Loops in Algol 60 and in Category Theory

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According to [6] 4.6.5, after exit from a for statement due to exhaustion of the for list, the value of the controlled variable is undefined.

Probably the idea lying behind this rule was that a controlled variable is intended to act merely as a counter inside the for statement and therefore not to be defined before entrance into it.

But, as a matter of fact, it results that the controlled variable may very well be defined before entrance into the for statement. In this case, the for statement causes, among other things, the cancellation of the controlled variable. E.g. the following for statement causes y to become x+y and x to be erased

<u>for</u> x:=x <u>while</u> x>0 <u>do</u> <u>begin</u> x:=x-1;y:=y+1 <u>end</u>

This feature of ALGOL 60 is certainly some kind of a curiosity, but it might have some significance if one considers that a quite analogous feature appears also in categorial function theory. In [1], in fact, a repetition operator is defined which transforms any function $f:N^{r+1}\rightarrow N^{r+1}$ into a function $f^{\nabla}:N^{r+1}\rightarrow N^{r}$ such that

$$f^{\nabla}(n_0 n_1 \dots n_r) = n_1^{(k)} \dots n_r^{(k)}$$

if there are sequences

$$n_0^{(i)} n_1^{(i)} \dots n_r^{(i)} \quad \text{with } 0\geq i\geq k$$

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$$n_0^{(0)} n_1^{(0)} \dots n_r^{(0)} = n_0 n_1 \dots n_r$$

$$n_0^{(i)} n_1^{(i)} \dots n_r^{(i)} = f(n_0^{(i-1)} n_1^{(i-1)} \dots n_r^{(i-1)}) \qquad \text{for } i>0$$

$$n_0^{(i)} \neq 0 \text{ while } i>k, \text{ whereas } n_0^{(k)} = 0 \text{ (see also [3].}$$

Recursive functions of type $N^r \to N^s$ are introduced in [1], [2], [4] and [5] as algebraic counterparts (in category theory) of a statement (of a programming language) insofar as they transform a sequence of numbers (the values of the variables defined before entrance into the statement) into a sequence of numbers (the values of the variables defined after exit from the statement).

From this point of view the statement of the example above corresponds precisely to the function

$$(P \times S)^{\nabla}$$

where P is the predecessor function, S the successor function and $P \times S$ is their cartesian product, and in general

<u>for</u> $x_0 := x_0$ <u>while</u> $x_0 > 0$ <u>do</u> F

corresponds to the function f^{∇}, provided the statement F corresponds to f. In particular the statement

<u>for</u> $x := x$ <u>while</u> $x > 0$ <u>do</u> $x := x-1$

causes just the cancellation of x.

Correspondingly it holds that

$$P^{\nabla} = \Pi$$

where $\Pi : N \to N$ is the cancellation function (see [1] and [4]) which maps from the sequence of one integer to the null sequence of intergers and f^{∇} is defined also for $f : N \to N$.

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