points later a flattening and drop–this may only be apparent–it may not be really a plateau.

R. J. Slutz: I have not felt that the details within three or four points were really reliable.

J. Levy: This may need some action by the tube manufacturer in his test program if it can be correlated with particular lots.

R. E. Wilson (Radio Corporation of America): Dr. Alexander, you explained that your pulses are distributed at low impedance. I would be interested in knowing what the pulse level is and what the maximum cross-talk level measures?

S. N. Alexander: By our definition the cross-talk level is zero, simply because it is below the clipping level. This clipping level was set by the inherent disturbance signals that occur in the germanium diode gating circuits. We found that when we had completed the design of these circuits so that they would discriminate against the inherent disturbance signals, the cross-talk from other circuits was less than these disturbance signals. It is very hard to define an impedance level in nonlinear circuits, but the average volt-ampere impedance level in SEAC is of the order of 300 ohms and the signal level of the order of 17 volts. With clamping and disconnecting circuits used throughout, the grids of the tubes simply see no signals unless the driving source can override the clamping and disconnect diodes and get above the clipping level.

I might say this is an example of the virtue of operating with telegraphy—all the old troubles of telephony technique go out the window. You have new kinds of troubles—different kinds of troubles—but you do get the advantage of zero disturbance up to a certain threshold.

J. Paivinen (Burroughs Adding Machine Company): Would you care to indicate something of the minimum performance characteristics assumed in your pulse circuit design and the possible effects on any bottomed operation that might be useable in the design of the machine with the *6AN5*.

R. J. Slutz: The *6AN5* we test for a minimum, at 60 volts on the plate and the screen, of 25 milliamperes plate current. In addition, we reject if there is a change in plate current of 25 per cent for a drop in filament voltage of 10 per cent. We also reject it if the change in plate current is 15 per cent for a drop of filament voltage of 10 per cent, and the lower of these two values of plate current is less than 25 milliamperes. The complete specifications are given in the paper.

The bottomed operation gives a uniformity of pulse output regardless of the strength of the tube. The plate current on test at 60 volts on the plate and screen may vary from 25 to 50 milliamperes, but the pulse out of that typical stage will vary at most by no more than about 5 to 10 per cent between these two tubes.

Because the tube is operating bottomed we do not attain standard plate dissipation. The limitation in dissipation is for the screen grid; the plate therefore is running cooler than the allowed manufacturer's rating. We believe that this probably gives us a trifle better life than would be the case if both the plate and screen were run at full plate dissipation.

J. F. Lash (General Motors Research Laboratory): I wonder if you could tell me approximately what is the maximum footage of magnetic tape that you run into the cells of the tape handling mechanism.

R. J. Slutz: A regular reel of tape, about 1,200 feet, is used in each bin.

Computing Machines in Aircraft Engineering

CHARLES R. STRANG

I HAVE been specifically asked to present a user's critical view of computing machinery with emphasis on its limitations.

This is an inversion of the situation in which aircraft manufacturers usually find themselves. We are normally the supplier rather than the user, and the users of our products rarely need this much encouragement to present their views of us very critically indeed.

Aircraft, like computing machines, are complicated to design and difficult to build. You will find those who struggle with aircraft design problems understanding and sympathetic with the difficulties involved in computing machine design.

We have gone far enough to see that there are special problems in making really full scale use of machine computing in our engineering work. There are marked differences between our work and the more academic or scientific applications for which many of the present machines were developed. I will try to convey an understanding of what our work is like.

Before I do make such comments as I have in mind, I should perhaps give some idea of how much of a user of computing equipment the Douglas Company has actually been, so that you may judge how to evaluate my remarks. In considering these data it should be kept in mind that we take a rather hard-boiled engineering view of our own work. We do not fancy ourselves as scientists and we do not undertake mathematical investigations for the sheer intellectual joy of doing so. Furthermore I do not wish to leave the impression that everything we do is dependent on large scale calculation. I suppose about 15 to 20 per cent our total work is mathematical in nature. Much of that work is a miscellany of casual calculations too small to benefit from high speed computing machinery. On the other hand, a great deal of our mathematical work tends to be concerned with operations that occur early in the formative stages of the design when much of what follows can only be tentative until the calculations are well advanced. Most of the remainder is concerned with formal demonstration that the design complies with all its requirements.

A few miscellaneous numbers may serve to give some idea of the scale of present-day aircraft engineering and manufacturing operations. For example, if we were to commit ourselves today to a 4-engine jet transport development program (which, incidentally, the newspapers say we should do) the cost up to first flight would not be less than $25,000,000, and probably more. The business risk involved is very much greater than that figure because competitive sales prices have to be set at a level such that a considerable number of airplanes must be sold before the break-even point is reached. As an example of engineering

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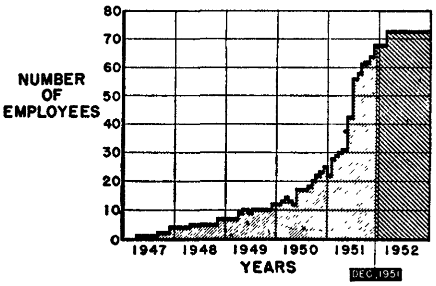


Figure 1. Growth in number of personnel needed for computing equipment

effort involved, the engineering man-hours devoted to the *DC-6* up to first flight was about 1,295,000 hours and up to now totals about 3,275,000 hours. The *DC-6* itself was a development of the *DC-4* on which engineering time totals some 3,850,000 hours. A grand total of 7,125,000 man-hours poured into the development of a specific type! There are over 250 engineering personnel working on the *DC-6* right now out of a total of something over 3,000.

Douglas Use of Computing Equipment

Against this background, Figure 1 shows the growth in number of personnel whose time was fully devoted to manipulating such computing equipment as we have had installed on our premises. It does not include the time of the engineering personnel who were the customers for the computing services and who participated in its performance. Nor does it include the staffs of outside equipment when working on our problems. Obviously, beyond this present date, we can only estimate the probable number of people whose services will be required. We have done so on the basis of the additional equipment on order and still to be received up through the end of 1952. As a parenthetical remark, the majority of these people have backgrounds in classical mathematics and physics. In the present shortage of trained engineering personnel it is important to us that computing machinery has enabled us to take real advantage of the services of a group of people whose training would otherwise have been of limited value to us.

Figure 2 shows the growth of the floor space that has had to be devoted to the installation of computing machines in our Southern California plants.

In talking about computing machines, one always seems to get around to the subject of power consumption. To remain in the tradition, Figure 3 plots the total power requirements of the various equipment as anticipated up through the end of 1952. These considerations are minor by comparison with the question of dollars involved. Figure 4 indicates the order of the direct cost in terms of actual machine cost or rental, salaries and directly chargeable items, but not including general plant overhead, as anticipated up through the end of 1952.

There has unquestionably been an appreciable dollar saving in the accomplishment of work by machine versus manual methods. This direct saving is only one

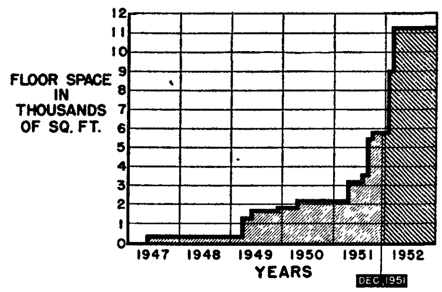


Figure 2. Growth of floor space needed for installation of computing machines in Southern California plants

of several reasons for our interest in computing machinery. As we shall see later, refined engineering design is a repetitive process. Where machine computing makes it economically feasible to accomplish a closer approach to the ideal, the value realized in terms of a better design has a magnitude hard to determine in

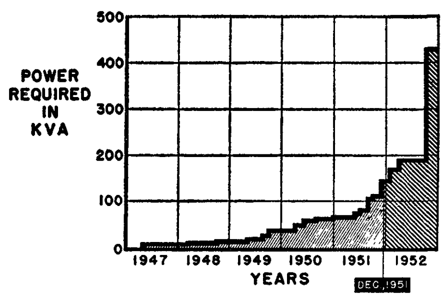


Figure 3. Total anticipated power requirements for equipment through 1952

dollars and cents. Most vital of all is the saving in elapsed time. As I mentioned previously, much of the work to be done during the formative stages of a design is only tentative. It is subject to change and deprived of final status until a large and growing volume of calculations are performed. When that work can proceed on a more firm basis, much waste of engineering time is avoided. Here a real saving is accomplished in the effectiveness of the work of hundreds of engineering personnel who may have had nothing whatever to do with the computing machines. In another sense, we are engaged in a military race. You will have to put your own dollars and cents value on winning versus losing.

Types of Machines Used

Up to this time the equipment actually installed in our plants has been a changing combination of IBM punched card tabulating machinery. However, Figures 1 through 4 reflect the fact that an electrical analogue machine of the type developed at California Institute of Technology by Dr. McCann and his associates, and being built by the William Miller Company of Pasadena under the guidance of Dr. McCann, is scheduled to go into service in January 1952. The nature of that equipment and some of the techniques developed for its use have appeared in AIEE and IRE papers by Dr. McCann.[1–3] The curves also anticipate the installation of a new Reeves Electric Analogue Computer. It is scheduled to take over in March 1952 the work that now is being done on another similar installation outside of the Douglas Company. The large jump in dollars and power consumption shown at the end of 1952 reflect the hope that at that time the two IBM Defense Calculators currently on order will be available and actually go into service. Equipment-wise this adds up, at the end of 1952, to

2 Defense Calculators (IBM)

5 Card Programmed Calculators (IBM)

1 electrical analogue (William Miller Company)

1 REAC (Reeves Instrument Company)

Miscellaneous IBM 604 electronic calculators and associated equipment

This equipment will be distributed, as our present equipment is now, among the three Douglas Company plants in the Los Angeles area.

In addition to these facilities which are operated as an integral part of our en-

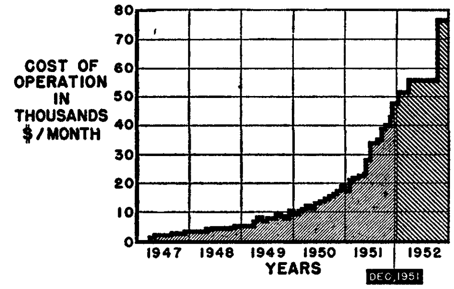


Figure 4. Cost in terms of machine cost or rental, salaries, and directly chargeable items not including plant overhead

gineering departments, we have had a number of projects which for one reason or another were carried out on various outside computing facilities. In such cases, of course, the projects were conducted largely by the staffs associated with those facilities. These have included a number of investigations, some of them extremely interesting in their nature, on the

MIT Meteor

Analogue (California Institute of Technology)

General Electric Mechanical Differential Analyzer (University of California at Los Angeles)

Thermal Analyzer (University of California at Los Angeles)

Bell Telephone Company G.P A.C.

Project Typhoon

Project Cyclone

REAC

As a company we have designed and built several devices in the general category of computing equipment. Also, in the normal course of our work on several missile projects, we have been exposed to the computing equipment phases of the guidance systems involved.

Problems Solved

A tabulation of projects is a dull and dry way of conveying the idea of what we have been doing with these facilities, but at least it is a compact way of doing so. Figure 5 lists the projects in broad general classifications which indicate their physical nature, together with the nature of the controlling mathematical procedures involved. The list is not complete and does not by any means represent all of the applications which the total scope of our engineering work will ultimately require.

Up to this point, I have presented in a necessarily superficial way enough data to show something of the scale of our work on, and our familiarity with, computing devices. I also have indicated the rate at which that phase of our work is growing, how it compares with the total engineering operation, and something of what has so far been done with it.

To those of you to whom the scale of these operations seems large, the question will naturally arise as to what is so particularly difficult about airframe design as to really need so much engineering and computing. The answer is sardonically implied in one of our favorite definitions of our own products, which declares that an airplane is a thing that almost doesn't work—and a missile is a thing that almost does! The difficulties involved are amply attested to by the unfortunate fact that most airplane designs are failures. In our philosophy a successful airplane is one that contributes more to society than it costs. We believe that when a particular design satisfies this definition its history will be characterized by long production life, repeated reorders, and wide application. There have been thousands of different airplanes designed and built in the last 45 years and pitifully few that could be considered successful by such a standard. Furthermore the technical difficulty involved goes up by leaps and bounds with increasing flight speeds. Unhappily the cost involved goes up to some power of these complications and the penalties for inadequate or misdirected engineering effort go up accordingly.

The Engineering Method

I am aware that the more scientifically minded sometimes tend to be horrified and disappointed with us as engineers when they discover how we operate. Unfortunately the present state of the engineering art does not permit us to solve directly for a design of anything to do any stipulated job. We have to work the other way around. A design is proposed that might do the job. All our techniques are such that they pertain to the performance of the proposed design. This performance is then compared with the desired and, in general, is found wanting. The proposed design is changed, the technique applied again, and the new performance compared with the desired, and so on ad nauseum. This is true whether the problem involved is large or small. Whether it is the design of a structural member to carry a load, a supercharger vane to move enough air, or an airplane to possess a given rate of climb with one engine out, gears up, and flaps down, for example. Furthermore the total number of variables involved are so great that no one has yet proposed a computing facility that could handle them all as one problem, even if we had reached the stage where we could express them as one problem.

The user soon finds that one of his biggest problems is to develop a proper sense of proportion. He finds himself suddenly in possession of relatively tremendous mathematical power. If we are typical examples, he is intellectually unprepared for mature use of that power. The state of his art is not built on availability of such power, but on the contrary, consists of a bag of tricks to enable him to get along without it. He is hard put to direct it always either to that backlog of work he would have to do anyway by other and slower methods—or to such new applications as truly benefit his design purpose. The temptation to waste that power away on fancy business, and investigations he could well do without, is tremendous.

Digital Versus Analogue Machines

Although your interest here lies primarily in digital machines, we have found it desirable to use both physical and mathematical analogues as well as digital machines and expect to continue to use them indefinitely. As users we take a dim view of futile arguments as to the relative merits of digital versus analogue machines, or mathematical analogues versus true physical analogues. We think there is room, and need, for all. Our interests would be better served if the proponents

Figure 5. Some typical aircraft engineering problems solved using automatic computing equipment

METHOD OF SOLUTION → / ↓ DESIGN PROBLEM	SIMULTANEOUS EQUATIONS	DIFFERENTIAL EQUATIONS	MATRIX ALGEBRA	HARMONIC ANALYSIS	STATISTICS & PROBABILITY	NON-LINEAR ALGEBRIC EQUATIONS
ACOUSTICAL STUDIES						
AERODYNAMIC PERFORMANCE	■					
AERODYNAMIC STABILITY			■			
AEROELASTIC STUDIES	■					
AIRFOIL PRESSURE DISTRIBUTIONS						
AUTOPILOT DESIGN						
CATAPULT LAUNCH ANALYSIS						
CONTINUOUS BEAM ANALYSIS	■		■			
CONTROL SYSTEM TRANSFER FUNCTIONS						
FLUTTER ANALYSIS	■		■			
FUSELAGE & WING SECTION ANALYSES	■		■			
LANDING GEAR SPIN-UP ANALYSIS						
LOFTING CALCULATIONS						
MISCELLANEOUS CURVE FITTING	■		■			
MISCELLANEOUS DATA REDUCTION	■		■		■	
MISSILE TACTICAL EMPLOYMENT STUDIES					■	
RADOME DESIGN			■			
SUPERCHARGER VANE DESIGN						
THERMODYNAMIC ANALYSIS						
TRAJECTORIES OF AIRPLANES & MISSILES						
WING SPANWISE LIFT DISTRIBUTION	■					

of each took a generous view of the advantages of the other and tried to incorporate comparable advantages in their own design.

I have selected two particular applications to tell you about in a descriptive nontechnical way to convey to you something of the situation into which computing machinery must fit to become an integral fully effective part of an operating engineering department.

The first application selected as an example is a flutter problem. The flat plane of a wing or tail surface forced through the air edgewise wants to wave like a flag, that is, to flutter. Deflection modes of wings, fuselage, and tail surfaces may want to interact with and mutually reinforce each other. Since they are meanwhile deriving energy from the passing air stream, a sufficiently high speed will cause the oscillations to build up to the point that something fails. The mechanism is very much as if the aerodynamic forces interacting with the elastic and inertia forces within the structures had constituted themselves into a mechanical analogy of what is going on electrically in an oscillating electrical circuit. Or it could be compared to the stability of a closed loop servo system. It is our purpose as designers to keep the critical speed at which this whole process becomes catastrophic safely above any speed at which the particular airplane or missile will ever fly.

The flutter problem is one of aeronautical engineering's most difficult problems to deal with mathematically. It was one of the first on which we brought digital machine computing methods to bear. During the last six years the mathematical procedures involved have been adapted to IBM punched card tabulating and computing equipment to the point that about 90 to 95 per cent of the work is mechanized. This has benefitted us enormously at a time when much higher flight speeds and the thin airplane and missile surfaces typical of high speed design have greatly increased the probability of flutter. Our small group of flutter engineers aided by machine computing have dealt with more flutter investigations in the last few years than those engineers would normally have seen in a lifetime.

Figure 6 gives a typical formal mathematical statement of the problem. I am told that these may be described as simultaneous, 4th order, integro-partial differential equations. From the hard-boiled engineering point of view I referred to earlier, it is simpler to describe them as a mathematical mess. The airplane in question had been carried into an advanced phase of its design with some eight months of work by several hundred design personnel involved when a preliminary conventional flutter analysis indicated that the critical flutter speed was far too low. This was an unusual situation in every respect. The traditional procedure until recently would have been for the design to be completed, the prototype airplane built, and natural frequencies and modes determined by forced vibration of the prototype. Months of calculations based on these data would then be carried out. These would normally be completed at about the time the airplane was ready for actual flight. At this point it would be found that the flutter speed was not critical. No one except the flutter engineers who had been toiling industriously away in the background for months would even be aware that anybody had considered the problem. However, this particular design was progressive and far from conventional. Therefore every effort had been made to carry out a preliminary analysis, even though the work had to be based on sheer estimates of the parameters involved. To find that the design might be heir to flutter trouble this early in the game was an accomplishment in itself, but a horrible state of affairs.

To meet the emergency and supplement the usual approach it was decided to deal with the problem in all its ramifications on the analogue machine at California In-

Figure 6. Typical wing bending-torsion flutter equations

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y}{\partial x^2}\right]+\frac{m\,\partial^2 y}{\partial t^2}\pm S_\alpha\frac{\partial^2\alpha}{\partial t^2}=-\pi\rho\, b^2(x)\frac{\partial^2 y}{\partial t^2}-\pi\rho V\, b^2(x)\frac{\partial\alpha}{\partial t}-2\pi\rho V\int_0^t\phi(x,t-\tau)\left[b(x)\frac{\partial^2 y}{\partial\tau^2}+b^2(x)\frac{\partial^2\alpha}{\partial\tau^2}+Vb(x)\frac{\partial\alpha}{\partial\tau}\right]d\tau+f(x,t)$$

$$\frac{\partial}{\partial x}\left[GJ\frac{\partial\alpha}{\partial x}\right]\pm S_\alpha\frac{\partial^2 y}{\partial\tau^2}+I_\alpha\frac{\partial^2\alpha}{\partial\tau^2}=-\pi\rho\frac{b^3(x)}{2}\frac{\partial^2 y}{\partial t^2}-\pi\rho\frac{b^4(x)3}{8}\frac{\partial^2\alpha}{\partial t^2}-\pi\rho V\, b^3(\gamma)\frac{\partial\alpha}{\partial t}+g(x,t)$$

L E G E N D

EI = BENDING RIGIDITY OF WING	V = FORWARD VELOCITY OF WING
GJ = TORSIONAL RIGIDITY OF WING	$\phi(x,t-\tau)$ = LAG FUNCTION DESCRIBING GROWTH OF LIFT ON WING
m = MASS PER UNIT LENGTH OF WING	$f(x,t)$ = ARBITRARY FORCING FUNCTION
S_α = UNBALANCE MOMENT PER UNIT LENGTH OF WING	$g(x,t)$ = ARBITRARY FORCING FUNCTION
I_α = MASS MOMENT OF INERTIA PER UNIT LENGTH OF WING	$\alpha(x)$ = ANGULAR COORDINATE DESCRIBING TORSION OF WING
ρ = DENSITY OF AIR	$y(x)$ = VERTICAL COORDINATE DESCRIBING VERTICAL TRANSLATION OF WING
$b(x)$ = SEMI-CHORD OF WING	

THE EQUATIONS FOR THE MECHANICAL SYSTEM ARE SOLVED BY WRITING THEM IN THE FOLLOWING FINITE DIFFERENCE FORM, WHICH IS ALSO A SET OF EQUATIONS DESCRIBING AN ELECTRICAL NETWORK.

(1) ELASTIC

$$\left.\frac{\partial\frac{\partial y}{\partial t}}{\partial x}\right|_{x=x_{n+\frac{1}{2}}}=\frac{\left.\frac{\partial y}{\partial t}\right|_{n+1}-\left.\frac{\partial y}{\partial t}\right|_{n}}{\Delta x_{n+\frac{1}{2}}}\qquad EI_n\left.\frac{\partial\theta}{\partial x}\right|_{x=x_n}=-EI_n\frac{\int\left(\left.\frac{\partial\theta}{\partial t}\right|_{x=n+\frac{1}{2}}-\left.\frac{\partial\theta}{\partial t}\right|_{x=n-\frac{1}{2}}\right)dt}{\Delta x_n}$$

$$\left.\frac{\partial M}{\partial x}\right|_{x=x_{n+\frac{1}{2}}}=\frac{M_{n+1}-M_n}{\Delta x_{n+\frac{1}{2}}}\qquad\left.\frac{\partial Q}{\partial x}\right|_{x=x_n}=\frac{S_{n+\frac{1}{2}}-S_{n-\frac{1}{2}}}{\Delta x_n}$$

$$\left.\frac{\partial\alpha}{\partial x}\right|_{x=x+\frac{1}{2}}=\frac{\alpha_{n+1}-\alpha_n}{\Delta x_{n+\frac{1}{2}}}\qquad\left.\frac{\partial\alpha}{\partial x}\right|_{x=x_{n-\frac{1}{2}}}=\frac{\alpha_n-\alpha_{n-1}}{\Delta x_{n-\frac{1}{2}}}$$

$$\frac{\partial}{\partial x}\left(GJ\frac{\partial\alpha}{\partial x}\right)_{x=x_n}=\left[GJ_{\left(x_{n+\frac{1}{2}}\right)}\frac{\int\left(\frac{\partial\alpha_{n+1}}{\partial t}-\frac{\partial\alpha_n}{\partial t}\right)dt}{\Delta x_{n+\frac{1}{2}}}+GJ_{\left(x_{n-\frac{1}{2}}\right)}\frac{\int\left(\frac{\partial\alpha_{n-1}}{\partial t}-\frac{\partial\alpha_n}{\partial t}\right)dt}{\Delta x_{n-\frac{1}{2}}}\right]\frac{1}{\Delta x_n}$$

(2) MASS

$\Delta x m_n$ = MASS OF nth CELL

$\Delta x S_\alpha$ = UNBALANCE MOMENT OF nth CELL

$\Delta x I_\alpha$ = MOMENT OF INERTIA OF nth CELL

(3) AERODYNAMIC APPARENT MASS COEFF.

$$\Delta x_n\Pi\rho b^2(x_n),\ \Delta x_n\Pi\rho b^4(x_n)\tfrac{3}{8}\quad \Delta x_n\Pi\frac{\rho b^3(x_n)}{2}$$

(4) AERODYNAMIC DAMPING COEFF.

$$\Delta x\Pi\rho V b^2(x_n),\ \Delta x\Pi\rho V b^3(x_n)$$

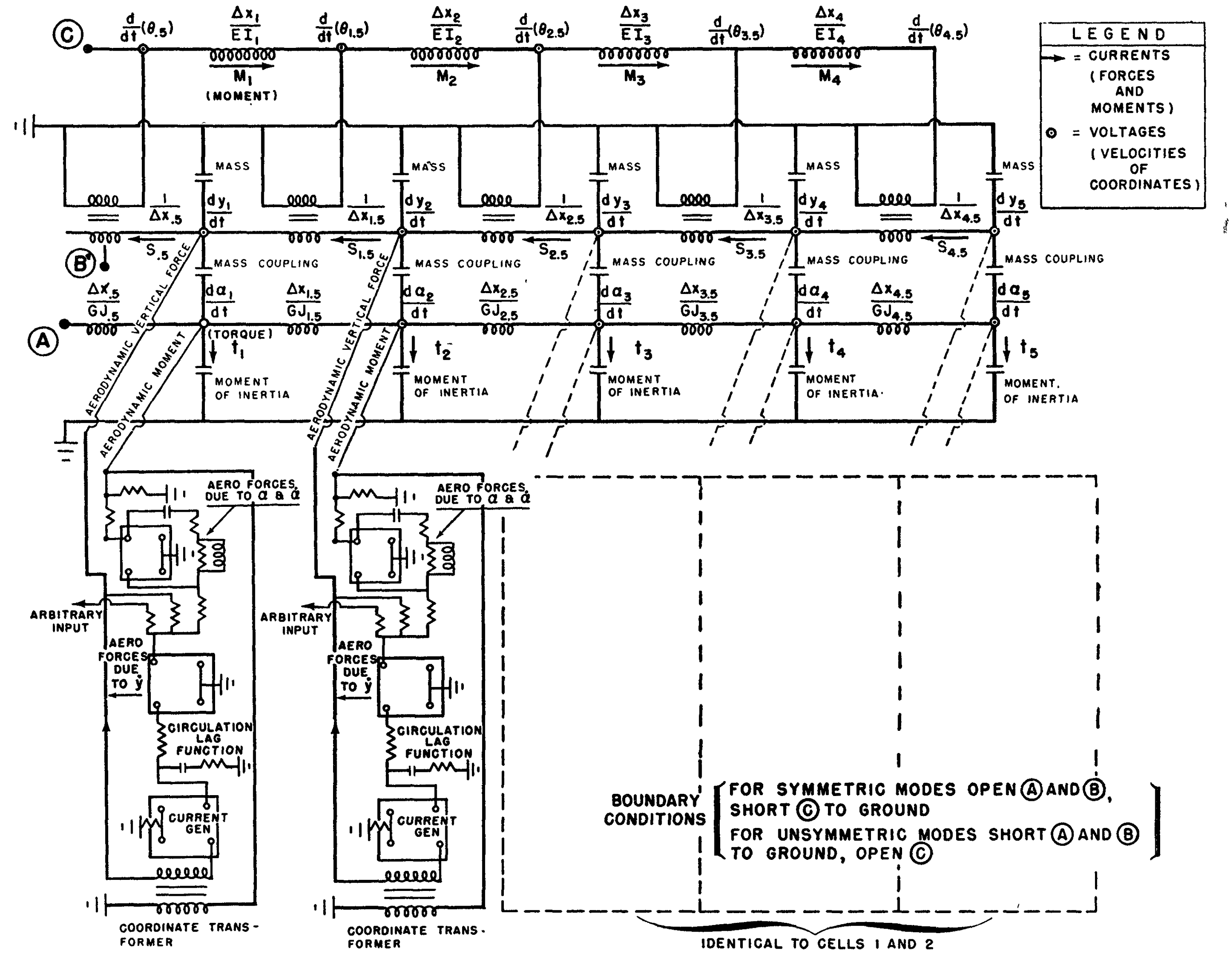
(5) AERODYNAMIC FORCES (CIRCULATORY)

THE INTEGRAL IN THE EQUATIONS OF MOTION MAY BE WRITTEN IN THE FOLLOWING OPERATIONAL FORM:

$$\left[\Delta x\,2\Pi\rho V\int_0^t\phi(x_n,t-\tau)\left(b(x_n)\frac{\partial^2 y}{\partial\tau^2}+b^2(x_n)\frac{\partial^2\alpha}{\partial\tau^2}+Vb(x_n)\frac{\partial\alpha}{\partial\tau}\right)d\tau\right]=$$

$$\Delta x\,2\Pi\rho V\left(\phi(x_n,p)\right)\left(b(x_n)py+b^2(x_n)p\alpha+Vb(x_n)\alpha\right)$$

$$\left(\phi(x_n,t)\right)\cong\frac{a_1+a_2\frac{b(x_n)}{V}p}{a_3+a_4\frac{b(x_n)}{V}p}$$



stitute of Technology. It was necessary to supplement the machine with additional equipment which had to be designed for the purpose and this of course took time to do. Figure 7 indicates the general nature of the circuitry involved. During the preliminary analysis stages, the analogue confirmed the predictions of the conventional analysis that flutter was inevitable. Design changes to affect a cure were considered to the tune of 126 separate investigations of mathematical systems similar to those shown by Figure 6 except that the actual case was appreciably more complex. After design decisions based on these 126 solutions had firmed up the redesign, the nearly final version was again put back into the machine and 50 more solutions covering numerous variations were made. It was a time consuming project to carry out this enormous volume of work. It did necessitate changes in the configuration of the airplane and the addition of structural weight. This latter is a thing that an aeronautical engineer resorts to with

Figure 7. Circuit for typical wing bending-torsion flutter problem

a reluctance that is the essence of his profession. It is conceivable that the entire project might have been an utter failure without the accomplishment of the work that I have described in such an over-simplified fashion. On the other hand, while it was being accomplished the entire project was delayed to one degree or another, schedules were thrown off, costs were going up and time was going by. The management's reaction was to arrange for the construction of a similar analogue computer so that it could be brought to bear on such problems at the earliest possible moment. This is the equipment that I mentioned earlier as scheduled for completion in January 1952.

Two basic points I would like you to extract from this short story—one is the number of times that the solution had to be carried through—176 times—the other is a sense of the compulsion under which the men were working who carried out the project.

A Catapult Problem

The second case that I am about to describe is a physical system not so mysterious to deal with but equally typical of our problems. It has to do with solving the equations of motion of a catapulted airplane at the instant it is airborne. It is typical of that class of problems which are fundamentally simple but become odious mathematically when many necessary details are taken into account. When a mission requires a naval airplane operating from a carrier to take off at weights greater than it can take off under its own power, some assistance, usually in the form of a catapult, is used to attain flight speed. Conventional tail wheel airplanes rolling down the deck with the tail wheel in contact are in a tail-down high-lift attitude suitable for flight at the speeds at which they reach the take-off end of the deck. No particular problem except the attainment of the necessary speed is involved. More modern nose-wheel type airplanes, on the contrary, reach take-off at a flatter angle, which the pilot has to correct as he leaves the deck. When nose-wheel type airplanes began to be used on carriers, the carriers had become pretty big. Flight decks were about 70 feet above the water. The pilot had about two seconds to collect his wits, get his airplane into flight attitude, and be on his way. This is obviously a pretty marginal operation, but it has been a highly successful one.

Figure 8 is a diagram of a tricycle gear airplane intended for such operations. The design was carried out, the prototype airplane was built and put through our manufacturer's flight test. It was then delivered to the Navy for continued flight testing, including simulated carrier flying. By this time production versions of the airplane were moving down the assembly line. This was the situation when it was decided that this would be the airplane on which carrier operations in bad weather and night flying, or both, should be undertaken. Now this was a horse of a different, and very dark, color. Flying off into the rain and darkness must be done on instruments. When subjected to catapulting accelerations, the instrument gyros must be caged. The pilot who in good weather day flying had two seconds to collect his wits and clear an ocean he could see 70 feet below him—now has the same two seconds to collect his wits, uncage his gyros, focus his attention on his flight instrument group—and fly off into the stormy darkness on instruments without dropping into an ocean that he can not see! It turns out, incredibly enough, that the boys can do that, too—if the airplane isn't pitching unpleasantly at the moment it is catapulted. Unhappily, the Navy flight tests showed that this was exactly what it was doing. It was absolutely necessary that some combination of dimensions and forces be found which would automatically deliver the airplane at the end of the run possessed of an angular motion that would help the pilot through his critical first two seconds.

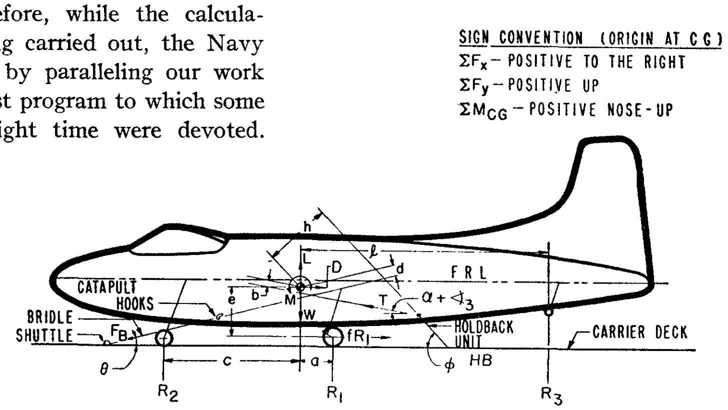
This now brings us to our problem, and to the equations of motion given on Figure 8. These describe, simply enough, the fact that such an airplane, during the few seconds of its catapulting run and under the influence of all the forces acting during that time, tends to rock back and forth alternately between nose wheel and tail wheel. Each of these introduce forces which are functions of the load-stroke characteristics of their respective shock absorbing systems. These forces, incidentally, not only inject mathematical discontinuities but involve a hysteresis loop that is the result of the fact that the load-stroke curve is not the same when the shock strut is moving in as it is when the shock strut is moving out. In other words, these discontinuous forces are nonlinear. Lift, drag, thrust, and acceleration forces are, of course, changing continuously. Taking all these mathematical horrors into account expands the simple equations of motion shown on Figure 8 into the state of affairs shown by Figure 9. These would obviously be long and arduous to solve by manual methods, as we found out when we tried to do it on a much simpler version than this.

The problem was set up for solution on an International Business Machine Card-Programmed Calculator, fortunately equipped with two storage banks. The procedures involved have been described in detail in a published paper by John Lowe,[4] who is the supervisorof Computing Engineering at the Santa Monica plant of Douglas Aircraft. Investigating the effect of changes in the location of the catapulting force, the characteristics of the shock absorbing systems, the physical location of the gears, different wind speeds over the deck, et cetera, made it necessary to carry out that procedure some 55 times. The design features that were incorporated in the airplane as a result of these calculations were checked against flight tests carried out by the Navy, and the results were in excellent agreement. The modifications to the design were successful in delivering the airplane off the deck automatically under conditions the pilots are well able to handle. In fact it was so successful that we now have to go through the whole procedure for four other airplanes! In the meantime, I think it is interesting and significant that the difficulties involved were such that we could not be assured of success. Therefore, while the calculations were being carried out, the Navy hedged its bet by paralleling our work with a flight test program to which some 50 hours of flight time were devoted. Fifty hours is a great deal of flying when you are only interested in the first few seconds of each flight. When weighing computing costs, it is fair to remember that a 50-hour flight test on this kind of equipment will buy a lot of computing machines. Now again I would like you to observe the same two basic points—the number of times that the solutions had to be carried out, and the sense of urgency under which the men worked who carried it out.

You may ask why the catapult problem was not put on the analogue. It will not be necessary even to argue the relative merits of the two types of equipment. The fact is that for the flutter job I described first we had already contracted for all the analogue time available to us. Both these problems were going on at the same time. They had two of the most powerful machines in the Los Angeles area tied up almost completely for weeks. These two problems were not even on the same airplane, and they were not the only airplanes we were working on. Hundreds of other problems were given slight treatment or none during that period.

To describe a really full-scale use of an adequate computing facility in a large engineering effort it would be necessary to multiply such situations as I have just described by perhaps several hundred times.

These are fed into the computing facility from a number of engineering specialty groups. The computing facility finds itself taking them all on at once like a master chess player playing many games simultaneously. Moving from one to another continually but returning periodically to the later developments of each problem in turn. Our operations to date are very very far from such a scale. Fortunately we have had six years to come



FORCE AND MOMENT ARM

(1) $\Sigma F_x = m x + fR_1 + D - T \cos(\alpha + \measuredangle_3) + HB \cos\phi - F_B \cos\theta = 0$

(2) $\Sigma F_y = R_1 + R_2 + R_3 + L + T \sin(\alpha + \measuredangle_3) - W - F_B \sin\theta - HB \sin\phi - m\dot{Y} = 0$

(3) $\Sigma M_{CG} = bT + cR_2 + dF_B - M - \ell R_3 - aR_1 - efR_1 + h(HB) - I\alpha = 0$

Figure 8. Catapult take-off diagram

$$4)\ \left[\frac{y - K_{25} + K_7\cos\alpha - K_6\sin\alpha + K_{14}\sin(\angle_4+\alpha)}{\sin(\angle_4+\alpha)}\right]^2 + K_{50} = K_{51}\bar{y} - \bar{y}^2 - \left[\frac{\delta_1 - \bar{y}\cos(\angle_4+\alpha)}{\sin(\angle_4+\alpha)}\right]^2 - \frac{2\left[y - K_{25} + K_7\cos\alpha - K_6\sin\alpha + K_{14}\sin(\angle_4+\alpha)\right]\left[\delta_1 - \bar{y}\cos(\angle_4+\alpha)\right]}{\sin^2(\angle_4+\alpha)}$$

$$5)\ F_B = \frac{K_4 + K_3(K'_5) - K_2\cos(\alpha+\angle_3) + \text{H.B.}\cos\phi + K_1\left[K_8+K_9t\right]^2}{\cos\theta}$$

$$6)\ \ddot{y} = \frac{K'_5 + K'_{15} + K_{13}\left[(K_8+K_9t)^2(\alpha+\angle_1) - K_{29}(K_8+K_9t)(\dot{y})\right] + K_2\sin(\alpha+\angle_3) - K_{16} - F_B\sin\theta - \text{H.B.}\sin\phi}{K_{17}}$$

$$7)\ \ddot{\alpha} = \frac{K_{20} + \left[y\cos\theta - \sin\theta\,(K_{12}\cos\theta + K_{11}\sin\alpha + K_{10}\cos\alpha)\right]\left[F_B\right] - \left[K_7\sin\alpha + K_{19}\cos\alpha - \overset{\equiv}{y}\sin(\alpha+\angle_5) + \frac{K_{24}}{\cot(\angle_5+\alpha)}\right]\left[K'_{15}\right]}{K_{22}}$$

$$\frac{-\left[K_7\sin\alpha + K_6\cos\alpha - K_{14}\cos(\angle_4+\alpha) - K_{21}\sin(\angle_4+\alpha) + K_{32}\cos(\angle_4+\alpha+T)\right]\left[K'_5\right] - \left[y - (K_{25}-\delta_1)\right]\left[K_3\right]\left[K'_5\right] + \left[\text{H.B.}\right]\left[h\right]}{K_{22}}$$

$$\frac{-K_{23}\left[(K_8+K_9t)^2(\alpha-\angle_2) - K_{29}(K_8+K_9t)(\dot{y})\right] - K_{31}(K_8+K_9t)(\dot{\alpha})}{K_{22}}$$

$$\theta = \sin^{-1}\left[\frac{y - K_{11}\cos\alpha + K_{10}\sin\alpha}{K_{12}}\right]$$

$$\phi = \sin^{-1}\left[\frac{y - K_{26}\sin\alpha - K_{27}\cos\alpha}{K_{28}}\right]$$

$$\overset{\equiv}{y} = \frac{K_7\cos\alpha - K_{19}\sin\alpha + y}{\cos(\angle_5+\alpha)}$$

$$\bar{\bar{y}} = \frac{K_{18}\sin\alpha + K_7\cos + y}{\cos(\angle_6+\alpha)}$$

$$\text{H.B.}\cos\phi = K_{30} + K_{34}t$$

$$K'_5 = K_5(1 \pm K_{35}) \quad \text{WHEN } \bar{y}_n < \bar{y}_{n-1}\ K_{35} \text{ is POSITIVE}; \quad \text{WHEN } \bar{y}_n > \bar{y}_{n-1}\ K_{35} \text{ IS NEGATIVE}$$

$$K'_{15} = K_{15}(1 \pm K_{36}) \quad \text{WHEN } \overset{\equiv}{y}_n < \overset{\equiv}{y}_{n-1}\ K_{36} \text{ is POSITIVE}; \quad \text{WHEN } \overset{\equiv}{y}_n > \overset{\equiv}{y}_{n-1}\ K_{36} \text{ IS NEGATIVE}$$

$$K'_{15} = K_{60}(K_{52} - \overset{\equiv}{y}) + K_{61} \quad \text{WHEN } \overset{\equiv}{y} < K_{52}$$

$$h = K_{26}\sin(\phi-\alpha) - K_{27}\cos(\phi-\alpha)$$

$$\beta = \alpha - \frac{y(K_{29})}{(K_8+K_9t)}$$

$$\sin T = \frac{\bar{y} - K_{21}}{K_{32}}$$

$$\alpha_{n+1} = \alpha_n + \dot{\alpha}_n\Delta t + \frac{\ddot{\alpha}_n(\Delta t)^2}{2}$$

$$\dot{\alpha}_{n+1} = \alpha_n + \ddot{\alpha}_n\Delta t$$

$$y_{n+1} = y_n + \dot{y}_n\Delta t + \frac{\ddot{y}_n(\Delta t)^2}{2}$$

$$\dot{y}_{n+1} = \dot{y}_n + \ddot{y}_n\Delta t$$

Figure 9. Catapult take-off analysis

as far as we have. I think we needed that preparation to arrive at a point where we have any hope of effectively using the relatively tremendous power of the equipment we are to start using over the next 12 months. How far that equipment will take us towards realizing such an increase in the number of problems we can deal with I cannot guess. However, I doubt very seriously whether it will make it possible to do all that we would like to be able to do. The reason for that, as you can see from what I have said, lies entirely in the fact that for some years to come our concern in realizing full utility from big machines will not be in solving a few big problems but will be in the more agile handling of enormous numbers of problems. Some of them will be larger, some smaller, than I have discussed here—but on the average of about that order.

Some Machine Design Objectives

The design of any machine is a matter of compromise and balance of its functions for the purpose intended. I am sure this balance has been earnestly sought in the design of all big computing machines. Almost everything I have said has been calculated to show the importance, to such users as ourselves, of really flexible input and output equipment.

It seems to me that most of the machines I know about are much cleverer in their internal operating philosophy than they are in their input and output devices. In our case, as we have seen, normal usage requires many repetitions of a given procedure, each time with changes in some of the initial parameters. We typically find on examination that nine times out of ten we have no further interest in the results but must try again. We must be able to completely remove the problem from and later return it to the machines at will. In such a case, obviously the design balance has to be shifted to a much heavier emphasis on the input and output equipment.

I am not saying that the input equipment and coding must be simple, though that would be nice. I am saying that it should be possible to perform those operations without tying up the computer itself.

Obviously, mechanization of the coding process by coding machines like Dr. Aiken has been developing are very much to the point. In the case of the output equipment the same sort of reasoning applies. We would not be interested in permanent records of most runs. We are interested in knowing immediately what the results have been. Some device that permits scanning of selected values from the internal storage, such as has recently been done on a display cathode-ray tube on Project Whirlwind, is excellent. Then we are in a position to examine the effects of the last change. We can either determine what to change next, or decide that we are satisfied and that we do want a permanent record. Alternatively, we can decide that we want that problem completely off the machine to take time to study what to do next. If the machine has a printer output, it should be possible to free the main machine by clearing its storage into an auxiliary storage and printing out from there. But we should ordinarily not have to do that to find out what is in the storage.

Conclusion

In conclusion, our engineering use of computing machinery has progressively increased in scope and magnitude during these past six years. At the beginning of that period we applied machine methods on a very modest scale. We did so in the hope that it would be the eventual means of breaking our major design bottleneck—the ever growing volume of mathematical investigation demanded by modern aircraft. Machine computing has been at least partially successful in accomplishing that purpose. The scale of our operations has grown naturally from its tentative beginnings to the point that machine computing is definitely indispensable now. It is becoming increasingly vital at a startling rate.

Computing machines are themselves an engineering product. It is entirely likely that, in their ultimate development, the engineering profession itself will be the biggest user of that product.

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A Review of the Bell Laboratories' Digital Computer Developments

E. G. ANDREWS

THE Bell Telephone Laboratories have designed and built seven digital computers. They are all electro-mechanical types using telephone systems relays and teletype transmitting and recording devices as their principal apparatus elements.

This succession of developments had its origin in 1938 in the mind of Dr. George R. Stibitz, then a research mathematician with the Bell Laboratories Stibitz observed that in one laboratories' development area, a considerable portion of computing effort involved complex number arithmetic computations. Another development area was designing dial systems using relays and crossbar switches as the principal apparatus. Stibitz recognized that the design techniques employed by this latter group were directly applicable to a systems design which could also produce a computing system for the first group. He designed such a system. He called it a "complex number computer."

Mr. Samuel B. Williams, a telephone systems design engineer, supervised the engineering and manufacturing of this early computer. It is believed that he thus became the one who first produced an automatic digital computer for purely scientific computing.

Dr. Thornton C. Fry* publicized this creation before the Mathematical Society at Dartmouth in the fall of 1940.[1] Demonstration equipment consisting of a keyboard input device and a teletypewriter for recording answers was installed at the University. This equipment was connected by a telephone circuit with the computer in New York. Those attending the conference placed their own test problems on the keyboard at Hanover, N. H. The computer in New York made the computation and controlled the printing of the answer on the teletypewriter at Hanover. The complete operation required about 1 minute. This feat of remote control operation was not to be duplicated until a computer conference was held in Washington, D. C., 10 years later.

With this complex number computer as the pioneer and with the Model VI as its latest achievement, the Bell Laboratories computer development has spanned the pre-electronic computer development era. The seven computers now are known by Model numbers, with Model I being the designation of the complex number computer. Two Model V computers were built. Table I shows some statistical information about their size and use. The Models V and VI, although operating at electromechanical speed, offer several challenges to current electronic computers. While the same cannot be said of the Models I to IV, nevertheless, they have features of interest.

Model I

The Model I consisted of about 400 relays and ten crossbar switches. It was operated from any of three stations located in various parts of the Laboratories' 463 West Street building. The station equipment consisted of a number of push button keys for originating a problem and a teletypewriter for recording the answer. It is the only one of the seven computers to employ crossbar switches.

Fundamentally, the Model I could handle only two kinds of problems, multiplication and division of complex numbers. The results of successive such problems could be accumulated when the operator required it. This feature made it feasible to add and subtract complex numbers. By multiplying a number by +1 or −1 with the accumulator key operated, the number was added to, or subtracted from the previous accumulation.

This computer operated with binary coded decimal notation, with the decimal digits 0 to 9 being represented by the binary numbers 0011 to 1100. The input and output information consisted of eight place numbers, but the calculator carried operations out to ten places, the two extra places being used to improve accuracy when accumulating the results of several problems.

The Model I was in daily use until 1949 when it was removed from service due to obsolesence and to make way for its successor, the Model VI.

Model II

The Model II[2,3,4] was built for the National Defense Research Council and placed in operation in September 1943. It is truly a special purpose computer. Its purpose was to handle some specialized fire control computing for several months. But on the completion of this computing assignment, new problems arose. It is still in service.

It has about 440 relays It uses paper tape input and output. It has a flexible control provided by external

* Subsequent to the original publication of this paper, it has been brought to the attention of the author that the paper describing the computer was delivered at the meeting by G. R. Stibitz. The local arrangements for the remote control operation were supervised by T. C. Fry.

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