A nonlinear H-infinity control approach for autonomous truck and trailer systems

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Abstract: A nonlinear optimal control method is developed for autonomous truck and trailer systems. Actually,two cases are distinguished: (a) a truck and trailer system that is steered by the front wheels of its truck, (b) an autonomous fire-truck robot that is steered by both the front wheels of its truck and by the rear wheels of its trailer. The kinematic model of the autonomous vehicles undergoes linearization through Taylor series expansion. The linearization is computed at a temporary operating point that is defined at each time instant by the present value of the state vector and the last value of the control inputs vector. The linearization is based on the computation of Jacobian matrices. The modelling error due to approximate linearization is considered to be a perturbation that is compensated by the robustness of the control scheme. For the approximately linearized model of the autonomous vehicles an H-infinity feedback controller is designed. This requires the solution of an algebraic Riccati equation at each iteration of the control algorithm. The stability of the control loop is confirmed through Lyapunov analysis. It is shown that the control loop exhibits the H-infinity tracking performance which implies elevated robustness against modelling errors and external disturbances. Moreover, under moderate conditions the global asymptotic stability of the control loop is proven. Finally, to implement state estimation-based control for the autonomous vehicles, through the processing of a small number of sensor measurements, the H-infinity Kalman Filter is proposed.

Keywords: truck and trailer, autonomous fire-truck robot, autonomous vehicle, nonlinear H-infinity control, nonlinear optimal control, Riccati equation, asymptotic stability, H-infinity Kalman Filter

1 Introduction

As a consequence of the rapid development of intelligent transportation systems, the need to provide multi-body and articulated vehicles with self-steering features and autonomy has also emerged [1],[2-4]. Due to their complicated kinematic and dynamic model the problems of path planning and path following for the aforementioned types of vehicles is of elevated difficulty [5-10]. To achieve accurate tracking of reference paths and to assure stability for the vehicles' autonomous navigation system, nonlinear control approaches have been proposed [11-15]. In [16-20] one can find results on global linearization-based control of multi-body and articulated vehicles. In [21-23] the controller's design for the above mentioned type of vehicles is based on approximate linearization and the description of their kinematics or dynamics with the use of local models. Moreover, in [24-26] Lyapunov theory-based control methods are developed for such complicated vehicles. Apart from truck and trailer vehicles which are steered by the front wheels of their truck one can also considered different kinematic models where steering comes from both the front wheels of the truck and rear wheels of the trailer. Vehicles having such kinematic models exhibit improved maneuverability and a typical case is the autonomous fire-truck robot [27-31] .

In this article the problems of nonlinear optimal control and the problem of autonomous navigation of truck and trailer vehicles are considered. The kinematic model of the vehicles are formulated and the controller's design proceeds by carrying out an approximate linearization on these models around a time-varying equilibrium. The linearization procedure is based on Taylor series expansion for the vehicles' kinematic model and on the computation of the associated Jacobian matrices [32-34]. The linearization point (equilibrium) is updated at each time instant and is defined by the present value of the vehicles' state vector and the last value of the vehicles' control inputs vector. The modelling error which is due to approximate linearization and the cut-off of higher order terms in the Taylor series expansion is considered as a perturbation that is compensated by the robustness of the H-infinity control scheme [35-36].

For the linearized equivalent model of the truck and trailer vehicles an H-infinity feedback controller is designed. This is an optimal controller for the case of a system that is subject to model uncertainty and external perturbations [37-41]. H-infinity control stands for the solution of a mini-max differential game. Actually, the control inputs try to minimize a quadratic cost function associated with the deviation of the vehicle's state vector from its reference values, while the perturbations and model uncertainty terms try to maximize this cost function [42-43]. The feedback gain of the controller is based on the solution of an algebraic Riccati equation that is performed at each iteration of the control algorithm. The stability of the control loop is confirmed through Lyapunov analysis. First, it is shown that the H-infinity tracking performance criterion is satisfied. This signifies elevated robustness of the control loop against model uncertainty and exogenous disturbances. Moreover, under moderate conditions the global asymptotic stability of the control loop is proven. Finally, to implement feedback control for the autonomous truck and trailer systems when their state vectors are only partially measurable, the H-infinity Kalman Filter is proposed [44-45].

The article offers one of the most effective solutions to the nonlinear optimal control problem of (a) truck and trailer vehicles that are steered by the front wheels of their truck and (b) autonomous fire-truck robots that are steered by both the front wheels of the truck and the real wheels of the trailer. Popular approaches for industrial control such as MPC or NMPC may have questionable performance when applied to such control problems. Actually, MPC has been developed for linear dynamical systems and its use in the case of the nonlinear model of the truck and trailer systems will risk the control loop's destabilization. Besides, the convergence of NMPC is not assured either. The convergence of the method's iterative search for an optimum depends on initialization and specific parameters' selection, therefore under NMPC one cannot always guarantee a solution for the nonlinear optimal control problem of the truck and trailer system. Finally, comparing to local models-based optimal control the article's approach exhibits specific advantages: (1) in the local-models based approach linearization is performed around multiple operating points (equilibria) which are selected off-line and which are not updated in time, whereas in the article's approach there is linearization only around one single operating point which is updated at each iteration of the control algorithm, (ii) in the local-models approach there is need to perform solution of multiple Riccati equations associated with the individual models and this solution is performed offline. On the other side, in the article's approach there is need to solve one single Riccati equation and this solution is repeated at each time-step of the control algorithm. (iii) in local models-based control there is need to find a common solution for the individual Riccati equations, and one cannot assure that such a solution always exists. On the other side, in the article's approach there is need to obtain solution for one single Riccati equation and the existence of such a solution can be assured through suitable selection of the gains and coefficients that appear in it. In conclusion, comparing to local models-based control, the article's control method is computationally more efficient and is subject to less constraining assumptions.

The structure of the paper is as follows: in Section 2 the kimenatic model of the truck and trailer system that is steered by the front wheels of its truck is formulated. Moreover, through Taylor series expansion and the computation of Jacobian matrices an approximately linearized model of the vehicle is obtained. In Section 3 the kimenatic model of the autonomous fire-truck robot that is steered by both the front wheels of its truck and the rear wheels of its trailer is formulated. Moreover, through Taylor series expansion and the computation of Jacobian matrices an approximately linearized model of the vehicle is obtained. In Section 4 an H-infinity feedback controller is designed for the linearized equivalent model of the truck and trailer system. In Section 5 the stability of the H-infinity control method is proven through Lyapunov analysis. In Section 6 the H-infinity Kalman Filter is introduced for implementing state estimation-based control for the truck and trailer model. In Section 7 simulation tests are performed to further confirm the stability and robustness properties of the control scheme for the autonomous vehicle. Finally in Section 8 concluding remarks are stated.

2 Kinematic model of the truck and trailer

2.1 State-space description of the truck and trailer system

The kinematic model of the truck and trailer system which is steered by the front wheels of its truck is given by

$$\begin{pmatrix} \dot{x}^t \\ \dot{y}^t \\ \dot{\theta} \\ \dot{x}^i \\ \dot{y}^i \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} vcos(\theta) \\ vsin(\theta) \\ \omega \\ vcos(\theta-\psi)cos(\psi) \\ vcos(\theta-\psi)sin(\psi) \\ \frac{v}{L^i}sin(\theta-\psi) \end{pmatrix} \tag{1}$$

where $(x^t$,$y^t)$ are the cartesian coordinates of the truck in an inertial reference frame, θ is the heading angle of the truck formed by its transversal axis and the OX axis of the reference frame, ω is the turn rate of the truck (turn rate of the steering wheel), $(x^i$,$y^i)$ are the cartesian coordinates of the trailer, ψ^i is the heading angle of the trailer, v is the longitudinal speed of the truck, and β is the hitch point angle between the truck and the drawbar that connects the truck with the trailer. The parameters of the truck and trailer system are shown in Fig. 1.

In the diagram of Fig. 1, the following distances are defined: L^t is the distance between the front and the rear axis of the truck, L^c is the distance between the hitch point RJ and the rear axis of the trailer. while L^i is the length of the implement. The state vector of the truck and trailer system is defined as $x=[x^t,y^t,\theta,x^i,y^i,\psi]^T$ while the control inputs vector is defined as $u=[v,\omega]^T$ and thus consists of the velocity of the truck and the turn rate of the front steering wheel of the truck.

The kinematic model of the truck and trailer system is justified as follows: The velocity v of point RJ is first projected on the longitudinal axis of the trailer, thus giving $vcos(\theta-\psi)$ and next (a) it is projected on the OX axis thus giving $vcos(\theta-\psi)cos(\psi)$. This variable is the velocity of the trailer along the OX axis (b) it is projected on the OY axis thus giving $vcos(\theta-\psi)sin(\psi)$. Morover, the trailer performs a rotational motion round point RJ, with rotational speed denoted as $\dot{\psi}$. The linear velocity of point RJ that is parallel

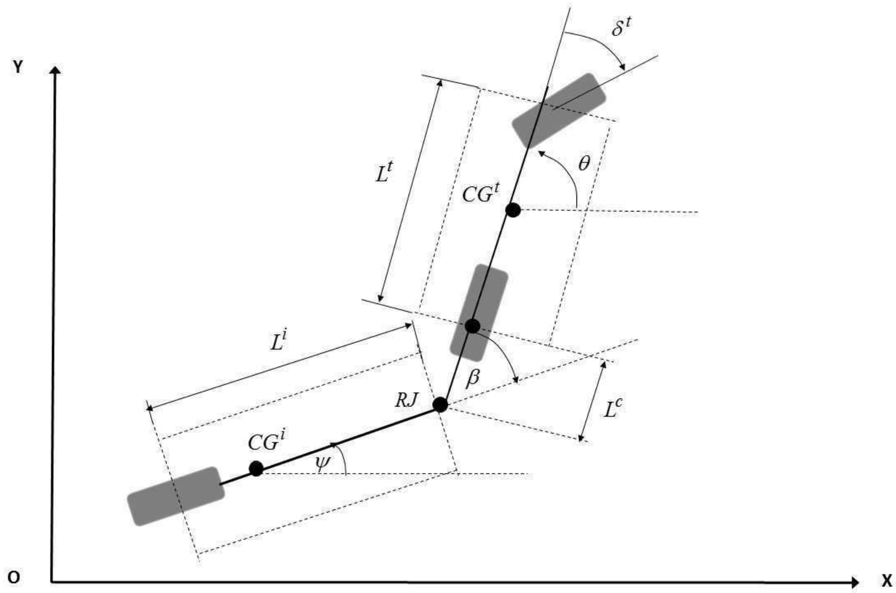


Figure 1: Kinematic model of the truck and trailer

to the transversal axis of the vehicle is given by $vsin(\theta - \psi)$. Thus, it holds: $\dot{\psi} = \frac{1}{L_i} vsin(\theta - \psi)$.

The kinematic model of the truck and trailer system is also written in the vector form:

$$\dot{x} = f(x, u) \tag{2}$$

where $x \in R^{6\times 1}$, $f \in R^{6\times 1}$ and $u \in R^{2\times 1}$. It also holds that $\beta = \theta - \psi$. With the previous definition of state variables one arrives at the following state-space description

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} u_1 cos(x_3) \\ u_1 sin(x_3) \\ u_2 \\ u_1 cos(x_3 - x_6) cos(x_6) \\ u_1 cos(x_3 - x_6) sin(x_6) \\ \frac{u_1}{L_i} sin(x_3 - x_6) \end{pmatrix} \tag{3}$$

2.2 Approximate linearization of the truck and trailer model

Approximate linearization is performed to the kinematic model of the truck and trailer system being steered by the fornt wheels of its truck, around a temporary equilibrium x^* which is re-computed at each iteration of the control algorithm. The method is based on Taylor series expansion and on the calculation of the associated Jacobian matrices, while the equilibrium consists of the present value of the system's state vector x^* and of the last value of the control inputs vector u^* that was exerted on it. Thus one has the linearization point (x^*, u^*). Using that the kinematic model of the system is $\dot{x} = f(x, u)$ the following linearized description is obtained

$$\dot{x} = Ax + Bu + \tilde{d} \tag{4}$$

where $\tilde{d}$ is the linearization error and the associated Jacobian matrices are:

$$A = \nabla_x f(x,u) \mid_{(x^*,u^*)} \quad B = \nabla_u f(x,u) \mid_{(x^*,u^*)} \tag{5}$$

The elements of the Jacobian matrices are

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_6} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \cdots & \frac{\partial f_6}{\partial x_6} \end{pmatrix} \quad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \cdots & \cdots \\ \cdots & \cdots \\ \frac{\partial f_6}{\partial u_1} & \frac{\partial f_6}{\partial u_2} \end{pmatrix} \tag{6}$$

With the previous definition of the Jacobian matrices one finds

The first row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_1}{\partial x_1} = 0$, $\frac{\partial f_1}{\partial x_2} = 0$, $\frac{\partial f_1}{\partial x_3} = -u_1 sin(x_3)$, $\frac{\partial f_1}{\partial x_4} = 0$, $\frac{\partial f_1}{\partial x_5} = 0$ and $\frac{\partial f_1}{\partial x_6} = 0$.

The second row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_2}{\partial x_1} = 0$, $\frac{\partial f_2}{\partial x_2} = 0$, $\frac{\partial f_2}{\partial x_3} = u_1 cos(x_3)$, $\frac{\partial f_2}{\partial x_4} = 0$, $\frac{\partial f_2}{\partial x_5} = 0$ and $\frac{\partial f_2}{\partial x_6} = 0$.

The third row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_3}{\partial x_1} = 0$, $\frac{\partial f_3}{\partial x_2} = 0$, $\frac{\partial f_3}{\partial x_3} = 0$, $\frac{\partial f_3}{\partial x_4} = 0$, $\frac{\partial f_3}{\partial x_5} = 0$ and $\frac{\partial f_3}{\partial x_6} = 0$.

The fourth row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_4}{\partial x_1} = 0$, $\frac{\partial f_4}{\partial x_2} = 0$, $\frac{\partial f_4}{\partial x_3} = -sin(x_3 - x_6)cos(x_6)u_1$, $\frac{\partial f_4}{\partial x_4} = 0$, $\frac{\partial f_4}{\partial x_5} = 0$ and $\frac{\partial f_4}{\partial x_6} = [sin(x_3 - x_6)cos(x_6) - cos(x_3 - x_6)sin(x_6)]u_1$.

The fifth row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_5}{\partial x_1} = 0$, $\frac{\partial f_5}{\partial x_2} = 0$, $\frac{\partial f_5}{\partial x_3} = -sin(x_3 - x_6)sin(x_6)u_1$, $\frac{\partial f_5}{\partial x_4} = 0$, $\frac{\partial f_5}{\partial x_5} = 0$ and $\frac{\partial f_5}{\partial x_6} = [sin(x_3 - x_6)sin(x_6) + cos(x_3 - x_6)cos(x_6)]u_1$.

The sixth row of the Jacobian matrix $A = \nabla_x f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_6}{\partial x_1} = 0$, $\frac{\partial f_6}{\partial x_2} = 0$, $\frac{\partial f_6}{\partial x_3} = \frac{1}{L_i} cos(x_3 - x_6)u_1$, $\frac{\partial f_6}{\partial x_4} = 0$, $\frac{\partial f_6}{\partial x_5} = 0$ and $\frac{\partial f_6}{\partial x_6} = -\frac{1}{L_i} cos(x_3 - x_6)u_1$.

In a similar manner one finds

The first row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_1}{\partial u_1} = cos(x_3)$, $\frac{\partial f_1}{\partial u_2} = 0$,

The second row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_2}{\partial u_1} = sin(x_3)$, $\frac{\partial f_2}{\partial u_2} = 0$,

The third row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_3}{\partial u_1} = 0$, $\frac{\partial f_3}{\partial u_2} = 1$,

The fourth row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_4}{\partial u_1} = cos(x_3 - x_6)cos(x_6)$, $\frac{\partial f_4}{\partial u_2} = 0$,

The fifth row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_5}{\partial u_1} = cos(x_3 - x_6)sin(x_6)$, $\frac{\partial f_6}{\partial u_2} = 0$,

The sixth row of the Jacobian matrix $B = \nabla_u f(x,u) \mid_{(x^*,u^*)}$ is $\frac{\partial f_6}{\partial u_1} = \frac{1}{L^i} sin(x_3 - x_6)$, $\frac{\partial f_6}{\partial u_2} = 0$,

3 Kinematic model of the autonomous fire-truck robot

3.1 State-space description of the autonomous fire-truck robot

The autonomous fire-truck comprises the truck (cab) and the trailer, as shown in Fig. 2. The vehicle is steered by both the front wheels of its truck and by the rear wheels of its trailer. The main parameters of the model of the autonomous fire-truck are as follows [27],[31]: (x_0, y_0): are the coordinates of the center of the front axle of the truck, (x_1, y_1) are the coordinates of the center of the rear axle of the truck, (ϕ_1) is the angle of the steering wheels of the truck with respect to the longitudinal axis of the truck, θ_1 is the angle between the longitudinal axis of the truck and the Ox axis of the inertial coordinates system, (x_2, y_2) are the coordinates of the center of the real axle of the trailer, ϕ_2 is the angle of the steering wheels of the trailer with respect to the longitudinal axis od the truck, θ_2 its the angle between the longitudinal axis of the trailer and the Ox axis of the inertial coordinates system. The length of the cab is denoted as L_0 and the length of the trailer is denoted as L_1.

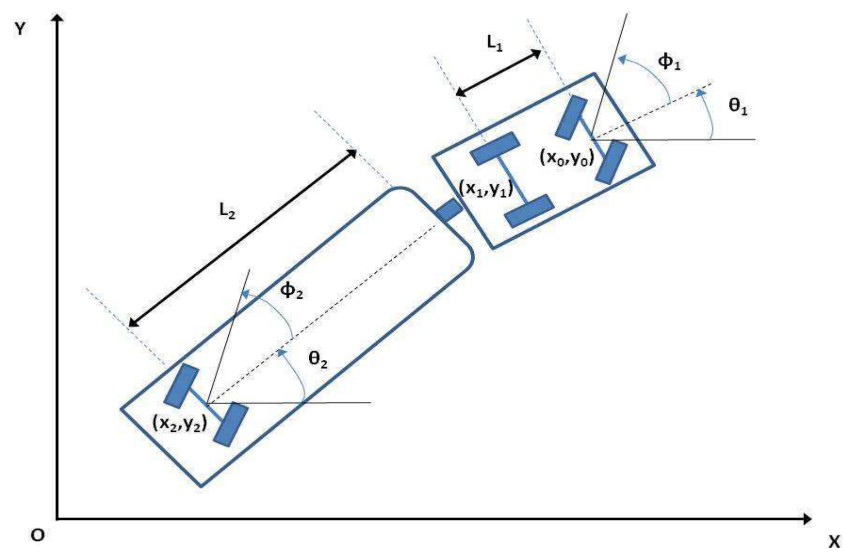


Figure 2: Diagram of the autonomous fire-truck robot which comprises the truck (cab) and the trailer

The kinematic model of the autonomous fire-truck is given by [27],[31]:

$$\dot{x} = g_1(x)u_1 + g_2(x)u_2 + g_3(x)u_3 \tag{7}$$

where the control inputs of the vehicle are defined as u_1: is the forward driving velocity of the truck v_f, u_2 is the steering speed of the front wheels of the cab $\dot{\phi}_1$ and u_3 is the steering speed of the rear wheels of the trailer $\dot{\phi}_2$. Vector fields $g_1(x) \in R^{6\times1}$, $g_2(x) \in R^{6\times1}$ and $g_3(x) \in R^{6\times1}$ are defined as:

$$g_1(x) = \begin{pmatrix} cos(\theta_1) \\ sin(\theta_1) \\ 0 \\ \frac{1}{L_o} tan(\phi_2) \\ 0 \\ -\frac{1}{L_1}\frac{1}{cos(\phi_2)} sin(\phi_2 - \theta_1 + \theta_2) \end{pmatrix} \quad g_2(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad g_3(x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{8}$$

The state vector of the system is defined as $x = [x, y, \phi_1, \theta_1, \phi_1, \theta_2]^T$ or $x = [x_1, x_2, x_3, x_4, x_5, x_6]^T$. Using this state variables' notation and the model of Eq. (7) and Eq. (8) one obtains the following form of the state-space equations

$$\dot{x}_1 = cos(x_4)u_1 \tag{9}$$

$$\dot{x}_2 = sin(x_4)u_1 \tag{10}$$

$$\dot{x}_3 = u_2 \tag{11}$$

$$\dot{x}_4 = \frac{1}{L_o} tan(x_3) \tag{12}$$

$$\dot{x}_5 = u_3 \tag{13}$$

$$\dot{x}_6 = -\frac{1}{L_1}\frac{1}{cos(x_5)} sin(x_5 - x_4 + x_6) \tag{14}$$

3.2 Approximate linearization of the autonomous fire-truck robot

By applying first-order Taylor series expansion in the kinematic model of the fire-truck

$$\dot{x} = g_1(x)u_1 + g_2(x)u_2 + g_3(x)u_3 \tag{15}$$

one obtains the following state-space description

$$\dot{x} = Ax + Bu + \tilde{d} \tag{16}$$

where matrices A and B rely on the computation of the system's Jacobians

$$A = \nabla_x g_1(x)u_1 \mid_{(x^*,u^*)} + \nabla_x g_2(x)u_1 \mid_{(x^*,u^*)} + \nabla_x g_3(x)u_3 \mid_{(x^*,u^*)} \tag{17}$$

$$B = [g_1(x) \ g_2(x) \ g_3(x)] \mid_{(x^*,u^*)} \tag{18}$$

First, the following Jacobian matrix is computed

$$\nabla_x g_1(x) = \nabla_x \begin{pmatrix} cos(x_4) \\ sin(x_4) \\ 0 \\ \frac{1}{L_0} tan(x_3) \\ 0 \\ -\frac{1}{L_1}\frac{1}{cos(x_5)} sin(x_5 - x_4 + x_6) \end{pmatrix} \tag{19}$$

1st row of the Jacobian matrix $\nabla_x g_1(x)$: $\frac{\partial g_{11}}{\partial x_1} = 0$, $\frac{\partial g_{11}}{\partial x_2} = 0$, $\frac{\partial g_{11}}{\partial x_3} = 0$, $\frac{\partial g_{11}}{\partial x_4} = -sin(x_4)$, $\frac{\partial g_{11}}{\partial x_5} = 0$ and $\frac{\partial g_{11}}{\partial x_6} = 0$.

2nd row of the Jacobian matrix $\nabla_x g_1(x)$: $\frac{\partial g_{12}}{\partial x_1}=0$, $\frac{\partial g_{12}}{\partial x_2}=0$, $\frac{\partial g_{12}}{\partial x_3}=0$, $\frac{\partial g_{12}}{\partial x_4}=cos(x_4)$, $\frac{\partial g_{12}}{\partial x_5}=0$ and $\frac{\partial g_{12}}{\partial x_6}=0$.

3rd row of the Jacobian matrix $\nabla_x g_1(x)$: $\frac{\partial g_{13}}{\partial x_1}=0$, $\frac{\partial g_{13}}{\partial x_2}=0$, $\frac{\partial g_{13}}{\partial x_3}=0$, $\frac{\partial g_{13}}{\partial x_4}=0$, $\frac{\partial g_{13}}{\partial x_5}=0$ and $\frac{\partial g_{13}}{\partial x_6}=0$.

4th row of the Jacobian matrix $\nabla_x g_1(x)$: $\frac{\partial g_{14}}{\partial x_1}=0$, $\frac{\partial g_{14}}{\partial x_2}=0$, $\frac{\partial g_{14}}{\partial x_3}=\frac{1}{L_0}\frac{1}{cos^2(x_3)}$, $\frac{\partial g_{14}}{\partial x_4}=0$, $\frac{\partial g_{14}}{\partial x_5}=0$ and $\frac{\partial g_{14}}{\partial x_6}=0$.

5th row of the Jacobian matrix $\nabla_x g_1(x)$: $\frac{\partial g_{15}}{\partial x_1}=0$, $\frac{\partial g_{15}}{\partial x_2}=0$, $\frac{\partial g_{15}}{\partial x_3}=0$, $\frac{\partial g_{15}}{\partial x_4}=0$, $\frac{\partial g_{15}}{\partial x_5}=0$ and $\frac{\partial g_{15}}{\partial x_6}=0$.

6th row of the Jacobian matrix $\nabla_x g_1(x)$: $\nabla_x g_1(x)$: $\frac{\partial g_{16}}{\partial x_1}=0$, $\frac{\partial g_{16}}{\partial x_2}=0$, $\frac{\partial g_{16}}{\partial x_3}=0$, $\frac{\partial g_{16}}{\partial x_4}=\frac{1}{L_1}\frac{1}{cos(x_5)}cos(x_5-x_4+x_6)$, $\frac{\partial g_{16}}{\partial x_5}=-\frac{1}{L_1}\frac{sin(x_5)}{cos^2(x_5)}sin(x_5-x_4+x_6)-\frac{1}{L_1}\frac{1}{cos(x_5)}cos(x_5-x_4+x_6)$ and $\frac{\partial g_{16}}{\partial x_6}=-\frac{1}{L_1}\frac{1}{cos(x_5)}cos(x_5-x_4+x_6)$.

Next, about the rest of the Jacobian matrices of the system one has: $\nabla_x g_2(x)=0{\in}R^{6\times 6}$ and $\nabla_x g_3(x)=0{\in}R^{6\times 6}$.

4 The nonlinear H-infinity control

4.1 Mini-max control and disturbance rejection

The generic nonlinear kinematic model of the truck and trailer systems is in the form

$$\dot{x}=f(x,u) \quad x{\in}R^n,\ u{\in}R^m \tag{20}$$

Linearization of the truck and trailer systems is performed at each iteration of the control algorithm round its present operating point $(x^*,u^*)=(x(t),u(t-T_s))$. The linearized equivalent of these systems is described by

$$\dot{x}=Ax+Bu+L\tilde{d} \quad x{\in}R^n,\ u{\in}R^m,\ \tilde{d}{\in}R^q \tag{21}$$

where matrices A and B are obtained from the computation of the Jacobians

$$A=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}|_{(x^*,u^*)} \quad B=\begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix}|_{(x^*,u^*)} \tag{22}$$

and vector $\tilde{d}$ denotes disturbance terms due to linearization errors. The problem of disturbance rejection for the linearized model that is described by

$$\begin{matrix} \dot{x}=Ax+Bu+L\tilde{d} \\ y=Cx \end{matrix} \tag{23}$$

where $x{\in}R^n$, $u{\in}R^m$, $\tilde{d}{\in}R^q$ and $y{\in}R^p$, cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term $\tilde{d}$. The disturbance term $\tilde{d}$ apart from modeling (parametric) uncertainty and external perturbation terms can also represent noise terms of any distribution.

In the H_∞ control approach, a feedback control scheme is designed for trajectory tracking by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner. The disturbances' effects are incorporated in the following quadratic cost function:

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2\tilde{d}^T(t)\tilde{d}(t)]dt, \quad r, \rho > 0 \tag{24}$$

The significance of the negative sign in the cost function's term that is associated with the perturbation variable $\tilde{d}(t)$ is that the disturbance tries to maximize the cost function $J(t)$ while the control signal $u(t)$ tries to mininize it. The physical meaning of the relation given above is that the control signal and the disturbances compete to each other within a mini-max differential game. This problem of min-max optimization can be written as

$$min_u max_{\tilde{d}} J(u, \tilde{d}) \tag{25}$$

The objective of the optimization procedure is to compute a control signal $u(t)$ which can compensate for the worst possible disturbance, that is externally imposed to the system of the truck and trailer system. However, the solution to the mini-max optimization problem is directly related to the value of the parameter ρ. This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

4.2 H-infinity feedback control

For the linearized system given by Eq. (23) the cost function of Eq. (24) is defined, where the coefficient r determines the penalization of the control input and the weight coefficient ρ determines the reward of the disturbances' effects. It is assumed that (i) The energy that is transferred from the disturbances signal $\tilde{d}(t)$ is bounded, that is $\int_0^\infty \tilde{d}^T(t)\tilde{d}(t)dt < \infty$, (ii) matrices $[A, B]$ and $[A, L]$ are stabilizable, (iii) matrix $[A, C]$ is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \tag{26}$$

with $K = \frac{1}{r}B^T P$, where P is a positive semi-definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P = 0 \tag{27}$$

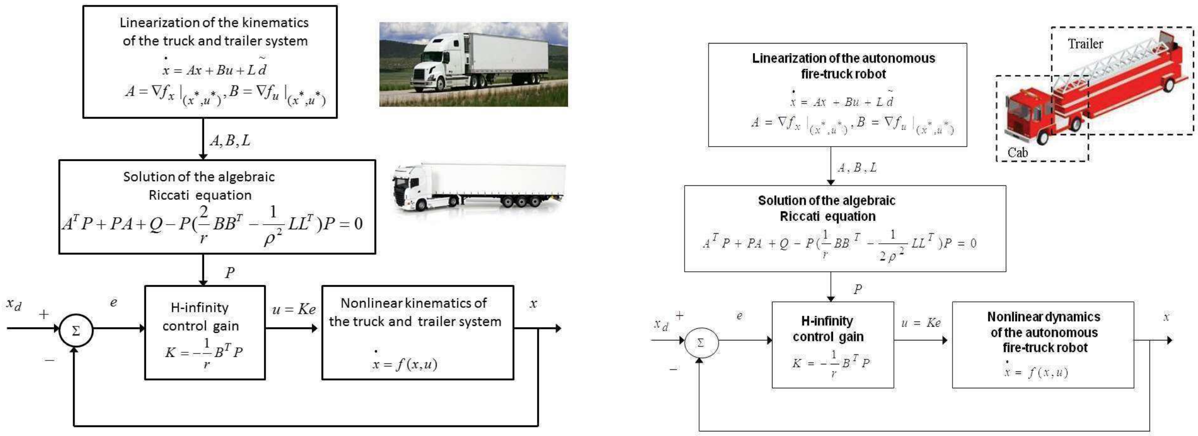
where Q is also a positive definite symmetric matrix. The worst case disturbance is given by $\tilde{d}(t) = \frac{1}{\rho^2}L^T Px(t)$. This equation is obtained by solving the optimal control problem for the case that the system receives as input only the disturbance $\tilde{d}(t)$. The diagrams of the considered control loop is depicted in Fig. 3.

4.3 The role of Riccati equation coefficients in H_∞ control robustness

The parameter ρ in Eq. (24), is an indication of the closed-loop system robustness. If the values of $\rho > 0$ are excessively decreased with respect to r, then the solution of the Riccati equation is no longer a positive definite matrix. Consequently there is a lower bound ρ_{min} of ρ for which the H_∞ control problem has a solution. The acceptable values of ρ lie in the interval $[\rho_{min}, \infty)$. If ρ_{min} is found and used in the design of the H_∞ controller, then the closed-loop system will have increased robustness. Unlike this, if a value $\rho > \rho_{min}$ is used, then an admissible stabilizing H_∞ controller will be derived but it will be a suboptimal one. The Hamiltonian matrix

$$H = \begin{pmatrix} A & -(\frac{1}{r}BB^T - \frac{1}{\rho^2}LL^T) \\ -Q & -A^T \end{pmatrix} \tag{28}$$

provides a criterion for the existence of a solution of the Riccati equation Eq. (27). A necessary condition for the solution of the algebraic Riccati equation to be a positive semi-definite symmetric matrix is that H



(a) **(b)**

Figure 3: (a) Diagram of the nonlinear optimal control scheme for the truck and trailer system that is steered by the front wheels of its truck, (b) Diagram of the nonlinear optimal control scheme for the autonomous fire-truck robot that is steered by both the front wheels of its truck and the real wheels of its trailer

has no imaginary eigenvalues [37].

5 Lyapunov stability analysis

Through Lyapunov stability analysis it will be shown that the proposed nonlinear control scheme assures H_∞ tracking performance for the control loop of the truck and trailer systems, that is (a) the truck and trailer that is steered by the front wheels of its truck, (b) the autonomous fire-truck robot that is steered by both the front wheels of its truck and by the rear wheels of its trailer. Moreover, under moderate conditions asymptotic stability is proven and convergence to the reference setpoints is achieved. The tracking error dynamics for the truck and trailer systems is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \tag{29}$$

where in such vehicles' case $L = I \in R^{6\times 6}$ with I being the identity matrix. Variable $\tilde{d}$ denotes model uncertainties and external disturbances of the truck and trailer models, as well as sensors' measurement noise. The following Lyapunov equation is considered

$$V = \frac{1}{2}e^T P e \tag{30}$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{gathered} \dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}eP\dot{e} \Rightarrow \\ \dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \end{gathered} \tag{31}$$

$$\begin{gathered} \dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \end{gathered} \tag{32}$$

$$\dot{V}=\frac{1}{2}e^TA^TPe+\frac{1}{2}u^TB^TPe+\frac{1}{2}\tilde{d}^TL^TPe+\\ \frac{1}{2}e^TPAe+\frac{1}{2}e^TPBu+\frac{1}{2}e^TPL\tilde{d} \tag{33}$$

The previous equation is rewritten as

$$\dot{V}=\frac{1}{2}e^T(A^TP+PA)e+(\frac{1}{2}u^TB^TPe+\frac{1}{2}e^TPBu)+\\ +(\frac{1}{2}\tilde{d}^TL^TPe+\frac{1}{2}e^TPL\tilde{d}) \tag{34}$$

Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP+PA=-Q+P(\frac{2}{r}BB^T-\frac{1}{\rho^2}LL^T)P \tag{35}$$

Moreover, the following feedback control law is applied to the system

$$u=-\frac{1}{r}B^TPe \tag{36}$$

By substituting Eq. (35) and Eq. (36) one obtains

$$\dot{V}=\frac{1}{2}e^T[-Q+P(\frac{2}{r}BB^T-\frac{1}{2\rho^2}LL^T)P]e+\\ +e^TPB(-\frac{1}{r}B^TPe)+e^TPL\tilde{d}\Rightarrow \tag{37}$$

$$\dot{V}=-\frac{1}{2}e^TQe+(\frac{2}{r}PBB^TPe-\frac{1}{2\rho^2}e^TPLL^T)Pe\\ -\frac{1}{r}e^TPBB^TPe)+e^TPL\tilde{d} \tag{38}$$

which after intermediate operations gives

$$\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPLL^TPe+e^TPL\tilde{d} \tag{39}$$

or, equivalently

$$\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPLL^TPe+\\ +\frac{1}{2}e^TPL\tilde{d}+\frac{1}{2}\tilde{d}^TL^TPe \tag{40}$$

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d}+\frac{1}{2}\tilde{d}L^TPe-\frac{1}{2\rho^2}e^TPLL^TPe\leq\frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{41}$$

Proof: The binomial $(\rho\alpha-\frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2a^2+\frac{1}{\rho^2}b^2-2ab\geq 0\Rightarrow\frac{1}{2}\rho^2a^2+\frac{1}{2\rho^2}b^2-ab\geq 0\Rightarrow\\ ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2\Rightarrow\frac{1}{2}ab+\frac{1}{2}ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2 \tag{42}$$

The following substitutions are carried out: $a=\tilde{d}$ and $b=e^TPL$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^TL^TPe+\frac{1}{2}e^TPL\tilde{d}-\frac{1}{2\rho^2}e^TPLL^TPe\leq\frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{43}$$

Eq. (43) is substituted in Eq. (40) and the inequality is enforced, thus giving

$$\dot{V}\leq-\frac{1}{2}e^TQe+\frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{44}$$

Eq. (44) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T\dot{V}(t)dt\leq-\frac{1}{2}\int_0^T||e||_Q^2dt+\frac{1}{2}\rho^2\int_0^T||\tilde{d}||^2dt\Rightarrow\\ 2V(T)+\int_0^T||e||_Q^2dt\leq 2V(0)+\rho^2\int_0^T||\tilde{d}||^2dt \tag{45}$$

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d \tag{46}$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d \tag{47}$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in Eq. (30) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e|e^T Pe \leq 2V(0) + \rho^2 M_d\}$.

According to the above and with the use of Barbalat's Lemma one obtains $lim_{t\to\infty} e(t) = 0$.

The outline of the global stability proof is that at each iteration of the control algorithm the state vector of the truck and trailer vehicles converges towards the temporary equilibrium and the temporary equilibrium in turn converges towards the reference trajectory [1]. Thus, the control scheme exhibits global asymptotic stability properties and not local stability. Assume the i-th iteration of the control algorithm and the i-th time interval about which a positive definite symmetric matrix P is obtained from the solution of the Riccati equation appearing in Eq. (35). By following the stages of the stability proof one arrives at Eq. (44) which shows that the H-infinity tracking performance criterion holds. By selecting the attenuation coefficient ρ to be sufficiently small and in particular to satisfy $\rho^2 < ||e||_Q^2 / ||\tilde{d}||^2$ one has that the first derivative of the Lyapunov function is upper bounded by 0. Therefore for the i-th time interval it is proven that the Lyapunov function defined in Eq (30) is a decreasing one. This signifies that between the beginning and the end of the i-th time interval there will be a drop of the value of the Lyapunov function and since matrix P is a positive definite one, the only way for this to happen is the Euclidean norm of the state vector error e to be decreasing. This means that comparing to the beginning of each time interval, the distance of the state vector error from 0 at the end of the time interval has diminished. Consequently as the iterations of the control algorithm advance the tracking error will approach zero, and this is a global asymptotic stability condition.

6 Robust state estimation with the use of the H_∞ Kalman Filter

The control loop for the truck and trailer systems can be implemented with the feedback of a partially measurable state vector and by processing only a small number of state variables. To reconstruct the missing information about the state vector of the autonomous vehicles it is proposed to use a filtering scheme and based on it to apply state estimation-based control [39]. The recursion of the H_∞ Kalman Filter, can be formulated in terms of a *measurement update* and a *time update* part

Measurement update:

$$\begin{array}{l} D(k) = [I - \theta W(k) P^-(k) + C^T(k) R(k)^{-1} C(k) P^-(k)]^{-1} \\ K(k) = P^-(k) D(k) C^T(k) R(k)^{-1} \\ \hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)] \end{array} \tag{48}$$

Time update:

$$\begin{array}{l} \hat{x}^-(k+1) = A(k)x(k) + B(k)u(k) \\ P^-(k+1) = A(k)P^-(k)D(k)A^T(k) + Q(k) \end{array} \tag{49}$$

where it is assumed that parameter θ is sufficiently small to assure that the covariance matrix $P^-(k) - \theta W(k) + C^T(k)R(k)^{-1}C(k)$ will be positive definite. When $\theta = 0$ the H_∞ Kalman Filter becomes equivalent to the standard Kalman Filter. One can measure only a part of the state vector of the system of the truck and trailer systems, such as the cartesian coordinates of the vehicle, and can estimate through filtering the rest of the state vector elements.

7 Simulation tests

7.1 Path tracking by the autonomous truck and trailer system

The performance of the proposed nonlinear optimal control scheme for the autonomous truck and trailer vehicle that is steered by the front wheels of its truck has been tested in the case of tracking of different reference setpoints. The control scheme exhibited fast and accurate tracking of the reference paths. The computation of the feedback control gain required the solution of the algebraic Riccati equation given in Eq. (27), at each iteration of the control algorithm. The obtained results are depicted in Fig. 4 to Fig. 8. The measurement units for the state variables of the vehicle's model were in the SI system (position coordinates measured in m). The H-infinity Kalman Filter has provided estimates of the state vector of the system by processing measurements of a subset of state variables, such as: $x_3 = \theta$, $x_4 = x^i$, $x_5 = y^i$ and $x_6 = \psi$. It can be noticed that the H-infinity controller achieved fast and accurate convergence to the reference setpoints for all elements of the vehicle's state-vector. Moreover, the variations of the control inputs, that is of the truck's velocity and of the truck's steering angle were smooth.

Yet computationally simple, the proposed H_∞ control scheme has an excellent performance. Comparing to the control of the truck and trailer system that can be based on global linearization methods the presented nonlinear H-infinity control scheme is equally efficient in setpoint tracking while also retaining optimal control features [39]. The tracking accuracy of the presented nonlinear optimal (H_∞) control method has been monitored in the case of several reference setpoints. The obtained results are given in Table I.

Table I: RMSE of the truck and trailer's state variables

path	RMSE θ	RMSE x^i	RMSE y^i	RMSE ψ
1	0.0001	0.0017	0.0017	0.0001
2	0.0023	0.0211	0.0084	0.0019
3	0.0004	0.0548	0.0831	0.0032
4	0.0054	0.0706	0.0984	0.0081
5	0.0084	0.0667	0.0998	0.0165

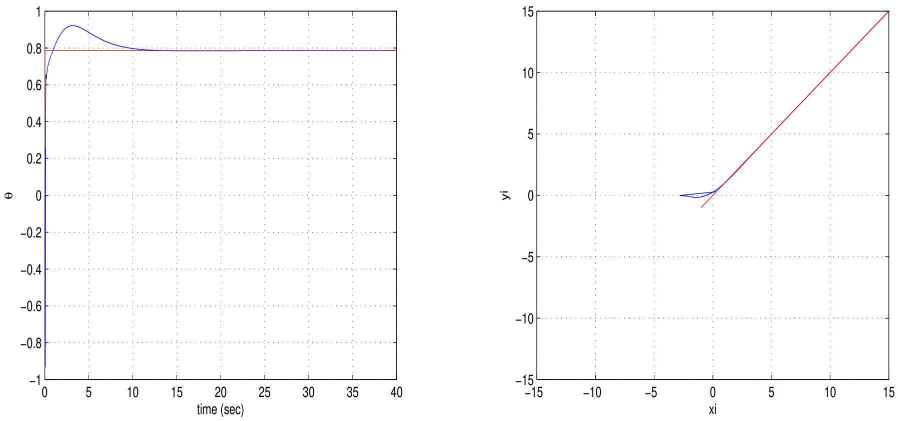
The tracking performance of the nonlinear H-infinity control method for the model of the truck and trailer system and under uncertainty, imposing a change equal to $\Delta a\%$ to the length of the implement L^i, is outlined in Table II. It can be noticed that despite model perturbations the tracking accuracy of the control method remained satisfactory.

Table II: RMSE of state variables under model disturbance

Δa	RMSE θ	RMSE x	RMSE y	RMSE ψ
0 %	0.0023	0.0211	0.0084	0.0019
25 %	0.0036	0.0217	0.0056	0.0018
50 %	0.0047	0.0223	0.0053	0.0018
75 %	0.0057	0.0228	0.0053	0.0018
100 %	0.0068	0.0232	0.0060	0.0018

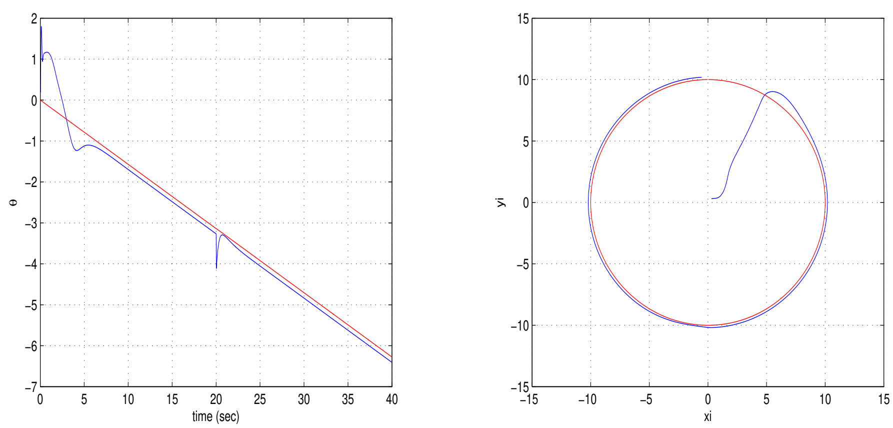
7.2 Path tracking by the autonomous fire-truck robot

The efficiency of the proposed nonlinear optimal control method for the model of the autonomous fire-truck robot that is steered by both the front wheels of its truck and the rear wheels of its trailer has been tested



(a) **(b)**

Figure 4: (a) tracking of reference setpoint 1 (red-line) by the heading angle θ of the truck (blue line), (b) tracking of reference path (red line) on the xy-plane by the center of the rear wheel axis of the trailer (blue line)



(a) **(b)**

Figure 5: (a) tracking of reference setpoint 2 (red-line) by the heading angle θ of the truck (blue line), (b) tracking of reference path (red line) on the xy-plane by the center of the rear wheel axis of the trailer (blue line)

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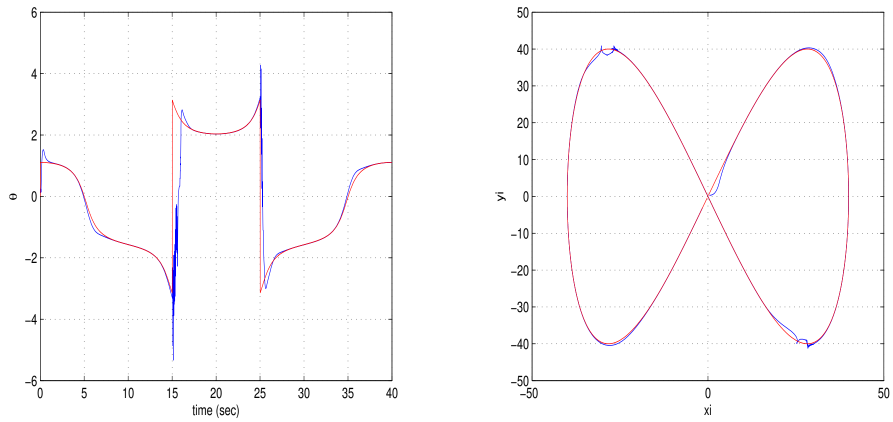
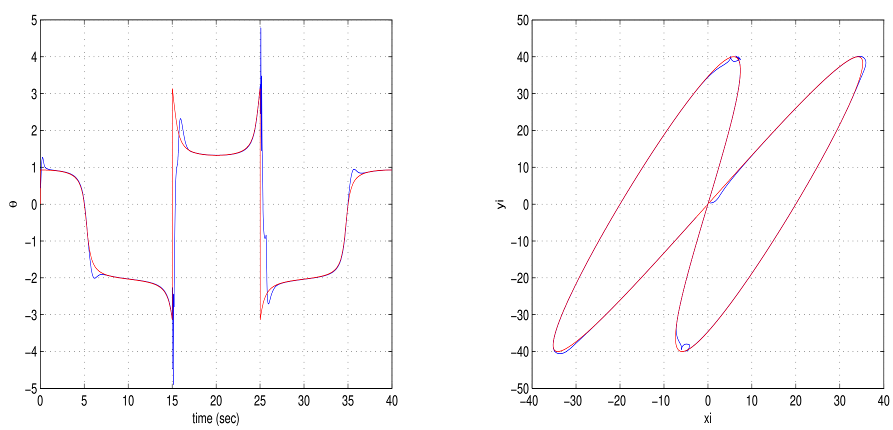


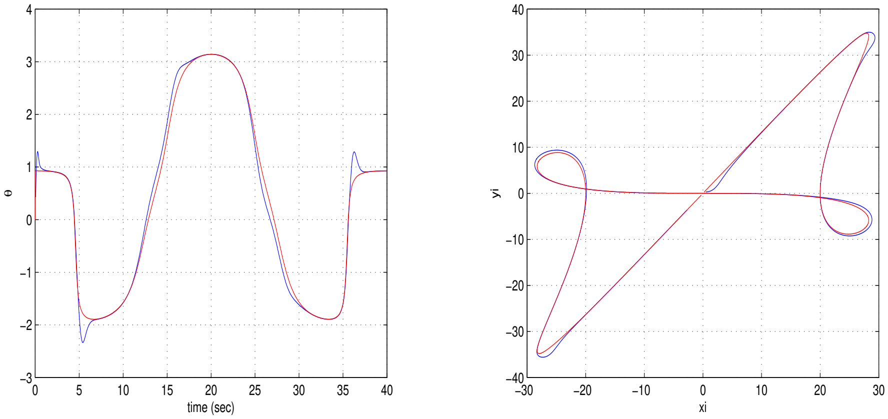
(a) **(b)**

Figure 6: (a) tracking of reference setpoint 3 (red-line) by the heading angle θ of the truck (blue line), (b) tracking of reference path (red line) on the xy-plane by the center of the rear wheel axis of the trailer (blue line)



(a) **(b)**

Figure 7: (a) tracking of reference setpoint 4 (red-line) by the heading angle θ of the truck (blue line), (b) tracking of reference path (red line) on the xy-plane by the center of the rear wheel axis of the trailer (blue line)



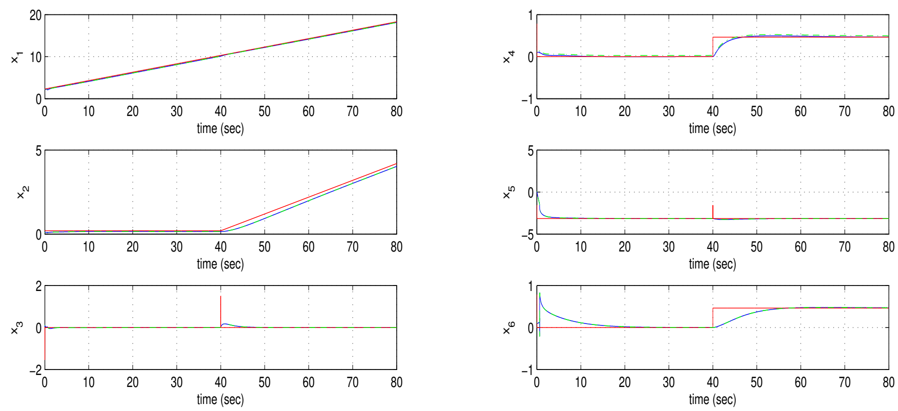
(a) **(b)**

Figure 8: (a) tracking of reference setpoint 5 (red-line) by the heading angle θ of the truck (blue line), (b) tracking of reference path (red line) on the xy-plane by the center of the rear wheel axis of the trailer (blue line)

through simulation experiments. The obtained results are depicted in Fig. 9 to Fig. 20.These demonstrate that fast and accurate tracking of the reference setpoints is achieved by all state vector elements of the robotic vehicle. The variations of the control inputs were moderate. For the computation of the control signal the algebraic Riccati equation of Eq. (35) had to be repetitively solved at each time-step of the control algorithm. The control inputs were applied on the initial nonlinear model of the vehicle and not on the equivalent linearized description of it that was obtained through the system's Jacobian matrices.

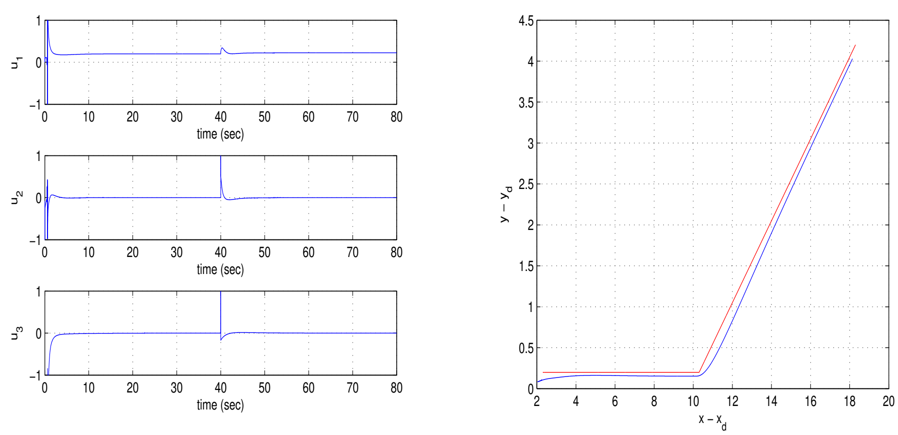
The transient performance of the control scheme relied on the control loop gains r and ρ and well as on the value of the diagonal elements of matrix Q. As explained above, the smallest value of the attenuation coefficient ρ for which the algebraic Riccati equation of Eq. (35) admits a solution, is the one that provides maximum robustness to the control system. It is also noted that by using the H-infinity Kalman Filter a state estimation-based implementation of the control method has been achieved. This allows the reliable functioning of the control loop after receiving measurements from a small number of sensors. Actually, the H-infinity Kalman Filter can be fed with measurements of the following state variables, such as: $x_1 = x$, $x_2 = y$ and $x_6 = \theta_2$. In the simulation diagrams, the real values of the state vector elements are depicted in blue, the estimated values are plotted in green and the related reference setpoints are printed in red.

As noted, the proposed nonlinear optimal control method for the truck and trailer model (that is steered by the front wheels of its truck), as well as for the autonomous fire-truck robot (that is steered by both the front wheels of its truck and the rear wheels of its trailer), was based on an approximate linearization of the vehicles' kinematics. The advantages that the proposed control method exhibits are outlined as follows: (i) it is applied directly on the nonlinear dynamical model of the truck and trailer systems and not on an equivalent linearized description of it, (ii) It avoids the elaborated linearizing transformations (diffeomorphisms) which can be met in global linearization-based control methods for autonomous vehicles (iii) the controller is designed according to optimal control principles which implies the best trade-off between precise tracking of the reference setpoints on the one side and moderate variations of the control inputs on the other side (iv) the control method exhibits robustness to parametric uncertainty, modelling errors as well as to external perturbations (v) the computational implementation of the control method is



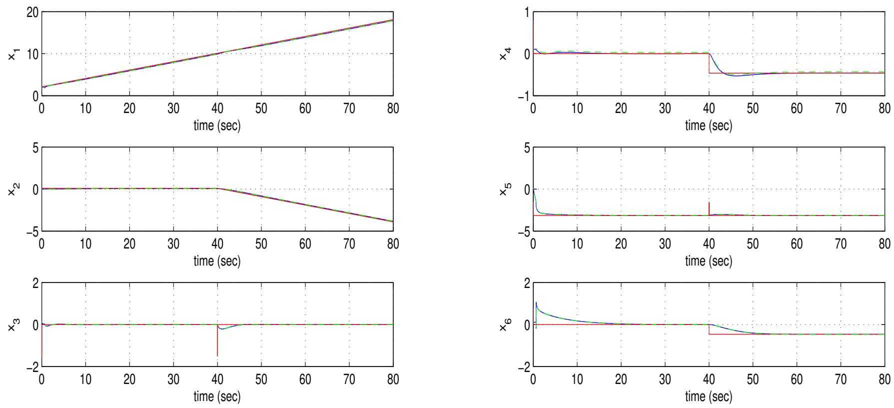
(a) **(b)**

Figure 9: Tracking of setpoint 1 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



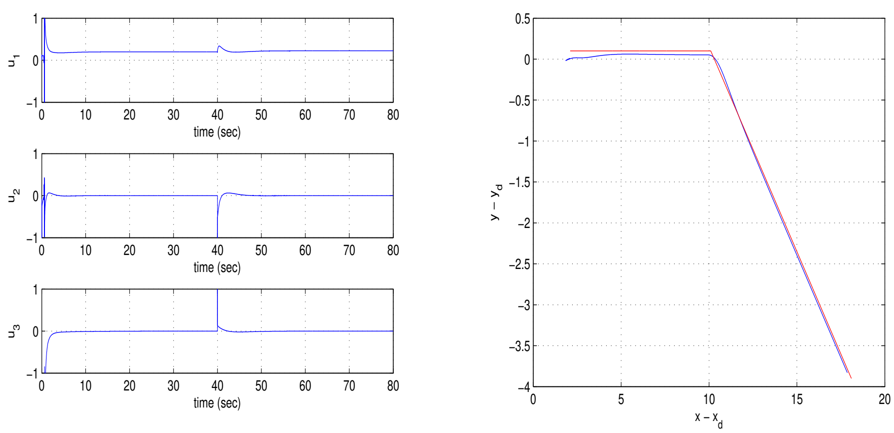
(a) **(b)**

Figure 10: Tracking of setpoint 1 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane



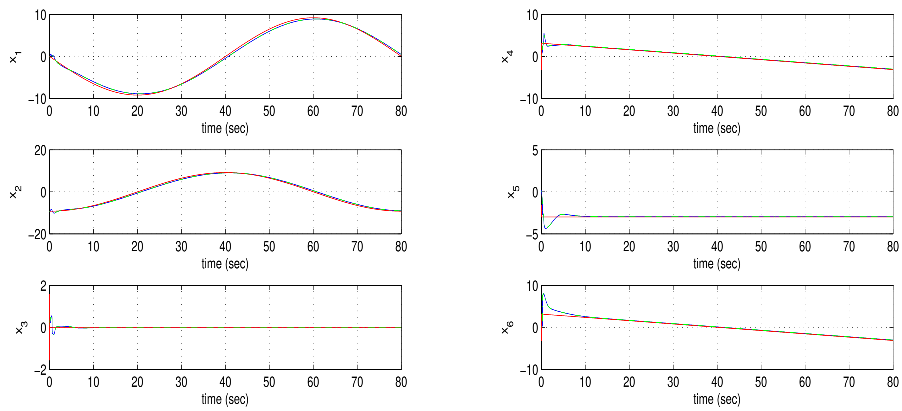
(a) **(b)**

Figure 11: Tracking of setpoint 2 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



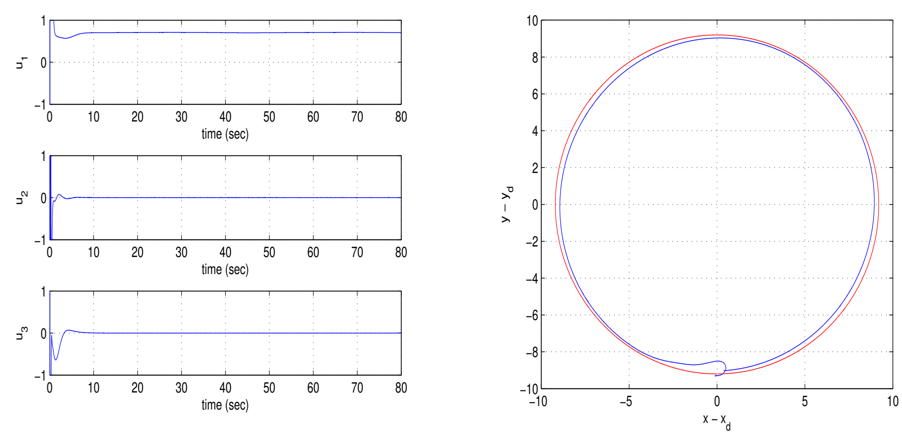
(a) **(b)**

Figure 12: Tracking of setpoint 2 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane



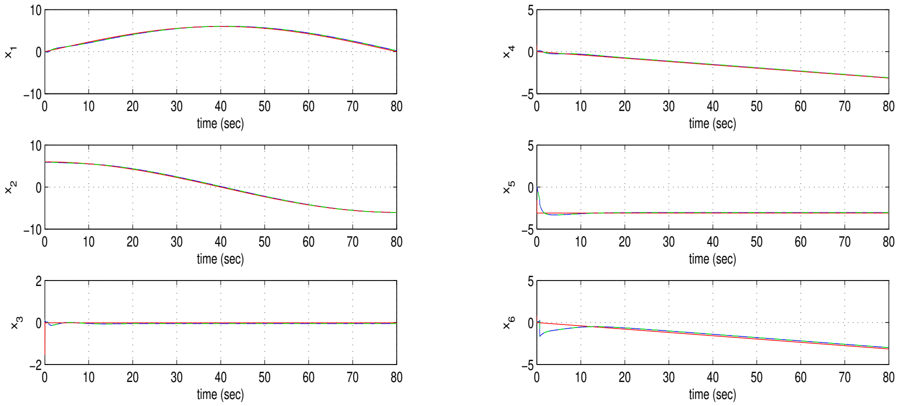
(a) **(b)**

Figure 13: Tracking of setpoint 3 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



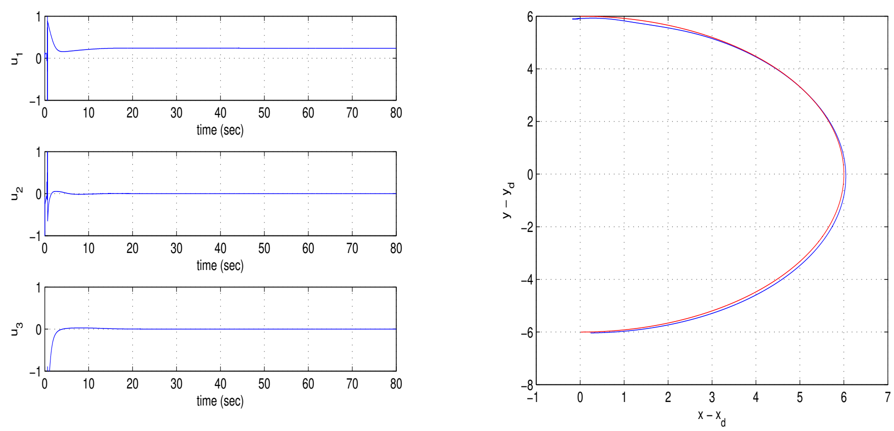
(a) **(b)**

Figure 14: Tracking of setpoint 3 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane



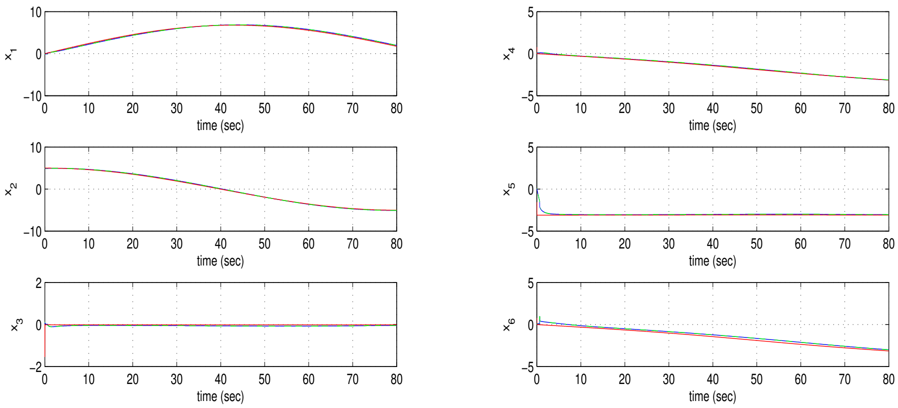
(a) **(b)**

Figure 15: Tracking of setpoint 4 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



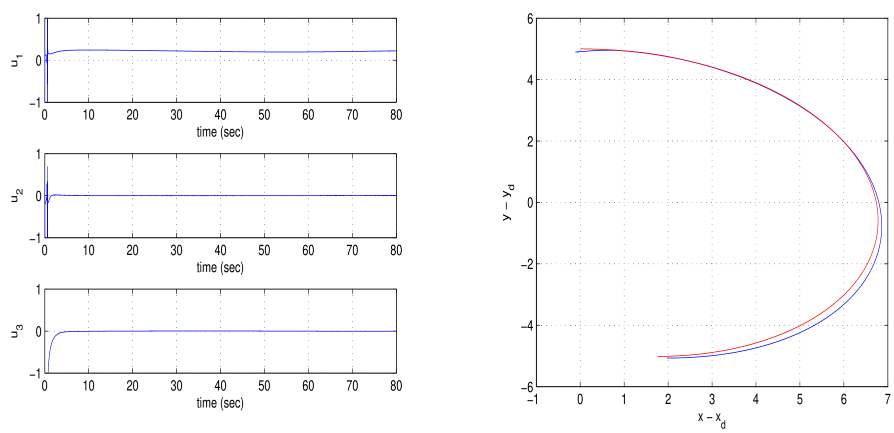
(a) **(b)**

Figure 16: Tracking of setpoint 4 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane



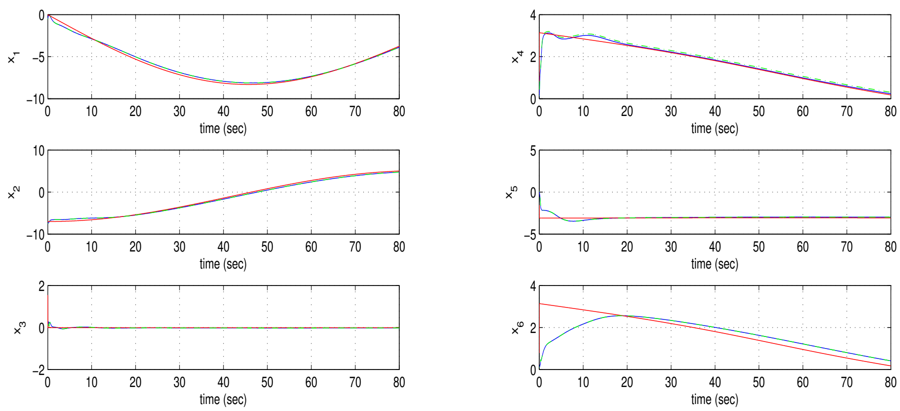
(a) **(b)**

Figure 17: Tracking of setpoint 5 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



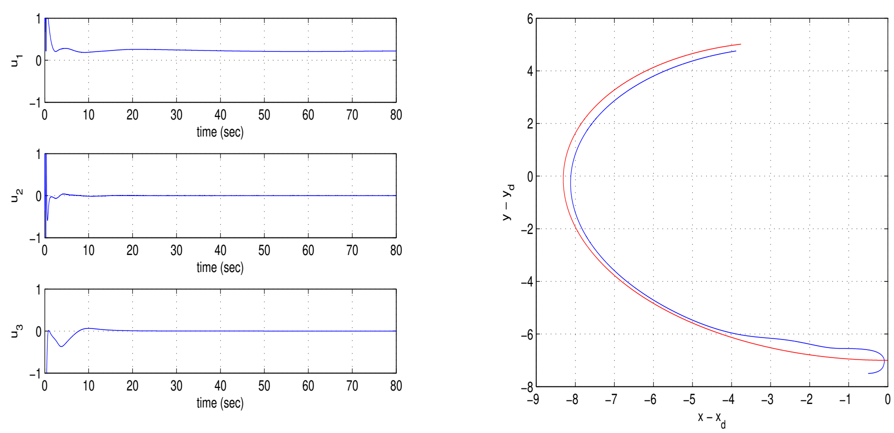
(a) **(b)**

Figure 18: Tracking of setpoint 5 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane



(a) **(b)**

Figure 19: Tracking of setpoint 6 for the autonomous fire-truck robot (a) convergence of state variables x_1 to x_3 to their reference setpoints (b) convergence of state variables x_4 to x_6 to their reference setpoints (red line: setpoint, blue line: real value, green line: estimated value)



(a) **(b)**

Figure 20: Tracking of setpoint 6 for the autonomous fire-truck robot (a) variation of the control inputs u_1 to u_3 (b) path followed by the autonomous fire-truck robot on the xy-plane

simple since it requires only the solution of an algebraic Riccati equation.

Remark 1: The nonlinear optimal approach proposed in this article for solving the control and trajectory tracking problem for the autonomous truck and trailer systems exhibits advantages to other optimal control methods such MPC and NMPC. For instance, MPC can be applied at specific operating points where the dynamic model of the truck and trailer systems is taken to be linear. However, in reality the model of the articulated vehicle is a nonlinear one therefore the MPC approach lacks a global stability proof and is likely to result into an unstable control loop. Besides, the NMPC control method is based on an iterative search of the optimum which is a procedure of non-confirmed convergence and again the global stability features of this method are questionable. Moreover, global linearization methods for the truck and trailer models cannot be directly applied and require elaborated and intuitive state variables transformations (diffeomorphisms). The latter approach may require the application of dynamic feedback linearization as it usually happens in the global linearization based control of underactuated dynamical systems.

Remark 2: H-infinity control is typically addressed to linear dynamical systems and stands for the solution of the optimal control problem under model uncertainty and external perturbations. However, H-infinity control cannot be applied to nonlinear dynamical systems. The article demonstrates that after approximate linearization for the system's dynamic model with the use Taylor series expansion, around a temporary operating point (equilibrium) which is recomputed at each iteration of the control method, then one can also solve the optimal (H-infinity) control problem for the linearized equivalent model of the system. The concept of the proposed control method is entirely novel and stands for a genuine contribution to the area of nonlinear control: (i) the model of the articulated vehicles is approximately linearized round a time-varying equilibrium which is re-computed at each step of the control method (ii) with the proposed H-infinity control law the system's state-space vector is made to converge to the temporary equilibrium (iii) the temporary equilibrium is made to convergence to the system's reference parh. Thus implicitly the system's state vector is also made to track the reference setpoint and the tracking error gets asymptotically eliminated.

Remark 3: The article's nonlinear optimal (H-infinity) control approach can be classified among purely nonlinear control methods. Actually, a control scheme is classified as nonlinear if it is addressed to systems with nonlinear dynamics, which are described by a nonlinear state-space model. Even if the control signal contains in parts of it linear feedback of the state vector's error the control method is classified as nonlinear if it can stabilize and eliminate the tracking error for the initial nonlinear dynamics of the system. A typical example of such a case of controllers are the so-called global linearization-based control schemes, such as Lie algebra-based control and differential flatness-theory-based control. Such controllers rely on the transformation of the nonlinear dynamics of the system into an equivalent linear form for which one can solve the control and stabilization problem through the design of a linear feedback-based controller. Next, with the use of an inverse nonlinear transformation one finds the control inputs which are applied to the initial nonlinear system. In a similar concept the article's nonlinear optimal (H-infinity) control method can be considered to be purely nonlinear. The method applies H-infinity control and the related linear state error feedback to the model that is obtained from the Taylor-series-based approximate linearization of the initial nonlinear state-space description of the system. The computed control inputs are applied directly to the initial nonlinear model of the system and as proven through Lyapunov analysis global asymptotic stability is achieved. Finally, there should be no comparison of the article's control method to PID control. Unlike PID control the method is of proven global stability, does not require heuristic tuning and remains reliable in the change of operating points as well as to external perturbations. PID control can be efficiently tuned for linear dynamical systems and only around local operating points while its use in the case of the nonlinear model of the truck and trailer systems will risk the control loop's stability.

Remark 4: (i) Autonomous navigation of the vehicles is practically achieved if control can be implemented through the vehicles' kinematic model. In such a case one computes the velocity of the vehicles and the angle of the steering wheels that make the cartesian coordinates (position) of a specific reference point on the vehicles (e.g. center of gravity) follow precisely the designated reference path. Once this first problem is solved one can use the dynamic model of the vehicles to compute the forces and torques that should be exerted on them (generated by its engine and their wheels) so to achieve convergence of the vehicles' velocity and heading angle to those values that are used as control inputs in the kinematics-based control problem of the first stage. Consequently, if one demonstrates solution of the control problem of such vehicles with the use of their kinematic model, the problem of their autonomous navigation is considered to have been solved in a complete manner. (ii) uncertainty in the kinematic model of the vehicles can be due to wrong information about its dimensions as well as due to additive input disturbance affecting the control inputs, of finally die to perturbations affecting the sensors' measurements.

Remark 5: The linearization of the vehicles' kinematic model is performed through first-order Taylor series expansion and through the computation of the associated Jacobian matrices. The partial derivatives which stand for the elements of these Jacobian matrices are computed off-line, while at each iteration of the control algorithm the numerical values of these terms are updated. Such a procedure does not incur much computational burden to the control scheme. The computation of the feedback gain of the H-infinity controller requires the solution of an algebraic Riccati equation which takes place at each iteration of the control method. The solution of this Riccati equation in Matlab and with the use of an i7 Intel processor takes place in miliseconds and in time interval which is much shorter than the samploing period of the control algorithm (0.1KHz). Consequently, the implementation of the proposed nonlinear optimal control method is time efficient.

8 Conclusions

The article has proposed a solution to the nonlinear optimal control problem of autonomous truck and trailer vehicles. Two different types of truck and trailer vehicles were examined: (a) truck and trailers which are steered by the front wheels of their truck (b) autonomous fire-truck robots which are steered by both the front wheels of their ruck and the rear wheels of their trailer. The kinematic model of the vehicles has undergone linearization through Taylor series expansion round a temporary equilibrium and with the computation of the associated Jacobian matrices. The equilibrium was redefined at each iteration of the control algorithm by the value of the vehicles' state vector and the last value of the vehicles' control input vector. The modelling error that was due to approximate linearization was considered to be a disturbance term that was compensated by the robustness of the control algorithm.

For the equivalent linearized model of the vehicles an H-infinity feedback controller was designed. The controller's feedback gain was calculated at each sampling instant through the solution of an algebraic Riccati equation. The H-inifnity controller is the optimal controller one can obtain for the vehicles' model under modelling uncertainty and external perturbations. The stability of the control scheme was analyzed with the use of the Lyapunov method. It has been demonstrated that the control loop satisfies the H-infinity tracking performance criterion and this is indicative of the controller's robustness. Moreover, it has been shown that under moderate conditions the global asymptotic stability of the control loop is assured. Finally, to implement state estimation-based control through the processing of measurements from a small number of sensors the use of the H-infinity Kalman Filter has been proposed.

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