ON COMMUTING GRAPHS OF GENERALIZED DIHEDRAL GROUPS

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ABSTRACT. For a group G and a subset X of G, the commuting graph of X, denoted by $\Gamma(G, X)$ is the graph whose vertex set is X and any two vertices u and v in X are adjacent if and only if they commute in G. In this article, certain properties of the commuting graph of generalized dihedral groups have been studied.

1. INTRODUCTION

Throughout the article, we consider the simple undirected graphs which are without loops or multiple edges. For a graph Γ , the vertex set and edge set are denoted by $V(\Gamma)$ and $E(\Gamma)$. If a vertex u is adjacent to a vertex v, then we denote it as $u \sim v$. The degree deg(v) of a vertex v in Γ is the number of edges incident to v. A graph Γ is said to be regular if and only if the degree of every vertex is equal. A graph Γ is complete graph if each vertex are adjacent with every other vertex of the graph Γ and denoted by K_n where n is the number of vertices in the graph.

A generalized dihedral group D(G) is the semi-direct product $G \rtimes_{\phi} C_2$ of an abelian group G with a cyclic group $C_2 = \{1, -1\}$, where homomorphism ϕ maps 1 and -1 to identity automorphism and inversion automorphism respectively. Therefore, the binary operation on D(G) is defined as follows

$$(g_1, c_1)(g_2, c_2) = (g_1 g_2^{c_1}, c_1 c_2),$$

where $g \in G$ and $c \in C_2$.

In this paper, G will denote a finite abelian group of order n. Then, G is isomorphic to the direct product $\mathbb{Z}_{m_1} \times \cdots \times \mathbb{Z}_{m_k}$ of cyclic groups, where $m_1 \cdots m_k = n$. In this paper, we will identify G with $\mathbb{Z}_{m_1} \times \cdots \times \mathbb{Z}_{m_k}$ and we place factors of direct product of 2-power order (if it exists) before the factors of odd order, that is

$$G = \underbrace{\mathbb{Z}_{m_1} \times \cdots \times \mathbb{Z}_{m_r}}_{\text{factors of 2-power order}} \times \underbrace{\mathbb{Z}_{m_{r+1}} \times \cdots \times \mathbb{Z}_{m_k}}_{\text{factors of 2-power order}}$$

In the above, $m_i = 2^t$ for some integer $t \ge 0$ and $1 \le i \le r$. By [4, Theorem 2.9, p. 9], we note that D(G) is abelian if and only if G is elementary abelian 2-group. Throughout the paper, by a generalized dihedral group D(G) we will always mean it to be non-abelian, otherwise it will be stated. Following are elementary results.

Lemma 1.1. ([4, Proposition 4.12, p.11]) Given any abelian group G, the element (g, 1) is in Z(D(G)) if and only if $g^2 = e \in G$.

Lemma 1.2. ([4, Proposition 4.13, p.12]) Given any abelian group G, the element (g, -1) is in the center of D(G) if and only if D(G) is abelian.

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Lemma 1.3. Let G be an abelian group. Then $(g_1, -1), (g_2, -1) \in D(G)$ commute if and only if $g_1^2 = g_2^2$.

Lemma 1.4. Let G be an abelian group. If $(g_1, 1) \in D(G)$ commute with $(g_2, -1) \in D(G)$, then $(g_1, 1) \in Z(D(G))$.

By Lemmas 1.1 and 1.2, one observes that $Z(D(G)) = \{(g,1) | g^2 = e\}$. This implies that $|Z(D(G))| = 2^r$. Let $G_1 = \{(g,1) \in D(G) | g \in G\}$ and $G_2 = \{(g,-1) \in D(G) | g \in G\}$. Note that $Z(D(G)) \subseteq G_1$. Further, we write $\Omega_1 = Z(D(G)), \Omega_2 = G_1 \setminus Z(D(G))$ and $\Omega_3 = G_2$. Let $g_1, g_2 \in G = \mathbb{Z}_{m_1} \times \cdots \mathbb{Z}_{m_r} \times \mathbb{Z}_{m_{r+1}} \times \cdots \times \mathbb{Z}_{m_k}$. Then $g_i = (g_{i1}, \cdots, g_{ik}), i = 1, 2$. Let $(g_1, -1)$ commutes with $(g_2, -1)$. Then, by Lemma 1.3, $g_{1j}^2 = g_{2j}^2$ for all $1 \leq j \leq k$. This implies that the number of elements in G_2 that commute with a fix element $(g_1, -1)$ is 2^r . Also, one notes that $g_1^2 = g_2^2$ defines an equivalence relation in G. Therefore, G_2 is partitioned into $\frac{n}{2^r}$ subsets. We denote these subsets by $B_1, \cdots, B_{\frac{n}{2^r}}$.

In the last twenty years, plenty of researchers have been attracted to study the graphs of algebraic structure. The study of algebraic structures, using the properties of graphs, has become an exciting research topic in these years, leading to many fascinating results and raising questions. The commuting graph of a group is studied by various author (see [1]-[3] and [6]-[10]). In [3], the certain properties of the commuting graph of dihedral group are studied. In this paper, we have studied those properties for the commuting graph of generalized dihedral group

2. Commuting Graph of D(G)

For a non-empty subset X of D(G), the commuting graph of X denoted by $\Gamma(X) = \mathfrak{C}(D(G), X)$ is a graph whose vertex set $V(\Gamma)$ is X and any two vertices u and v are adjacent $(u \sim v)$ if and only if uv = vu. If $m_i = 2 \forall i \in \{1, 2, \dots, k\}$ then D(G) is abelian hence $\mathfrak{C}(D(G), D(G)) = K_{2^n}$. Since $\Omega_2 \subseteq G_1$, each element of Ω_2 commutes with each other. Also, no element of Ω_2 commute with any element of G_2 . Therefore, we obtain the following.

Proposition 2.1. If X is subset of D(G), then

$$\mathfrak{C}(D(G), X) = \begin{cases} K_{2^r} & \text{if } X = \Omega_1 \\ K_{n-2^r} & \text{if } X = \Omega_2 \\ K_{2^r} & \text{if } X = B_i \ (1 \le i \le \frac{n}{2^p}) \end{cases}$$

The join $\Gamma' \vee \Gamma''$ of two graphs Γ' and Γ'' is a graph with vertex set $V(\Gamma') \cup V(\Gamma'')$ and an edge set $E(\Gamma') \cup E(\Gamma'') \cup \{u \sim v \mid u \in V(\Gamma') \& v \in \Gamma'(u)\}$ and the union $\Gamma' \cup \Gamma''$ of two graphs Γ' and Γ'' is a graph with vertex set $V(\Gamma') \cup V(\Gamma'')$ and an edge set $E(\Gamma') \cup E(\Gamma'')$. If a graph Γ is union of m complete graph K_n with n vertices, then we write $\Gamma = mK_n$.

Corollary 2.2. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G). Then $\Gamma = K_{2^r} \vee (K_{n-2^r} \cup \frac{n}{2^r} K_{2^r}).$

Proof. By Proposition 2.1, there is no edge between two vertices from distinct blocks B_i . Therefore, $\mathfrak{C}(D(G), \Omega_3) = \bigcup_{i=1}^{n/2^r} \mathfrak{C}(D(G), B_i) = \frac{n}{2^r} K_{2^r}$. Also note that there is no edge between vertices from Ω_2 and Ω_3 . Further, since Ω_1 is center of D(G), each vertex from $\Omega_2 \cup \Omega_3$ is adjacent with each vertex of Ω_1 . Hence $\mathfrak{C}(D(G), D(G)) = K_{2^r} \vee \{K_{n-2^r} \cup \frac{n}{2^r} K_{2^r}\}$.

In a simple undirected graph Γ , degree of a vertex v is the number of edges incident to that vertex and denoted by deg(v). Now, we have the following.

Corollary 2.3. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G) and v be a vertex of Γ . Then

$$deg(v) = \begin{cases} 2n-1 & if v \in \Omega_1 \\ n-1 & if v \in \Omega_2 \\ 2^{r+1}-1 & if v \in \Omega_3 \end{cases}$$

Corollary 2.4. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G) and v be a vertex of Γ . Then

$$|E(\Gamma)| = 3n2^{r-1} + \frac{n(n-2)}{2}$$

The coloring of a graph is an assignment of colors to the vertices of the graph so that no two adjacent vertices have the same color. The chromatic number of a graph is the smallest number of colors needed to color the graph and denoted by $\psi(\Gamma)$. We have the following proposition.

Proposition 2.5. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G). Then $\psi(\Gamma) = n$.

Proof. We start coloring from $\Omega_1 \cup \Omega_2 = G_1$. Since $\mathfrak{C}(D(G), \Omega_1 \cup \Omega_2)$ is complete graph of *n* vertices, we need *n* colors to get it colored. Therefore $(\Omega_1 \cup \Omega_2)$ is colored with *n* colors. To color vertices from Ω_3 , we move block wise. In each block there are 2^r vertices and these all are adjacent with vertices from Ω_1 but not adjacent with vertices from Ω_2 . Since $|\Omega_2| > 2^r$, we can choose any of 2^r colors that we used for Ω_2 . These colors again can be used to color another block as any block has no common edge with the rest of blocks. Hence, $\psi(\Gamma) = n$.

3. Detour Distance of Commuting graph of D(G)

The detour distance $d_D(u, v)$ between two vertices u and v in a graph Γ is the length of a longest u - v path in Γ . A u - v path of length $d_D(u, v)$ is called a u - v detour geodesic. The detour eccentricity $(ecc_D(v))$ of a vertex v in Γ is the maximum detour distance between v and any vertex of Γ . The minimum detour eccentricity among the vertices of Γ is called the detour radius of Γ , denoted by $rad_D(\Gamma)$. The detour diameter $diam_D(\Gamma)$ of a graph Γ is the maximum detour eccentricity in Γ . Now, we have the following.

Theorem 3.1. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G) with $|\Omega_1| = 2^r$. Then for each $v \in \Omega_1$

$$ecc_D(v) = \begin{cases} 2n-1 & \text{if } \frac{n}{2^r} < 2^r \\ n+2^r(2^r-1)-1 & \text{if } \frac{n}{2^r} \ge 2^r \end{cases}$$

and for each $v \in \Omega_2 \cup \Omega_3$

$$ecc_D(v) = \begin{cases} 2n-1 & \text{if } \frac{n}{2^r} \le 2^r \\ n+4^r-1 & \text{if } \frac{n}{2^r} > 2^r \end{cases}$$

Proof. First assume that $v \in \Omega_1$. If $\frac{n}{2r} < 2^r$, then the center Ω_1 has more elements than number of blocks. To get a maximum length path starting from $v \in \Omega_1$, one first covers all vertices of Ω_2 , for they all are adjacent. Therefore, one covers each vertex from Ω_2 without repetition. Now, one moves to any vertex of Ω_1 except vitself and from there one moves to one of the blocks and complete all the vertices of that block. Now, move to another vertices in Ω_1 from there we again cover a block. Repeating this process we cover all the blocks. If there are some vertices still left in Ω_1 , then they will be covered at last. Hence $ecc_D(v) = 2n - 1$. If $\frac{n}{2r} > 2^r$, then we do the same process as above but the vertices from Ω_1 exhausts before it cover all the blocks. Hence in this case the maximum blocks that can be covered is $2^r - 1$. Therefore, the total vertices which are covered are $n + 2^r(2^r - 1)$ by the longest path started from a vertex $v \in \Omega_1$. Hence, $ecc_D(v) = n + 2^r(2^r - 1) - 1$.

Now, assume that $v \in \Omega_2$ and $\frac{n}{2^r} \leq 2^r$. We proceed in the same manner as we did in the above case. We start from vertex $v \in \Omega_2$ and first cover all the vertices of Ω_2 and then move to one of the vertex of Ω_1 . Then we head to one of the block and cover it and again move to another vertices of Ω_1 . Keep on repeating the process we first exhaust with blocks and still left some vertices in Ω_1 which we cover at the end. Hence, $ecc_D(v) = 2n - 1$. If $\frac{n}{2^r} > 2^r$, then one can similarly observed that $ecc_D(v) = n + 4^r - 1$.

Finally, assume that $v \in \Omega_3$ and $\frac{n}{2^r} \leq 2^r$. We start from one vertex from a block and cover it and then moves to one of the vertex in Ω_1 and then covers whole Ω_2 . Then, as above $ecc_D(v) = 2n - 1$. If $\frac{n}{2^r} > 2^r$, then by the similar argument as above $ecc_D(v) = n + 4^r - 1$.

Corollary 3.2. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G) with $|\Omega_1| = 2^r$. Then

$$rad_D(\Gamma) = \begin{cases} 2n-1 & \text{if } \frac{n}{2^r} < 2^r \\ n+2^r(2^r-1)-1 & \text{if } \frac{n}{2^r} \ge 2^r \end{cases}$$

and

$$diam_D(\Gamma) = \begin{cases} 2n - 1 & \text{if } \frac{n}{2^r} \le 2^r \\ n + 4^r - 1 & \text{if } \frac{n}{2^r} > 2^r \end{cases}$$

4. Resolving Polynomial of Commuting Graph of D(G)

Let $\beta(G)$ the metric dimension of Γ and $\beta(\Gamma, x) = \sum_{i=\beta(\Gamma)}^{n} s_i x^i$ denote the resolving polynomial of Γ (see [3]).

Let u be a vertex of a graph Γ . Then, the set $N(u) = \{v \in V(\Gamma) \mid v \sim u \text{ in } \Gamma\}$ is called the open neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called the closed neighborhood of u. Two distinct vertices u and v of Γ are called twins if N[u] = N[v]or N(u) = N(v). A subset U of vertex set of Γ is called a twin-set in Γ if u, v are twins in Γ for every pair of distinct vertices $u, v \in U$. The following the remark from [3].

Remark 4.1. ([3, Remark 3.3, p. 2398]) If U is a twin-set in a connected graph Γ of order n with $|U| = l \ge 2$, then every resolving set for Γ contains at least l - 1 vertices of U.

Theorem 4.2. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph of D(G). Then

$$\beta(\Gamma) = \begin{cases} 2n - \frac{n}{2^r} - 2 & |\Omega_1| \ge 2\\ 2n - 3 & |\Omega_1| = 1 \end{cases}$$

Proof. Assume that $|\Omega_1| = 2^r \ge 2$. Note that $N[u] = N[v] = D(G) \ \forall u, v \in \Omega_1$ and there does not exist $w \in D(G) \setminus \Omega_1$ such that N[w] = D(G). Hence Ω_1 is a twin-set in Γ with $|\Omega_1| \ge 2$. Further $N[u] = N[v] = \Omega_1 \cup \Omega_2 \ \forall u, v \in \Omega_2$ and there does not exist $w \in D(G) \setminus \Omega_2$ such that $N[w] = \Omega_1 \cup \Omega_2$. Hence Ω_2 is a twin-set in Γ with $|\Omega_2| \ge 2$ as $|\Omega_2| \ge |\Omega_1|$. Similarly each block $B_i(1 \le i \le \frac{n}{2^r})$ is a twin-set as $N[u] = N[v] = \Omega_1 \cup B_i \ \forall u, v \in B_i(1 \le i \le \frac{n}{2^r})$ with $|B_i| \ge 2$. Hence all twin-sets in D(G) with cardinality greater than or equal to 2 are $\Omega_1, \Omega_2, B_1, B_{,2}, \cdots B_{\frac{n}{2^r}}$. Therefore, by Remark 4.1, $\beta(\Gamma) = 2n - \frac{n}{2^r} - 2$. Now assume that $\Omega_1 = 1$. One can note that Ω_2 is a twin-set with $|\Omega_2| \ge 2$ and Ω_3 is also a twin-set as $N(u) = N(v) = \Omega_1 \forall u, v \in \Omega_3$. Therefore, there are two twin-sets in this case. Hence $\beta(\Gamma) = |\Omega_2| - 1 + |\Omega_3| - 1 = (n-1) - 1 + n - 1 = 2n - 3$.

Now, we find the resolving polynomial $\beta(\Gamma, x)$ of the graph Γ .

Theorem 4.3. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph with $|\Omega_1| = 1$. Then

$$\beta(\Gamma,x) = x^{2n-3}(x^3 + 2nx^2 + (n^2 + n - 1)x + n(n-1))$$

Proof. By Theorem 4.2, $\beta(\Gamma, x) = 2n-3$. In order to find the resolving polynomial, we need to calculate $s_{2n-3}, s_{2n-2}, s_{2n-1}, s_{2n}$. By [3, Proposition 3.5, p. 2399], $s_{2n-1} = 2n$ and $s_{2n} = 1$.

The sets Ω_1, Ω_2 and Ω_3 are mutually disjoint out of which Ω_2 and Ω_3 are twins set. There for we have to pick $|\Omega_2| - 1$ vertices from Ω_2 and $|\Omega_3| - 1$ vertices from Ω_3 and no vertex from Ω_1 . Therefore, the total number of choices of resolving set for Γ of cardinality 2n - 3 is

$$s_{2n-3} = {}^{1}C_{0} \times {}^{n-1}C_{n-2} \times {}^{n}C_{n-1} = (n-1)n = n(n-1)$$

Now, we calculate s_{2n-2} . In this case, we have to choose one more vertices than 2n-3. This may be from Ω_1, Ω_2 or Ω_3 . Therefore, the total number of choices of resolving sets of cardinality 2n-2 is

$$s_{2n-2} = {}^{1}C_{1} \times {}^{n-1}C_{n-2} \times {}^{n}C_{n-1} + {}^{1}C_{0} \times {}^{n-1}C_{n-1} \times {}^{n}C_{n-1} + {}^{1}C_{0} \times {}^{n-1}C_{n-2} \times {}^{n}C_{n}$$
$$\implies s_{2n-2} = (n-1)n + n + n - 1 = n^{2} + n - 1$$

Theorem 4.4. Let $\Gamma = \mathfrak{C}(D(G), D(G))$ be the commuting graph with $|\Omega_1| \geq 2$. Then

$$\beta(\Gamma, x) = x^{2n - \frac{n}{2^r} - 2} \left((n - 2^r)(2^r)^{\frac{n}{2^r} + 1} + \sum_{i=2n - \frac{n}{2^r} - 1}^{2n - 2} s_i x^i + 2n x^{\frac{n}{2^r} + 1} + x^{\frac{n}{2^r} + 2} \right)$$

where

$$s_i = (n - 2^r)(2^r)^{2n-i-1} \times \frac{n}{2^r} + 1C_{2n-i-1} + (2^r)^{2n-i} \times \frac{n}{2^r} + 1C_{2n-i-1} + (2^r)^{2n-i} \times \frac{n}{2^r} + 1C_{2n-i-1} + (2^r)^{2n-i-1} + 1C_{2n-i-1} + (2^r)^{2n-i-1} \times \frac{n}{2^r} + 1C_{2n-i-1} + (2^r)^{2n-i-1} + (2^r)^{$$

Proof. We will calculate s_j , where $j = 2n - \frac{n}{2^r} - 2$ and $s_i (2n - \frac{n}{2^r} - 1 \le i \le 2n - 2)$.

We first calculate s_j , $j = 2n - \frac{n}{2^r} - 2$. Since $|\Omega_1| \ge 2$, all the twin-sets in D(G) with cardinality greater than or equal to 2 are $\Omega_2, \Omega_1, B_1, B_{,2}, \cdots B_{\frac{n}{2^r}}$. We have to choose all vertices except any one vertex from each of twin-sets. Therefore, the total ways to form such a resolving set is

$$s_{j} = {}^{n-2^{r}}C_{n-2^{r}-1} \times {}^{2^{r}}C_{2^{r}-1} \times \underbrace{\underbrace{2^{r}C_{2^{r}-1} \times \cdots \times {}^{2^{r}}C_{2^{r}-1}}_{\frac{n}{2^{r}}-\text{times}}$$
$$s_{j} = (n-2^{r}) \times 2^{r} \times \underbrace{2^{r} \cdots \times 2^{r}}_{\frac{n}{2^{r}}-\text{times}} = (n-2^{r})\left(2^{r}\right)^{\frac{n}{2^{r}}+1}$$

Now, we calculate s_i $(2n - \frac{n}{2r} - 1 \le i \le 2n - 2)$. Let us choose a resolving set of cardinality $2n - \frac{n}{2r} - 2 + t$ with $1 \le t \le \frac{n}{2r}$, that is, we choose t more vertices than $\beta(\Gamma) = 2n - \frac{n}{2r} - 2$. Observe that except Ω_2 , all the twin-sets $\Omega_1, B_1, B_2, \dots, B_{\frac{n}{2r}}$ have cardinality 2^r . We have to choose t more vertices. First, choose one of t

vertices from Ω_2 and the rest from $\left(\bigcup_{j=1}^{\frac{n}{2^r}} B_j\right) \cup \Omega_1$ and then choose all from $\left(\bigcup_{j=1}^{\frac{n}{2^r}} B_j\right) \cup \Omega_1$. Therefore, the total choices are

$$s_{2n-\frac{n}{2^r}-2+t} = {}^{n-2^r}C_{n-2^r} \times \left(\left(\underbrace{\underbrace{2^r C_{2^r} \times \cdots \times \frac{2^r C_{2^r}}{(t-1)\text{times}}}_{(t-1)\text{times}} \right) \times \left(\underbrace{\frac{n}{2^r}+1}_{2^r}C_{t-1} \right) \right)$$
$$\times \left(\underbrace{\underbrace{2^r C_{2^r-1} \times \cdots \times \frac{2^r C_{2^r-1}}{\frac{n}{2^r}-t+2}}_{t \text{ times}} \right) \times \left(\underbrace{\underbrace{2^r C_{2^r} \times \cdots \times \frac{2^r C_{2^r}}{t \text{ times}}}_{t \text{ times}} \right)$$
$$\times \left(\underbrace{\underbrace{2^r C_{2^r-1} \times \cdots \times \frac{2^r C_{2^r-1}}{t \text{ times}}}_{(\frac{n}{2^r}-t+1)\text{ times}} \right) \times \left(\underbrace{\frac{n}{2^r}+1}_{(\frac{n}{2^r}-t+1)\text{ times}} \right)$$

 $s_{2n-\frac{n}{2^{r}}-2+t} = (2^{r})^{\frac{n}{2^{r}}-t+2} \times \frac{n}{2^{r}}+1}C_{t-1} + (n-2^{r}) \times (2^{r})^{\frac{n}{2^{r}}-t+1} \times \frac{n}{2^{r}}+1}C_{t}$ Suppose $2n-\frac{n}{2^{r}}-2+t = i$. This implies $t = i-2n+\frac{n}{2^{r}}+2$ and $2n-\frac{n}{2^{r}}-1 \le i \le 2n-2$. Therefore, the total ways of choosing resolving set of cardinality i is

$$s_{i} = (2^{r})^{\frac{n}{2^{r}} - i + 2n - \frac{n}{2^{r}} - 2 + 2} \times \frac{n}{2^{r}} + 1C_{i-2n+\frac{n}{2^{r}} + 2 - 1} + (n-2^{r}) \times (2^{r})^{\frac{n}{2^{r}} - i + 2n - \frac{n}{2^{r}} - 2 + 1} \times \frac{n}{2^{r}} + 1C_{i-2n+\frac{n}{2^{r}} + 2}$$

$$s_{i} = (2^{r})^{2n-i} \times \frac{n}{2^{r}} + 1C_{i-2n+\frac{n}{2^{r}}+1} + (n-2^{r}) \times (2^{r})^{2n-i-1} \times \frac{n}{2^{r}} + 1C_{i-2n+\frac{n}{2^{r}}+2}$$

Since ${}^{n}C_{r} = {}^{n}C_{n-r},$
 $s_{i} = (2^{r})^{2n-i} \times \frac{n}{2^{r}} + 1C_{2n-i} + (n-2^{r}) \times (2^{r})^{2n-i-1} \times \frac{n}{2^{r}} + 1C_{2n-i-1}.$

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