analog and digital solutions, the one shown in this figure represents the most critical; that is, the mode having the lowest flutter speed. Also shown are the frequencies of oscillation for the analog and IBM at various stability and speed conditions. Comparison of the two solutions indicates that the frequency of oscillation at flutter agrees remarkably well. The flutter speed for the analog, however, is higher than the speed obtained by the digital setup. This difference can be explained by adjusting the digital calculations to correspond to the amount of inherent "structural" damping present in the analog. An estimate of the analog damping is approximately $g_s = 0.015$. The original digital calculations correspond, of course, to no structural damping. If a value of damping, $g_s = 0.015$, is used for the digital calculations, the revised digital stability curve corresponds almost identically to the analog curve. Thus the necessary correlation has been established. At this point the effect of numerous variations of structural or aerodynamic parameters on the critical flutter speed can be investigated in a relatively short time.

Approximately one to two weeks is required to obtain both the analog and digital correlations.

Concluding Remarks

This paper attempts to define the major phases of dynamic loads problems faced by the dynamicist at various design stages of the airplane. At each stage the nature of these problems was investigated and the following characteristics were found to exist for each:

1. The dynamic problems are becoming increasingly complex due to higher airplane speeds and increased flexibility due to thinner lifting surfaces necessary for optimum performance.
2. The amount of time required for obtaining accurate results which make good "engineering sense" is critically limited since accurate structural and aerodynamic data is not readily available.
3. As a result of the lack of data a large number of parameter variations are required in the preliminary design stage. Examination of the other two stages—shake test and final analysis—also indicates the requirement of a large number of parameter variations.
4. As a result of the increased complexity of the dynamics problem the results must be reasonably guaranteed to be free of errors.

In view of these characteristics a review of possible methods using high-speed computing machines—both digital and analog—was made and an approach was determined using both the analog and digital computing machines in an integrated manner. An example illustrating the use of both types of computers is given, indicating a means of rapidly correlating the results obtained as well as permitting the dynamicist to use the best features of each computer to meet the stringent requirements necessary for a successful analysis.

A General Digital Computer Program for Static Stress Analysis

P. H. DENKE AND I. V. BOLDT†

Summary—**In a digital computer installation devoted primarily to the solution of aircraft design problems, an attempt has been made to provide the engineer, whenever possible, with programs which are general enough to be applicable to a large class of the problems which he typically encounters. Such a general program has been devised for the stress and deformation analysis of aircraft structures. The mathematical formulation of the problem and the program for its solution are discussed, and some selected applications of the analysis to actual structures are presented.**

Introduction

THE EXISTENCE of the high performance digital computer and the introduction of matrix algebra into structural analysis have created conditions favoring the development of a general program for solving static stress problems. To be most useful such a program should be applicable to any structure without additional programming time, should require as input data no more than the basic geometric and elastic parameters which define the structure, should be expressed in terms familiar to the practicing stress analyst, should require a minimum of machine time with provision for rapidly taking into account changes of stiffness and loading, and should permit the accurate analysis of highly complex indeterminate structures. The following paper describes a program which was designed with these requirements in mind.

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Static Stress Analysis

The problem considered is the stress and deflection analysis of statically indeterminate structures. When either the Maxwell-Mohr or the least work approach is adopted, the solution of the problem usually requires the following steps: (1) Given the co-ordinates of the structure, compute certain geometric constants, such as direction cosines and moment arms of forces; (2) write the equations of equilibrium; (3) write the equations of continuity. The solution of the equations of equilibrium and continuity completes the analysis of the structure.

The equations appropriate to the analysis of indeterminate structures have been known for many years and have appeared in a variety of forms. However, the introduction of digital computing machinery has led to new developments, one of which is the use of matric notation. The matric formulation of the problem is rather recent and has occurred historically in what seems like reverse order; that is, the equations of continuity have come first. The matric continuity equations have appeared in the literature in several places.[1-3] Denke has given matric formulations of the Maxwell-Mohr equations which are general enough to account for the effects of thermal deformation and certain nonlinear effects.[4]

However, the matric formulation of the continuity conditions still leaves a large portion of the problem unsystematized and poorly adapted to machine computation. This fact was pointed out by Langefors,[5] who developed a method of generating the matrices required in the continuity equations for beamlike structures from matrices established for single cells. This matter was given further study by Denke,[4] who developed in very general terms the matric equations of equilibrium and certain linking equations which permit generation of the continuity matrices from the conditions of statics. This work virtually completes the matric formulation of the equations of equilibrium and continuity. There remains the geometric problem mentioned previously. In the following discussion the matric equations of continuity and equilibrium are reviewed, and a systematic method of generating the equilibrium matrices from the structural geometry is presented. These geometric equations allow machine computation to begin at an earlier stage of the solution, and greatly simplify the work of the analyst.

The Continuity Equations

Enough redundant constraints are removed (or "cut") to produce a statically determinate structure. Then in Denke's notation,[4] the internal forces and deflections of the indeterminate structure are given by

$$F = [I - f(f^TDf)^{-1}f^TD]F_0, \tag{1}$$

and

$$\Delta = f_\Delta{}^TD[I - f(f^TDf)^{-1}f^TD]F_0 \tag{2}$$

[1] B. Langefors, "Analysis of elastic structures by matrix transformation with special regard to semi-monocoque structures," *Jour. Aeronaut. Sci.*, vol. 19, pp. 451–458; July, 1952.

[2] L. B. Wehle and W. Lansing, "A method for reducing the analysis of complex redundant structures to a routine procedure," *Jour. Aeronaut. Sci.*, vol. 19, pp. 677–684; October, 1952.

[3] H. Falkenheimer, "Systematic analysis of redundant elastic structures by means of matrix calculus," *Jour. Aeronaut. Sci.*, vol. 20, p. 293; April, 1953.

[4] P. H. Denke, "A matric method of structural analysis," Proc. Second U. S. National Congress of Applied Mechanics, 1955.

[5] B. Langefors, "Matrix methods for redundant structures," *Jour. Aeronaut. Sci.*, vol. 20, p. 292; April, 1953.

where

F = a matrix of forces in the indeterminate structure,
F_0 = a matrix of forces in the determinate structure resulting from the external loading,
f = a matrix of forces in the determinate structure resulting from unit values of the redundants,
f^Δ = a matrix of forces in the determinate structure resulting from unit dummy loads,
Δ = a matrix of deflections of the indeterminate structure, and
D = a matrix of member flexibilities.

The Equilibrium Equations

The statically determinate structure is broken into free bodies. Associated with each free body are a number of degrees of freedom, and acting upon the various free bodies are a number of force pairs and reactions. Denke[4] explains the usage of the terms "degrees of freedom" and "force pairs."

The matric equilibrium equations are

$$Q_0 = -M^{-1}P_0, \quad q_\Delta = -M^{-1}P_\Delta, \quad q_x = -M^{-1}P_x, \tag{3}$$

where

M = a matrix of the components in the various degrees of freedom of unit values of the force pairs and reactions,
Q_0, q_Δ, q_x = matrices of statically determinate force pairs resulting from external loads, unit dummy loads, and unit redundants, respectively, and
P_0, P_Δ, P_x = matrices of components in the various degrees of freedom of the external loads, unit dummy loads, and unit redundants, respectively.

Eqs. (3) give the statically determinate internal loads in terms of the external loads and the matrix of "generalized co-ordinates" M.

The Linking Equations

The continuity matrices are related to the equilibrium matrices by the equations

$$F_0 = NQ_0 + H_0, \quad f_\Delta = Nq_\Delta + H_\Delta, \quad f = Nq_x + H_x. \tag{4}$$

These transformations are required because the force pairs used in solving the equilibrium problem are not necessarily the same as the member "forces" required in the continuity analysis. Note that the "forces" referred to in (1) to (4) are generalized forces and represent either forces or moments.

The Geometric Equations

Eqs. (1) to (4) comprise a nearly complete matric formulation of the indeterminate structures problem. In order to start the analysis, however, one must have the matrices D, N, H_0, H_Δ, H_x, M, P_0, P_Δ, and P_x. The matrix D contains the element flexibilities and is computed from basic data. The computation of the matrices

N, H_0, H_Δ, and H_x is simple and often trivial. The matrices M, P_0, P_Δ, and P_x can be computed by the method outlined in the following paragraphs.

At this stage it is convenient to introduce a second set of internal forces called "subordinate force pairs" related to the principal force pairs by simple transformations. It is necessary to make such a transformation when, for example, it is desired to utilize the notion of shear flow in a panel (the principal force pair) in place of the four forces acting on the edges (the subordinate force pairs). Similarly, a single external loading condition (corresponding to a column of P_0) can be thought of as a principal generalized force, while the individual external loads can be regarded as subordinate forces. Similar remarks apply to deflection forces and redundants. Subordinate forces and quantities associated with them are indicated by a prime (′) attached to the symbol.

By definition, then, the matric element m_{ij}' represents the component in the ith degree of freedom of a unit value of the jth subordinate force pair. These "generalized co-ordinates" may be classified as "translation-force," "rotation-moment," "rotation-force," or "translation-moment," depending on the type of degree of freedom and force pair. Fig. 1 shows a reference frame,

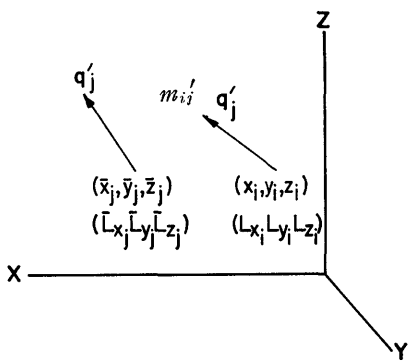

Fig. 1—Force pairs and degrees of freedom.

one member of a subordinate force pair, and the component of that force pair in a given degree of freedom. The figure shows the notation for co-ordinates and direction numbers. The co-ordinates are the co-ordinates of arbitrarily chosen points on the lines of action of the force pair and degree of freedom. The generalized co-ordinates are given as follows for the four cases.

1. Translation-Force

$$m_{ij}' = C_{ij}(l_{x_i}\bar{l}_{xj} + l_{y_i}\bar{l}_{yj} + l_{z_i}\bar{l}_{zj}). \tag{5}$$

2. Rotation-Moment

$$m_{ij}' = C_{ij}(l_{x_i}\bar{l}_{x_j} + l_{y_i}\bar{l}_{yj} + l_{z_i}\bar{l}_{zj}). \tag{6}$$

3. Rotation-Force

$$m_{ij}' = -C_{ij}\begin{vmatrix} l_{x_i} & l_{y_i} & l_{z_i} \\ x_i - \bar{x}_j & y_i - \bar{y}_j & z_i - \bar{z}_j \\ \bar{l}_{x_j} & \bar{l}_{y_j} & \bar{l}_{z_j} \end{vmatrix}. \tag{7}$$

4. Translation-Moment

$$m_{ij}' = 0. \tag{8}$$

In the above expressions the symbols l_x, l_y, etc., are direction cosines derived from the direction numbers defined in Fig. 1. The symbol C_{ij} is defined as follows:

$C_{ij} = 1$ if the ith degree of freedom contains a component of that member of the jth subordinate force pair for which direction numbers were defined.

$C_{ij} = -1$ if the ith degree of freedom contains a component of that member of the jth subordinate force pair for which direction numbers were not defined.

$C_{ij} = 0$ if the ith degree of freedom contains no component of the jth subordinate force pair.

The symbols C_{ij} are referred to as "topologic coefficients," since they define the manner in which the structure is connected. The m_{ij}' are the elements of the matrix M' thus:

$$M' = [m_{ij}'].$$

Let

$P_{0_{ij}}'$ = the component in the ith degree of freedom of the jth subordinate external load;

$P_{\Delta_{ij}}'$ = the component in the ith degree of freedom of the jth subordinate dummy (deflection) load.

$P_{x_{ij}}'$ = the component in the ith degree of freedom of the jth subordinate unit redundant force pair.

Evidently $P_{0_{ij}}'$, $P_{\Delta_{ij}}'$, $P_{x_{ij}}'$ are analogous to m_{ij}' and can be obtained by formulas similar to (5), (6), (7), and (8). These analogous formulas need not be written down. The following matrices are defined:

$$P_0' = [P_{0_{ij}}'] \qquad P_\Delta' = [P_{\Delta_{ij}}'] \qquad P_x' = [P_{x_{ij}}'].$$

The components of the principal forces in the various degrees of freedom may be obtained from the components of the subordinate forces by the following transformations:

$$\left.\begin{aligned} M &= M'K_m \\ P_0 &= P_0'K_0 \\ P_\Delta &= P_\Delta'K_\Delta \\ P_x &= P_x'K_x \end{aligned}\right\}; \tag{9}$$

where

$K_{m_{ij}}$ = contribution to the ith subordinate force pair of jth unit force pair,

$K_{0_{ij}}$ = value of ith subordinate external load in jth loading condition,

$K_{\Delta_{ij}}$ = contribution to ith subordinate dummy (deflection) load of jth unit deflection load,

$K_{x_{ij}}$ = contribution to ith subordinate redundant force-pair of jth unit redundant force-pair.

The Deflection Influence Matrix and Free Vibration

The deflection influence matrix is given by

$$\delta = f_\Delta^T D[I - f(f^T Df)^{-1} f^T D] f_\Delta. \quad (10)$$

The natural frequency ω and the modal column Δ can be obtained by standard methods from

$$(\omega^2 \delta M - I)\Delta = 0, \quad (11)$$

where M is a mass matrix.

Some Applications of the Program

Some of the structures to which the static stress program has been applied are shown in Figs. 2 to 6. Fig. 2 is a diagram of the root structure of a swept wing. This structure had 98 elastic elements, 127 statically determinate forces, and 28 redundants. The complete stress distribution was obtained for 28 separate loading conditions. Deflections were obtained at 28 points and the deflection influence matrix at the outboard end was computed for use in flutter analysis.

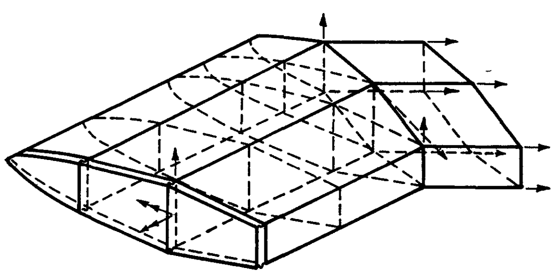

Fig. 2—Swept wing root structure.

Fig. 3 shows a doubly symmetric segment of fuselage structure having 5 frames and 18 stringers. Loads were applied to the cross beam at section AA. The purpose was to find the stresses in the center frame; the rest of the structure was included to provide a realistic condition of support. There were 180 statically determinate forces and 29 redundants. Stresses were also obtained in the adjacent frames and fuselage skin.

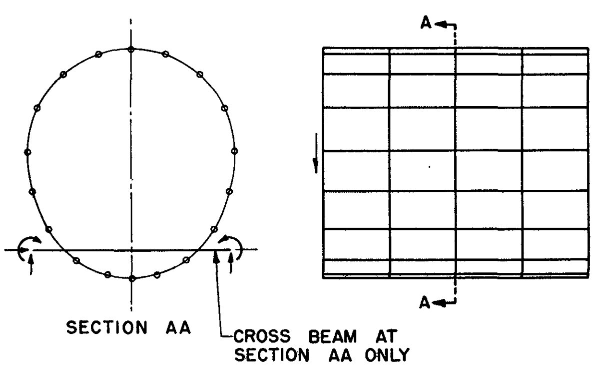

Fig. 3—Fuselage frame analysis.

Fig. 4 is a diagram of a noncircular, nonuniform fuselage frame for which natural modes and frequencies of vibration in the plane of the ring were computed. There were 24 elastic elements, 12 mass elements and 2 redundants. The solution required the determination of the eigenvalues of a 24th-order matrix. The figure shows the fifth mode of vibration. As a check on the method, the modes and frequencies of a uniform circular ring were also computed. The results checked values computed from Hoppe's formula with a maximum of 2 per cent error for the first five modes, even though only seven mass elements were used per 90 degrees of arc.

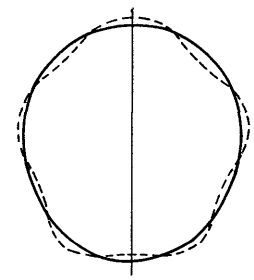

Fig. 4—Vibration of a noncircular, nonuniform ring.

A rib structure of the Vierendeel type is shown in Fig. 5. This rib was about 8 feet long, and was worthy of careful analysis. The upper part of the figure shows the structure and loading, the lower part shows the equivalent elastic framework and method of support. As indicated, support was provided by a number of springs representing the elastic restraint provided by surrounding structure. There were 118 statically determinate forces and 47 redundants. A complete stress and deflection analysis was obtained. The deflections are shown in the upper part of Fig. 5. Notice the complicated shape of the outline of the elastic framework. This complication is typical of the geometric detail which can be accounted for by digital methods.

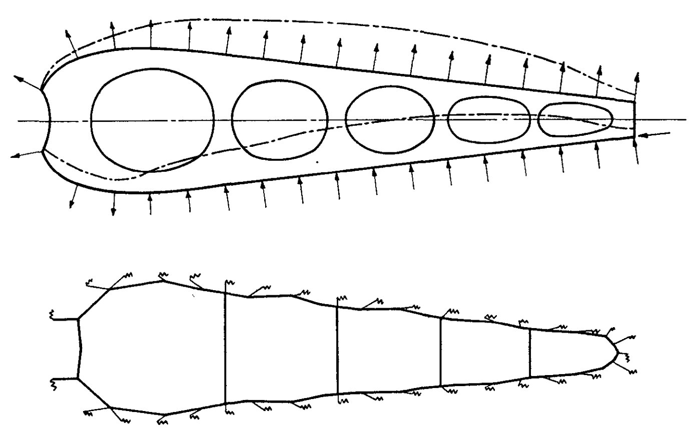

Fig. 5—Rib analysis.

Matrix Arithmetic for Systems of Large Order

The foregoing section has described a method for treating a large and important class of problems which arise in the design of airframes. It is a method with inherent appeal for its conciseness and generality, and for the explicit manner in which it exhibits the relationships existing among the physical quantities which characterize the structure.

The power of the method as a practical tool must, nevertheless, be finally measured in terms of the success with which the necessary matrix operations can be carried out when the system is one of large order. The remainder of this paper will be devoted to a description of the steps taken by the Computing Engineering group at Douglas, Santa Monica, to develop a digital computer program which is capable of performing these calculations rapidly and accurately.

The expression "a matrix of large order," is subject to varied interpretations, depending upon the circumstances in which it is encountered. It is natural that the computing analyst should assign to it a meaning which is primarily a function of the digital computer facilities at his disposal. The computer in use at Douglas, Santa Monica, is the IBM Model 701, providing 2,048 words of fast-access electrostatic memory, with auxiliary storage consisting of four magnetic drums and four magnetic tape units. Matrix operations are most conveniently and rapidly performed if they can be entirely contained within the electrostatic memory of this computer and the limiting dimensions for such operations may be put at 30×30. In what follows, the term "a matrix of large order," will imply one of dimensionality exceeding this limit; for such matrices, it is necessary at all times to make use of one or another of the auxiliary memory units.

Most of the matrix problems encountered by a group which serves as the computing facility for a large engineering department are of moderate dimensions. In order to provide solutions in the least possible elapsed time, and at small programming cost, it is necessary to make extensive use of standard matrix routines; this is especially true of jobs of an exploratory nature, and those which consist of a relatively small number of cases.

This method is often uneconomical, however, for a general program, and in particular for one requiring significant computing time for its solution. In such a case, a program more closely tailored to the characteristics of the problem may be desirable.

Programming the Static Stress Analysis

The static stress analysis program has provided the motivation for developing a specialized group of routines to perform large-order matrix arithmetic, in which the operands for a matrix operation are stored on two magnetic tapes, and the resultant is written on a third tape as it is generated. Each row or column of a matrix forms a separate record on the tape, and because only one such record need be retained in electrostatic memory at any time, it is manifestly possible to handle systems of considerable dimensions; a nominal limit of 300th order is imposed for these routines. Since many of the matrices, especially those of the equilibrium and linking equations, are characterized by a large proportion of zero elements, the principle has been adopted that only nonzero elements are retained, each with its row and column indexes.

The operation of inversion, or, more precisely, the solution of a system of simultaneous equations, occurs twice in the static stress analysis: first, in solving the equations of static equilibrium [see (3)] and second, in obtaining the redundant forces [see (1)], where the former is, in general, of much larger order than the redundant system. It is possible, however, to so order the degrees of freedom and forces that the coefficient matrix M of the equilibrium system is of triangular form, or so nearly triangular that premultiplication by an operator, which may be obtained by inspection, reduces it to this form. In such a case, the solution of the simultaneous equations is easily carried out in stepwise fashion with the matrices stored on tape.

It may be pointed out that a procedure which gives this triangular matrix is familiar to the stress analyst; in fact, it is equivalent to the method of joints, in the analysis of statically determinate pin-jointed trusses.

A solution for the redundant forces offers no such direct escape. The present program imposes a limitation of thirty or less on the number of such forces, and the inversion is accomplished in the electrostatic memory, using the well-known Jordan elimination method. However, a structure with nearly twice this number of redundants has been treated by the method of partitioning, and a modification of the program now in preparation is expected to permit consideration of as many as 110 redundants in the near future.

All matrix operations described above are carried out using standard floating binary point arithmetic; this device, although relatively slow on the Model 701, has been found indispensable in preserving the accuracy of the calculations.

The generation of the equilibrium matrices from their topologic equivalents is a recent addition to the program. It is a basically nonmatrix operation which materially reduces the amount and complexity of information which the stress engineer must provide when he prepares a problem for solution.

Computing Time

Computing times for the routines described above, when matrices are stored on tape in addressed-element form, are somewhat variable, depending upon the proportion of null elements. Approximately three hours are needed for the solution of a typical system with 225 degrees of freedom and the maximum of 30 redundant forces, including card input and printing operations. Some representative times for specific operations are given below:

Addition:	228×228 (full)	3.3 minutes
Multiplication:	$(228 \times 228) \cdot (228 \times 228)$	
	(2% full)	20 minutes
	$(100 \times 25) \cdot (25 \times 25)$ (full)	15 minutes
Inversion:	25×25 (full)	1 minute.

It is characteristic of digital techniques for large-order matrix systems that what may be called "bookkeeping" considerations assume an increasing importance. An operation such as transposition, which is trivial for a matrix retained entirely in electrostatic memory, involves significant amounts of sorting and collating when the matrix is stored on tape, and the time required quickly approaches that necessary for the multiplication of a comparable system.

Numerical Accuracy of the Method

There is a considerable body of recent literature dealing with the errors which occur in matrix and other calculations involving many repeated arithmetic operations upon approximate numbers such as those used in digital computation. No attempt will be made here to review the results of these studies. The formulas which have been developed to estimate bounds for such errors are a function of the individual case, and are often difficult to apply in practice.

statically determinate forces. This fact was pointed out by Hoff[6] in connection with the analysis of a plane ring.

A check of over-all accuracy can quickly be made by verifying the equilibrium of forces acting upon individual elements chosen as a sample from those which comprise the structure.

Fig. 6 reproduces a portion of an accuracy test of the inversion performed in determining the internal forces and deflections of a structure. This test is calculated as a routine part of the analysis, and the result, which should closely approximate the negative identity matrix, is printed. The matrix elements shown consist of a characteristic part of seven significant digits, followed by the power of ten by which this characteristic must be multiplied to obtain the true number. In the matrix of order 28×28, from which the figure was taken, the largest off-diagonal element was −0.00000208, and the inversion was considered satisfactory.

An element selected at random from a wing root study

MATRIX 9999

		COL.			COL.			COL.			COL.		
ROW	25	19	.2777670-	6-	20	.1341105	6-	21	.1263106	6-	22	.1447042-	6-
		25	.1000000-	1	26	.2680463	7-	27	.1527951	7-	28	.1015724	7-
ROW	26	1	.2305023	7-	2	.7404014	7-	3	.2260786-	6-	4	.2998490-	11-
		7	.2281740	6-	8	.7105427-	13-	9	.2421439-	7-	10	.7171184	7-
		13	.2328306-	8-	14	.1278977-	11-	15	.8509960	7-	16	.7881317-	7-
		19	.1899898	6-	20	.1755543-	6-	21	.1990702-	6-	22	.1946464	6-
		25	.5424954-	7-	26	.1000000-	1	27	.2962770-	7-	28	.1603621-	7-
ROW	27	1	.1580920	6-	2	.2079178	6-	3	.3736932-	7-	4	.7680967-	11-
		7	.7229391	7-	8	.1204370	11-	9	.2444722	6-	10	.3182795	6-
		13	.8032657	8-	14	.2234657-	11-	15	.1528533-	6-	16	.1281733	6-
		19	.2549496	6-	20	.3562309	7-	21	.3539026-	7-	22	.2491288	7-
		25	.2750312-	7-	26	.2660090-	7-	27	.1000000-	1	28	.2927845-	7-
ROW	28	1	.1965091-	6-	2	.2665911-	6-	3	.5925540	7-	4	.8213874	11-
		7	.9080395-	7-	8	.4511946-	12-	9	.1860317-	6-	10	.2288725-	6-
		13	.2922025-	7-	14	.3179679	11-	15	.7811468	7-	16	.7642666-	7-
		19	.2814922-	6-	20	.1164153-	8-	21	.2561137	7-	22	.7369090-	7-
		25	.4583853	8-	26	.4452886-	8-	27	.1577428	7-	28	.1000000-	1

Fig. 6—Portion of inversion check. $-I = -(f^T D f)^{-1}(f^T D f)$.

It is, however, possible for the analyst to ensure a satisfactory degree of computational accuracy by appealing to certain physical principles of structural analysis. First, as has been pointed out, the force pairs and degrees of freedom for the statically determinate structure should be numbered in such an order that the M matrix is approximately or actually triangular, since this procedure not only simplifies the solution but also tends to provide a strongly nonsingular matrix. The second principle may be stated as follows: If the structure which remains after relaxation of the redundant constraints is physically strong, then the solution of the continuity problem tends to be mathematically strong, because the redundants are small compared to the

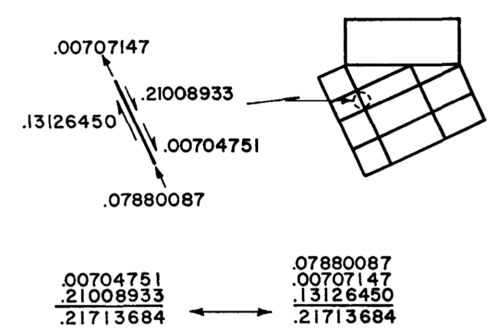

Fig. 7—Equilibrium check.

[6] N. J. Hoff, "Stress analysis of rings for monocoque fuselages," *Jour. Aeronaut. Sci.*, vol. 9, p. 245; May, 1942.

is presented in Fig. 7, showing the forces which act upon it, as well as a numerical verification that the equilibrium conditions are satisfied.

It is perhaps superfluous to add that the final test of results obtained with any such mathematical model must be their reasonableness, based upon experience with the behavior of similar structures and upon experimental verification whenever this is possible.

Conclusions

A general digital-computer program for static stress analysis has been described. This program is applicable to a large class of structures without additional programming time, and relieves the stress analyst of most of the computational burden. After the analyst has idealized the structure, and has made it statically determinate, his task consists of a straightforward tabulation of elastic and geometric properties. Once a unit load solution has been obtained, multiple load conditions can be investigated at little additional cost, and the effect of varied elastic properties can be studied by rerepeating only the solution of the continuity equations.

Extensive use of magnetic tapes makes possible the large-order matrix operations characteristic of the stress analysis problem. The program is designed throughout for economy of computation.

Aircraft Performance Studied on an Electronic Analog Computer

L. B. WADEL and C. C. WAN†

Introduction

PERFORMANCE analysis of an airplane consists of the study of various operating characteristics and operating limits. These characteristics may be divided essentially into three major groups as follows:

1. *Performance at Constant Altitudes.* These include the conventional performance characteristics such as maximum speed, rate of climb, and equilibrium turn performance. All of these items may be determined by comparison of "thrust available" and "thrust required" for steady-state operations, based on constant airplane weight and constant altitude.

2. *Performance at Varying Altitudes.* For certain portions of a typical flight program, fuel economy and elapsed time become factors of major importance. It is necessary to establish a theoretical optimum operating schedule which would serve as an upper bound to the attainable performance. Optimum-climb programs for an interceptor and optimum-cruise profiles for a long range bomber belong to this category. Although these problems may be rigorously resolved by means of the calculus of variations, equivalent methods requiring only steady-state considerations can be formulated to yield the required solutions.

3. *Range Performance.* The maximum range, or radius of action, of an airplane for a specific mission profile is one of the most important items in performance analysis. Here, the variation of operating altitudes and airplane weight must be properly accounted for. Generally, this can be achieved through appropriate combination of results obtained from the first two groups.

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During the preliminary design of a prototype aircraft, a fairly exhaustive study of range performance is usually carried out for a range of design parameters. The proper selection of compromises among these design parameters is made on the basis of the results obtained from such a study. Similar performance calculations would be made also for guided missiles, but different properties will be emphasized. For production aircraft, performance data must be presented in the pilot's handbook. A wide range of variations in atmospheric conditions and mission types is usually included in this presentation.

It may readily be realized that a large amount of computational effort is necessary to provide a complete assessment of the pertinent performance characteristics of an aircraft. It is, therefore, desirable to mechanize performance calculations as much as possible. Several exploratory analog computer studies have been made, with promising results, of certain performance characteristics of a high speed fighter aircraft. The method of approach and the nature of the results obtained during these studies are described in this paper.

The Electronic Analog Computer

The general-purpose electronic analog computer (electronic differential analyzer) as we know it today was developed in the years immediately following World War II; its original mission was to solve the ordinary differential equations which describe the dynamic behavior of guided missiles. The key to such computers is the operational or computing amplifier. This is a high-gain dc amplifier which, when supplied with proper precision input and feedback impedances (usually resistors and capacitors), performs summation, integration, sign-changing, or other more specialized linear operations.