Erratum to “Splitting an Operator: Algebraic Modularity Results for logics with Fixpoint Femantics”

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In [Vennekens et al. 2006], we defined a class of stratified auto-epistemic theories and made the claim that the models of such theories under a number of different semantics (namely, (partial) expansions, (partial) extensions, Kripke-Kleene model and well-founded model) can be constructed in an incremental way, following the stratification of the theory. However, it turns out that this result only holds as long as the constructed models are *consistent*. This can be demonstrated by the following example.

Let T be the theory $\{\neg p; q \wedge \neg Kp\}$. This T is stratifiable with respect to the partition $\Sigma_0 = \{p\}$, $\Sigma_1 = \{q\}$ of its alphabet. The least precise partial expansion, i.e., the Kripke-Kleene model, of this theory is the pair of possible world structures $(\{\{q\}\}, \{\})$. However, the stratified construction would first consider only the formula $\neg p$ in alphabet Σ_0 and construct the exact pair $(\{\{\}\}, \{\{\}\})$, i.e., p would be known to be false. Next, it would consider the formula $q \wedge \neg Kp$ in alphabet Σ_1 and substitute Kp by its truth value in the possible world structure $\{\{\}\}$ for Σ_0, thus arriving at $q \wedge \neg \mathbf{f}$. The only partial expansion of this formula is the exact pair $(\{\{q\}\}, \{\{q\}\})$. Combining these possible world structures for Σ_0 and Σ_1, the stratified construction would finally yield $(\{\{q\}\}, \{\{q\}\})$, i.e., p is known to be false and q is known to be true, which is clearly not equal to the Kripke-Kleene model of T.

The origin of this error lies in our definition of the operator $\tilde{\mathcal{D}}_T^u$ (Definition 4.15 on page 790). This was defined as mapping every pair $(\tilde{P}, \tilde{S}) \in \tilde{\mathcal{B}}_\Sigma$ to the possible world structure $\tilde{Q} \in \tilde{\mathcal{W}}_\Sigma$ for which, for all $i \in I$:

$$\tilde{Q}(i) = \{X \in \mathcal{I}_{\Sigma_i} \mid \forall \phi \in T_i : \mathcal{H}_{\overline{\kappa}(\tilde{P},\tilde{S}),X}(\phi) = \mathbf{t}\}.$$

In the middle of page 791, we then claimed that each such operator is by construction stratifiable. This is however not the case, because the value of $\tilde{\mathcal{D}}_T^u(\tilde{P}, \tilde{S})$ in the ith level is not necessarily independent of the value of $\tilde{P}$ in the levels $j \succ i$. Indeed, if for some j, $\tilde{P}(j) = \{\}$, then this will always cause $\kappa(\tilde{P})$ to be $\{\}$, even if $\kappa(\tilde{P}|_{\preceq i}) \neq \{\}$.

This problem can be solved by defining $\tilde{\mathcal{D}}_T^u$ in the following way instead:

$$\tilde{\mathcal{D}}_T^u(\tilde{P}, \tilde{S}) = \{X \in \mathcal{I}_{\Sigma_i} \mid \forall \phi \in T_i : \mathcal{H}_{\overline{\kappa}(\tilde{P}|_{\preceq i},\tilde{S}|_{\preceq i}),X}(\phi) = \mathbf{t}\}.$$

That is, $\tilde{\mathcal{D}}_T^u(\tilde{P}, \tilde{S})$ is the set $Mod(T_i\langle\overline{\kappa}(\tilde{P}|_{\preceq i}, \tilde{S}|_{\preceq i})\rangle)$ of all models of the propositional theory $T_i\langle\overline{\kappa}(\tilde{P}|_{\preceq i}, \tilde{S}|_{\preceq i})\rangle$. This operator is now indeed stratifiable. In the corrected version of our paper[1] use this operator to show that *consistent* partial expansions and partial extensions can be incrementally constructed, that is, we prove the following theorem.

THEOREM 1. *Let T be a stratifiable auto-epistemic theory. . An element (P,S) of $\mathcal{B}_\Sigma$ is a consistent partial expansion (or consistent partial extension, respectively) of T iff there exists a $(\tilde{P},\tilde{S}) \in \tilde{\mathcal{B}}_\Sigma$, such that $\overline{\kappa}(\tilde{P},\tilde{S}) = (P,S)$ and for all $i \in I$, $(\tilde{P},\tilde{S})(i)$ is a consistent fixpoint (or consistent stable fixpoint) of the component operator $(\tilde{\mathcal{D}}_T)_i^{(\tilde{P},\tilde{S})|_{\prec i}}$.*

These component operators $(\tilde{\mathcal{D}}_T)_i^{(\tilde{P},\tilde{S})|_{\prec i}}$ can be constructed from the original theory T as outlined in our paper. In particular, this theorem also implies that a possible world structure P is a consistent expansion (consistent extension, respectively) of T if and only if there exists a $\tilde{P} \in \tilde{\mathcal{W}}_\Sigma$, such that $\overline{\kappa}(\tilde{P}) = P$ and for all $i \in I$, $\tilde{P}(i)$ is a consistent expansion (consistent extension) of the theory $[T_i]\langle(\tilde{P},\tilde{P})|_{\prec i}\rangle$.

We now define a class of theories, for which (partial) expansions or (partial) extensions cannot be inconsistent.

Definition 2. A theory T is *permaconsistent* if every propositional theory T' that can be constructed from T by replacing all occurrences of modal literals by $\mathbf{t}$ or $\mathbf{f}$ is consistent.

For such theories, all (partial) expansions and (partial) extensions can be incrementally constructed. We therefore obtain the following theorem.

THEOREM 3. *Let T be a stratifiable and permaconsistent theory. An element (P,S) of $\mathcal{B}_\Sigma$ is an expansion, a partial expansion, an extension, a partial extension, the Kripke-Kleene model or the well-founded model of a stratifiable theory T iff there exists a $(\tilde{P},\tilde{S}) \in \tilde{\mathcal{B}}_\Sigma$, such that $\overline{\kappa}(\tilde{P},\tilde{S}) = (P,S)$ and for all $i \in I$, $(\tilde{P},\tilde{S})(i)$ is, respectively, an exact fixpoint, a fixpoint, an exact stable fixpoint, a stable fixpoint, the least fixpoint or the least stable fixpoint of the component operator $(\tilde{\mathcal{D}}_T)_i^{(\tilde{P},\tilde{S})|_{\prec i}}$.*

In our original paper, we claimed that our results generalize work by Gelfond and Przymusinska [1992] and Niemelä and Rintanen [1994] on stratification in auto-epistemic logic. Because the theories considered in these works are all permaconsistent, this claim is still valid, as Theorem 3 shows. From the above results for auto-epistemic logic, corresponding theorems for default logic can be derived, in the same way as in our original paper. Our claim that these results generalize work from Turner [1996] is also still valid. Indeed, Turner's theorem considers only consistent extensions and is therefore generalized by a result that follows from Theorem 1.

REFERENCES

GELFOND, M. AND PRZYMUSINSKA, H. 1992. On consistency and completeness of autoepistemic theories. *Fundamenta Informaticae 16,* 1, 59–92.

NIEMELÄ, I. AND RINTANEN, J. 1994. On the impact of stratification on the complexity of nonmonotonic reasoning. *Journal of Applied Non-Classical Logics 4,* 2.

[1]This can be downloaded at the ACM Digital Library.

Turner, H. 1996. Splitting a default theory. In *Proc. Thirteenth National Conference on Artificial Intelligence and the Eighth Innovative Applications of Artificial Intelligence Conference*. AAAI Press, 645–651.

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