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ANALOG-DIGITAL TECHNIQUES IN AUTOPILOT DESIGN

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Abstract

The analytical and computer techniques employed in the design of aircraft and missile autopilot systems at Douglas Aircraft Company are presented. Roles assigned to digital and analog computation are discussed with the associated reasons for such assignment. The mutual support provided by both types of computation is stressed. Typical examples are used for illustration.

Introduction

One frequently hears discussed, with various degrees of conviction, the relative merits of digital and analog computation. In some fields of endeavor such discussion may be warranted; however, in the design of autopilots at Douglas Aircraft Company, Santa Monica Division, both types have been fitted into design techniques[1]. The mutual support of the design problem by both yields results which use of only one type could not do. It is believed that this mutual support has enough application to other design problems that its description as applied to autopilot design may help others in their problems.

Illustration of the points involved is best accomplished through discussion of a specific design problem. The problem selected here is that of automatically controlling a guided missile so that it will respond accurately to lateral acceleration commands from a guidance computer. Target interception may be made under a wide variety of altitude and speed conditions so it is required that the controlled missile operate satisfactorily over a rather wide band of flight conditions. The problem considered here is not that of hardware design but rather the system design leading to hardware specifications.

Preliminary Analytical Preparation

Preliminary analytical work prepares the way for eventual solution of the autopilot design problem. This usually consists of preparation of the system block diagram, description of the component elements by their respective equations relating input and output, derivation of the complete system equations relating missile response to input signals or disturbances and a listing of the system pecularities or conditions not well described by the preceding

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1 Douglas Aircraft Company has used digital International Business Machine equipment for the last six years, and Reeves Electronic Analog equipment for the last two years, and the Cal. Tech Electrical Analog Computer for four years.

equations. This latter is important in that most of the preliminary analytical work is based upon linear theory and it is important to know the limits involved in order that the effects may be considered in later machine computation.

System Description

Autopilot systems are based upon the use of closed loop or feedback intelligence. In such a system the desired behavior is compared with the actual behavior and any difference, i.e., error, used to drive the system into more exact correspondence. In the sample at hand, this is accomplished by the outer loop of the block diagram shown in Figure 1. The achieved acceleration is compared with the commanded acceleration in the electrical network and, neglecting the other two inputs to the network for the moment, any error taken to the power servo. This servo functions to move the fin in the direction necessary to reduce the measured error. The resulting fin motion imposes forces and moments on the missile causing an incremental change in the lateral (translational) acceleration, the resulting acceleration being measured by the accelerometer for comparison with the commanded acceleration.

The internal rate loop and fin loop feedbacks to the network are required to provide system damping, preventing excessive transient overshoots and oscillations while responding to input commands. The rate loop feedback is accomplished by measuring missile angular rate (about the center of gravity) with a single degree of freedom, spring restrained gyroscope. The fin loop feedback is accomplished by measuring fin actuator travel with a linear potentiometer. All three feedback signals, as well as the input signal, are shaped and summed in the desired manner by the electrical network. The resulting signal which is actually fed to the power servo not only moves the fin in the direction of reducing an acceleration error but provides for moving it in a manner leading to well damped missile motions. The missile structure block in Figure 1 represents measuring instrument pickup caused by structural resonances. Although this pickup contributes nothing to the satisfactory operation of the system, it must be included in the design analysis.

In order to provide a summary of the nomenclature used in the example, the symbols used to represent each individual block are indicated on Figure 1. Specification of each component element by a symbol of the form, F_{oi}, has the meaning:

$$\text{Output} = (F_{oi})(\text{Input})$$

In general, the function F_{oi} is nothing more than the differential equation specifying the component's behavior. When this differential equation is linear, the function becomes the common transfer function of the component.

Preliminary Design Phase

This phase of the design is carried out by direct analytical syntheses and by simplified analysis procedures that require no more complex equipment than desk calculators. The system indicated by the block diagram is simplified to the barest essentials. For example, the effects of the missile structure are neglected completely. The measuring instruments and the power servo are assumed to be ideal components. The frequency characteristics of the electrical network are neglected. In fact, the only dynamics considered are those resulting from forces and moments caused by missile wing deflection and angle of attack.

The important design information obtained in this phase consists of the following:

1. Approximate component gains

2. Measuring instrument ranges

3. Torque and speed requirements of the power servo

4. Desirable aerodynamic characteristics

5. Approximate system performance characteristics

The above information, coupled with engineering experience, is usually sufficient to permit the procurement of hardware, at least for the initial test vehicles.

Frequency Analysis Phase

After the preliminary work has been completed, it is desirable to perform an extensive frequency analysis of the system considering as many of the system characteristics as possible.

The frequency analysis phase is considered to be an extremely important part of any control system design. It furnishes a very clear insight into the manner in which the various components affect the stability and basic response of the system. Virtually all of the design of the electrical shaping network is accomplished through the frequency analysis process. In addition, it is very valuable in tracing down troubles that may be encountered in the flight testing of the design.

This phase of the analysis lends itself well to the use of digital computing equipment because of the large number of routine calculations involved. For the sample problem, the functional operation of each component is represented by a transfer function. If some of the component differential equations are nonlinear, these equations are linearized around certain operating points to permit the derivation of their transfer functions.

The eighteen component transfer functions are represented by expressions of the following form:

$$F_{oi} = \frac{A_0 + A_1P + A_2P^2 + A_3P^3 + A_4P^4 \ldots}{B_0 + B_1P + B_2P^2 + B_3P^3 + B_4P^4 \ldots}$$

where operator notation has been employed by replacing $\frac{d}{dt}$ with P. The order of the polynomials required to represent the component transfer functions varies from zero for the fin potentiometer (that is $F_{\dot{d}\delta}$ is independent of P) to as high as ten or more for some of the electrical network transfer functions. The coefficients of the polynomial terms are determined by the differential equation representing the components.

The system equations which are derived from the block diagram yield six transfer functions of the following type:

$$\frac{n_o}{n_i} = \frac{F_{Sc}F_{\delta E}F_{n\delta}}{1 + F_{\delta E}F_{d\delta}F_{Dd} + F_{\delta E}F_{h\dot{\theta}}F_{Hh}(F_{\dot{\theta}\delta} + F_{\dot{\theta}\delta_B}) + F_{\delta E}F_{an}F_{Aa}(F_{n_a\delta} + F_{n\delta_B})}$$

In order to begin the frequency analysis, a trial electrical network is devised (the first trial network is usually one containing only resistors), and the polynomial term coefficients of all of the transfer functions determined from the basic data. Examples of this basic data are wind tunnel test results, resistor and condenser values, instrument natural frequencies and damping ratios, data obtained by experimental measurements on system components, etc. Then each of the component transfer functions are evaluated as complex number functions of frequency[2]. These complex numbers representing the component transfer function are then combined to obtain the products and sums indicated by the system transfer function shown above. The results are then plotted in the form of Nyquist diagrams and system transfer functions. The Nyquist diagram, which indicates the stability of the system, and the system transfer function, which indicates the basic transient response of the system, are then examined for inadequacies. As a result of the knowledge gained in the previous trial, the electrical network or one or more of the component transfer functions are modified and the cycle repeated. An autopilot system that operates over a wide range of aerodynamic conditions (i.e. a number of sets of aerodynamic transfer functions must be considered) may require that the above cycle be repeated several hundred times before a sufficiently optimum design is achieved.

Our digital equipment has been programmed to perform the above transfer function calculations. This has proved very economical when it is considered that the equipment can perform one complete cycle of the above calculation in about 2.5 machine hours as compared to about 100 man hours required for a manually computed solution. Selective storage of parts of the computation reduces the average time per cycle to about 1.25 machine hours. In addition, the machine results are much more dependable than the manually computed results when the problem is this complex.

The ability to perform a complete frequency analysis of a complex system in such a short time becomes a tremendous asset if, as sometimes happens, the test program shows that a design change is required. The change can be completed in a matter of hours instead of weeks, thus greatly expediting flight test schedules.

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2 This is accomplished by the well known substitution, $P = j2\pi f$, where $j = \sqrt{-1}$, and f is frequency in cps.

Every attempt is made to eliminate the necessity of processing the data that goes into the machine or the results computed by the machine because it is felt that the machine itself is the most efficient processor. The program for the digital equipment is arranged to use the basic data in the form in which it is most easily available; the results are printed out in the form most easily plotted.

For one complete cycle in the sample problem, the input to the machine consists of approximately 200 numbers. The machine performs some 20,000 operations[3] with these numbers, and then prints out slightly more than 1500 numbers for plotting. A survey is presently being made to determine the feasibility of using an automatic plotter for these results. A sample Nyquist diagram and overall transfer function as obtained from digital equipment is shown in Figures 2 and 3.

Analog equipment does not appear to be as well suited as the digital equipment to perform frequency analyses of this type from the standpoint of setup time, running time or accuracy of results. In fact, it is estimated that an analog solution would be more time consuming than the hand solution, because of the difficulty in obtaining the large number of amplitude ratio and phase measurements.

An additional insight into the stability of a system is achieved if the roots of the characteristic[4] equation can be found. This equation, being of about the 20th order, is difficult to obtain as a polynomial in P and, once obtained, considerable difficulty is encountered in solving for its roots. A digital program to compute, from the same basic data required for the transfer function process, the characteristic equation and its roots has been devised. Setup is not quite complete at this time, but it is estimated that the running time will average 3 hours per solution.

Transient Analysis

Throughout the frequency analysis phase of the design, the transient response of the system must be considered. The transfer function gives a qualitative view of the transient, but at times it is advisable to actually compute the transient response of the linearized system. This can be done conveniently by the following Fourier series process. First, the input function is represented by a Fourier series[5]. This series is then modified by the values of the system transfer function to obtain the series that represents the transient response. This resulting series is summed up to obtain the transient response. A sample transient is shown in Figure 4.

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3 These computations are performed on a "floating decimal" scheme which permits eight significant figures to be carried on all numbers, within the magnitude range from 10^{-50} to 10^{+50}.

4 The characteristic equation is obtained by setting the denominator of the transfer function $\frac{n_o}{n_i}$ equal to zero.

5 A digital process has been programmed to compute the first 40 harmonic terms of a general periodic input in an average time of 3 machine hours. This process has proved very valuable in the reduction of flight test data.

This process has also been programmed for computation on the digital equipment. For example, to obtain the transient response to a step function input, the values of the transfer function at 40 to 200 discrete frequencies are used as inputs to the machine. (This data is the same data that was computed by the machine in the transfer function process described previously.) When 40 frequencies are used, the machine performs some 40,000 operations to obtain 50 points on the transient response in 2 machine hours. It is estimated that this operation performed by manual calculation would require approximately 60 man hours.

The transient analysis performed by the above process is very restricted for two reasons. First, the theory of the transfer function approach requires that the system be represented by linearized differential equations whose coefficients do not vary with time; second, the required digital machine time becomes prohibitive if large numbers of transients are desired, (1,000 to 5,000 may sometimes be required).

It is in the performance of an exhaustive transient analysis of the system that analog computer equipment has proved so useful. Employment of analog equipment permits a very complete study of nonlinear phenomenon and 3-axis coupling problems, and it also permits realistic evaluation of transients when component characteristics change appreciably during the time of the transient. For example, the aerodynamic parameters change throughout the flight of a missile because the velocity and/or altitude is continually changing. These complicated analyses can be conducted by solving the set of simultaneous differential equations (nonlinear or time varying) digitally, but this is very time consuming. It should be mentioned, however, that a digital solution of this sort is often required as a check problem[6] for an analog setup.

One important system nonlinearity is that associated with the aerodynamic restoring moment as the missile rotates away from zero angle of attack, i.e., the aerodynamic spring. A typical variation is shown in Figure 5. The dashed lines on the figure show the linear approximations made for use in the digital frequency analysis. Analog equipment permits the inclusion of the actual characteristic. A similar nonlinearity is associated with most power servos and is analyzed in the same manner.

Another important nonlinearity is associated with measuring instrument pickoff granularity. This problem is illustrated in Figure 6 for a wire wound potentiometer pickoff. Again, the dashed line represents the linear approximation used in the digital frequency analysis. The true characteristic is almost impossible to include in a digital analysis, but it may be included readily in an analog analysis by driving wire wound potentiometers with a computer servo. A sample transient showing the effect of too coarse pickoff granularity is shown in Figure 7.

Aerodynamic cross coupling phenomenon, because of their nonlinear nature, are also very difficult to study by digital techniques, but they may be studied readily by means of analog equipment.

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6 The ratio of digital to analog machine running time may be of the order of 1000 to 1 for these check problems.

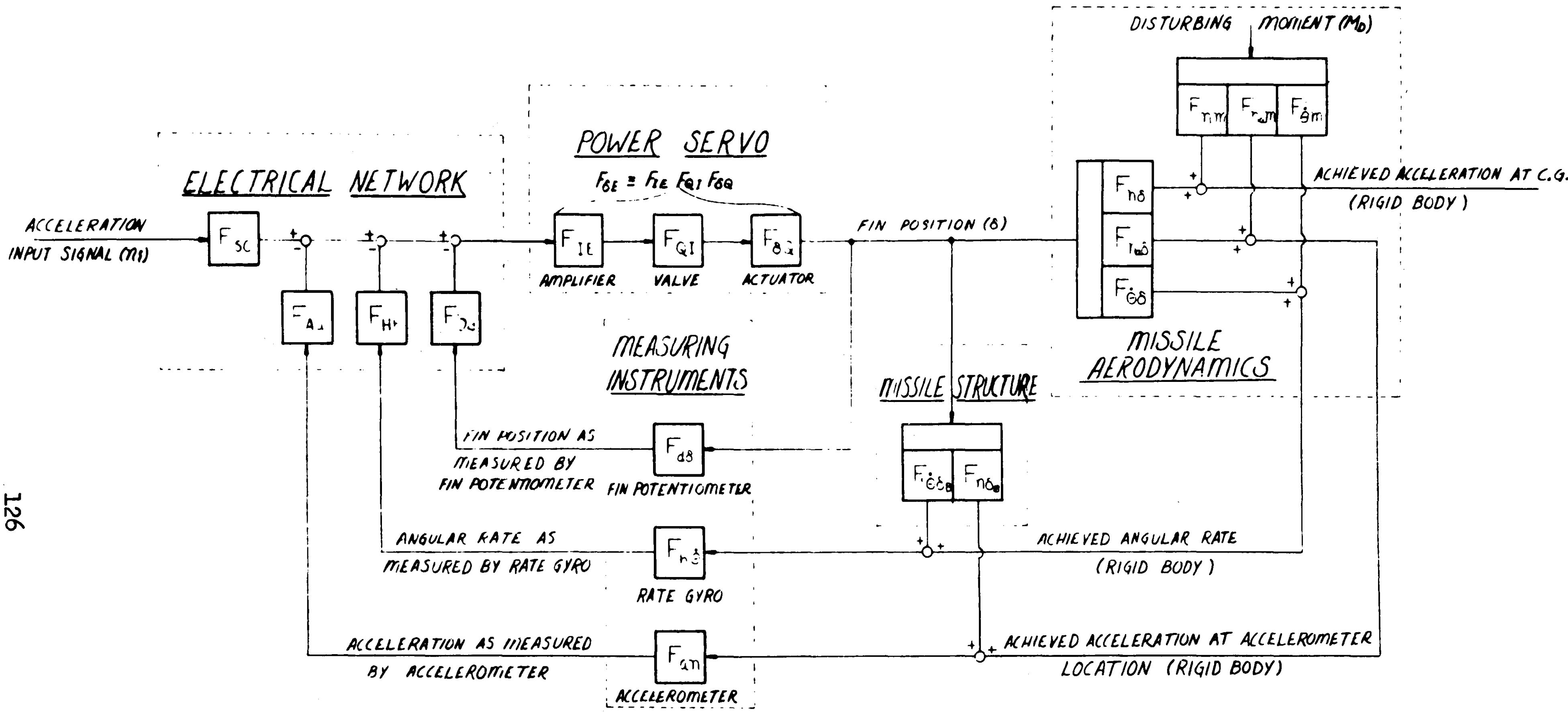
Another important advantage of using analog equipment can be realized if a real time scale can be employed in the analog (i.e., the analog works at the same speed as the actual system). The aerodynamics and measuring instruments can be represented by analog equipment. The output voltage of these analogs can be fedback through the actual electrical network to drive the power servo. The power servo in turn drives the analogs, thus obtaining a very realistic simulation. Transients may be run very quickly on such a piece of equipment, but it is not as flexible as a more general setup. It does have a distinct advantage in that hardware which is difficult to represent by differential equations can be used in its exact role in the system. However, some analyses performed with analog equipment require that the system be analoged on an extended time scale so that the dynamics of the computer servos do not distort the results. This extended time scale prevents the use of actual components in the analog.

Conclusions

The use of both digital and analog equipment permits a much more optimum design than could possibly be achieved in the absence of such equipment. Digital equipment has some advantage in the accuracy of solutions, but the accuracy of analog equipment is sufficient for most autopilot engineering purposes. Digital equipment has a distinct advantage in that it permits the use of familiar analysis techniques which have been developed in communication engineering; the digital equipment simply performs the labor involved in applying these techniques to complicated autopilot systems. The frequency analysis performed on the digital equipment seems to give a better qualitative understanding of how the various parameters effect the system stability and basic response characteristics and provides a firm general design foundation; the transient analysis performed on analog equipment seems to be the only practical way of studying certain nonlinear phenomenon. However, digital equipment is usually required to solve check problems for the analog equipment.

It must be recognized that the computer tools available to the system designer have a major influence upon the nature of the completed design (i.e., hardware specifications). Access to digital computers with the consequent thorough system frequency analysis permits a more optimum selection of component dynamic characteristics. However, a design that is completed on digital equipment only, tends to be as linear as possible; the use of analog equipment may permit the designer to exploit some of the advantages of nonlinear systems. The design that is completed on the basis of linear theory tends to be over conservative in regard to allowable tolerances, whereas the use of analog equipment generally permits a more realistic evaluation of the tolerances, and may greatly alleviate some production problems. In addition, the use of analog equipment gives the designer the distinct advantage of using actual components in the solution of problems.

The use of digital and analog equipment tends to result in a much more complete design then would have been obtained without this equipment, but the cost of the design is usually increased because so many more cases and conditions are investigated. The real saving is made in the flight test program which tends to be shorter, more successful, and to require fewer test vehicles.



LATERAL ACCELERATION CONTROL SYSTEM BLOCK DIAGRAM

FIGURE 1

NYQUIST DIAGRAM

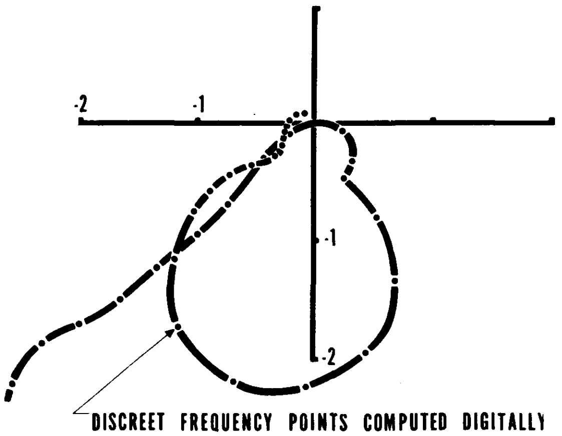


Fig. 2

SYSTEM TRANSFER FUNCTION

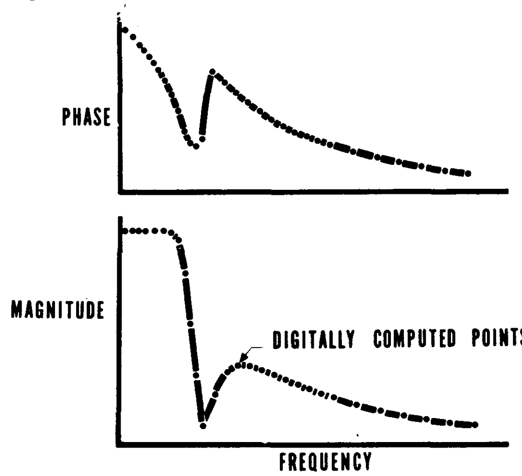


Fig. 3

TRANSIENT RESPONSE

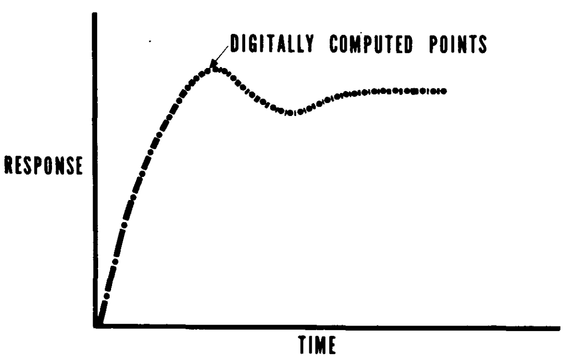


Fig. 4

AERODYNAMIC RESTORING MOMENT

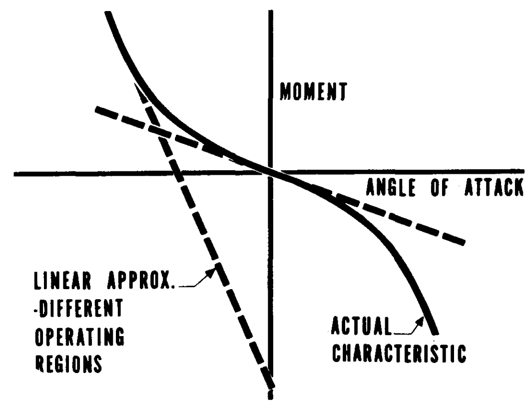


Fig. 5

POTENTIOMETER GRANULARITY

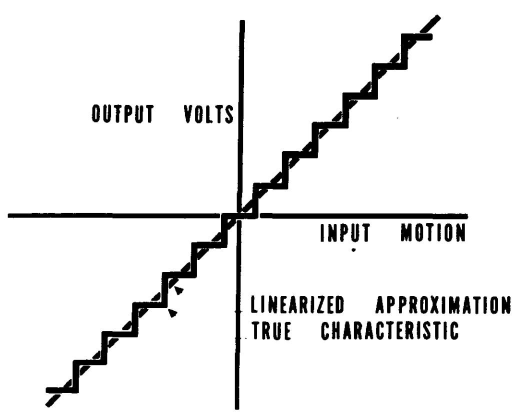


Fig. 6

ANALOG COMPUTED TRANSIENT

POTENTIOMETER GRANULARITY TOO COARSE

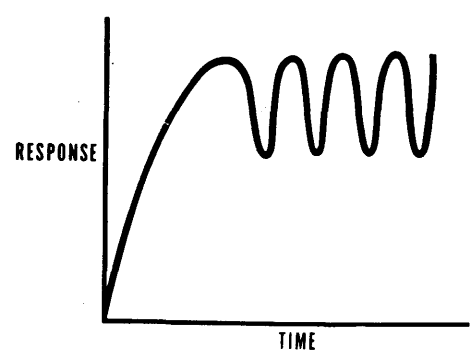


Fig. 7