Determination of Minutiae Scores for Fingerprint Image Applications *

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Abstract

Many Automatic Fingerprint Identification Systems (AFIS) are based on minutiae matching. Minutiae are the terminations and bifurcations of the ridge lines in a fingerprint image. A fingerprint image that has undergone binarization, followed by thinning, in order to extract the minutiae, contains hundreds of minutiae, all of which are not so vivid and obvious in the original image. Thus, the set of minutiae that are well-defined and more prominent than the rest should have given higher relevance and importance in the process of minutiae matching.

In this work, a method to assign a score value to each of the extracted minutiae is proposed, based on some topographical properties of a minutia. The score associated to a minutia signifies its genuineness and prominence. A minutia with a higher score value should be given higher priority in the matching scheme to yield better results.

1. Introduction

A fingerprint image I essentially consists of a set of minutiae on the x-y plane. Minutiae are the terminations and bifurcations of ridge lines in a fingerprint image. The ridge lines, appearing in the foreground of the gray-scale topography, are separated by valley lines appearing in the background. In a fingerprint image, there exists a striking duality in the sense that the valley lines also have minutiae (terminations and bifurcations) and flow patterns similar to the ridge lines [4, 5]. The ridge and valley characteristics, such as ridge and valley flow directions, inter-ridge and inter-valley distances, ridge and valley breaks, etc., are very useful properties that indicate the validity criteria of a minutia detected by any algorithm. These parameters have been used extensively in a number of earlier works. For enhancing a gray-level fingerprint image, orientation of ridges is used for designing a filter by O'Gorman and Nickerson [11], and, for using directional images by Mehtre et al [10]. In a work by Hung [5], ridge enhancement is done based on ridge directions, and noise removal and pattern purification are performed with the help of both ridge and valley characteristics.

A gray-scale fingerprint image often undergoes binarization, followed by thinning, in the preprocessing stage, in order to extract the minutia points [2, 6]. During preprocessing, apart from spurs, bridges or loops, several spurious and misleading lines appear in the thinned image because of the noise present in the original gray-scale image. These lines are mere aberrations that often give rise to poor or not-so-obvious minutiae, thereby delaying the process of minutiae matching, or reporting a poor fingerprint match. Spurs, bridges, and loops are easily detectable in a less noisy region. In a substantially large noisy part of an image, several criss-crosses may arise that are not always detectable as bridges or loops. There may also exist some minutiae in a noise-free region (apparently, by the naked eye) that are feebly recognizable in the gray-scale image because of erratic gray-value pattern in that locality. As a result, an ambiguity may arise regarding the inclusion or exclusion of a minutia depending on its visual clarity in the original gray-scale image.

In order to circumvent this uncertainty, we propose a methodology of assigning a score value to each minutia, after elimination of spurs, bridges, and loops. Each minutia is assigned a score in the scale [1, 100] depending on its topographical characteristics in the skeletonized ternary image (ridge, valley, and background), which in turn, are derived from its visual prominence in the original gray-scale image.

2. Score-based fingerprint matching

Let A be the set of minutiae, called *data set*, existing in the fingerprint database, and B be the *query set* of minutiae that has to be checked for a match with some element of A. The existing matching schemes do not discriminate among the

*This work is funded by a grant from Intel Corp., USA (PO #CAC042717000)

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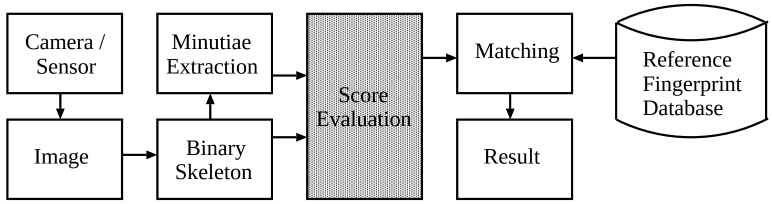

Figure 1: Generic structure of an AFIS with minutiae scores.

minutiae apropos their quality either in the data set or in the query set. A match is reported if the coordinates, types and angles of minutiae of query set B are found to be agreeing with those of data set A under certain transformations like translation, rotation, or scaling [4, 6, 7, 8, 12].

In order to consider the relative quality of a minutia in a fingerprint image as a practical matching criterion, we define a minutia point P as a 5-tuple, $P = \langle x, y, t, \theta, s\rangle$, where, (x, y) = coordinates of P, t = type of minutia (a bifurcation minutia or a termination minutia as considered by the Federal Bureau of Investigation and adopted in most AFIS), θ = angle made by the tangent to the corresponding ridge at the point (x, y), and s = an integer score associated with the minutia P.

The score values are normalized within a scale of 1 to 100, where, a minutia with score nearing 100 is of the highest significance compared to any other minutia with a lower score value. In other words, if a minutia P_1 has a score s_1, and another minutia P_2 has a score s_2, where $s_1 < s_2$, then P_1 is a less dependable minutia than P_2.

While applying a matching procedure based on fingerprint minutiae, the scores of minutiae of A and those of B can be used to tell about how good or bad the match is. If a minutia (x_{a_i}, y_{a_i}) with score s_{a_i} in set A is a potential match with a minutia (x_{b_i}, y_{b_i}) with score s_{b_i} in set B, the difference $s_{a_i} \sim s_{b_i}$ indicates the quality of matching of (x_{a_i}, y_{a_i}) and (x_{b_i}, y_{b_i}). For a matching between A and B with n minutiae, $n \geq 3$, we define the *matching index* MI as follows:

$$MI \;=\; 100 - \frac{1}{n} \sum_{\substack{(x_{a_i}, y_{a_i}) \in A \\ (x_{b_i}, y_{b_i}) \in B}} |s_{a_i} - s_{b_i}|$$

Since $1 \leq s_{a_i} \leq 100$ and $1 \leq s_{b_i} \leq 100$ for $1 \leq i \leq n$, so $0 \leq |s_{a_i} - s_{b_i}| \leq 100$, and therefore, MI also lies in the range [0,100]. A high value of MI implies a strong match between A and B, whereas, a low value indicates a poor one.

The concept of score can be also exploited to expedite the matching procedure between a query set B and a data set A. The problem is to check for a matching in A, if at all exists, in the fingerprint image database, with respect to the query set B. In that case, a small subset B_1 of minutiae with leading score values in the query set B should be considered first to check for a match with the data set A. If the match between A and B_1 is satisfactory, a next level match can be tried between A and a larger subset of B. This may be continued till there is a total match between A and B. At any intermediate matching stage involving, say, A and B_1, if the match is not satisfactory, the remaining set of minutiae, i.e., $B - B_1$ need not be tried for, thus saving the matching time for an unsuccessful case. A score-based generic structure of an AFIS is shown in Fig. 1.

3. Evaluation of score

The score s of a minutia P is estimated based on the following properties:
- pattern of ridge flow in and around P;
- pattern of valley flow in and around P;
- noise level in the locality of P.

If the ridge and valley lines in the local neighbourhood of P have a smooth nature of flow, the corresponding minutia P will have a genuine contribution in the fingerprint matching. On the contrary, if in some region, the ridge and valley lines have an erratic or uneven nature of flow, a minutia P' in that region should not predominate the matching procedure. The former minutia (P), being located in a tidy region, contributes more confidence in the matching procedure than the latter (P') which is located in a noisy region.
For a minutia $P(x, y)$, the score is given by the equation

$$s = s_{ri} + s_{va} + s_{no} \tag{1}$$

where, s_{ri}, s_{va} and s_{no} are the score components due to ridge flow, valley flow, and noise level respectively in the local neighborhood of P. The components s_{ri} and s_{va} denote measures of perfectness of ridge and valley flow respectively, that are evaluated based on some distances estimated in the local ridge and valley topography around the minutia P. To take into account the noise of the region in and around P, the component s_{no} is estimated in a local window centered at P. Noise imparts a negative effect on the score.

3.1. Score of a bifurcation minutia

Let λ be the average inter-ridge distance of a fingerprint image. First, we find the three neighbor pixels N_1, N_2, N_3 of P, considering 8-neighborhood. N_1, N_2, N_3 are the three starting pixels of the ridges r_1, r_2, r_3 respectively, incident at P. We explore a walk along each of r_1, r_2, r_3 starting from N_1, N_2, N_3 respectively, each walk being of length λ.

Let these walks be named as w_1, w_2, and w_3 respectively. If during some walk $w_i, 1 \leq i \leq 3$, any bifurcation or termination minutia is encountered, the walk is halted. Let, $l_i, 1 \leq i \leq 3$, denote the length of the walk w_i. Let, l_{min} be the minimum of $l_i, 1 \leq i \leq 3$, and μ be the number of walks whose lengths are less than λ. If P is a minutia of good quality, then each l_i should be at least $\lambda/2$, and at least two of them should be λ. So, if $l_{min} < \lambda/2$ or, $\mu \geq 2$, we assign 0 to $score$ and return from this point. Else, if $l_{min} < \lambda$, then we walk for a length l_{min} along each of the three ridges r_1, r_2, r_3 starting from N_1, N_2, N_3 respectively, so that after the (re-)walks, each of the points Q_1, Q_2, Q_3, reached on the three ridges r_1, r_2, r_3 respectively, is at equal distance from P (Fig. 2).

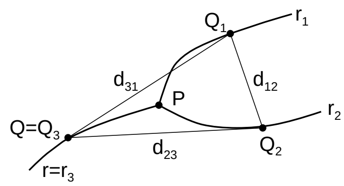

Figure 2: Ridges incident at the bifurcation minutia P.

In this scenario, we need to identify the ridge line that bifurcates at P. In Fig. 2, the three ridges are shown as r_1, r_2, and r, where, w.l.g., r (=r_3) has been depicted as the pre-bifurcated ridge, and r_1, r_2 are its two bifurcations at P. To identify the pre-bifurcated ridge, we define $d_{min} = min(d_{12}, d_{23}, d_{31})$, where, d_{ij}= L_2-distance between Q_i and Q_j, $1 \leq i, j \leq 3, i \neq j$. If Q_1 and Q_2 are on the two bifurcated ridges r_1 and r_2, then $d_{12} < d_{23}$ and $d_{12} < d_{31}$. However, this condition may fail if P is a poor minutia candidate, viz., when the ridges incident at P are of uneven nature, and it is difficult to ascertain the pre-bifurcated ridge among r_1, r_2, r_3. Hence, if $d_{min} > 3l_{min}/2$, we assign 0 to $score$, and return.

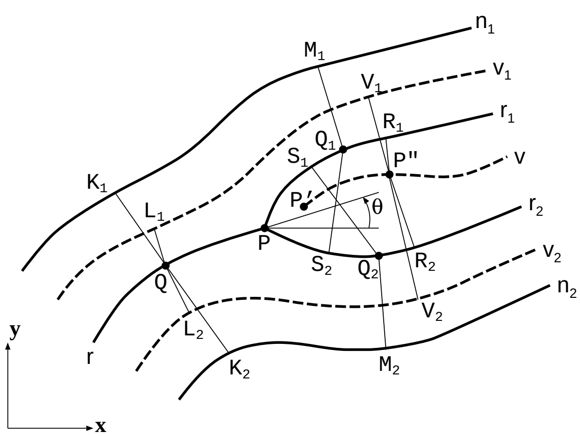

Figure 3: Ridge and valley characteristics around a bifurcation minutia.

In order to compute the score s_{ri} for a bifurcation minutia P, we define the following distances, vide Fig. 3.
d_{qn_1} = distance from Q to neighbor ridge n_1 = QK_1;
d_{qn_2} = distance from Q to neighbor ridge n_2 = QK_2;
$d_{q_1n_1}$ = distance from Q_1 to neighbor ridge n_1 = Q_1M_1;
$d_{q_2n_2}$ = distance from Q_2 to neighbor ridge n_2 = Q_2M_2;
$d_{q_1r_2}$ = distance from Q_1 to bifurcated ridge r_2 = Q_1S_2;
$d_{q_2r_1}$ = distance from Q_2 to bifurcated ridge r_1 = Q_2S_1;

For a good minutia, the above distances should be close to λ. So, s_{ri} is assigned to P depending on the closeness of $d_{BM,ri} \in \{d_{qn_1}, d_{qn_2}, d_{q_1n_1}, d_{q_2n_2}, d_{q_1r_2}, d_{q_2r_1}\}$ w.r.t. λ. Thus, for a bifurcation minutia P, the score w.r.t. the ridge characteristics can be chosen as:

$$s_{ri} = \alpha_{ri} \sum_{d_{BM,ri}} (\lambda - |\lambda - d_{BM,ri}|); \qquad (2)$$

where, α_{ri} is the *ridge score multiplier for bifurcation minutiae*.

Similarly, the score s_{va} for the bifurcation minutia P is based on the following set of distances.
d_{qv_1} = distance from Q to neighbor valley v_1 = QL_1;
d_{qv_2} = distance from Q to neighbor valley v_2 = QL_2;
$d_{pp'}$ = distance from P to valley termination minutia P', if any, lying near P in between r_1 and r_2 = PP';
$d_{p''r_1}$ = distance from P'' to bifurcated ridge r_1 = $P''R_1$;
$d_{p''r_2}$ = distance from P'' to bifurcated ridge r_2 = $P''R_2$;
$d_{p''v_1}$ = distance from P'' to neighbor valley v_1 = $P''V_1$;
$d_{p''v_2}$ = distance from P'' to neighbor valley v_2 = $P''V_2$;
where, P'' is the point along the valley v at a distance λ from P', or, a bifurcation or termination of v appearing within the target walk-length of λ.

While the parameter $\{d_{BM,ri}\}$ represents some kind of inter-ridge distance, we define other distance measures with a subtle difference. Distances in the set $\{d^1_{BM,va}\}$ = $\{d_{p''v_1}, d_{p''v_2}\}$ are inter-valley distances, which should be ideally close to λ. The other set $\{d^2_{BM,va}\}$ = $\{d_{qv_1}, d_{qv_2}, d_{pp'}, d_{p''r_1}, d_{p''r_2}\}$ contains distances from a ridge point to a valley line, or from a valley point to a ridge line, and therefore, requires a flexibility in their contribution to s_{va}. Hence, distances in the set $\{d^1_{BM,va}\}$ are very much similar to $\{d_{BM,ri}\}$ as far as the estimation of s_{va} is concerned. Their contribution to score may be chosen as:

$$s^1_{va} = \alpha_{va} \sum_{d^1_{BM,va}} (\lambda - \left|\lambda - d^1_{BM,va}\right|); \qquad (3)$$

And, that due to $\{d^2_{BM,va}\}$ is

$$s^2_{va} = \sum_{d^2_{BM,va}} s_{d^2_{BM,va}} \qquad (4)$$

where, $s_{d^2_{BM,va}}$ is chosen as:

$$s_{d^2_{BM,va}} = \begin{cases} \alpha_{va}\lambda & \text{if} \quad \lambda/4 \leq d^2_{BM,va} \leq 3\lambda/4 \\ \alpha_{va}(d^2_{BM,va} - \lambda/4) & \text{if} \quad d^2_{BM,va} < \lambda/4 \\ \alpha_{va}(3\lambda/4 - d^2_{BM,va}) & \text{if} \quad d^2_{BM,va} > 3\lambda/4 \end{cases} \tag{5}$$

and α_{va} is the *valley score multiplier for a bifurcation minutia*.

3.2. Score of a termination minutia

Let P be a termination minutia and N be the adjacent ridge pixel of P, considering 8-neighborhood. Since P is a termination minutia, there will be only one ridge line, say r, incident at P [Fig. 4]. We walk along r starting from N, for a length λ, and designate the walk as w. Let l denote the length of the walk. Since a skeletonized fingerprint image should be devoid of spurs and bridges, l should always be equal to λ. Let Q be the point on the ridge r reached

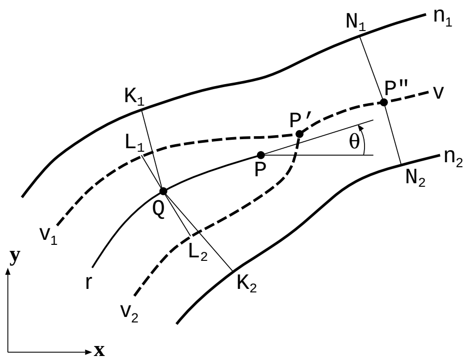

Figure 4: Ridge and valley characteristics around a termination minutia.

after the walk w. For estimation of the score s_{ri} for the termination minutia with respect to ridge lines in the region containing P, we define the set $\{d_{TM,ri}\}$ of following distances.
d_{qn_1} = distance from Q to neighbor ridge n_1 = QK_1;
d_{qn_2} = distance from Q to neighbor ridge n_2 = QK_2;

For P to be a termination minutia of good quality, the above distances, should be close to λ. These distances are basically inter-ridge distances similar to $\{d_{BM,ri}\}$ in the case of bifurcation minutiae. Hence, the score s_{ri} is assigned to P based on the following equation that resembles with Eqn. 2 in form:

$$s_{ri} = \beta_{ri} \sum_{d_{TM,ri}} (\lambda - |\lambda - d_{TM,ri}|); \tag{6}$$

where, β_{ri} is the *ridge score multiplier for termination minutiae*.

Similarly, the score s_{va} for the termination minutia P is based on the set $\{d_{TM,va}\}$ of following distances.
d_{qv_1} = distance from Q to neighbor valley v_1 = QL_1;
d_{qv_2} = distance from Q to neighbor valley v_2 = QL_2;
$d_{pp'}$ = distance from P to valley termination minutia P', if any, lying near P in between n_1 and n_2 = PP';
$d_{p''n_1}$ = distance from P'' to neighbor ridge n_1 = $P''N_1$;
$d_{p''n_2}$ = distance from P'' to neighbor ridge n_2 = $P''N_2$;
where, P'' is the point along the valley v at a distance λ from P', or, a bifurcation or termination of v appearing within the target walk-length of λ.

The above set of distances are measured either from a ridge point to a valley line or from a valley point to a ridge line. Hence, their contribution to score s_{va} is given by:

$$s_{va} = \sum_{d_{TM,va}} s_{d_{TM,va}} \tag{7}$$

where, $s_{d_{TM,va}}$ is chosen as:

$$s_{d_{TM,va}} = \begin{cases} \beta_{va}\lambda & \text{if} \quad \lambda/4 \leq d_{TM,va} \leq 3\lambda/4 \\ \beta_{va}(d_{TM,va} - \lambda/4) & \text{if} \quad d_{TM,va} < \lambda/4 \\ \beta_{va}(3\lambda/4 - d_{TM,va}) & \text{if} \quad d_{TM,va} > 3\lambda/4 \end{cases} \tag{8}$$

and β_{va} is the *valley score multiplier for a termination minutia*.

3.3. Estimation of noise

Let P be a bifurcation or termination minutia having a positive score after the evaluation of s_{ri} and s_{va}. If P does not have a positive score, we need not evaluate s_{no}, since s_{no} will contribute a negative score to P; finally we will consider only the set of minutiae with postive scores. Consider a circular window W of radius $R = N\lambda$ around $P(x, y)$, vide Fig. 5. Let $\{Q_i | Q_i \text{ lies within } W; i = 1, 2, \ldots, \eta\}$ be the set of points, with each point Q_i satisfying any one of the following 3 properties (Fig. 5):
(i) Q_i is a ridge minutia with s_{ri} + s_{va} = 0;
(ii) Q_i is a non-minutia ridge point having three or more ridges incident upon it;
(iii) Q_i is either a valley bifurcation or a valley termination minutia.

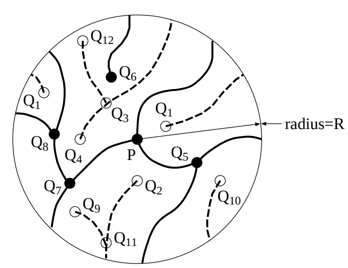

Figure 5: Contributing points $\{Q_1, Q_2, \ldots, Q_{12}\}$ in a noisy window W centered around the minutia P.

The above definition enables us to use $|\{Q_i\}|$ = η as a measure of noise level in the window W centered around P. We define another parameter ν, called the *noise factor*, which is used to find the *noise threshold* τ_{noise} given in the equation below, that will indicate whether or not a window W associated with a minutia P is noisy:

$$\tau_{noise} = \nu N \tag{9}$$

If η is higher than τ_{noise} in W corresponding to P, the noise level in W is considered high enough and each point Q_i, $i = 1, 2, \ldots, \eta$, is accounted one by one for their individual contribution to the noise-induced (negative) score s_{no} of P. Thus, Eqn. 10 can be used to find s^i_{no} attributed by each Q_i, and Eqn. 11 sums up the individual scores to compute the total score due to noise.

$$s^i_{no} = \gamma(R - L_2(P, Q_i)) \tag{10}$$

$$s_{no} = \begin{cases} 0 & \text{if} \quad \eta \leq \tau_{noise} \\ \sum_{i=1}^{\eta} s^i_{no} & \text{if} \quad \eta > \tau_{noise} \end{cases} \tag{11}$$

where, γ is the *noise score multiplier*.

In Eqn. 10, L_2-distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by:

$$L_2((x_1, y_1), (x_2, y_2)) = \sqrt{\{(x_1 - y_1)^2 + (x_2 - y_2)^2\}} \tag{12}$$

4. Experiments and results

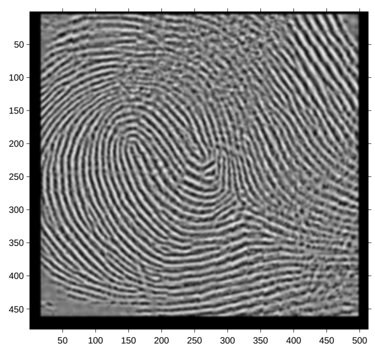

Figure 6: A sample fingerprint image from NIST 14 sdb.

We used several fingerprint images from the NIST Special Database 14 [1] and NIST Special Database 4 [13]. In order to keep the minutiae scores of the order of 100 prior to normalization, the value of α_{ri} (= α_{va}) has been chosen as 1.00.

For evaluating the score of a bifurcation minutia, we need to compute 6 distances in the set $\{d_{BM,ri}\}$, measured w.r.t. different ridge lines, and 7 distances in the set $\{d_{BM,ri}\}$, measured w.r.t. different valley lines. For finding the score of a termination minutia, we need 2 and 3 such distances, in the sets $\{d_{TM,ri}\}$ and $\{d_{TM,va}\}$, respectively. Thus, in order to have parity in the score values of bifurcation and termination minutiae, we choose β_{ri} = (6/2)α_{ri} = 3.0 and β_{va} = (7/3)α_{va} = 2.33.

Table 1: Score Values of Minutiae

sl.no.	x	y	Type	Angle	Score
1	441	379	BM	93	1
2	118	304	BM	245	11
3	90	136	BM	285	13
4	429	40	BM	19	16
5	345	210	BM	250	23
6	76	81	BM	315	35
7	342	381	BM	267	44
8	424	55	BM	267	50
9	246	261	BM	71	55
10	261	219	BM	41	55
11	408	82	BM	246	58
12	425	164	BM	269	58
13	50	195	BM	258	59
14	435	91	BM	267	62
15	251	234	BM	248	64
16	390	77	BM	235	64
17	409	205	BM	252	66
18	406	143	BM	252	66
19	347	118	BM	229	68
20	362	115	BM	40	71
21	187	88	BM	328	76
22	56	127	BM	91	76
23	128	91	BM	304	77
24	407	114	BM	251	80
25	115	55	BM	305	85
26	294	87	BM	14	90
27	173	432	BM	215	91
28	433	209	BM	277	92
29	146	414	BM	23	92
30	149	388	BM	217	95
31	317	282	BM	108	96
32	356	290	BM	286	97
33	330	352	BM	254	99
34	297	197	BM	39	100
35	154	473	TM	15	17
36	117	337	TM	224	30
37	144	215	TM	223	33
38	150	115	TM	116	33
39	388	202	TM	90	35
40	82	400	TM	41	41
41	177	242	TM	34	43
42	412	430	TM	114	73
43	42	205	TM	270	80
44	327	91	TM	22	90
45	115	450	TM	216	98

In the estimation of noise-based score, ν is a controlling paramater that decides the effect of noise on the score. From Eqn. 9, it is evident that a higher value of ν will enforce a lesser impact of noise in the score. On the basis of our experimental results, we have emperically chosen N = 2, ν = 3, and, $\gamma = 1/\nu$ = 0.33.

In Fig. 6, a sample fingerprint image of size 480×512 is shown. The corresponding ternary skeleton image is shown in Fig. 7, where the darker lines represent the ridges and the faint lines are valleys. The minutiae having positive scores are shown in Fig. 7, with the darkness of a minutia being proportional to its score. Table 1 includes the scores (positive values only) of the bifurcation minutiae (BM), followed by those of the termination minutiae (TM), arranged in ascending orders. The bifurcation minutia at (297, 197) has the maximum score 100, which is well justified by its visual clarity in the image shown in Fig. 6 and the topographical orderliness in its neighborhood in Fig. 7. On the other hand, the minutia at (441, 379) is located in a highly noise-affected region. Scores of some minutiae are written beside the corresponding minutiae in Fig. 7.

The proposed method is implemented in C on a Sun_Ultra 5_10, Sparc, 233 MHz, the OS being the SunOS Release 5.7 Generic. The total CPU time for the evaluation of scores of all minutiae in a ternary skeletonized fingerprint image was found to be around 0.03 to 0.07 sec.

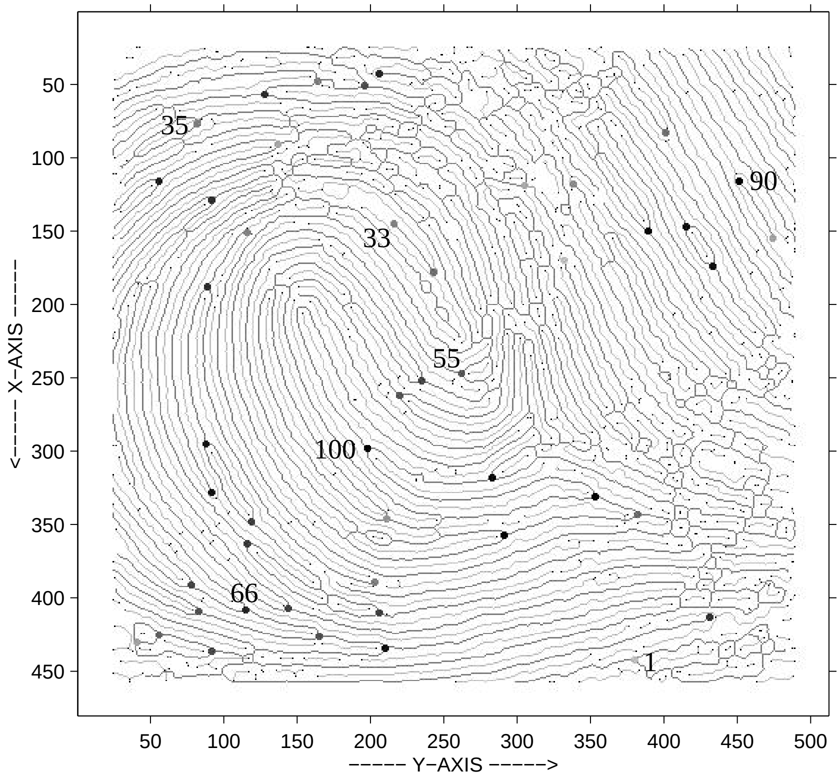

Figure 7: Minutiae shown with darkness proportional to scores.

5. Conclusions and future works

A method of scaling to assess a minutia for fingerprint matching is reported in this paper. Development of a faster and realistic fingerprint matching technique based on the proposed method is currently in progress. Some of the empirical formulae mentioned in this paper may require further refinements for more accurate matching result. In reality, the score of a minutia in a query image may be drastically different from that of the database image. If the scores vary widely, then the confidence in matching may reduce significantly. These anomalies have to be resolved to ensure a matching result.

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