Mathematical Modeling of Incentive Policies in P2P Systems

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ABSTRACT

In order to stimulate cooperation among nodes in P2P systems, some form of incentive mechanism is necessary so as to encourage service contribution. Hence, designing and evaluating the stability, robustness and performance of incentive policies is extremely critical. In this paper, we propose a general mathematical framework to evaluate the stability and evolution of a family of shared history based incentive policies. To illustrate the utility of the framework, we present two incentive policies and show why one incentive policy can lead to a total system collapse while the other is stable and operates at the optimal point. One can use this mathematical framework to design and analyze various incentive policies and verify whether they match the design objectives of the underlying P2P systems.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Distributed Systems; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Design, Economics, Theory

Keywords

incentive, stability, strategy, learning, peer-to-peer, reciprocative

1. INTRODUCTION

The emergence of the Internet as a global platform for communication has sparkled the research and development of many large-scale networked systems and applications. Often, these systems require individual devices or nodes to *cooperate* so as to achieve good performance. For example, (a) in wireless ad-hoc networks, wireless nodes rely on other nodes to forward their packets so as to reach the destination nodes, (b) in P2P file sharing networks, peers rely on each other to perform uploading services so they can complete the file downloading in a reasonable amount of time, (c) in P2P streaming applications, each peer relies on other peers to provide local storage and uploading services so that peers can have the appropriate playback rate. All these networked systems hinge on an important factor: *cooperation is necessary* for the system to perform properly. However, individual devices or nodes have different and sometimes, competing interests, and cooperation implies a higher operating cost. Therefore, it is paramount to have some form of incentive mechanisms so as to encourage cooperation.

There are a number of proposals on how to provide incentives for networks or distributed systems. One proposal is that a node needs to pay for the services it receives, and payment can be made via the micro-payment approach [4]. However, this requires a centralized server to mediate all the transactions in the system.

Without using a centralized server, peers have to collect information themselves so as to make a proper decision. There are two categories of such incentive mechanisms. The first one is called the *private history based mechanism:* a peer provides a service to a requester based on the requesters' past generosity to this peer. This incentive mechanism is easy to implement but for a large system, it is very unlikely that two peers have many previous encounters, especially for a dynamic, high churn system. Hence, nodes may not have the necessary information to make a proper decision. Moreover, one has to address the issue of asymmetric interest [1]. Another category is the *shared history based mechanism:* peers can use others' past experience to infer a requester's reputation. This approach is scalable and robust. There has been some simulation study on this approach [1], but in general, there is a lack of rigorous mathematical analysis to understand why this approach is robust or stable for that matter.

The aim of this paper is to provide a general mathematical framework to study and evaluate a large class of incentive policies for networking applications. The contributions of our paper are:

- We provide a general mathematical framework to analyze the incentive protocols (Section 2).
- To illustrate the mathematical framework, we present two incentive policies (Section 3) and derive their performance measures and stability conditions (Section 4).
- We carry out performance evaluation to illustrate the performance gain of using strategy adaptation and under what situations the system may collapse (Section 5).

The outline of this paper is as follows. In Section 2, we present the mathematical framework in analyzing incentive policies for P2P systems. In Section 3, we present two incentive policies and show how to apply the mathematical framework to analyze them. In Section 4, we derive the performance measures (e.g., system gain, expected gain for individual strategy) as well as the stability conditions for the two given incentive policies. In Section 5, we present the performance evaluation on these two policies, in particular, demonstrate the evolution of the systems and state the reasons why one policy can lead to complete system collapse while the other is robust and operates at a good performance point. Related work is given in Section 6 and Section 7 concludes.

2. MATHEMATICAL MODEL

In this section, we present a general mathematical model to study different incentive policies for P2P systems. The goals of the model are twofold: (a) to study whether a given incentive policy is stable[1], and (b) to study the performance measures (e.g., expected services received, expected services contributed, . . .,etc) for a given incentive policy.

For mathematical tractability, we make the following assumptions

- **Finite strategies:** Given an incentive policy $\mathcal{P}$ which has a finite strategy set $\mathcal{P} = \{s_1, s_2, \ldots, s_n\}$ where s_i is the i^{th} strategy. All users in a P2P system can use any $s \in \mathcal{P}$. A peer using strategy s_i is of type s_i.

- **Service model:** The system runs in discrete time slots. At the beginning of each time slot, each peer randomly selects another peer in the system and requests for a service. Let $g_i(j)$ be the probability that a peer of type s_i will provide a service to a peer of type s_j. Here we assume random selection for three reasons: (1) This is a natural behavior when peers have little information about others. (2) It is easy to implement and mathematically tractable. (3) It can balance the load among peers.

- **Gain and loss model:** For each time slot, a peer gains $\alpha > 0$ points when it receives a service from another peer, while loses β points when it provides a service to another. Without loss of generality, one can normalize β by setting $\beta = 1$.

- **Learning model:** At the end of a time slot, a peer can choose to switch (or adapt) to another strategy $s' \in \mathcal{P}$. To decide which strategy to switch to, a peer needs to "*learn*" from other peers. Let $\mathcal{G}_i(t)$ be the expected gain (we will formally define this in later sections) of using strategy s_i at time slot t, and $s_h(t)$ is the strategy that has the highest performance among all $s \in \mathcal{P}$ at the end of time slot t. Then a peer using strategy s_i will switch to strategy s_h at time slot $t+1$ with probability $\gamma(\mathcal{G}_h(t) - \mathcal{G}_i(t))$, where $\gamma > 0$ is the learning rate. Note that this is a mathematical abstraction and there are many ways to implement this learning process, e.g., tit-for-tat in the BT protocol or inferring from the reputation of peers are examples of such adaptations.

We now present the mathematical model. Let $x_i(t)$ be the fraction of type s_i peers at time t. If a peer is of type s_i, the expected services it receives, denoted by $E[R_i(t)]$, can be simply expressed as:

$$E[R_i(t)] \;=\; \sum_{j=1}^{n} x_j(t) g_j(i) \qquad \text{for } i = 1, \ldots, n. \tag{1}$$

Let us now derive $E[S_i(t)]$, the expected number of services provided by type s_i peer at time t. Assume that at time t, there are $N(t)$ number of peers in the system. Consider a generic s_i peer and $\mathcal{N}$ be the set representing the other $N(t)-1$ peers. Let $k^* \in \mathcal{N}$, the probability that s_i peer will provide a service to this peer k^* is $\mathcal{V}$, where:

$$\begin{aligned}\mathcal{V} &= \text{Prob}[k^* \text{ selects } s_i \text{ peer}]\text{Prob}[s_i \text{ peer will serve } k^*] \\ &= \left[\frac{1}{N(t)-1}\right]\left[\sum_{j=1}^{n} \text{Prob}[k^* \text{ is of type } s_j] g_i(j)\right],\end{aligned}$$

and

$$\text{Prob}[k^* \text{ is of type } s_j] = \begin{cases} \frac{x_j(t)N(t)}{N(t)-1} & \text{for } s_j \neq s_i, \\ \frac{x_i(t)N(t)-1}{N(t)-1} & \text{for } s_j = s_i. \end{cases}$$

Since $|\mathcal{N}| = N(t)-1$, the expected number of services provided by this type s_i peer in one time slot is

$$E[S_i(t)] = [N(t)-1]\,\mathcal{V}.$$

Combining the above expressions and by assuming that $N(t)$ is sufficiently large, we have

$$E[S_i(t)] \;\approx\; \sum_{j=1}^{n} x_j(t) g_i(j) \quad \text{for } i = 1, 2, \ldots, n. \tag{2}$$

Since a peer receives α points for each service it receives and loses $\beta = 1$ point for each service it provides, the expected gain per slot at time t is $\mathcal{G}_i(t)$:

$$\mathcal{G}_i(t) \;=\; \alpha E[R_i(t)] - E[S_i(t)] \quad i = 1, 2, \ldots, n. \tag{3}$$

We can put the above expression in matrix form and derive $\mathcal{G}(t)$, the expected gain per slot for the whole P2P system at time t as

$$\mathcal{G}(t) \;=\; \sum_{i=1}^{n} x_i(t)\mathcal{G}_i(t) = (\alpha - 1)\boldsymbol{x}^T(t) G \boldsymbol{x}(t), \tag{4}$$

where $\boldsymbol{x}(t)$ is a column vector of $(x_1(t), \ldots, x_n(t))$ and G is an $n \times n$ matrix with $G_{ij} = g_i(j)$.

In short, given an incentive policy $\mathcal{P}$, we can determine G (e.g., probabilities that different type of peers will help each other). Based on the given learning model, we can obtain the evolution of $\boldsymbol{x}(t)$ and answer important questions like whether the given incentive policy $\mathcal{P}$ will lead to a stable system. Also, given $\boldsymbol{x}(t)$, we can compute the individual expected gain $\mathcal{G}_i$ and the system gain $\mathcal{G}$ via Eq. (3) and (4).

Let us now proceed to describe how to model the dynamics (or evolution) of the system $\boldsymbol{x}(t)$. Let $s_h(t)$ be the strategy that has the highest performance among all $s \in \mathcal{P}$ at the end of time slot t. Then peers using strategy s_i will switch to strategy s_h at time slot $t+1$ with probability $\gamma(\mathcal{G}_h(t) - \mathcal{G}_i(t))$, where $\gamma > 0$ is the learning rate. We can express this dynamics of $\boldsymbol{x}(t)$ via the following difference equations:

$$\begin{aligned}x_h(t+1) &= x_h(t) + \gamma \sum_{i=1, i\neq h}^{n} x_i(t)\left(\mathcal{G}_h(t) - \mathcal{G}_i(t)\right), \\ x_i(t+1) &= x_i(t) - \gamma x_i(t)\left(\mathcal{G}_h(t) - \mathcal{G}_i(t)\right), \; i \neq h.\end{aligned}$$

For computational efficiency, one can transform the above difference equations to a continuous model. Informally, the transformation can be carried out by assuming that (1) the peer-request process is a Poisson process with rate equal to 1, (2) the number of learning events is a Poisson process with rate γ_e. (3) In each learning event, the learning rate is γ_l. If we let $\gamma = \gamma_e\gamma_l$, then the continuous

[1]Informally, stability implies there is a high contribution level in the system.

model will be:

$$\dot{x}_h = \gamma \sum_{i \neq h} x_i(t) \left(\mathcal{G}_h(t) - \mathcal{G}_i(t)\right)$$
$$= \gamma \left(\mathcal{G}_h(t) - \sum_{i=1}^{n} x_i(t)\mathcal{G}_i(t)\right) = \gamma \left(\mathcal{G}_h(t) - \mathcal{G}(t)\right) \quad (5)$$
$$\dot{x}_i = -\gamma x_i(t) \left(\mathcal{G}_h(t) - \mathcal{G}_i(t)\right), \quad i \neq h. \quad (6)$$

In summary, the above equations allow one to study the evolution and stability of any incentive policy. Let us now illustrate some incentive policies.

3. INCENTIVE POLICIES

In a typical P2P system, one can classify peers according to their *behavior* upon receiving a request:

- *cooperator:* a peer has a cooperative behavior when it serves other peers unconditionally.
- *defector:* a peer has a defective behavior when it refuses to serve any request from other peers.
- *reciprocator:* a peer has a reciprocative behavior when it serves according to the requester's contribution level. In short, it tries to make the system fair.

Note that these behavior can be easily implemented into a P2P software. Of course, an interesting question is how to design a proper incentive policy so as to keep the system scalable and robust. In the following, we will study two incentive policies.

3.1 Image Incentive Policy

Let us consider the *image incentive policy* $\mathcal{P}_{image}$. Under this policy, when a reciprocative peer receives a request for service, this peer checks the requester's reputation, and it will only provide service with the same probability as this requester serves other peers. One can view this as a "*probabilistic*" version of the tit-for-tat strategy in BitTorrent. Therefore, if the requester is a cooperator (defector, reciprocator), this peer will behave exactly like a cooperator (defector, reciprocator), and that is why we coined it the image strategy.

Image incentive policy $\mathcal{P}_{image}$ has three pure strategies: (1) s_1, or pure cooperation, (2) s_2, or image reciprocation, (3) s_3, or pure defection. To model this incentive policy, we have to derive $g_i(j)$, which is the probability that a peer of type s_i will serve a peer of type s_j. Based on the definition of the image strategy, it is easy to see that $g_1(j) = 1$ and $g_3(j) = 0$ for $j \in \{1, 2, 3\}$, $g_2(1) = 1$ and $g_2(3) = 0$. One can derive $g_2(2)$, which is:

$$\begin{aligned}
g_2(2) &= \text{Prob[a reciprocator will grant a request]} \\
&= \sum_{i=1}^{3} \text{Prob[the requester is of type } s_i] \times \\
&\quad\ \text{Prob[granting the request|type } s_i \text{ requests]} \\
&= x_1(t)g_2(1) + x_2(t)g_2(2) + x_3(t)g_2(3) \\
&= x_1(t) + x_2(t)g_2(2).
\end{aligned}$$

Solving the above equation, we have

$$g_2(2) = \frac{x_1(t)}{1 - x_2(t)}. \quad (7)$$

3.2 Proportional Incentive Policy

We consider a different incentive policy which was proposed by authors in [1], in which results were obtained only via simulation. Reciprocative peers serve the requester with the probability equal to the requester's contribution/consumption ratio, or $E[S_j]/E[R_j]$. In case the ratio is larger than one, the probability to serve the request is set to one. If the requester is a cooperator, its ratio is larger than one, thus, we set $g_2(1) = 1$. If the requester is a defector, its ratio is zero, hence $g_2(3) = 0$. As for $g_2(2)$, we have:

$$\begin{aligned}
E[R_2(t)] &= x_1(t)g_1(2) + x_2(t)g_2(2) + x_3(t)g_3(2) \\
&= x_1(t) + x_2(t)g_2(2), \\
E[S_2(t)] &= x_1(t)g_2(1) + x_2(t)g_2(2) + x_3(t)g_2(3) \\
&= x_1(t) + x_2(t)g_2(2).
\end{aligned}$$

Since $E[R_2(t)] = E[S_2(t)]$, we have $g_2(2) = 1$.

Comparing to image strategy, proportional strategy takes into account the services consumed by requesters. Even if a requester always grants services, it is still not fair for the system if it requests much more services than it provides.

4. PERFORMANCE AND STABILITY

In this section, we analyze and compare the performance and stability of the two incentive policies described in the previous section.

4.1 Image incentive policy

Since we have derived $g_i(j)$ of $\mathcal{P}_{image}$ in Section 3.1, substituting them into Equation (3) and (4), we have:

$$\mathcal{G}_1(t) = \alpha(x_1(t) + x_2(t)) - 1, \quad (8)$$
$$\mathcal{G}_2(t) = (\alpha - 1)\frac{x_1(t)}{1 - x_2(t)}, \quad (9)$$
$$\mathcal{G}_3(t) = \alpha x_1(t), \quad (10)$$
$$\mathcal{G}(t) = (\alpha - 1)\frac{x_1(t)}{1 - x_2(t)}. \quad (11)$$

Let us consider their respective differences:

$$\begin{aligned}
\mathcal{G}_3(t) - \mathcal{G}_1(t) &= 1 - \alpha x_2(t), \\
\mathcal{G}_3(t) - \mathcal{G}_2(t) &= \frac{x_1(t)(1 - \alpha x_2(t))}{1 - x_2(t)}, \\
\mathcal{G}_2(t) - \mathcal{G}_1(t) &= \frac{(1 - \alpha x_2(t))(1 - x_1(t) - x_2(t))}{1 - x_2(t)}.
\end{aligned}$$

We have the following important observations:

- **Case A:** when $x_2(t) > 1/\alpha$, we have $\mathcal{G}_1(t) > \mathcal{G}_2(t) > \mathcal{G}_3(t)$, or cooperators achieve the best performance. Therefore defectors and reciprocative peers will continue to adapt their strategies to cooperative strategy until $x_2(t) = 1/\alpha$.
- **Case B:** when $x_2(t) = 1/\alpha$, the performance of three strategies are the same and there is no more strategy adaptation.
- **Case C:** when $x_2(t) < 1/\alpha$, $\mathcal{G}_3(t) > \mathcal{G}_2(t) > \mathcal{G}_1(t)$, and defectors have the best performance, therefore, cooperators and reciprocative peers will adapt their strategies to the defective strategy. Since $x_2(t) < 1/\alpha$ will continue to hold, the population of cooperators and reciprocative peers will keep decreasing until defectors dominate the system. At this time, the P2P system collapses (or unstable) since there is no service exchange.

Therefore, the system has two equilibria: B and C respectively. In B, the fraction of reciprocative peers $x_2(t)$ will stay at the level $1/\alpha$. In C, the P2P system will be dominated by defectors. However, point B is *not* a stable equilibrium. Suppose the system is at B, we disturb $x_2(t)$ a little bit (e.g., peers arrival or departure and they are of defective behavior). If the disturbance is positive, the system will go to case A and then drop back to B. But if the disturbance is negative, the P2P system will go to C and never return to B. Since we cannot control peers arrival or departure, the system will eventually collapse. In summary, the image incentive policy is *unstable*.

4.2 Proportional incentive policy

For this incentive policy, $g_i(j)$ are derived in Section 3.2. The performance of the three strategies are:

$$\begin{aligned}
\mathcal{G}_1(t) &= \alpha(x_1(t)+x_2(t))-1,\\
\mathcal{G}_2(t) &= (\alpha-1)(x_1(t)+x_2(t)),\\
\mathcal{G}_3(t) &= \alpha x_1(t),\\
\mathcal{G}(t) &= (\alpha-1)(x_1(t)+x_1(t)x_2(t)+x_2^2(t)).
\end{aligned}$$

Let us consider their respective differences:

$$\begin{aligned}
\mathcal{G}_3(t)-\mathcal{G}_2(t) &= x_1(t)-(\alpha-1)x_2(t),\\
\mathcal{G}_2(t)-\mathcal{G}_1(t) &= 1-x_1(t)-x_2(t)\geq 0,\\
\mathcal{G}_3(t)-\mathcal{G}_1(t) &= 1-\alpha x_2(t).
\end{aligned}$$

Note that under the proportional incentive policy, reciprocative behavior is *always better* than cooperative behavior, and we have the following cases:

- **Case A:** when $x_2(t) > \frac{1}{\alpha-1}x_1(t)$, $\mathcal{G}_2(t) > \mathcal{G}_3(t)$, so the fraction of reciprocative peers $x_2(t)$ will keep increasing until they dominate the P2P system. In this situation, the performance of system gains $\mathcal{G}(t)$ reaches the maximum at $\alpha-1$ and the system stabilizes at this point.
- **Case B:** when $x_2(t)=\frac{1}{\alpha-1}x_1(t)$, $\mathcal{G}_3(t)=\mathcal{G}_2(t)>\mathcal{G}_1(t)$, so only cooperative peers will continue to adapt to either s_2 or s_3. In this case, $x_1(t)$ will decrease but $x_2(t)$ will not decrease. So eventually the system will go back to case A.
- **Case C:** when $x_2(t) < \frac{1}{\alpha-1}x_1(t)$, defective behavior has the highest performance so peers will adapt to this strategy. However, since s_2 has a higher performance than s_1, $x_1(t)$ will decrease at a faster rate than $x_2(t)$ until the system reaches $x_2(t)=\frac{1}{\alpha-1}x_1(t)$ and the system will go to case B.

In summary, the system operates in case A, where the fraction of reciprocative peers dominates the system. Moreover, the system achieves the optimal overall performance at this point. This mathematical result agrees with the observation by [1] which was obtained only via simulation.

5. PERFORMANCE EVALUATION

In this section, we present the mathematical results such as system evolution and stability, performance gain of each strategy and the overall system gain. The parameters we used are shown in Table 1.

Evolution of the Image Incentive Policy: Figure 1 to 3 illustrate the population dynamics of the image incentive policy with different initially conditions $\boldsymbol{x}(0)$. Again, $x_i(t)$ represents the *fraction* of peers using strategy s_i at time t, with s_1, s_2 and s_3 being cooperative, reciprocative and defective strategy respectively. In Figure 1, when the fraction of reciprocator at $t=0$ is $x_2(0) < 1/\alpha$. One can observe that the fraction of cooperators, $x_1(t)$, gradually drops to zero and the system collapses. This implies that the system is *unstable* since no one wants to contribute. In Figure 2, when $x_2(0) > 1/\alpha$, the system seems to be stable at first since there is a significant increase of the fraction of cooperator. And at $t=2500^-$, we have $\boldsymbol{x}(t)=(0.80, 0.16, 0.04)^T$. However, at $t=2500$, we introduce a small disturbance where some new peers arrive and they all use the cooperative strategy, so we have $\boldsymbol{x}(2500)=(0.83, 0.14, 0.03)^T$. Although there are more cooperative peers, this small disturbance cause the fraction of reciprocative peers to drop below the threshold $1/\alpha$, causing the system to collapse. As we observe at $t=5000$, there are a significant fraction of defectors in the system. In Figure 3, initially $x_2(0)$ is larger than the threshold $1/\alpha$ and there are more cooperators in the system initially. There are two small disturbances (e.g., some new cooperative peers arrive) at $t=1500$ where $\boldsymbol{x}(t)$ goes from $(0.782, 0.205, 0.013)^T$ to $(0.805, 0.174, 0.021)^T$, and at $t=3000$, where $\boldsymbol{x}(t)$ goes from $(0.822, 0.171, 0.007)^T$ to $(0.842, 0.141, 0.016)^T$. From the figure, we can observe that the system eventually collapses and defectors dominate the system. The performance gains for these three cases are listed in Table 2.

α	gain per service received	7
β	cost per service provided	1
γ	learning rate	0. 004
T	# of time slots	6000

Table 1: Parameters used in the evaluation

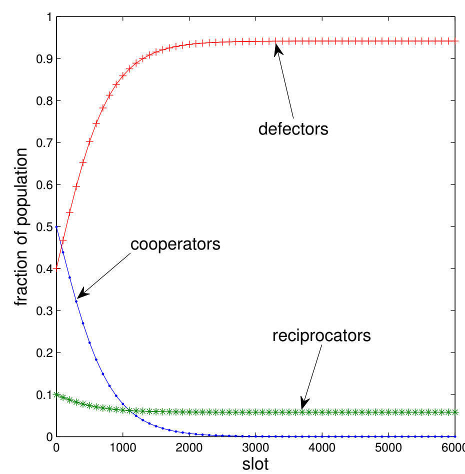

Figure 1: Evolution of $\mathcal{P}_{image}$, $\boldsymbol{x}(0)=(0.5, 0.1, 0.4)^T$.

$\boldsymbol{x}(0)$	$\mathcal{G}_0$	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}$
$(0.5, 0.1, 0.4)^T$	-0.60	0	0	0
$(0.2, 0.3, 0.5)^T$	-0.55	0.045	0.050	0.045
$(0.4, 0.3, 0.3)^T$	-0.55	0.049	0.054	0.049

Table 2: Performance Gains of $\mathcal{P}_{image}$

Evolution of the Proportional Incentive Policy: Figure 4 to 6 illustrate the population dynamics of the proportional incentive policy with different initially conditions $\boldsymbol{x}(0)$. In Figure 4, we have

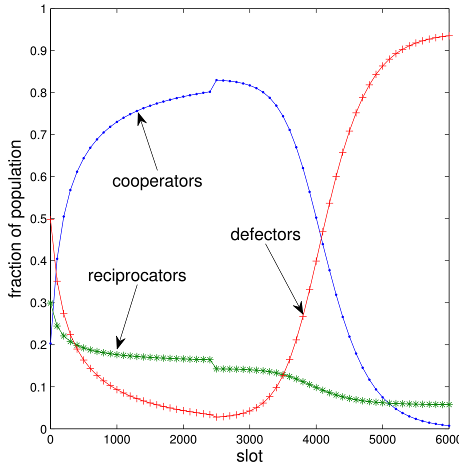

Figure 2: Evolution of $\mathcal{P}_{image}$, $\boldsymbol{x}(0) = (0.2, 0.3, 0.5)^T$, disturbance at $t = 2500$.

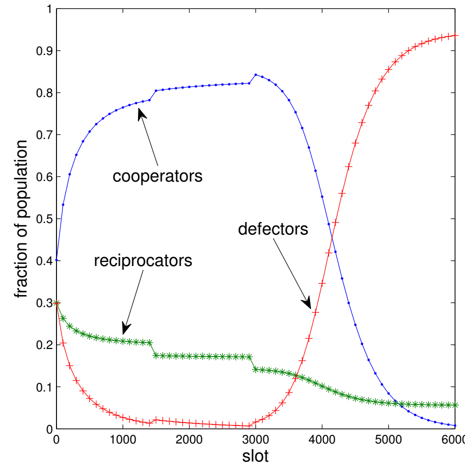

Figure 3: Evolution of $\mathcal{P}_{image}$, $\boldsymbol{x}(0) = (0.4, 0.3, 0.3)^T$, disturbance at $t = 1500$ and $= 3000$.

$x_2(0) < x_1(0)/(\alpha - 1)$, and we observe that the system will be stabilized. At the equilibrium, there will be no defector and there remains a significant fraction of cooperators. In Figure 5, we have $\boldsymbol{x}(0) = (0.10, 0.05, 0.85)^T$, so we start the system with a large fraction of defectors and we also introduce a small disturbance at $t = 200$. Note that even under these adverse conditions, $\mathcal{P}_{prop}$ is very robust. At the equilibrium, there are still some cooperators in the system while the fraction of defectors will converge to zero. In Figure 6, we set $x_2(0) > x_1/(\alpha - 1)$ and we introduce two disturbances at slot $t = 300$ and $t = 700$ respectively. Once again, we can observe the robustness of the incentive policy wherein peers with defective behavior will diminish. The performance for these three cases are listed in Table 3. In all these three cases, the defective strategy will be abandoned by peers and finally, the system operates with optimal performance of $\alpha - 1$.

$\boldsymbol{x}(0)$	$\mathcal{G}_0$	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}$
$(0.5, 0.05, 0.45)^T$	6.0	6.0	1.2	6.0
$(0.1, 0.05, 0.85)^T$	6.0	6.0	0.6	6.0
$(0.4, 0.3, 0.3)^T$	6.0	6.0	2.7	6.0

Table 3: Performance Gains of $\mathcal{P}_{ratio}$

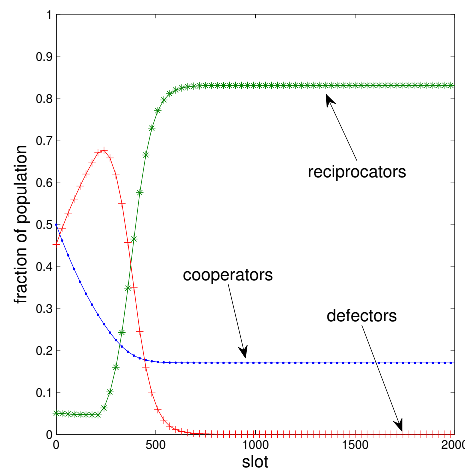

Figure 4: Evolution of $\mathcal{P}_{prop}$,$\boldsymbol{x}(0) = (0.5, 0.05, 0.45)^T$.

6. RELATED WORK

Various incentive techniques have been proposed with Micropayment [4] being the earliest ones. It relies on a central server and uses virtual currency to provide incentive for resource sharing. Since then, much efforts are focused on incentive mechanisms for P2P systems [6–8]. Shared history based incentives can overcome the scalability problem of private history based mechanisms, while DHT technology [9] can be used to implement the shared history incentive mechanism in a distributed fashion. One shared history based incentive is the reciprocative strategy [1, 5]. It makes decisions according to the reputation of requesters. There are also some models developed to help in understanding and designing incentives mechanism. A motivating model in [2, 3] assumes that each peer has a internal generosity. In [10], authors show that a proportional response can lead to market equilibria.

7. CONCLUSION

We present a general mathematical framework to model the evolution and performance of incentive policies. Peers are assumed to be rational and are able to learn about the behavior of other peers. We compare two incentive policies and show that why the image incentive policy may lead to a complete system collapse, while the proportional incentive policy, which takes into account of service consumption and contribution, can lead to a robust and scalable system. The mathematical framework is general to analyze the stability and performance of other incentive mechanisms.

8. REFERENCES

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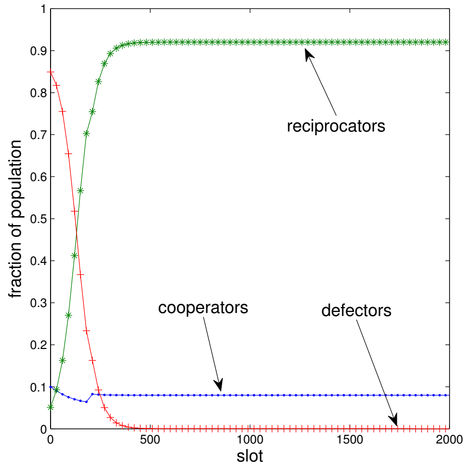

Figure 5: Evolution of $\mathcal{P}_{prop}$, $\boldsymbol{x}(0) = (0.1, 0.05, 0.85)^T$, disturbance at $a = 500$.

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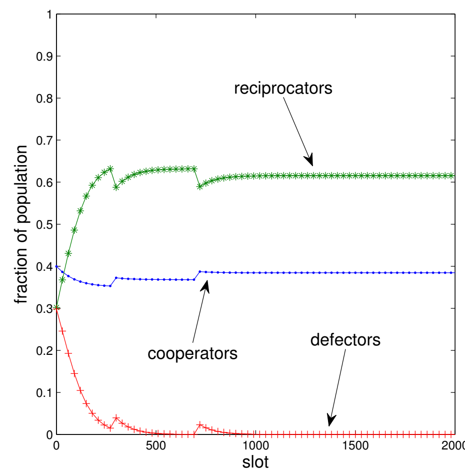

Figure 6: Evolution of $\mathcal{P}_{prop}$, $\boldsymbol{x}(0) = (0.4, 0.3, 0.3)^T$, disturbance at $t = 300$ and $t = 700$.