.

**JEL codes:** C73, D43, L13.

**Keywords:** Tacit collusion; duopoly games; unknown discount factor.

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# 1 Introduction

In recent years, the analysis of tacit collusion became an object of interest for industrial economists.<sup>1</sup> While explicit collusion requires that competitors directly communicate with each other to coordinate their actions, tacit collusion implies coordination without communication, and it is a primary concern of antitrust authorities. It usually takes the form of “concerted practices”, which implies a collusive behaviour without any formal agreement or decision. In addition, the rapid advancements in machine learning and the increasing algorithmic competition in online markets made the analysis of tacit collusion particularly relevant. Indeed, algorithms may be employed to elicit how competitors set their market strategies, by observing the competitor’s behaviour, or analysing the code of other algorithms. Hence, algorithms help to achieve coordination and in turn tacit collusion (Ezrahi and Stucke, 2015, Gal, 2019, and Schwalbe, 2019, among others).<sup>2</sup>

To model tacit collusion, one important question is how firms coordinate their actions without setting explicit agreements. Indeed, starting a collusive behaviour also represents the signal for the competitor of the willingness to coordinate, at the risk that the rival will not answer in kind. By contrast, waiting for the competitor to signal its intent to collude will delay the time where a collusive profit is reached. In this view, Harrington and Zhao (2012) examine tacit collusion in an infinitely repeated prisoners’ dilemma where a firm’s discount factor is private information. They find that, the longer cooperation takes to occur, the lower the probability of cooperation in future periods. In this context, common wisdom suggests that the belief on the competitor’s patience plays a crucial role to reach a cooperative solution. In a similar setting, Lefez (2017) models a transition phase which allows to sustain collusion and where prices gradually increase over time before reaching the highest sustainable level.

Following the issue of coordination, a relevant question is whether tacit collusion is

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<sup>1</sup>An overview of the literature can be found in Green *et al.* (2014).

<sup>2</sup>Calvano *et al.* (2019) survey the developments on the literature on algorithmic pricing in industrial organization.

sustainable in the presence of changes in the market opportunities. Such changes may be due to a demand or supply shocks, which affect all the industry.<sup>3</sup> For instance, a market characterised by a stable demand may be one where it is easy to sustain collusion, whereas demand instability might hinder it (Motta, 2004). Also demand growth is one of the main relevant factors to explain collusion (Ivaldi *et al.*, 2003).

The uncertainty on profits opportunities translates into the uncertainty on the competitor's patience: a rival may sustain collusion under certain market conditions but not under others. The implicit assumption is that firms may discount profit differently, which seems the case in the real world (Haan *et al.*, 2009). This may occur for different reasons: one is the fact that small firms could be financially constrained and face higher interest rates. The second is related to time preference of managers: they may strongly discount future profits if they are close to retirement or if they expect to change company soon. Or they may have a longer-term discount factor at the beginning of their appointment (Harrington, 1989). In addition, when collusion is tacit, a change of events complicates coordination. This paper is devoted to investigate this aspect.

We analyse tacit collusion through an infinitely repeated duopoly game where a firm's discount factor is private information, and where two states of the world randomly occur over time. The difference among states reflects a difference in terms of payoffs. In particular, in one state (the “good state”) the incentive of collusion is stronger than in the other (the “bad state”). Moreover, the payoffs in the different states of the world are such that a not too patient firm may collude in one state of the world and defect in the other. This entails the presence of potentially three classes of firms:<sup>4</sup> “patient”, adopting a cooperative behaviour in both states; “impatient” never cooperative; and the “mildly patient”, who cooperate in the good state but defect in the bad state.

When the game starts, each firm tries to learn the competitor's class through its actions.

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<sup>3</sup>Similarly, Garrod (2012) analyses the implementation of a price matching punishment when costs stochastically fluctuate in two states of the world.

<sup>4</sup>Throughout the paper, a firm's type is determined by its discount factor, while a firm's class depends on whether its discount factor lies above or below certain thresholds.

In this initial phase of the game (*learning phase*), the firm’s belief about the competitor’s class is updated at any period. Once the competitor’s class is revealed, collusion may emerge (*collusion phase*). Along the paper, we first describe the game with public information. We then introduce incomplete information by artificially assuming that the learning phase lasts one period. This exercise has expositional purposes, as it highlights the features of this game in a simpler setting. Finally, we let the length of the learning phase be determined in equilibrium.

The assumption of different states influences the strategic role of beliefs. Suppose that, in the learning phase, a patient firm has a strong belief that the competitor is patient too. Suppose also the competitor is in fact mildly patient, and behaves like (or “pretends to be”) a patient firm. If the patient firm agreed to play a cooperative strategy in the collusion phase, the mildly patient competitor would defect in the bad state. Therefore, the mildly patient competitor has an incentive in pretending to be patient. Throughout the paper, we will refer to this effect as “faking patience”.

Using the standard wording of signalling games, in the presence of faking patience, the equilibrium is pooling between a patient and a mildly patient firm in the good state, and separating in the bad state.

We show that the faking patience effect increases with the firm’s belief about the competitor’s patience. In turn, since firms rationally predict this kind of behaviour, a strong belief in patience surprisingly will not lead to a fully collusive equilibrium. In particular, in the simplifying case with one learning phase period, the equilibrium strategy will exhibit cooperation in the good state and non-cooperation in the bad state of the world. Conversely with endogenous learning phase, this effect delays the beginning of the collusion phase, and again it occurs for high beliefs on the competitor’s patience.

The starting point of our analysis is Harrington and Zhao (2012). Like the present paper, they analyse tacit collusion in an infinitely repeated prisoner’s dilemma with incomplete information on the discount factors, where the game develops in a learning phase and in a

collusion phase. Harrington and Zhao (2012) assume that every firm’s state is persistent across time (it never changes). By contrast we allow them to change state of the world in each period. This is what happens in several industries, where states cannot be assumed to be persistent across time but, instead, vary from one period to the next. In addition, this approach allows us to investigate the effects of shocks in the endurance of the collusion. Moreover, given the presence of only one state of the world, Harrington and Zhao (2012) focus on mixed strategies in the learning phase: otherwise, with pure strategies, the competitor’s type will be immediately known. Conversely, we focus on pure strategy to highlight the features of the equilibria, albeit our analysis can be easily extended to the mixed strategy case.<sup>5</sup>

**Related literature** The analysis of tacit collusion has focused on its sustainability by introducing “price matching punishments” strategies (Lu and Wright, 2010, Garrod, 2012), its emergence in experimental markets (Fonseca and Normann, 2012) or when consumers may experiment products before purchase (Piccolo and Pignataro, 2018), and the choice between cartel formation and tacit collusion (Garrod and Olczak, 2017).

Together with the literature on tacit collusion, the present paper is also related to the theory of repeated games, where incomplete information has been examined in several elements of the game (Mailath and Samuelson, 2006). Strands of the literature analysed uncertainty about discount factors (Bodoh-Creed, 2019, Kartal, 2018, Kranton, 1996), technology (Athey and Bagwell, 2008, *inter alia*) payoffs (Peski, 2014, 2008, Fudenberg and Yamamoto, 2011, 2010, Wiseman, 2005, *inter alia*) or actions (Abreu *et al.*, 1990, Fudenberg *et al.*, 1994, Ghosh and Ray, 1996, Kandori and Obara, 2006, *inter alia*). The present paper is mainly linked to those contributions that focused on uncertainty about the competitor’s discount factor. In Watson (2002) and (1999), players are in a partnership, and in each period choose the level of interaction among each other and whether to cooperate. The level of interaction

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<sup>5</sup>The analysis with mixed strategies is available upon request. Yet, this extension would require further analytical complications without adding much insight.

can be seen, for instance, as an investment in a joint project. In equilibrium, players “start small” (i.e., make a low investment) to learn about the rival’s patience.

This paper is also connected to Rotemberg and Saloner (1986), Haltiwanger and Harrington (1991) and Bagwell and Staiger (1997), who investigate the relationship between collusion and the business cycle. These papers examine collusive pricing in markets with demand shocks and wonder if and when collusion is procyclical or countercyclical. For instance, in Rotemberg and Saloner (1986), the stochastic nature of cooperation is modeled by incorporating a random variable in the inverse demand function which also leads to the existence of different states of nature playing a significant role in firms’ competition over time. Following these contributions, a change of states here may be interpreted as a demand shock. Moreover, although Rotemberg and Saloner (1986) refer to “implicit collusion”, firms’ coordination is not explicitly modelled.

Yet, our results are consistent with the findings of Rotemberg and Saloner (1986): economic boosts obstruct collusion. In addition, the paper shows the presence of situations in which tacit collusion comes up systematically during downturns, while it is abandoned in the favourable economic cycles. This is represented by the equilibrium where a semi-cooperative strategy is played. In other words, albeit Rotemberg and Saloner (1986)’s wisdom tells that economic fluctuations hinder collusion, we show that certain cooperative relationships can be flexible to economic shocks, and come back in more opportune periods. This is easier when collusion is tacit, so no explicit agreement is taken and can be broken, and yet collusion in fact occurs.

The remainder of the paper is organised as follows. Section 2 introduces the model. Section 3 develops the analysis where discount factors are unknown. Section 4 shows the cooperative results, while Section 5 concludes. All formal proofs can be found in Appendix B.

## 2 The model

Consider a two-player, two-state prisoner's dilemma

$$\begin{array}{cc}
 & \begin{array}{cc} C & D \end{array} \\
 \begin{array}{c} 1 \\ \\ \\ \end{array} & \begin{array}{cc} C & D \\ \hline C & \begin{array}{c} 1, 1 \end{array} \\ D & \begin{array}{c} (1 + g^s), -l \end{array} \end{array} \end{array} \quad (1)$$

with players  $i \in \{1, 2\}$ , states  $s \in \{\ell, h\}$  and payoff functions  $u_1^s, u_2^s$  given in matrix (1), in which  $g^s > 0$  for every  $s$  and  $l > 0$ . The action set of any player  $i$  in any state  $s$  is  $X = \{C, D\}$ , where action  $C$  stands for “cooperate”, while action  $D$  is “defect”. Our parameter normalisation is standard in the analysis of prisoners' dilemma, see Kandori (1992), Ellison (1994), and, more recently, Camera and Gioffré (2014, 2017), among others. We also assume

$$g^s - l < 1 \text{ for every } s \in \{\ell, h\},$$

which is sometimes part of the definition of Prisoners' Dilemma (Roth and Keith, 1978). The latter condition entails that, if players maximise the sum of their payoffs, they prefer the action profile  $(C, C)$  to profiles  $(D, C)$  or  $(C, D)$  in any state. This condition is not strictly necessary but aims at focusing on players who try to sustain  $(C, C)$  in each period.

As standard in the literature, a player may be interpreted as a firm competing in a symmetric Cournot or Bertrand duopoly:<sup>6</sup> to do so, condition  $g^s > l$  ensures that the prisoners' dilemma is indeed a representation of a duopoly game.<sup>7</sup> The condition implies that the

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<sup>6</sup>Harrington and Zhao (2012) provide two detailed examples that a prisoner's dilemma represents duopoly competition.

<sup>7</sup>Condition  $g^s > l$  amounts to

$$u_i^s(D_i, C_j) - u_i^s(C_i, C_j) > u_i^s(D_i, D_j) - u_i^s(C_i, D_j),$$

for every  $s$  and  $i, j \in \{1, 2\}$ . This condition tells that the gain to deviate from collusion is at least as large as the gain to deviating when the competitor also plays  $D$ .



incentive to defect is stronger against a cooperator than against a defector.<sup>8</sup> Following this interpretation, taking action  $C$  represents setting the collusive price, while action  $D$  implies setting the competitive price.

The prisoners' dilemma is infinitely repeated. Time is discrete and, in each period  $t = 1, 2, \dots$ , one of two states of nature can be realised. We assume perfect monitoring, so that the past actions and states are common knowledge. On the other hand, a firm's discount factor is private information.

Payoffs are symmetric among firms, but change according to the state of the world. Without loss of generality, we are interested in a situation where the incentive to deviate is stronger in one state, say state  $s = h$  ("high" incentive) than in the other  $s = \ell$  ( $\ell$  "low" incentive), so that  $g^h > g^\ell$ . This approach wants to depict the effect of demand evolution on collusion, which depends on the kind of demand shocks. A classic example regards the situation where a positive demand shock may lure firms to break collusion to reap an unusually high profit (Motta 2004, Rotemberg and Saloner 1986).

States evolve according to a Markov process. In the present analysis, the transition from state  $\ell$  to state  $h$  can be interpreted as a boom in demand, and *vice versa*. The game may start with any initial state  $s \in \{\ell, h\}$ . The transition from a particular state does not depend on the current period or the action implemented, nor on the current state: the probabilities that the game transits from any state  $s$  to state  $\ell$  and  $h$  are equal to  $p \in (0, 1)$  and  $1 - p$ , respectively.<sup>9</sup> Throughout the paper, we will make the following assumption:

**Assumption 1** *Let  $p > \frac{g^\ell}{g^h}$ .*

Assumption 1 helps to focus on the most interesting case, where all possible equilibrium configurations occur (see footnote in Section 2.1.2). The economic interpretation is that the incentive to collude in the  $h$  state is sufficiently high.

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<sup>8</sup>Heller and Mohlin (2017) call a prisoner's dilemma "offensive" if this condition holds.

<sup>9</sup>The analysis can be extended by considering the state transition based on the current state. However, this generalization does not add much to the results.

## 2.1 Publicly known discount factors

First, we consider the game where the firms' discount factor is public information. This analysis is convenient to later define the firms' classes according to their intrinsic degree of patience (see Section 2.1.3). Moreover, the strategy profiles for the game with complete information are also used in the second phase of the game with unknown discount factors, once that firms are aware of the level of patience of their competitors (see Section 3 for details).

### 2.1.1 Strategy profiles

In this section we consider the strategy profiles that will be examined in Section 2.1.2 and the conditions for which these are subgame perfect. We focus the analysis on pure strategies. The strategy profile is denoted by  $\sigma = (\sigma_i : i \in \{1, 2\})$ , where strategy  $\sigma_i$  determines firm  $i$ 's action for every period and every state depending on the history of the period.

We restrict our analysis considering three strategy profiles in which firms (i) play action  $D$  in any state forever (*non-cooperative strategy* profile  $\sigma_{\mathbf{n}}$ ), (ii) cooperate only in state  $\ell$ , defect in state  $h$  and transit to playing action  $D$  forever if they observe a deviation from the described behaviour in the history (*semi-cooperative strategy* profile  $\sigma_{\mathbf{sc}}$ ),<sup>10</sup> (iii) cooperate by playing action  $C$  in any state and transit to playing action  $D$  forever if they observe deviation in the history (*cooperative strategy* profile  $\sigma_{\mathbf{c}}$ ). The formal definition of these strategy profiles is given in Appendix A.

### 2.1.2 Expected payoffs

Let  $\delta_i$  denote the discount factor of a firm  $i$ 's payoff. We are interested in finding the conditions under which the pure strategy profiles described above are subgame perfect.

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<sup>10</sup>A semi-cooperative strategy type is also found in Fershtman and Pakes (2000), who model collusion in a setting with heterogeneous firms and entry, and where different states of the world may occur. They find that, in periods of low demand, there may be an incentive to adopt a noncollusive strategy. Unlike the present analysis, this strategy aims at driving competitors out of the industry.

The strategy profile is subgame perfect if a vector of restricted strategies form the Nash equilibrium in the subgame in every period and every state of the game.

In each period, the payoff function of any firm  $i \in \{1, 2\}$  in state  $s$  is  $u^s : X \times X \rightarrow \mathbb{R}$ , given by the payoff matrix (1). A firm's discounted payoff in an infinitely repeated Prisoner's dilemma when a strategy profile  $\sigma$  is implemented is

$$V(\sigma, \delta_i) = \sum_{t=1}^{\infty} \delta_i^{t-1} \Pi^{t-1} U_{i,t}^s(\sigma),$$

where  $V(\sigma, \delta_i) = (V^\ell(\sigma, \delta_i), V^h(\sigma, \delta_i))'$ ,  $U_{i,t}^s(\sigma) = (u_{i,t}^\ell(\sigma), u_{i,t}^h(\sigma))'$  is a vector, with  $()'$  representing the transpose vector,  $u_{i,t}^s(\sigma)$  is the payoff of the firm  $i$  in period  $t$  and state  $s$ , corresponding to the strategy profile  $\sigma$ . Finally,  $\Pi$  is the transition matrix

$$\Pi = \begin{pmatrix} p & 1-p \\ p & 1-p \end{pmatrix}.$$

We also define vector  $\mathbf{p} = (p, 1-p)$ . We denote the discounted payoffs of firm  $i$  in equilibria  $\sigma_{\mathbf{n}}$ ,  $\sigma_{\mathbf{sc}}$  and  $\sigma_{\mathbf{c}}$  as  $V_{\mathbf{n}}^s(\delta_i)$ ,  $V_{\mathbf{sc}}^s(\delta_i)$  and  $V_{\mathbf{c}}^s(\delta_i)$ , respectively, where subscripts  $\mathbf{n}$ ,  $\mathbf{sc}$  and  $\mathbf{c}$  stand for “non-cooperative”, “semi-cooperative” and “cooperative” equilibrium, while superscript  $s \in \{\ell, h\}$  indicates the state of the game in the *first* period. In Appendix A, we provide the formal derivation and we prove the following preliminary result.

**Lemma 1** *For every  $\delta_i \in (0, 1)$ ,  $V_{\mathbf{c}}(\delta_i) > V_{\mathbf{sc}}(\delta_i) > V_{\mathbf{n}}(\delta_i)$ .*

In words, each component of vector  $V_{\mathbf{c}}(\delta_i)$  ( $V_{\mathbf{sc}}(\delta_i)$ ) is larger than the corresponding component of vector  $V_{\mathbf{sc}}(\delta_i)$  ( $V_{\mathbf{n}}(\delta_i)$ ). We are now in a position to examine the critical value of  $\delta$  for which each strategy profile is a subgame perfect equilibrium (see Appendix A).

We also define

$$\tilde{\delta} \equiv \frac{g^\ell}{p + g^\ell}, \quad (2)$$

$$\hat{\delta} \equiv \frac{g^h}{1 + g^h}, \quad (3)$$

where  $\tilde{\delta}, \hat{\delta} \in (0, 1)$ , and  $\hat{\delta} > \tilde{\delta}$  by Assumption 1.<sup>11</sup> The next proposition summarises the conditions on the discount factors for which each particular strategy profile is a subgame perfect Nash equilibrium (SPNE).

**Proposition 1** *Let firm  $i$ 's discount factor  $\delta_i$  be public information, and let Assumption 1 hold. A cooperative strategy profile is SPNE iff  $\delta_i \geq \hat{\delta}$  for every  $i \in \{1, 2\}$ . A semi-cooperative strategy profile is SPNE iff  $\delta_i \geq \tilde{\delta}$  for every  $i \in \{1, 2\}$ . A non-cooperative strategy profile is SPNE for every  $\delta_i \in (0, 1)$ ,  $i \in \{1, 2\}$ .*

### 2.1.3 Firm's classes

Based on Proposition 1 and Assumption 1, we define the firms' classes according to their discount factors, as follows.

**Definition 1** *Define a firm's class as  $y_i \in \{I, M, P\}$ . A firm  $i$  belongs to class*

1. *I (impatient) if  $\delta_i = \delta_I \in (0, \tilde{\delta})$ ,*
2. *M (mildly patient) if  $\delta_i = \delta_M \in [\tilde{\delta}, \hat{\delta})$ , and*
3. *P (patient) if  $\delta_i = \delta_P \in [\hat{\delta}, 1)$ .*

Figure 1 shows a firm's class according to Definition 1, while Table 1 depicts the possible equilibria according to which firms' classes are competing in the duopoly. The cooperative

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<sup>11</sup>If  $\tilde{\delta} \geq \hat{\delta}$ , the interval  $(\tilde{\delta}, 1]$ , where the semi-cooperative strategy profile is SPNE, is contained into  $(\hat{\delta}, 1]$ , where also the cooperative strategy profile is SPNE. Hence, by Lemma 1, a semi-cooperative strategy profile is never played because the firms' payoffs in  $\sigma_{\mathbf{sc}}$  are lower than in  $\sigma_{\mathbf{c}}$ .

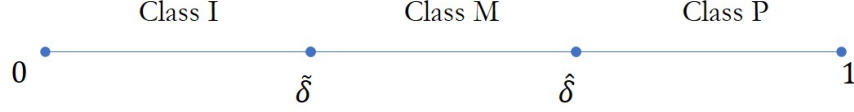


Figure 1: A firm's class according to its discount factor

strategy profile is SPNE if and only if both firms are of class  $P$ . Otherwise, a firm which is not of class  $P$  will deviate from the cooperative strategy profile because defection is profitable. A semi-cooperative strategy profile is SPNE if (i) both firms are of class  $P$ , (ii) both firms are of class  $M$  or (iii) if one is of class  $P$  and the other is of class  $M$ . If at least one of two firms is of class  $I$ , then neither a cooperative nor a semi-cooperative strategy profile is SPNE.

Table 1. Equilibria with publicly known discount factor

Firms' classes	Equilibrium strategies
both $I$ ; $I$ and $M$ ; $I$ and $P$	$\sigma_n$
both $M$ and $M$ and $P$	$\sigma_n, \sigma_{sc}$
both $P$	$\sigma_n, \sigma_{sc}, \sigma_c$

#### 2.1.4 Example

Here we propose a numerical simulation. Consider the game represented by matrices

$$\begin{array}{c}
 \begin{array}{cc} & \begin{array}{cc} & 2 \\ & C \quad D \end{array} \\ s = \ell : & \begin{array}{cc} 1 & C \quad D \\ C & \begin{array}{|c|c|} \hline 1, 1 & -0.3, 1.4 \\ \hline 1.4, -0.3 & 0, 0 \\ \hline \end{array} \\ D & \end{array} \end{array}, & \begin{array}{cc} & \begin{array}{cc} & 2 \\ & C \quad D \end{array} \\ s = h : & \begin{array}{cc} 1 & C \quad D \\ C & \begin{array}{|c|c|} \hline 1, 1 & -0.3, 2 \\ \hline 2, -0.3 & 0, 0 \\ \hline \end{array} \\ D & \end{array} \end{array}
 \end{array}$$

Let the probabilities of transition from state  $s$  to state  $\ell$  and state  $h$  be  $p = 0.7$  and  $1 - p = 0.3$ , respectively. Assumption 1 is also true: we obtain the discount factors  $\tilde{\delta} \approx 0.363$  and  $\hat{\delta} = 0.5$ . Thus, let firms  $M$  and  $P$  have discount factors  $\delta_M = 0.45$  and  $\delta_P = 0.9$  respectively. We will use this ongoing example over the paper to ease the exposition of our

results.

### 3 Unknown discount factors

We now turn the analysis on the case where a firm's discount factor is private information. Possibly, this game may exhibit several classes of equilibria. In what follows, we focus on a specific class that seems the most natural, given the scope of the analysis. As long as the competitor's patience is unknown, a firm with a certain level of patience will play both to (i) infer the competitor's class, and (ii) to signal its availability to cooperate. Once the competitor's class is known, a firm may elaborate a collusive strategy, if possible.

Following Harrington and Zhao (2012), this situation is modelled in a game that develops in two phases. The first phase is *learning*, where the firms' discount factors are private information, so that they try to recognize the competitor's class. In this phase, firms' strategies are Markovian: they are based on beliefs on the competitor's class, and not on the game history. The second phase is *collusion*, where the competitor's class is known: in this part of the game, any firm uses a strategy from the set  $\{\sigma_{\mathbf{c},i}, \sigma_{\mathbf{sc},i}, \sigma_{\mathbf{n},i}\}$  in the same way as in the case with public information on the discount factors.

#### 3.1 Learning phase

In this section, we describe how the process of learning the competitor's class takes place. Let  $T$  be the last period of the learning phase. In what follows, we focus on the case in which  $T$  is finite. In period  $t \in T$ , a firm believes the competitor to be of class  $P$  with probability  $\alpha_t$ , to be of class  $M$  with probability  $\beta_t$ , and to be of class  $I$  with probability  $\gamma_t = 1 - \alpha_t - \beta_t$ . We define the strategies for every  $t = 1, \dots, T$ : a firm chooses its strategy in period  $t$  based on symmetric beliefs  $\alpha_t$  and  $\beta_t$ . We assume that the initial beliefs about the other firm's class in period 1,  $\alpha_1 \in (0, 1)$  and  $\beta_1 \in (0, 1)$  are given and known, and that  $\alpha_1 + \beta_1 \in (0, 1)$ .

We denote a firm  $i$ 's strategy in the learning phase as  $\psi_i(y_i) \in \Psi(y_i)$  as a function of its class. The set of Markovian strategies of a firm of class  $P$  is  $\Psi(P) = \{q_t^s, t = 1, \dots, T, s = \ell, h\}$ , where  $q_t^s : [0, 1] \times [0, 1] \rightarrow \{0, 1\}$  is a function of  $\alpha_t$  and  $\beta_t$ . Given the focus on pure strategy, “0” corresponds to choosing action  $D$  while “1” corresponds to choosing action  $C$ .

Conversely, the set of Markovian strategies of a firm of class  $M$  is  $\Psi(M) = \{r_t^s, t = 1, \dots, T, s = \ell, h\}$ . Similar to class- $P$  firms, strategy  $r_t^\ell : [0, 1] \times [0, 1] \rightarrow \{0, 1\}$  is a function of  $\alpha_t$  and  $\beta_t$ , where “0” corresponds to action  $D$  while “1” corresponds to action  $C$ .

Finally, we define the set of Markovian strategies for an impatient firm  $i$ ,  $y_i = I$  as  $\Psi(I) = \{z_t^s, t = 1, \dots, T, s = \ell, h\}$ , where  $z_t^s : [0, 1] \times [0, 1] \rightarrow 0$  for every period  $t$  and every state  $s$ . Strategy  $z_t^s$  prescribes a firm of class  $I$  to choose action  $D$  in any state  $s = \ell, h$  with probability 1 in any period.

Hence a firm  $i$ 's pure strategy  $\psi(y_i)$  determines a probability of choosing action  $C$  in every period  $t$  and state  $s$  as follows, depending on the firm's class:

$$\psi(y_i) = \begin{cases} q_t^s \in \{0, 1\} & \text{if } y_i = P, \quad s = \ell, h; \\ r_t^s \in \{0, 1\} & \text{if } y_i = M, \quad s = \ell; \\ r_t^s = 0 & \text{if } y_i = M, \quad s = h; \\ z_t^s = 0 & \text{if } y_i = I, \quad s = \ell, h. \end{cases}$$

Based on the definition of the firms' strategies in the learning phase, we may state that, if a firm chooses action  $C$  in state  $h$ , it has revealed its class as  $P$ , because  $P$  is the only class who may choose action  $C$  in state  $h$  with positive probability. Conversely, if a firm chooses action  $C$  in state  $\ell$ , it may be identified as a  $P$  or  $M$  class because only firms of these two classes may choose action  $C$  in state  $\ell$  with positive probability.

We use Bayes rule to update beliefs  $\alpha_t$  and  $\beta_t$  over time,  $t = 2, \dots, T + 1$ . The rule of defining beliefs for period  $t + 1$  depends on the state  $s$  which appeared in period  $t$ . If state  $s$  is realized at period  $t$ , then in the next period  $t + 1$ , the belief that the competitor is of

class  $P$  is

$$\alpha_{t+1} = \begin{cases} \alpha_{t+1}^\ell, & \text{if } s = \ell, \\ \alpha_{t+1}^h, & \text{if } s = h. \end{cases}$$

The same rule applies to  $\beta_{t+1}$  and  $\gamma_{t+1}$ . Updating follows the standard procedure when strategies are pure (see Appendix A for details).

### 3.2 Collusion phase

The collusion phase begins when a competitor's class is public information.<sup>12</sup> To be more precise, it begins when a firm interested in collusion has sufficient information on the competitor's class. In particular, a firm of class  $P$  needs full information on the competitor's class, this because it chooses among two colluding strategy (cooperative and semi-cooperative).

On the other hand, a firm of class  $M$  is going to play a semi-cooperative strategy irrespective on whether the competitor is of class  $M$  or  $P$ : therefore its collusion phase starts when it knows that the competitor is not of class  $I$ . Firms  $I$  defect regardless of the phase of the game.

We now determine how firms choose their strategies in the collusion phase. Here, the strategy of firm  $i$  is a mapping from a firm's class and beliefs on the other firm's class by the end of the learning phase to the set  $\{\sigma_{\mathbf{c},i}, \sigma_{\mathbf{sc},i}, \sigma_{\mathbf{n},i}\}$ .

At the end of the learning phase, the beliefs that the competitor is of class  $P$  or  $M$  are  $\alpha_{T+1}$  and  $\beta_{T+1}$ , respectively. Thus the strategy of firm  $i$  is a function of class  $y_i$  and beliefs  $\alpha_{T+1}, \beta_{T+1}$  such that:

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<sup>12</sup>Naturally, the collusion phase starts as soon as one of the players reveals her type (Harrington and Zhao, 2012).



$$\sigma_i(y_i, \alpha_{T+1}, \beta_{T+1}) = \begin{cases} \sigma_{\mathbf{c},i}, & \text{if } y_i = P, \quad \alpha_{T+1} = 1, \\ \sigma_{\mathbf{sc},i}, & \text{if } y_i = P, \quad \beta_{T+1} = 1, \\ & \text{or } y_i = M, \quad \alpha_{T+1} + \beta_{T+1} = 1, \\ \sigma_{\mathbf{n},i}, & \text{if } y_i \in \{P, M\}, \quad \alpha_{T+1} + \beta_{T+1} = 0, \\ & \text{or } y_i = I. \end{cases} \quad (4)$$

The first two lines of strategy (4) are intuitive: a firm of class  $P$  plays cooperation or semi cooperation if it is sure that the competitor is of class  $P$  or  $M$ , respectively.

The third line deserves some explanation: to an  $M$ -class firm, it does not matter if the competitor is  $P$  or  $M$ : it will play semi cooperation in both cases. What matters if the competitor is of class  $I$ , as in this case the firm will choose non cooperation. Therefore, an  $M$ -class firm will start the collusion phase and play semi cooperation as long as it is sure that the competitor is not of class  $I$  ( $\gamma_{T+1} = 0$ , which implies  $\alpha_{T+1} + \beta_{T+1} = 1$ ).

In the last two lines, non cooperation is played by firms of  $P$  or  $M$  class if they are sure than the competitor is of  $I$  class, and always by firms of class  $I$ .

### 3.3 Payoff and equilibrium concept

The payoff of firm  $i = 1, 2$  of class  $y_i \in \{I, M, P\}$  is the sum of its payoffs in the two phases of the game and it is a function of her class, initial beliefs and the strategies of firm 1 and 2 in the learning phase:<sup>13</sup>

$$\begin{aligned} \Phi_i(\psi_i, \psi_j, \alpha_1, \beta_1 | y_i) &= \sum_{t=1}^T \delta_i^{t-1} \Pi^{t-1} U_i(\psi_{i,t}, \psi_{j,t}) \\ &+ \delta_i^T \Pi^T V(\delta_i, (\sigma_1(l_1, \alpha_{T+1}, \beta_{T+1}), \sigma_2(l_2, \alpha_{T+1}, \beta_{T+1}))), \end{aligned} \quad (5)$$

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<sup>13</sup>We omit players' strategies in the collusion phase as the arguments of the function because they are uniquely defined by the rule (4) given the strategies  $\psi_i, \psi_j$ .

where  $\psi_i \in \Psi(y_i)$ . The first part in the RHS of (5) is the payoff of the learning phase, while the second part is the payoff of the collusion phase. In (5),  $\sigma_i(l_i, \alpha_{T+1}, \beta_{T+1})$  and  $\sigma_j(l_j, \alpha_{T+1}, \beta_{T+1})$  are defined by (4).

The strategy set of a firm in the two-phase game consists of the strategy in the learning and in the collusion phases. The solution concept is close to Markov Perfect Bayesian Equilibrium (MPBE) with the following modification. The strategy of any firm is Markovian<sup>14</sup> only during the learning phase when firms' classes are not common knowledge, and in the collusion phase firms' strategies are from the set  $\{\sigma_{\mathbf{c},i}, \sigma_{\mathbf{sc},i}, \sigma_{\mathbf{n},i}\}$ , as described in (4). To avoid confusion, we use the name of the solution concept as Partial Markov Perfect Bayesian Equilibrium (PMPBE) given in Harrington and Zhao (2012).

**Definition 2** *A strategy profile  $\psi^* = (\psi_1^*, \psi_2^*)$  is PMPBE if, for each  $i \in \{1, 2\}$ ,  $y_i \in \{M, P\}$  and  $\psi_i \in \Psi(y_i)$ , the following inequality holds:*

$$\Phi_i(\psi^*, \alpha_1, \beta_1 | y_i) \geq \Phi_i((\psi_i, \psi_{-i}^*), \alpha_1, \beta_1 | y_i), \quad (6)$$

where  $\alpha_1 \in (0, 1)$ ,  $\beta_1 \in (0, 1)$ ,  $\alpha_1 + \beta_1 \in (0, 1)$ .

## 4 Cooperative outcomes

In this section we characterise the equilibria according to which at the second phase of the game the cooperative or semi-cooperative strategy profiles may occur (depending on the firms' classes). For completeness, the equilibria yielding a non-cooperative outcome are outlined in Appendix A.

In the first part of the section, we consider the case where the learning phase is limited to one period. This restriction is strong, but allows us to highlight some features of the

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<sup>14</sup>The Markov property is that the strategy in the learning phase in any time period  $t$  depends only on the beliefs on the competitor's class, while it does not depend on the period and the history of the game.

equilibria that may be then found, in the second part, in the more general version where the length of the learning phase is endogenously determined.

#### 4.1 One-period learning phase

In this section we limit the length of the learning phase  $T$  to one period. We present first this simplifying case for expositional purposes, as it helps highlighting the role of beliefs in this problem. Qualitatively similar results are obtained when we relax this assumption, but the analysis and the equilibrium conditions are more cumbersome. Nonetheless, the exogenous duration of the learning phase may be dictated by external conditions. For instance, extraordinary market conditions might force to anticipate the collusive behaviour, even if coordination is not fully completed.

Assuming a one-period learning phase also requires imposing some restrictions on the strategy in the collusion phase, as it might be not possible to tell the competitor's class after only one learning period. These restrictions are necessarily discretionary: to fix ideas, we assume that firms adopt a somewhat "prudent" strategy: if the competitor's class is unknown after  $T = 1$ , a  $P$ -class firm plays  $\sigma_{\text{sc}}$  if it is sure that the competitor is not of class  $I$ , and a  $P$ - or  $M$ -class firm plays  $\sigma_{\text{n}}$  if it is not completely sure that the competitor is either patient or mildly patient.

The assumptions of a prudent behaviour requires a modification in the strategy choice of the collusion phase as follows:

$$\sigma_i(y_i, \alpha_{T+1}, \beta_{T+1}) = \begin{cases} \sigma_{\text{c},i}, & \text{if } y_i = P, \quad \alpha_{T+1} = 1, \\ \sigma_{\text{sc},i}, & \text{if } y_i = P, \quad \alpha_{T+1} + \beta_{T+1} = 1, \quad \leftarrow \\ & \text{or } y_i = M, \quad \alpha_{T+1} + \beta_{T+1} = 1, \\ \sigma_{\text{n},i}, & \text{if } y_i \in \{P, M\}, \quad \alpha_{T+1} + \beta_{T+1} < 1, \quad \leftarrow \\ & \text{or } y_i = I. \end{cases} \quad (7)$$

Notice that, compared to equation (4), the only lines that differ are those indicated by arrows  $\leftarrow$ . The results would be qualitatively similar by assuming different behaviours.

We find the conditions when a PMPBE exists in the learning phase. We sort the equilibria by the type of equilibria adopted in the learning phase. For convenience, thresholds  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  are defined in the proof (see Appendix B), where  $A_1 < A_2$ .

**Proposition 2** *Suppose  $T = 1$ . Then the following equilibria exist:*

1. *If the initial state is  $s = \ell$ :*

1.i  $(q_1^\ell, r_1^\ell) = (1, 0)$  is a PMPBE for  $\alpha_1 \in [A_1; A_2]$ .

1.ii  $(q_1^\ell, r_1^\ell) = (1, 1)$  is a PMPBE for  $\alpha_1 + \beta_1 \geq A_3$ .

2. *If the initial state is  $s = h$ :*

2.i  $(q_1^h) = 1$  is a PMPBE for  $\alpha_1 \geq A_4$ .

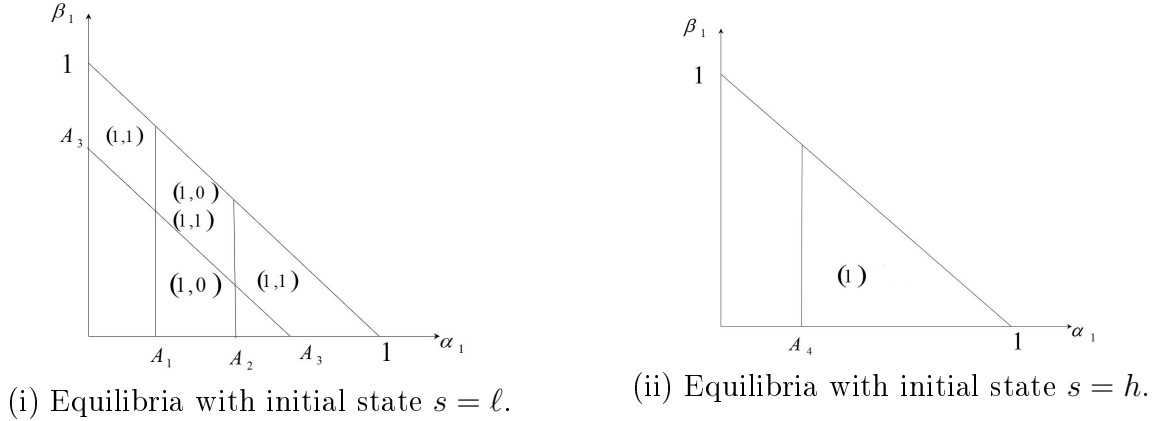


Figure 2: Equilibrium region.

**Example.** *Going back to the numerical simulation in Section 2.1.4, we have  $A_1 \approx 0.03$ ,  $A_2 \approx 0.39$ ,  $A_3 \approx 0.63$  and  $A_4 \approx 0.03$ .*

Figure 2 depicts the regions of PMPBE for initial state  $\ell$  and  $h$ , in the space of initial beliefs  $(\alpha_1, \beta_1)$ . In the blank regions, only a non-cooperative equilibrium occurs (see Appendix A).

The rule of updating beliefs are outlined in Appendix A and helps to understand the strategy profile in the collusion phase. Suppose, for instance, that the game starts with state  $\ell$  and profile  $(q_1^\ell, r_1^\ell) = (1, 0)$  is chosen. If action  $C$  is observed, the updated beliefs are  $\alpha_2 = 1$ ,  $\beta_2 = \gamma_2 = 0$ , thus it is possible to recognize the competitor's class as  $P$ . The equilibrium is separating between one  $P$ - and an  $M$ -class firm.

Hence, if two firms of class  $P$  meet, the equilibrium  $(q_1^\ell, r_1^\ell) = (1, 0)$  leads to the cooperative strategy profile  $\sigma_c$  in the collusion phase. If even one of the two firms is not of class  $P$ , equilibrium  $(q_1^\ell, r_1^\ell) = (1, 0)$  implies that the non-cooperative strategy profile  $\sigma_n$  will be implemented in the collusion phase. Indeed, since firms belonging to the mildly patient and the impatient class adopt the same strategy, a patient firm cannot recognise from the learning phase if the competitor is a mildly patient one, thus the semi-cooperative strategy is never used in the collusion phase.

When the game starts at state  $\ell$  and profile  $(q_1^\ell, r_1^\ell) = (1, 1)$  is implemented, i.e., firms of class  $M$  and  $P$  cooperate with probability 1, the beliefs of a competitor's class after observing  $C$  are:

$$\alpha_2 = \frac{\alpha_1}{\alpha_1 + \beta_1}, \quad \beta_2 = \frac{\beta_1}{\alpha_1 + \beta_1}, \quad \gamma_2 = 0.$$

In this case there are positive probabilities that the competitor is either  $P$  or  $M$ : the equilibrium is pooling.

Hence the strategy of firms  $P$  or  $M$  during the collusion phase is semi-cooperative ( $\sigma_{sc,i}$ ) according to the rule outlined in equation (7), which allows cooperation in future states  $\ell$  and deviation in future states  $h$ . This result emerges as a firm does not recognize whether the competitor is of class  $P$  or  $M$ .

When the game starts with state  $h$ , a firm of class  $M$  defect like an  $I$ -class firm, thus it cannot be identified. Hence the belief  $\beta_1$  does not play any role in determining the equilibrium. However, a class  $P$  competitor is identified with certainty. Hence if the firms are both  $P$ , they choose cooperative strategies  $\sigma_{c,i}$  in the collusion phase.

The next corollary compares the equilibrium payoffs in the parameter ranges where

multiple equilibria occur, as a refinement in the equilibrium choice.

**Corollary 1** *Suppose the game starts from state  $s = \ell$ , and  $\alpha_1, \beta_1$  satisfy the conditions:  $\alpha_1 \in [A_1, A_2]$  and  $\alpha_1 + \beta_1 \in [A_3, 1]$ . Then the payoff of an  $M$ -class firm in equilibrium  $(q_1^\ell, r_1^\ell) = (1, 1)$  is weakly greater than its payoff in equilibrium  $(q_1^\ell, r_1^\ell) = (1, 0)$ .*

Corollary 1 suggests equilibrium  $(1, 1)$  as a refinement of multiple equilibria in state  $\ell$ . This result intuitively suggests that, when the beliefs that the competitor is  $P$  or  $M$  are both low (lower than  $A_3$ ), it is unlikely to reach a result of full cooperation. Indeed, the outcome is a semi-cooperative strategy profile in the collusion phase.

The next proposition summarises some comparative statics on the equilibrium payoffs with respect to beliefs.

**Proposition 3** *The equilibrium payoffs of classes  $P$  and  $M$  firms are increasing functions of  $\alpha_1$ . The payoffs of  $P$  and  $M$  class firms in equilibrium  $(q_1^\ell, r_1^\ell) = (1, 1)$  are increasing functions of  $\beta_1$ .*

Proposition 3, together with Corollary 1, state a surprising result: a strong belief that the competitor is of class  $P$  does not lead to a cooperative strategy profile (fully collusive equilibrium) in the collusion phase. This is immediately evident by looking at Figure 2. A high  $\alpha_1$  gives a strong incentive to an  $M$ -class firm to fake patience, that is, it induces to act as a  $P$  class to lure the competitor into choosing a cooperative strategy in the second phase.

Indeed, if a cooperative strategy is played by a firm of class  $P$  and state  $h$  occurs at some period, then the  $M$ -class firm would deviate from cooperation, thus tricking her competitor. Given that firms are aware of the “faking patience” effect, a semi-cooperative equilibrium occurs: cooperation in state  $\ell$ , non-cooperation in state  $h$ .

## 4.2 Endogenous learning phase

In this section we generalise the previous results by endogenising the duration of the learning phase. Several equilibria emerge: in what follows, we aim at showing that the faking patience effect may occur for some configurations. For the sake of exposition, we focus our attention to those strategies that allow to identify the class of any firm in the shortest number of periods among the set of strategies that we consider.

Notice that this case qualitatively encompasses all those equilibria in which the learning phase lasts whatever number of periods, while the state of the world remains the same but in the last period. This is because, even though the beliefs update during the learning phase, yet it is necessary to alternate in the states of the world to fully distinguish the competitor's type, and thus begin the collusion phase.

### 4.2.1 Initial state $s = \ell$

A natural structure of the strategy profiles satisfying our requirement is the following. In the first period, firms of both class  $P$  and  $M$  use strategy  $C$  to verify whether the competitor is an  $I$ -class firm. If so, the game transits to state  $\ell$  or  $h$  in which firms of class  $P$  and  $M$  use different strategies to reveal their class in period 2, i.e., their classes will be identified with probability 1.

Accordingly, assume that, in the first period, firms of classes  $P$  and  $M$  adopt strategies  $q_1^\ell = r_1^\ell = 1$ . In period 2 and

- $s = \ell$ , strategies are  $q_2^\ell = 1, r_2^\ell = 0$ ;
- $s = h$ , firm  $P$ 's strategy is  $q_2^h = 1$ .

Using these strategies, firms' classes are revealed not later than in period 2. The following proposition summarises the conditions on the initial beliefs for which the described strategies form a PMPBE. To ease the exposition, coefficients  $A_5, A_6, A_7, A_8, A_9$  and  $A_{10}$  are defined in the proof (see Appendix B), with  $A_5, A_7, A_8 > 0$ .

**Proposition 4** *Let the initial state be  $s = \ell$ , and suppose that the following conditions hold:*

$$\left\{ \begin{array}{ll} i. & l \leq \min \{A_5\alpha_1 + A_6\beta_1, A_7\alpha_1 + A_8\beta_1\} \\ ii. & \frac{\beta_1}{\alpha_1} \in [A_9, A_{10}] \end{array} \right.$$

*Then the following strategies are PMPBE:*

$$P : (q_1^\ell, q_2^\ell, q_2^h) = (1, 1, 1), \quad M : (r_1^\ell, r_2^\ell) = (1, 0).$$

The equilibrium described in Proposition 4 shows the emergence of faking patience in the first period, where a firm of class  $M$  cooperates and, by doing so, does not reveal herself. On the other hand, in the second period the  $M$ -class firm would defect in state  $h$ , and by doing so it reveals its type and the learning phase ends afterwards, by playing the semi-cooperative equilibrium in the collusion phase. Intuitively, the faking patience effect is also what delays entering the collusion phase. Proposition 4 may help explaining the results whenever the learning phase lasts more than two periods. Suppose that a firm of type  $M$  keeps playing  $r_t^\ell = 1$  for all periods  $t$  until a change of state takes place. In this case the learning phase goes on until state  $h$  occurs.

Unlike the example where the learning phase lasts one period, the conditions of Proposition 4 are harder to interpret. We may however take a closer look at coefficients of  $\alpha_1$  in condition  $i$ . As shown in Appendix B, they are unambiguously positive, suggesting that an increase in  $\alpha_1$  increases the chance that the two conditions hold.

In words, the higher the belief that the competitor is patient, the higher the change of faking patience, the less likely the reaching of full cooperation. By contrast, the coefficients of  $\beta_1$  in  $i$ . are ambiguous, as well as those of  $\alpha_1$  and  $\beta_1$  in  $ii$ . To fix ideas, in Section 4.2.3 we verify this intuition through our numerical simulation in Section 4.2.3.



#### 4.2.2 Initial state $s = h$

We examine the strategy profile according to which a firm of class  $P$  chooses action  $C$  in the initial state,  $q_1^h = 1$ . Therefore, if at least one of two firms chooses action  $C$  in period 1, the learning phase is over, and the collusion phase starts from period 2. If both firms choose action  $D$ , then the learning phase transmits to period 2, and firms can be either of type  $M$  or  $I$ , according to the following beliefs:

$$\alpha_2 = 0, \quad \beta_2 = \frac{\beta_1}{1 - \alpha_1}, \quad \gamma_2 = \frac{1 - \alpha_1 - \beta_1}{1 - \alpha_1}.$$

Notice that these classes of firms keep playing  $D$  until state  $\ell$  is realized because of the Markovian property of the strategies.

Next, consider the case where state  $\ell$  occurs in the second period. A firm of class  $I$  keeps playing  $D$ . On the other hand, a firm of class  $M$  may choose action  $C$  (strategy  $r_2^\ell = 1$ ) or action  $D$  (strategy  $r_2^\ell = 0$ ). If it uses strategy  $r_2^\ell = 0$ , the beliefs remain the same and the strategy  $r_t^\ell$  will be equal 0 until infinity because of the Markovian property<sup>15</sup>. Thus we focus on the conditions for which strategy  $r_2^\ell = 1$  is a part of PMPBE.

**Proposition 5** *Let the game start with state  $s = h$ , and suppose*

$$\begin{cases} \beta_1 \geq \frac{l(1 - \alpha_1)}{l - g^\ell + \frac{\delta_{MP}}{1 - \delta_M}}, \\ \beta_1 \leq \frac{\alpha_1 \left( l - g^h + \frac{\delta_P}{1 - \delta_P} \right) - l}{l - g^\ell + \frac{\delta_{PP}}{1 - \delta_P}}. \end{cases}$$

*Then the following strategies are PMPBE:*

$$P : (q_1^h) = (1), \quad M : (r_t^h, r_{t+1}^\ell) = (0, 1),$$

*from  $t = 1$  onwards until  $s = h$ .*

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<sup>15</sup>Hence, this strategy of player  $M$  is a part of a PMBE with infinite learning phase.

### 4.2.3 Example

We highlight the features of the equilibria in Propositions 4 and 5 by extending the numerical simulation proposed in Section 2.1.4. If the game starts from state  $s = \ell$ , conditions of Proposition 4 amounts to

$$\begin{cases} 0.3 \leq \min \{8.9\alpha_1 + 5.3\beta_1, 0.86\alpha_1 + 0.15\beta_1\}, \\ \frac{\beta_1}{\alpha_1} \leq \min \{7.66, 5.66\}, \\ \frac{\beta_1}{\alpha_1} \geq -0.395, \end{cases}$$

then the strategy profile  $q_1^\ell = 1$ ,  $r_1^\ell = 1$  in period 1 and  $q_2^h = 1$  in period 2 is PMPBE.

The region of  $(\alpha_1, \beta_1)$  where the system is satisfied (yellow color) is depicted on Figure 3, case (i), and it is where the “faking patience effect” delays to reach cooperation or semi-cooperation. Similar to the case of one-period learning phase, the region of existence exhibits a combination of high values of  $\alpha_1$  and  $\beta_1$ . If the game starts from state  $s = h$ , the conditions

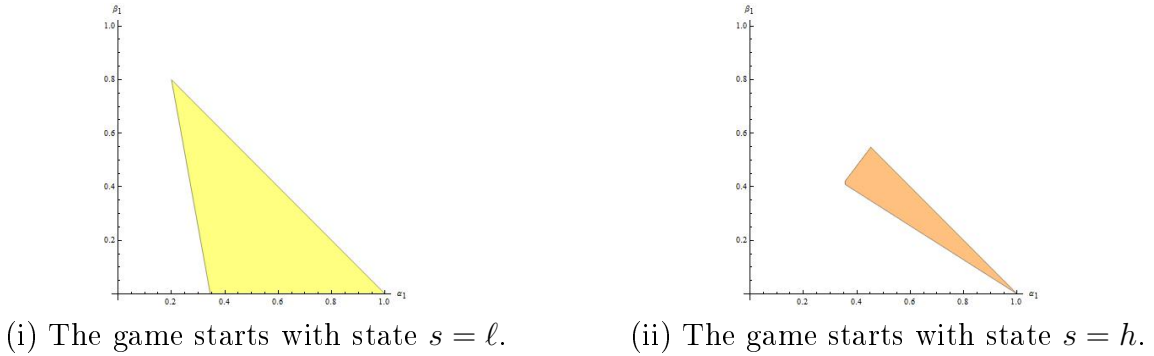


Figure 3: Equilibrium region.

of Proposition 5 are equivalent to

$$\begin{cases} \frac{1 - \alpha_1 - \beta_1}{\beta_1} \leq 0.57, \\ 8.3\alpha_1 - 6.3\beta_1 \geq 0.3. \end{cases}$$

The range of parameters for which the strategy profile  $q_1^h = 1$  and  $r_t^\ell = 1$  for every  $t > 1$

in the learning phase given in Proposition 5 is PMPBE in the game starting with  $s = h$  is depicted in Figure 3, case (ii) (orange area).

## 5 Concluding remarks

In this paper we have analysed tacit collusion in an infinitely repeated prisoners' dilemma where a firm's discount factor is private information. We have shown that the presence of different states of the world drastically affects the strategic role of beliefs. A competitor that shifts from cooperation to deviation according to the state of the world has an incentive in faking patience in the good state. Since this behaviour is expected and increases with the belief in patience, the latter loses its role in determining cooperation. In case when the length of the learning phase is endogenously determined, the faking patience effect may still emerge by hampering coordination and delaying collusion.

These results are relevant for managers who are willing to engage in colluding behaviour. From the regulator though, it is hard to draw policy conclusions, given that tacit collusion is not illegal,<sup>16</sup> and anyway it is hard to be detected and proved in court. Thus the main message of the paper is the fact that, contrary of what one would expect from the standard wisdom (Motta, 2003), the belief that the competitor is patient might in fact delays collusion.

An interesting extension might investigate the implementation of different strategy concepts. In the present analysis, we have considered grim trigger strategies. These seemed to be natural in the presence of incomplete information on the other player's discount factor (Maor and Solan, 2015). Future research may analyse equilibria using another trigger strategies such as tit-for-tat strategies (Axelrod and Hamilton, 1981), in which at every current stage the firm chooses an action that the competitor played at the previous stage. In this case though, the profile of these strategies is not subgame perfect. Alternatively, the

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<sup>16</sup>In reality, this is not so clear. There are a few cases where firms have been prosecuted for colluding implicitly. Yet, collusion was difficult to prove. Examples are the GE and Westinghouse Harvard case, and the paper pulp case in Europe.

trigger strategies with limited number of punishing periods can also be used to construct the punishment of a deviating player.

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# Appendices

## Appendix A

### Strategy profiles

The strategy profile is given by

$$\sigma = (\sigma_i : i \in \{1, 2\}). \quad (8)$$

In (8),  $\sigma_i = \{\sigma_{i,t}^s\}_{t=1}^\infty$ , where  $\sigma_{i,t+1}^s : H(t) \rightarrow X$  is an action of firm  $i$  in period  $t+1$  and state  $s \in \{\ell, h\}$ .  $H(t) = ((s(1), x(1)), \dots, (s(t), x(t)))$  is a history of period  $t$ , where  $s(t)$  is the state in period  $t$  and  $x(t)$  is the action profile played in state  $s(t)$  in period  $t$ .

**Definition 3** A non-cooperative strategy of firm  $i$  is denoted as  $\sigma_{\mathbf{n},i} = \{\sigma_{i,t}^s\}_{t=1,\dots,\infty}^{s=\ell,h}$  such  $\sigma_{i,t+1}^s(H(t)) = D$  for every  $s = \ell, h$ ,  $t = 1, \dots, \infty$  and any history  $H(t)$ .

We call the profile  $\sigma_{\mathbf{n}} = (\sigma_{\mathbf{n},i} : i \in \{1, 2\})$  as non-cooperative strategy profile.

**Definition 4** A semi-cooperative strategy of firm  $i$  is denoted as  $\sigma_{\mathbf{sc},i} = \{\sigma_{i,t}^s\}_{t=1,\dots,\infty}^{s=\ell,h}$  such that

$$\sigma_{i,t+1}^s(H(t)) = \begin{cases} C, & \text{if } s = \ell \text{ and } H(t) = H_{\mathbf{sc}}(t), \\ D, & \text{otherwise,} \end{cases}$$

while  $H_{\mathbf{sc}}(t)$  is a history of period  $t$  containing only the elements  $(\ell, (C, C))$  and  $(h, (D, D))$ .

We call the profile  $\sigma_{\mathbf{sc}} = (\sigma_{\mathbf{sc},i} : i \in \{1, 2\})$  as semi-cooperative strategy profile, according to which firms choose action  $C$  in state  $\ell$  and action  $D$  in state  $h$  if the deviation from history  $H_{\mathbf{sc}}(t)$  is not observed. Otherwise, firms switch to playing action  $D$  in any state forever.

**Definition 5** A cooperative strategy of firm  $i$  is denoted as  $\sigma_{\mathbf{c},i} = \{\sigma_{i,t}^s\}_{t=1,\dots,\infty}^{s=\ell,h}$  such that

$$\sigma_{i,t+1}^s(H(t)) = \begin{cases} C, & \text{if } H(t) = H_{\mathbf{c}}(t), \\ D, & \text{otherwise,} \end{cases}$$

and  $H_{\mathbf{c}}(t) = ((s(1), (C, C)), \dots, (s(t), (C, C)))$  is a history at period  $t$  according to which both firms choose action  $C$  in all periods before  $t + 1$ .

We call the profile  $\sigma_{\mathbf{c}} = (\sigma_{\mathbf{c},i} : i \in \{1, 2\})$  as *cooperative strategy profile*, which prescribes firms to choose action  $C$  in period  $t + 1$  if the history shows past cooperation (i.e., no deviations are observed in the previous periods). If a firm observes deviation from action profile  $(C, C)$ , then it chooses action  $D$  forever.

### Expected payoffs

In this section we derive the value of the expected payoffs. For convenience, define

$$\tilde{\Pi}(\delta_i) \equiv \frac{1}{1 - \delta_i} \begin{pmatrix} 1 - \delta_i(1 - p) & \delta_i(1 - p) \\ \delta_i p & 1 - \delta_i p \end{pmatrix}.$$

We can easily calculate the firm's payoff in any equilibria  $\sigma_{\mathbf{n}}$ ,  $\sigma_{\mathbf{c}}$  or  $\sigma_{\mathbf{sc}}$ :

1. The discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{n}}$  is

$$V_{\mathbf{n}}(\delta_i) = \begin{pmatrix} V_{\mathbf{n}}^{\ell}(\delta_i) \\ V_{\mathbf{n}}^h(\delta_i) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (9)$$

2. The discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{c}}$

$$V_{\mathbf{c}}(\delta_i) = \begin{pmatrix} V_{\mathbf{c}}^{\ell}(\delta_i) \\ V_{\mathbf{c}}^h(\delta_i) \end{pmatrix} = \tilde{P}(\delta_i) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{1 - \delta_i} \\ \frac{1}{1 - \delta_i} \end{pmatrix}. \quad (10)$$

3. The discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{sc}}$

$$V_{\mathbf{sc}}(\delta_i) = \begin{pmatrix} V_{\mathbf{sc}}^\ell(\delta_i) \\ V_{\mathbf{sc}}^h(\delta_i) \end{pmatrix} = \tilde{P}(\delta_i) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1-\delta_i(1-p)}{1-\delta_i} \\ \frac{\delta_i p}{1-\delta_i} \end{pmatrix}. \quad (11)$$

We obtain these formulas by calculating the payoff of firm  $i$  according to the profile definitions. The discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{n}}$  is

$$V_{\mathbf{n}}(\delta_i) = \begin{pmatrix} V_{\mathbf{n}}^\ell(\delta_i) \\ V_{\mathbf{n}}^h(\delta_i) \end{pmatrix} = \begin{pmatrix} \delta_i \mathbf{P} V_{\mathbf{n}}(\delta_i) \\ \delta_i \mathbf{P} V_{\mathbf{n}}(\delta_i) \end{pmatrix}$$

or in vectorial form,  $V_{\mathbf{n}}(\delta_i) = \delta \mathbf{P} V_{\mathbf{n}}(\delta_i)$ . This equation gives:

$$V_{\mathbf{n}}(\delta_i) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Second, we calculate the discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{c}}$  that is:

$$V_{\mathbf{c}}(\delta_i) = \begin{pmatrix} V_{\mathbf{c}}^\ell(\delta_i) \\ V_{\mathbf{c}}^h(\delta_i) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \delta_i \begin{pmatrix} p & 1-p \\ p & 1-p \end{pmatrix} \begin{pmatrix} V_{\mathbf{c}}^\ell(\delta_i) \\ V_{\mathbf{c}}^h(\delta_i) \end{pmatrix}.$$

Rewriting this equation in vectorial form, we obtain equation (10).

Third, we calculate the discounted payoff of firm  $i$  in equilibrium  $\sigma_{\mathbf{sc}}$  that is:

$$V_{\mathbf{sc}}(\delta_i) = \begin{pmatrix} V_{\mathbf{sc}}^\ell(\delta_i) \\ V_{\mathbf{sc}}^h(\delta_i) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \delta_i \begin{pmatrix} p & 1-p \\ p & 1-p \end{pmatrix} \begin{pmatrix} V_{\mathbf{sc}}^\ell(\delta_i) \\ V_{\mathbf{sc}}^h(\delta_i) \end{pmatrix}.$$

Rewriting this equation in a vectorial form, we obtain equation (11).

Notice that, in expressions (9), (10) and (11), matrix  $\tilde{P}(\delta_i)$  is the same.

## Bayesian updating

First, consider the updating rule for state  $s = \ell$ . If a firm chooses  $C$  in period  $t$ , it is identified as class<sup>17</sup>

$$\begin{cases} P & \text{with prob. } \alpha_{t+1}^\ell = \frac{\alpha_t q_t^\ell}{\alpha_t q_t^\ell + \beta_t r_t^\ell}; \\ M & \text{with prob. } \beta_{t+1}^\ell = 1 - \alpha_{t+1}^\ell = \frac{\beta_t r_t^\ell}{\alpha_t q_t^\ell + \beta_t r_t^\ell}. \end{cases} \quad (12)$$

Instead, a class  $I$  firm is the only one that never plays  $C$  in this state. Thus detecting collusion in  $s = \ell$  entails that the competitor does not belong to this class for certainty,  $\gamma_{t+1}^\ell = 0$ .

If a firm chooses  $D$  in state  $\ell$ , it is identified as class<sup>18</sup>

$$\begin{cases} I & \text{with prob. } \gamma_{t+1}^\ell = 1 - \alpha_{t+1}^\ell - \beta_{t+1}^\ell = \frac{1 - \alpha_t - \beta_t}{1 - \alpha_t q_t^\ell - \beta_t r_t^\ell}; \\ P & \text{with prob. } \alpha_{t+1}^\ell = \frac{\alpha_t(1 - q_t^\ell)}{1 - \alpha_t q_t^\ell - \beta_t r_t^\ell}; \\ M & \text{with prob. } \beta_{t+1}^\ell = \frac{\beta_t(1 - r_t^\ell)}{1 - \alpha_t q_t^\ell - \beta_t r_t^\ell}. \end{cases} \quad (13)$$

Next, consider the updating rule for state  $h$ . As already discussed, if a firm chooses  $C$ , it is identified as class  $P$  with probability 1: only patient firms collude in state  $h$ . On the other hand, if a firm chooses  $D$ , it is identified as class<sup>19</sup>

$$\begin{cases} I & \text{with prob. } \gamma_{t+1}^h = 1 - \alpha_{t+1}^h - \beta_{t+1}^h = \frac{1 - \alpha_t - \beta_t}{1 - \alpha_t q_t^h}; \\ P & \text{with prob. } \alpha_{t+1}^h = \frac{\alpha_t(1 - q_t^h)}{1 - \alpha_t q_t^h}; \\ M & \text{with prob. } \beta_{t+1}^h = \frac{\beta_t}{1 - \alpha_t q_t^h}. \end{cases} \quad (14)$$

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<sup>17</sup>The probabilities  $\alpha_{t+1}^\ell$  and  $\beta_{t+1}^\ell$  are defined if  $\alpha_t q_t^\ell + \beta_t r_t^\ell \neq 0$ . If  $\alpha_t q_t^\ell + \beta_t r_t^\ell = 0$ , it is impossible to observe action  $C$  in state  $s = \ell$ .

<sup>18</sup>The probabilities  $\alpha_{t+1}^\ell$ ,  $\beta_{t+1}^\ell$  and  $\gamma_{t+1}^\ell$  are defined if  $\alpha_t q_t^\ell + \beta_t r_t^\ell \neq 1$ . If  $\alpha_t q_t^\ell + \beta_t r_t^\ell = 1$ , it is impossible to observe action  $D$  in state  $s = \ell$ .

<sup>19</sup>The probabilities  $\alpha_{t+1}^h$ ,  $\beta_{t+1}^h$  and  $\gamma_{t+1}^h$  are defined if  $\alpha_t q_t^h \neq 1$ . If  $\alpha_t q_t^h = 1$  ( $\alpha_t = q_t^h = 1$ ), it is impossible to observe action  $D$  in state  $h$ .

## Non-cooperative outcomes

In this section we show the non-cooperative results. In this case, the non-cooperative strategy profile  $\sigma_{\mathbf{n}}$  is formed by rule (4) in the collusion phase of the game. We classify the equilibria according to which the non-cooperative strategy profile is played in the collusion phase regardless of the firms' classes. Like in the main text, we consider first the case where the learning phase lasts one period: the results are summarised in the following proposition.

**Proposition 6** *Suppose  $T = 1$ . Then the strategy profiles  $(q_1^\ell, r_1^\ell) = (0, 0)$  and  $(q_1^h) = (0)$  are PMPBE if the initial state is  $s = \ell$  and  $s = h$  respectively.*

**Proof.** Consider the initial state  $\ell$  and the strategy profile  $(q_1^\ell; r_1^\ell) = (0, 0)$ . A firm  $P$  obtains the following payoff if it does not deviate from  $(0, 0)$ :

$$\delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P). \quad (15)$$

If it deviates from profile  $(0, 0)$  ( $q_1^\ell = 1$ ), it gets:

$$\delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P) - l, \quad (16)$$

so that (15) is always greater than (16). A deviation of a class  $M$  cannot be profitable either.

Consider the initial state  $h$  and the strategy profile  $(q_1^h) = (0)$ . A firm  $P$  obtains the following payoff if it does not deviate from  $(0)$ :

$$\delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P). \quad (17)$$

If it deviates from profile  $(0)$  ( $q_1^h = 1$ ), it gets:

$$\delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P) - l. \quad (18)$$

Therefore, the strategy profile  $(q_1^h, r_1^h) = (0, 0)$  when the game starts from  $\ell$  and  $(q_1^h) = (0)$  when the game starts from  $h$  are PMPBE. ■

We now turn to the case where the learning phase is endogenously determined. Let the initial state be  $h$ . Consider the strategy  $q_1^h = 0$  of firm  $P$  in period 1 in state  $s = h$ . In this case, all firms use action  $D$  and, after this period, the beliefs are not updated:  $\alpha_2 = \alpha_1$ ,  $\beta_2 = \beta_1$ ,  $\gamma_2 = \gamma_1$ . If in any further periods only state  $s = h$  is realized, then the strategy of firm of type  $P$  is  $q_t^h = 0$  because of the Markovian property of the strategy.

The beliefs can be changed only if state  $s = \ell$  is realized in the game. Let state  $\ell$  be realized in period  $t > 1$ . If in this state firms use strategies  $q_1^\ell = 0$  and  $r_1^\ell = 0$ , then the beliefs do not change and again  $\alpha_t = \alpha_1$ ,  $\beta_t = \beta_1$ . Therefore, using the Markovian property, we get by induction  $q_t^\ell = r_t^\ell = 0$  for any  $t$ . These strategies determine a subgame perfect equilibrium with infinite learning phase when firms always adopt action  $D$  in any state. The existence of a similar PMPBE can be proved when the game starts from state  $\ell$  and firms use actions  $D$  in this state and then in the firstly appeared state  $h$  they also use actions  $D$ . The ongoing discussion can be summarised as follows.

**Proposition 7** *For any initial probabilities  $\alpha_1 > 0$ ,  $\beta_1 > 0$  such that  $\alpha_1 + \beta_1 < 1$ , there always exists PMPBE in which the firms' strategies for both initial states  $\ell$  and  $h$  are as follows:  $q_t^\ell = r_t^\ell = 0$  and  $q_t^h = 0$ ,  $t = 1, 2, \dots$  (firms of all classes choose action  $D$  in any state forever). In this case the learning phase lasts forever.*

## Appendix B. Proofs

### Proof of Lemma 1

This is easily derived by expected firms' payoffs  $V_{\mathbf{n}}(\delta_i)$ ,  $V_{\mathbf{c}}(\delta_i)$  and  $V_{\mathbf{sc}}(\delta_i)$  given in Appendix A.

### Proof of Proposition 1

First, we prove that cooperative strategy profile is SPNE iff  $\delta_i \geq \hat{\delta} = \frac{g^h}{1+g^h}$ . This is equivalent to finding conditions under which the deviation from any state from strategy profile  $(C, C)$  is not profitable, i.e.

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \delta_i PV_{\mathbf{c}} \geq \begin{pmatrix} 1 + g^\ell \\ 1 + g^h \end{pmatrix} + \delta_i PV_{\mathbf{n}},$$

which implies

$$\begin{pmatrix} g^\ell \\ g^h \end{pmatrix} \leq \begin{pmatrix} \frac{\delta_i}{1-\delta_i} \\ \frac{\delta_i}{1-\delta_i} \end{pmatrix}.$$

Taking into account that  $g^\ell < g^h$ , we obtain

$$\delta_i \geq \frac{g^h}{1+g^h} = \hat{\delta}.$$

Second, we prove that semi-cooperative strategy profile is SPNE iff  $\delta_i \geq \tilde{\delta} = \frac{g^\ell}{p+g^\ell}$ . This is equivalent to finding conditions under which the deviation from state  $\ell$  from strategy profile  $(C, C)$  is not profitable:

$$1 + \delta_i \mathbf{p}V_{\mathbf{sc}} \geq 1 + g^\ell + \delta_i \mathbf{p}V_{\mathbf{n}},$$

which implies

$$g^\ell \leq p \frac{\delta_i}{1-\delta_i}.$$

We obtain

$$\delta_i \geq \frac{g^\ell}{p+g^\ell} = \tilde{\delta}.$$

### Proof of Proposition 2

Before proving the proposition, it is convenient to introduce the following strategy profile, as it may emerge in the case of deviation. We will next proceed with the proof.

## Deviating strategy profile

**Definition 6** A “deviating strategy profile” is denoted as  $\sigma_{\mathbf{d}} = (\sigma_{\mathbf{d},i}, \sigma_{\mathbf{c},j})$ , where  $\sigma_{\mathbf{d},i} = \{\sigma_{i,t}^s\}_{t=1,\dots,\infty}^{s=\ell,h}$  such that

$$\sigma_{i,t+1}^s(H(t)) = \begin{cases} C, & \text{if } s = \ell \text{ and } H(t) = H_{\mathbf{c}}(t) \\ D, & \text{if } s = \ell \text{ and } H(t) \neq H_{\mathbf{c}}(t) \\ D, & \text{if } s = h \end{cases}.$$

In this profile, firm  $j$  plays strategy  $\sigma_{\mathbf{c},j}$  given by Definition 5 while firm  $i$  applies strategy  $\sigma_{\mathbf{d},i}$ . This profile may occur when firm  $j$  has a belief that the competitor  $i$  will play cooperatively while it will in fact defect in state  $s = h$ . In turn, when firm  $j$  observes a deviation from the cooperative strategy profile, it reacts with  $D$  in all stages afterwards according to strategy  $\sigma_{\mathbf{c},j}$ .

Denote by  $V_{\mathbf{d}}(\delta_i)$  an expected payoff of deviating firm  $i$  in strategy profile  $\sigma_{\mathbf{d}}$ . We compute the expected payoff  $V_{\mathbf{d},i}(\delta_i)$  of a deviating firm  $i$  which is:

$$V_{\mathbf{d}}(\delta_i) = \begin{pmatrix} V_{\mathbf{d}}^{\ell}(\delta_i) \\ V_{\mathbf{d}}^h(\delta_i) \end{pmatrix},$$

where  $V_{\mathbf{d}}^s(\delta_i)$  is the payoff of firm  $i$  in the subgame starting from state  $s$ . If the subgame starts from state  $\ell$ , firm  $i$  gets

$$V_{\mathbf{d}}^{\ell}(\delta_i) = 1 + \delta(pV_{\mathbf{d}}^{\ell}(\delta_i) + (1-p)V_{\mathbf{d}}^h(\delta_i)).$$

If the subgame starts from state  $h$ , firm  $i$  defects and gets  $(1 + g^h)$ . Then it will be punished by playing  $(D, D)$  in any state from the next stage until infinity. Its total payoff will be

$$V_{\mathbf{d}}^h(\delta_i) = (1 + g^h) + \delta \mathbf{p} V_{\mathbf{n}}(\delta_i) = 1 + g^h.$$



From these two equations we obtain

$$V_{\mathbf{d}}(\delta_i) = \left( \frac{1 + \delta_i(1-p)(1+g^h)}{1 - \delta_i p^l} \right).$$

**Initial state**  $s = \ell$ . **Strategy profile**  $(q_1^\ell, r_1^\ell) = (1, 0)$  Begin from a firm of class  $P$ . If it does not deviate from  $(1, 0)$ , it gets

$$\alpha_1(1 + \delta_P \mathbf{P}V_{\mathbf{c}}(\delta_P)) + (1 - \alpha_1)(\delta_P \mathbf{P}V_{\mathbf{n}}(\delta_P) - l). \quad (19)$$

If it deviates from profile  $(1, 0)$  ( $q_1^\ell = 0$ ), it gets:

$$\alpha_1(1 + g^\ell + \delta_P \mathbf{P}V_{\mathbf{n}}(\delta_P)) + (1 - \alpha_1)\delta_P \mathbf{P}V_{\mathbf{n}}(\delta_P). \quad (20)$$

The deviation is not profitable if (19) is larger or equal to (20), taking into account  $\delta_P \geq \hat{\delta}$ .

Now consider the firm of class  $M$ . Its payoff in profile  $(1, 0)$  is

$$\alpha_1(1 + g^\ell + \delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M)) + (1 - \alpha_1)\delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M). \quad (21)$$

If it deviates from profile  $(1, 0)$  (playing  $r_1^\ell = 1$ ), it gets:

$$\alpha_1(1 + \delta_M \mathbf{P}V_{\mathbf{d}}(\delta_M)) + (1 - \alpha_1)(\delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M) - l), \quad (22)$$

where  $V_{\mathbf{d}}(\delta_M)$  is the payoff of an  $M$ -class firm when it cooperates in  $s = \ell$  and defects in state  $s = h$  (which is profitable according to its discount factor). The deviation is not profitable if (21) is larger or equal than (22), taking into account inequality  $\tilde{\delta} \leq \delta_M \leq \hat{\delta}$  from Proposition 1. The strategy profile  $(1, 0)$  is a PMPBE when one of the following systems

has a solution:

$$\begin{cases} \alpha_1 \geq \frac{l}{l - g^\ell + \frac{\delta_P}{1-\delta_P}}, \\ l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M) \leq 0, \end{cases}$$

or

$$\begin{cases} \alpha_1 \geq \frac{l}{l - g^\ell + \frac{\delta_P}{1-\delta_P}}, \\ l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M) > 0, \\ \alpha_1 \leq \frac{l}{l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M)}. \end{cases}$$

Now we need to verify the sign of expression:

$$l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M). \quad (23)$$

First, consider expression  $\delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M)$ . We can easily conclude that

$$\mathbf{p}V_{\mathbf{d}}(\delta_M) > \mathbf{p}V_{\mathbf{sc}}(\delta_M),$$

and equivalently

$$\delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M) > \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M). \quad (24)$$

Therefore

$$\begin{aligned} l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{d}}(\delta_M) &\geq l \\ \text{if } \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M) &\geq g^\ell, \end{aligned}$$

which is always verified for  $\delta_M$ . Therefore, the expression (23) is positive.

Simplifying the systems and considering  $\delta_P \geq \hat{\delta}$ , we obtain condition

$$\alpha_1 \in [A_1, A_2], \quad (25)$$

where

$$A_1 \equiv \frac{l}{l - g^\ell + \frac{\delta_P}{1 - \delta_P}},$$

and

$$A_2 \equiv \frac{l}{l - g^\ell + \frac{\delta_M(1+g^h) - pg^h}{1 - \delta_{Mp}}},$$

and  $0 < A_1 < A_2$ .

**Initial state**  $s = \ell$ . **Strategy profile**  $(q_1^\ell, r_1^\ell) = (1, 1)$  Again, we begin from a firm of class  $P$ . If it does not deviate from strategy  $(1, 1)$ , it gets:

$$(\alpha_1 + \beta_1)(1 + \delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P)) + (1 - \alpha_1 - \beta_1)(\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P) - l). \quad (26)$$

If it deviates from  $(1, 1)$  ( $q_1 = 0$ ), it gets:

$$(\alpha_1 + \beta_1)(1 + g^\ell + \delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P)) + (1 - \alpha_1 - \beta_1)\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P). \quad (27)$$

The deviation is not profitable if (26) is larger than or equal to (27), taking into account  $\delta_P \geq \hat{\delta}$  from Proposition 1.

Consider next a firm of class  $M$ . Its payoff in profile  $(1, 1)$  is

$$(\alpha_1 + \beta_1)(1 + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)) + (1 - \alpha_1 - \beta_1)(\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M) - l). \quad (28)$$

If it deviates from profile  $(1, 1)$  ( $r_1^\ell = 0$ ) it gets:

$$(\alpha_1 + \beta_1)(1 + g^\ell + \delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M)) + (1 - \alpha_1 - \beta_1)\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M). \quad (29)$$

The deviation is not profitable if payoff (28) is larger than or equal to (29), taking into account  $\tilde{\delta} \leq \delta_M \leq \hat{\delta}$ . Thus, the strategy profile  $(1, 1)$  is a PMPBE if the following system

has a solution:

$$\begin{cases} (\alpha_1 + \beta_1) [l - g^\ell + \delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P)] \geq l, \\ (\alpha_1 + \beta_1) [l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)] \geq l. \end{cases}$$

Since  $\delta_M < \delta_P$ , the system is equivalent to the following inequality:

$$\beta_1 \geq A_3 - \alpha_1. \quad (30)$$

where

$$A_3 \equiv \frac{l}{l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)} = \frac{l}{l - g^\ell + \frac{\delta_M P}{(1 - \delta_M)}}.$$

**Initial state  $s = h$ . Strategy profile  $(q_1^h) = (1)$**  If the game starts in state  $h$ , the strategy profile  $(q_1^h) = (1)$  is a PMPBE if the following inequality holds:

$$\alpha_1 [l - g^h + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)] \geq l.$$

Since  $\delta_P \geq \hat{\delta}$ , then  $\delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P) \geq g^h$ , so that:

$$\alpha_1 \geq \frac{l}{l - g^h + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)} = \frac{l}{l - g^h + \frac{\delta_P}{1 - \delta_P}} \equiv A_4. \quad (31)$$

### Proof of Corollary 1

The payoff of an  $M$  firm in profile  $(1, 0)$  is

$$\alpha_1(1 + g^\ell + \delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M)) + (1 - \alpha_1)\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M)$$

and in profile  $(1, 1)$  is

$$(\alpha_1 + \beta_1)(1 + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)) + (1 - \alpha_1 - \beta_1)(\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M) - l).$$

The payoff of an  $M$  firm in profile  $(1, 1)$  is not less than his payoff in profile  $(1, 0)$  if

$$\alpha_1 [g^\ell - l - \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)] + \beta_1 [-\delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M) - l - 1] + l \leq 0.$$

or

$$(\alpha_1 + \beta_1)(l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)) \geq l - \beta_1 (1 + g^\ell). \quad (32)$$

Taking into account that  $\alpha_1 + \beta_1 \geq A_3$ , we may state that

$$(\alpha_1 + \beta_1) [l - g^\ell + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)] \geq l.$$

The latter inequality guarantees that (32) is satisfied because  $(1 + g^\ell) > 0$ .

### Proof of Proposition 3

Consider the payoffs of firms of class  $P$  and  $M$  as functions of parameter  $\alpha_1$ . By Proposition 2, there are three equilibria:

1. Equilibrium  $(q_1^\ell, r_1^\ell) = (1, 0)$ : the payoff of a firm of class  $P$  is

$$\alpha_1(1 + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)) + (1 - \alpha_1)(\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P) - l) = \alpha_1(1 + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)) - l(1 - \alpha_1).$$

It is a linear function of  $\alpha_1$  with coefficient  $1 + l + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)$  which is positive because  $1 > -l$  for every  $\delta \in (0, 1)$ .

The payoff of an  $M$ -class firm is

$$\alpha_1(1 + g^\ell + \delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M)) + (1 - \alpha_1)\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M) = \alpha_1(1 + g^\ell).$$

It is also a linear function of  $\alpha_1$  with coefficient  $1 + g^\ell > 0$ .

2. Equilibrium  $(q_1^\ell, r_1^\ell) = (1, 1)$ : we begin with the firm of class  $P$ . Its payoff is

$$(\alpha_1 + \beta_1)(1 + \delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P)) + (1 - \alpha_1 - \beta_1)(\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P) - l).$$

It is a linear function of  $\alpha_1$  with coefficient  $1 + l + \delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P)$  which is positive for every  $\delta \in (0, 1)$ .

Then, the payoff of a firm of class  $M$  is

$$(\alpha_1 + \beta_1)(1 + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)) + (1 - \alpha_1 - \beta_1)(\delta_M \mathbf{p}V_{\mathbf{n}}(\delta_M) - l).$$

This is a linear function of  $\alpha_1$  with coefficient  $1 + l + \delta_M \mathbf{p}V_{\mathbf{sc}}(\delta_M)$  which is positive for every  $\delta \in (0, 1)$ .

The derivatives of the payoffs of the  $P$  and  $M$  firms with respect to  $\beta_1$  equal the corresponding derivatives subject to  $\alpha_1$ . Therefore, the payoffs are also increasing functions of  $\beta_1$ .

3. Equilibrium  $(q_1) = (1)$  in initial state  $s = h$ : the payoff of the firm of class  $P$  is

$$\alpha_1(1 + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)) + (1 - \alpha_1)(\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P) - l).$$

It is a linear function of  $\alpha_1$  with coefficient  $1 + l + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)$  which is positive for every  $\delta \in (0, 1)$ .

#### **Proof of Proposition 4**

**Period 1, state  $\ell$ .** If firm  $P$  follows the described strategy  $q_1^\ell = 1$ , his payoff will be

$$\begin{aligned} & \alpha_1 [1 + \delta_P \mathbf{p}V_{\mathbf{c}}(\delta_P)] + \beta_1 [1 + \delta_P p(\delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P) - l) + \delta_P (1 - p)(\delta_P \mathbf{p}V_{\mathbf{sc}}(\delta_P) - l)] \\ & + (1 - \alpha_1 - \beta_1) [\delta_P \mathbf{p}V_{\mathbf{n}}(\delta_P) - l]. \end{aligned}$$

If he deviates to strategy  $q_1^\ell = 0$ , his class will be identified as  $I$  and his payoff will be

$$\alpha_1 [1 + g^\ell + \delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P)] + \beta_1 [1 + g^\ell + \delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P)] + (1 - \alpha_1 - \beta_1) \delta_P \mathbf{p} V_{\mathbf{n}}(\delta_P).$$

Remembering that  $\mathbf{p} = (p, 1 - p)$ , the deviation of firm  $P$  in period 1 is not profitable if

$$\alpha_1 [l - g^\ell + \delta_P \mathbf{p} V_{\mathbf{c}}(\delta_P)] + \beta_1 [l(1 - \delta_P) - g^\ell - \delta_P p + \delta_P \mathbf{p} V_{\mathbf{sc}}(\delta_P)] \geq l.$$

We call

$$\begin{aligned} A_5 &\equiv [l - g^\ell + \delta_P \mathbf{p} V_{\mathbf{c}}(\delta_P)] = l - g^\ell + \frac{\delta_P}{1 - \delta_P}, \\ A_6 &\equiv l - g^\ell - l\delta_P + \frac{\delta_P^2 p}{1 - \delta_P} \end{aligned}$$

It is easy to verify that  $A_5 > 0$ .

If firm  $M$  follows the described strategy  $r_1^\ell = 1$ , his payoff will be

$$\begin{aligned} &\alpha_1 [1 + \delta_M p(1 + g^\ell + \delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_M)) + \delta_M(1 - p)(1 + g^h + \delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_M))] \\ &+ \beta_1 [1 + \delta_M p \delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_P) + \delta_M(1 - p)(\delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_M))] \\ &+ (1 - \alpha_1 - \beta_1) [\delta_M \mathbf{p} V_{\mathbf{n}}(\delta_M) - l]. \end{aligned}$$

If he deviates to strategy  $r_1^\ell = 0$ , his class will be identified as  $I$  and his payoff will be

$$\alpha_1 [1 + g^\ell + \delta_M \mathbf{p} V_{\mathbf{n}}(\delta_M)] + \beta_1 [1 + g^\ell + \delta_M \mathbf{p} V_{\mathbf{n}}(\delta_M)] + (1 - \alpha_1 - \beta_1) \delta_M \mathbf{p} V_{\mathbf{n}}(\delta_M).$$

The deviation of firm  $M$  in period 1 is not profitable if

$$\begin{aligned} &\alpha_1 [l - g^\ell + \delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_M) + \delta_M p g^\ell + \delta_M(1 - p)(1 + g^h)] \\ &+ \beta_1 [l - g^\ell + \delta_M \mathbf{p} V_{\mathbf{sc}}(\delta_M) - \delta_M p] \geq l. \end{aligned}$$

We call

$$\begin{aligned} A_7 &\equiv l - g^\ell + \delta_M \left[ \frac{p}{1 - \delta_M} + pg^\ell + (1 - p)(1 + g^h) \right], \\ A_8 &\equiv l - g^\ell + \frac{\delta_M^2 p}{1 - \delta_M}. \end{aligned}$$

It is easy to notice that  $A_7, A_8 > 0$ .

**Period 2. State  $\ell$ .** If in period 1 the firms' classes are not revealed, i. e. only action  $C$  was observed, then the learning phase continues and the updated beliefs are

$$\alpha_2 = \frac{\alpha_1}{\alpha_1 + \beta_1}, \quad \beta_2 = \frac{\beta_1}{\alpha_1 + \beta_1}.$$

If firm  $P$  uses strategy  $q_2^\ell = 1$ , his payoff will be

$$\alpha_2 [1 + \delta_P \mathbf{P} V_{\mathbf{c}}(\delta_P)] + \beta_2 [\delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P) - l].$$

If he deviates to strategy  $q_2^\ell = 0$ , his class will be identified as  $M$  and his payoff will be

$$\alpha_2 [(1 + g^\ell) + \delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P)] + \beta_2 [0 + \delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P)].$$

The deviation of firm  $P$  in period 2, state  $\ell$ , is not profitable if

$$\alpha_2 [\delta_P \mathbf{P} (V_{\mathbf{c}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - g^\ell] - \beta_2 l \geq 0,$$

taking into account the expressions of  $\alpha_2$  and  $\beta_2$ , we obtain condition

$$\frac{\beta_1}{\alpha_1} \leq \frac{\delta_P \mathbf{P} (V_{\mathbf{c}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - g^\ell}{l}.$$



where the RHS can be simplified as

$$\frac{\delta_P \mathbf{P}(V_{\mathbf{c}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - g^\ell}{l} = \frac{1}{l} \left( \frac{\delta_P(1-p)}{1-\delta_P} - g^\ell \right). \quad (33)$$

If firm  $M$  uses strategy  $r_2^\ell = 0$ , his payoff will be

$$\alpha_2 [1 + g^\ell + \delta_M \mathbf{P} V_{\mathbf{sc}}(\delta_M)] + \beta_2 [\delta_M \mathbf{P} V_{\mathbf{sc}}(\delta_M)].$$

If he deviates to strategy  $r_2^\ell = 1$ , his class will be identified as  $P$  and his payoff will be

$$\alpha_2 [1 + \delta_M \mathbf{P} V_{\mathbf{d}}(\delta_M)] + \beta_2 [\delta_M \mathbf{P} V_{\mathbf{sc}}(\delta_M) - l].$$

The deviation of firm  $M$  in period 2, state  $\ell$ , is not profitable if

$$\alpha_2 [g^\ell + \delta_M \mathbf{P}(V_{\mathbf{sc}}(\delta_M) - V_{\mathbf{d}}(\delta_M))] + \beta_2 l \geq 0,$$

taking into account the expressions of  $\alpha_2$  and  $\beta_2$ , we obtain condition

$$\frac{\beta_1}{\alpha_1} \geq A_9 \equiv \frac{\delta_M \mathbf{P}(V_{\mathbf{d}}(\delta_M) - V_{\mathbf{sc}}(\delta_M)) - g^\ell}{l}.$$

**Period 2. State  $h$ .** If in period 2, a  $P$ -class firm uses strategy  $q_2^h = 1$ , his payoff will be

$$\alpha_2 [1 + \delta_P \mathbf{P} V_{\mathbf{c}}(\delta_P)] + \beta_2 [\delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P) - l].$$

If he deviates to strategy  $q_2^h = 0$ , his class will be identified as  $M$  and his payoff will be

$$\alpha_2 [1 + g^h + \delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P)] + \beta_2 \delta_P \mathbf{P} V_{\mathbf{sc}}(\delta_P).$$

The deviation of firm  $P$  in period 2, state  $h$ , is not profitable if

$$\alpha_2 [\delta_P \mathbf{P}(V_{\mathbf{c}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - g^h] - \beta_2 l \geq 0.$$

Taking into account the expressions of  $\alpha_2$  and  $\beta_2$ , we obtain condition

$$\frac{\beta_1}{\alpha_1} \leq \frac{\delta_P \mathbf{P}(V_{\mathbf{c}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - g^h}{l},$$

where the RHS can be simplified as

$$A_{10} \equiv \frac{1}{l} \left( \frac{\delta_P(1-p)}{1-\delta_P} - g^h \right) \quad (34)$$

Comparing equations (33) and (34) we get

$$\frac{1}{l} \left( \frac{\delta_P(1-p)}{1-\delta_P} - g^\ell \right) > \frac{1}{l} \left( \frac{\delta_P(1-p)}{1-\delta_P} - g^h \right),$$

So that  $\frac{\beta_1}{\alpha_1} \leq A_{10}$  is a sufficient condition.

Combining all conditions in the system we prove the proposition.

### **Proof of Proposition 5**

The deviation to strategy  $r_2^\ell = 0$  is not profitable when

$$\beta_2(1 + \delta_M \mathbf{P}V_{\mathbf{sc}}(\delta_M)) + \gamma_2(\delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M) - l) \geq \beta_2(1 + g^\ell + \delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M)) + \gamma_2(\delta_M \mathbf{P}V_{\mathbf{n}}(\delta_M)).$$

Taking into account that  $\beta_2 = \frac{\beta_1}{1-\alpha_1}$  and  $\gamma_2 = \frac{1-\alpha_1-\beta_1}{1-\alpha_1}$ , we obtain:

$$\beta_1 \geq \frac{l(1-\alpha_1)}{l - g^\ell + \delta_M \mathbf{P}V_{\mathbf{sc}}(\delta_M)}.$$

The fact that an  $M$ -class firm adopts  $C$  in period 2 and state  $\ell$  affects in turn the choice

of a competitor of class  $P$ : we now examine the condition under which  $q_1^h = 1$  is a part of PMPBE if the  $M$  firm chooses  $r_2^\ell = 1$  in period 2 and state  $\ell$ . The deviation of firm  $P$  to strategy  $q_1^h = 0$  is not profitable if

$$\alpha_1 [\delta_P \mathbf{P}(V_{\mathbf{c}}(\delta_P) - V_{\mathbf{n}}(\delta_P)) - g^h] + \beta_1 \delta_P \mathbf{P}(V_{\mathbf{n}}(\delta_P) - V_{\mathbf{sc}}(\delta_P)) - (1 - \alpha_1)l \geq 0,$$

which is equivalent to inequality

$$\beta_1 \leq \frac{\alpha_1 [l - g^h + \delta_P \mathbf{P}V_{\mathbf{c}}(\delta_P)] - l}{\delta_P \mathbf{P}V_{\mathbf{sc}}(\delta_P)}.$$