# Phase diagram in Bonabeau social hierarchy model with individually different abilities 

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#### Abstract

The 1995 model of Bonabeau et al is generalized by giving each individual a different ability to win or lose a fight. We also introduce different groups such that the losers of fights between different groups are eliminated. The overall phase diagram for the first-order transition between egalitarian and hierarchical societies does not change much by these generalizations.


Keywords: Sociophysics, phase transition, different abilities

## 1 Introduction

The model of Bonabeau et al [1] described how a difference between powerful and powerless levels of society can self-organize out of randomness and the memory of past power fights. With suitable modifications [2] a first-order phase transition was found: at low concentrations the society remained egalitarian, while at high concentrations some individuals became "more equal" than others. These inequalities are measured by the probability to win the fights arising whenever one individual wants to move onto the place occupied by another; initially we have an egalitarian society where everybody has the winning probability $1 / 2$. As in some biological species [3], the individuals
keep a memory of past fights such that past victories re-inforce the probability to win again. The present note makes the individuals unequal from the beginning and checks how this modification changes the results.

## 2 Old Model

A $L \times L$ square lattice is filled randomly with $N=p L^{2}$ individuals such that no two or more share one site. Then they diffuse randomly to nearestneighbour sites. If one individual $i$ wants to occupy the site on which already another individual $k$ sits, a fight breaks out which is won by one of the two, who then moves into the contested site; the loser instead moves into the site previously occupied by the winner. The probability for $i$ to win is

$$
q=1 /[1+\exp (\sigma \cdot(h(k)-h(i))]
$$

where $h(i)$ is the number of past victories minus the number of past losses of $i$ except that at each time step all $h$ are diminished by a factor $1-f$; this memory factor $f$, which is the same for all individuals, indicates how fast past events are forgotten. The inequality $\sigma$ is the standard deviation in the probabilities $q$ :

$$
\sigma^{2}=<q^{2}>-<q>^{2} ;
$$

during the first 10 iterations we set $\sigma=1$. With this feedback through $\sigma$ a sharp first-order phase transition (discontinuity) was found [2] near $p=0.32$ for $f=0.1$ : For higher concentrations $p$ a non-zero social inequality $\sigma$ was obtained while for lower $p$ this $\sigma$ vanished to zero.

## 3 New Models

First we gave each individual $i$ different histories $h(i)$ to start with; then initially the preferred individuals for low concentrations could enlarge their advantage but after some time they lost it and no correlations were seen between initial and final $h$ (not shown).

Second, we gave each individual different abilities to win a fight by replacing $h(i)$ with $h(i)+a(i) z$ where $z$ is a random number between 0 and 1 , evaluated for each fight and each $i$ again and again, while the ability $a(i)$ stays with each individual $i$ forever and is initially distributed randomly between -5 and +5 . (Setting $z=1$ does not change much.)


Figure 1: Search for phase transition with permanent random abilities, at $f=$ 0.1. For the more complicated models discussed later the order parameter did not always go to unity if $\sigma \rightarrow 0$, and thus $\sigma$ which gave similar figures was used as a criterion.

Third, we divided the whole population into $G$ groups, initially equally large. If two individuals of the same group fight, the above rules apply; if two individuals of different groups fight, the loser vanishes and is replaced by a replica of the winner with the same $h$ and $a$; thus the winner is duplicated and occupies both sites 4].

Fourth, a feedback [5] for the special case $G=2$ is introduced such that the majority wins more easily than the minority. Thus the above $h(i)+a(i) z$ is replaced by $h(i)+[a(i)+10 m] z$ where $m$ with $-1<m<+1$ is the magnetization, i.e. the normalized difference $\left(N_{1}-N_{2}\right) /\left(N_{1}+N_{2}\right)$ between the numbers $N_{1}, N_{2}$ of individuals in the two groups.

The similarity between the final power $h(i)+a(i)$ and its initial value $a(i)$ is given by the order parameter

$$
\Psi \propto \sum_{i}[h(i)+a(i)] a(i)
$$



Figure 2: Stationary values for order parameter and inequality, from simulations like in Fig.1, using medium and large lattices.
which we normalize to unity at the beginning.

## 4 Results

Usually we made up to 5000 iterations with up to about $5000 \times 5000$ sites. For the second model, Fig. 1 shows how the order parameter initially increases due to re-inforcement of individuals with higher abilities. However, if the concentration is low such that fights occur seldomly, then the positive effects of these abilities are mostly forgotten until the next fight, and everybody has the same chance of $1 / 2$ to win or lose. This happens for the 8 leftmost curves, $p=0.10,0.11, \ldots, 0.17$, in Fig.1. For $p=0.18$ the system needs a long time to find this egalitarian state. For $p=0.19$ and 0.20 we see two plateaus in the later times, with no indication of a decay. With additional runs at $p=0.181,0.182, \ldots$ and 5000 iterations we found a stable order paraneter $\Psi>1$ and inequality $\sigma>0$ only at $p \geq 0.182$. Fig. 2 shows the long-time


Figure 3: Phase diagram with one group (part a) and two groups (part b); in the latter case also the results without different abilities are shown.
values of $\Psi$ and $\sigma$, indicating a jump (first-order transition) of both quantities near $p=0.182$. All these simulations were made with the traditional value $f=0.1$.

Varying the memory factor $f$ we find the phase diagram of Fig.3a; for $f>1 / 3$ the model always leads to egalitarian results since past fights are forgotten too fast. Fig.3b shows the analogous phase diagram in the third model with two groups, also for the case without different abilities.

With five groups, Fig. 4 shows a behaviour similar to one or two groups, Fig.3. We also give here as the lower data the results for the standard model [2] without different abilities and only one group; again, the behaviour seems


Figure 4: Phase diagram for five groups $(+)$; the x indicate the standard model without different groups and without different abilities.
similar. Thus for all models except the fourth one the results are similar: The transition line starts at the origin and increases roughly linearly to a fully occupied lattice, $p=1$, at a memory decay factor between $1 / 5$ and $1 / 3$; for larger $f$ (shorter memory) the society remains egalitarian.

In the fourth model with feedback the concept of a transition is less clear. The larger the lattice is, the smaller is the normalized fluctuating magnetization $m$ appearing in $h(i)+[a(i)+10 m] z$ and the higher is the critical concentration at which the egalitarian society is destroyed. The earlier transition points were determined by gradually increasing $p$, Fig.1; in the feedback version, however, it happened that when at some $p$ the inequality $\sigma$ went to zero, at some slightly higher $p$ it became nonzero, and then zero again for still higher $p$. In that case, for large $L=4001$ and one sample only, some intermediate $p$ value was taken as transition point. For smaller $101 \leq L \leq 1001$, hundred samples were simulated and the transition point defined as the concentration $p$ where about 50 samples ended with $\sigma=0$


Figure 5: Size effects in the feedback model, indicating a roughly logarithmic increase of the transition $p$ with $L$.
and the others with nonzero $\sigma$. Fig. 5 shows an increase of the transition $p$ roughly logarithmic in system size; and for infinite system size the whole feedback effect as simulated here would vanish and the results agree with those of Fig.3b.

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