Education

FINAL PROBLEM SET EXCERPTS

CS 292-S (Algebraic Algorithms)

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1. Integrate

$$\log(\log(x) + 1) + 1/(\log(x) + 1)$$

with respect to x using the Risch integration algorithm. Show all work.

ANSWER: $x \log(\log x + 1) + c)$

2. Using Richardson's Zero-equivalence algorithm, prove that

$$\log((e^{2x} - 1)/(e^{x}+1) + 1) - x = 0$$

Show all work. You may not use the fact that

$$e^{2x} = (e^{x})^{2}.$$

3. Compute the first three terms in the Taylor series expansion of $f(x)$ about $x = 0$ when

$$f(x) = e^{x}/(\sin^{3}(x) + \cos^{3}(x))$$

by doing the appropriate arithmetic on power series for e^x, $\sin(x)$, $\cos(x)$. Prove, as a first step, the ratio of two power series

$$U = u_0 + u_1.x + \ldots, \quad V = v_0 + v_1.x + \ldots$$

is W where

$$w_n = (u_n - \textstyle\sum_{0\le k<n} w_k v_{n-k})/v_0$$

$n = 0, 1, \ldots$ (where $v_0 \neq 0$).

ANSWER: $1 + x + 2x^2 + \ldots$

4. How is the cost of evaluation of a polynomial at a point affected by representation in factored form.

ANSWER: Depends. Consider $(x-1)^8$, $x^8 - 1$.

5. Propose as many credible interpretations of the following expressions as you can. Which would you prefer?

a) DIFF(SUM(F(X),X,0,INF),X);

b) DIFF(SUM(X+I,I,0,INF),X);

c) DIFF(SUM(Y↑I,I,0,X),X);

d) DIFF(SUM(I↑X,I,0,INF),X);

e) SUM(F(I),I,0,-2);

ANSWER:

a) 0.

c) $Y^{X+1} \log(Y)/(Y-1)$

e) -F(-1) preserves some useful properties.

6. Develop a quick test for a multivariate polynomial being "square-free". This test will return one of three indications:

a) square free
b) not square-free
c) cannot tell without more work.

Use no operations which produce coefficients larger than some (machine-dependent, in general) bound.

7. a) Prove 3 is a primitive eighth root of unity in Z_{41}.

b) Using a balanced representation for the integers mod 41, compute an 8 point Discrete Fourier Transform on the coefficients of

$$p = 3x^2 - 4x + 1,$$

a polynomial over Z_{41}.

Then cube each term in the transformed sequence, and compute the inverse transform, (i.e., $p^3 \bmod 41$). For the forward transform, use two techniques: multiplication by the Vandermonde matrix, and the "fast" algorithm. For the inverse, use whichever technique you find simpler. Show all essential computation.