"go back to the drawing board." It is possible, and, on the basis of this experience, eminently practical, to pick oneself up off the floor. Several times, in one or the other of these circumstances, a corrective set of instructions has been programmed, keypunched, and a set of correct answers run off within the hour. Second, at certain unpredictable and unannounced times, problems of moderate difficulty are presented to the computing section for immediate solution. These are problems of extremely high priority with a machine solution time of 1/2 to 3 hours and a total elapsed time from problem presentation to final solution of 4 to 24 hours. This time scale does not permit the normal cycle of algebraic manipulation, programming, debugging, test cases, and solution. The *702* is simple enough in its order structure to permit coding on an "on the spot" basis. Note particularly that corrections or other machine instructions can be entered directly via the card reader or console.

The last-mentioned advantage is part of the final point: The *702* is decimal and alphabetic, which makes for ease of communication between the machine and the programmers, and the machine and the console operators. Further, although numerical analysts work largely with a 10-character vocabulary, the fact that the *702* has 47 characters available is frequently useful, if only for dressing up printed reports.

Lest it be thought that the picture is too rosy, let some of the disadvantages to using a variable-word-length machine for scientific computing be stated. (1) The chief disadvantage probably lies in the economics of the operation: A pure numeric, fixed word-length, parallel machine with equivalent circuit speeds would perform each operation at greater speed, hence lower cost, particularly on long production runs where the word length happened to fit the problem's needs or could be made to fit. This statement would apply with even greater force to a machine with built-in floating point. (2) For some types of logical mathematical work, there is an advantage to using a binary machine, and all present variable-word-length machines are decimal. (3) The variable-word-length principle itself is responsible for programming slips that might not otherwise be made, in that word boundaries occasionally slip. For example, it is easy to store a 4-digit word at a place in memory where a 3-digit bucket has been set up; this defaces the word to the left. It is hoped that much of this sort of nuisance will be eliminated through improvements in the next automatic programming system.

The dual-purpose installation has only one serious drawback, which comes about from the diametrically opposite nature of the work of the two groups when they are sharing one machine. By and large, commercial problems tend toward the routine, "chiseled-in-granite," repeating problems; numerical analysis problems tend to be on-demand one-time (or few-time) affairs. This can lead to questions of priority; fortunately for this thesis, there are questions of priority with any kind or kinds of work on any type of machine.

On the whole, however, it is felt that the advantages far outweigh these somewhat tenuous disadvantages. Attention should be focused on the possibility of adding numerical analysis work to a commercial installation and/or the attractiveness of a dual-purpose installation. In any installation involving commercial problems, the record-keeping (which is the essence of present-day commercial work) gets done; there may then be an opportunity to work on commercial problem solving. When that situation evolves, it may be quite a help to have a scientific group at hand. Under a common leader, the two groups become compatible and ideas cross-fertilize each other.

Unusual Problems and Their Solutions by Digital Computer Techniques

L. ROSENFELD

TEN years ago the primary role of an electronic computer was to develop mathematical tables and to solve the scientific problems which arose during the research and development work being done at that time for the military and other government agencies. Since then, and particularly of late, the emergence of the computer as a powerful and versatile tool for the management of most major business and industrial corporations in assisting them to carry out their normal business operations has been witnessed. These would include such standard operations as payroll accounting, inventory control, production scheduling, invoicing, billing, and cost accounting.

It is natural that company management should now seek other fruitful areas where the computer might be utilized for profitable advantage. By and large, with the possible exception of those companies which are large enough to afford to have on their staff a group similar to an operations research group, the places where management will seek additional areas of work for its computing facility will be chiefly limited to those areas which are normally under the cognizance of the comptroller or finance officer. Usually, management will not think in terms of using the computer as a device which could possibly aid in improving a company system or subsystem which is part of the way by which part or all of the business of the company is carried out. For example, management of a large trucking company probably would not use the computer to help determine the most economical time to replace worn-out vehicles though it might use it to bill its customers; similarly an automobile manufacturer probably would not use the computer to predict what his spare parts inventory should be for the next 10 or 20 years although he would use it to maintain his inventory.

This paper is essentially a brief description of three case histories of what would generally be defined as operations research

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problems. Basically, there were two reasons why it was written: One was to indicate to company management several examples of a large number of vastly different operational problems which were or are being solved with the aid of electronic computers. These problems initially would not have been considered by management as problems which would readily lend themselves for solution by electronic computation. For corporate management, then, this paper perhaps will serve the function of exciting its imagination to suggest other problems or other areas where the computer can be effectively used, and give it the necessary incentive and encouragement to do so.

There was a second reason for writing this paper, namely, to point out to the computer specialist and to the numerical analyst another area where the capabilities of the computer are limited by the state of the art, i.e., for want of the mathematical tools and techniques to solve the problem efficiently. By this is meant that in each of the three case histories which this paper discusses there did not exist any practical set of mathematically defined rules to improve an existing solution except by trial and error. This subject will be discussed in more detail in another section.

Problem I. The Determination of an Optimal Trucking Route Through a Given Traffic Congestion Pattern

This problem can be classified as a large-scale traveling salesmen problem. However, the enormous number of points involved prohibited applying any of the standard methods for solving this type of problem. A large trucking firm, X, with a fleet of more than 250 vehicles, contemplated relocating its main warehouse and dispatching depot from a southerly point, B, to a southwesterly point, A, of a metropolitan midwestern city, Y. An approximate annual savings of $20,000 would be realized because of the relocation. However, more than 60 per cent of the volume of business of company X was the transportation of goods to points in areas or cities to the northeast of city Y. The question arose whether the increase in cost both in additional time and wear of equipment from originating in their new location would more than wipe out the possible annual savings of $20,000.

A survey was taken by a Traffic Planning group which determined the time involved to travel from a point of intersection of two roads to another point of intersection. The total number of roadways considered included the following:

(*a*). Six dual highways which permitted commercial highways.
(*b*). More than 40 2-way streets.
(*c*). More than 130 1-way streets.
(*d*). Two toll bridges.

Consequently, if R_j is defined as one of these roads and P_{ij} as the point of intersection of roads R_i and R_j, more than 5,000 ΔT's were enumerated where ΔT_{ij} equals the time elapsed to traverse from P_{ij} to P_{ij+1}.

In the first approximation to this problem, only the additional cost to traverse the distance from A to B was considered. This reduced the problem by a factor of 100 and hence, a modified simplex method to obtain answers to this approximation was going to be applied. However, the minimal increased costs that were incurred to traverse the distance from A to B were more than $20,000, nullifying this approach completely.

Essentially, the problem was to determine the costs incurred to travel from A to points $D_1, \ldots, D_n$ which were on the perimeter of the city's limits since to travel from D_j to the ultimate destination was the cost common to both the old and new dispatch points. Moreover, the time involved to traverse from B to D_j was known from past experience.

After the initial failure, it was decided to approach this problem from a different point of view which was in essence a trial and error method. The basic problem then, since labor costs and, to a large extent, wear of equipment, were a linear function of time, was to find the minimum time required to traverse the pattern from A to D_j (for discussion, D_j will be considered to be D_1).

If every P_{ik} and D_j is assigned an x and y co-ordinate, a sequence of segments (a segment denoting the distance between two successive P_{ik}'s) will be a possible path for destination D_j (in this case D_1) if every segment of the sequence does not have either an x or y component in a direction opposite to the x and y components of the vector $\mathbf{AD}_1$.

If t denotes a measure of time, $S(t)$ is defined to be the set of points P_{ij} which can be reached by possible paths to D_1 in time t. For sufficiently small values of t, obviously D_1 cannot be reached so that in this case only those points P_{ij} of a possible path to D_1 that can be reached in time t_1 are considered. Hence,

$$S(t)=(P_{i_1j_1}, P_{i_2j_2}, \ldots, P_{i_rj_r})$$

where the $P_{i_kj_k}$ are points of parts of a possible path.

Let $P^*_{ij}(t)$ be a point such that $P^*_{ij}(t)$ satisfies the following conditions:

(*a*). $P^*_{ij}(t)$ belongs to the set of points $S(t)$.
(*b*). $\mathbf{AP}^*_{ij}(t) \cdot \mathbf{AD}_1 \geqq \mathbf{AP}_{ij} \cdot \mathbf{AD}_1$ for all P_{ij} in $S(t)$.
(*c*). $l(AP^*_{ij}(t)) \leqq l(AP_{ij})$ for those P_{ij} satisfying equality in (*b*).

where $l(AP_{ij})$ denotes the length of path to go from A to P_{ij}.

If $P^*_{ij}(t)$ is redesignated by A_1 the problem of getting from A_1 to D_1 can now be considered. Proceeding as above, a $P_{ij}(2t)$ is obtained and redesignated by A_2. Iterating in such a manner an A_r is ultimately obtained which is, in fact, D_1. In effect by trial and error a sequence of paths which minimizes the objective function as a function of a time increment t has been determined. Of course, now the size of the increment can be varied to obtain a minimum-minimum solution for a given set of increments $t_1, t_2, \ldots, t_n$.

This solution was coded for and worked out on the IBM *701* for each D_j and for five different increments t. The weighted additional costs, the weighting factors being a function of the percentage of the volume of traffic destined for D_j compared to the total volume of business, were determined. The smallest additional cost for all points D_j and for the t's used was approximately $14,000 more than justifying the contemplated relocation.

Problem II. Baseball Forecasting

This problem is of interest to management from the point of view that there exists a large number of business and industrial problems whose solutions require the development of a strategy or a set of rules or criteria which can be applied in a given situation. Generally, the situation or problem varies as a function of time. Moreover, if this variation could be predicted on the average and a set of rules or strategy developed accordingly, then a greater expected return or achievement might result in contrast to just establishing a fixed rule which would be satisfactory for all situations.

Briefly, the problem of baseball forecasting[1] was whether a strategy could be developed which would enable one to wager on major league baseball results with a greater expectancy of winning than of losing. If one wishes to win one unit whenever his selection on a given game is the correct one, then he must risk an amount R units, $R>1$ if his choice has the greater likelihood of winning, i.e., the favorite team, and an amount r units, $r<1$ if his choice has the lesser likelihood of winning, i.e., the underdog team. The assigned risk costs are the handicappers' estimate of the opposing teams' relative worth and these risk costs are assigned prior to the start of a game.

Major league baseball consists of two distinct leagues with eight teams in each league. Each team plays 154 games per season playing the other seven teams in the league 22 times, 11 of which are on the home field, while the other 11 are played on their opponent's field. The data which were available for analysis consisted of the results and associated risk costs for each of more than 6,000 major league baseball games played during the period of the 5 years from 1950 through 1954. These data were put on International Business Machines Corporation (IBM) punch cards and for each team, the following information was listed:

(*a*). Team t_i's opponent, i.e., t_j.
(*b*). The risk cost for team t_i.
(*c*). The game number.
(*d*). Whether t_i was the home team or not.
(*e*). The outcome of the game.

A number of parameters or variables which might play an important role in the development of the strategy were enumerated. Among these were·

(*a*). For a given game, whether a team t_i was the home team or not.
(*b*). For a given game, whether t_i was an underdog or not.
(*c*). The frequency of m consecutive losses for each team and for $m \leqq 15$.
(*d*). The frequency of m consecutive wins for each team and for $m \leqq 15$.
(*e*). The ratio of the average risk costs of team t_i is t_j to the number of times that t_i beats t_j at home games.
(*f*). The ratio of the average risk costs of team t_i versus t_j to the quotient. The percentage wins of t_i divided by the percentage wins of t_j.
(*g*). The ratio of the average risk cost of a team t_i to the percentage wins of t_i.

All in all more than 15 parameters were listed and analyzed but the foregoing 7 turned out to be the most significant ones.

Routines were written for the IBM *650* which tabulated a number of frequency or sample distributions of the various parameters as a function of either a team t_i the number of games played, the risk costs, etc. From these distributions a number of observations were made, and the important variables selected for future study. At this time a routine was written to determine the multiple linear regression coefficients of the least squares regression hyperplane, where the variable approximated by the linear form was the number of units won in a season. These coefficients indicated the relative importance of each of the variables each of which was acting simultaneously.

The next phase consisted in determining the strategy of wagering. It was decided that a decision for a particular game should be conditional on what had transpired previous to that game. The so-called system of progression was adopted whereby every time a particular team on which there had been a wager won, a unit would be won, and if the team lost, enough would be wagered the next time in order to (1) recuperate the previous losses since the last win and (2) win the initial one unit. However, it was apparent that under these conditions if no limit was set on this progression, there would be times when a risk as high as 500 units or more would be necessary in order to recuperate previous successive losses and to win the initial one unit. Consequently, a new variable had to be introduced, a so-called cutoff point which was the limit above which the progression would cease and it would be assumed that the successive losses were irretrievable. At this time one unit would begin anew to be won.

Twenty strategies were enumerated, each utilizing the significant parameters determined by the distribution tables and weighted accordingly by the multiple linear correlation coefficients. Each strategy was programmed for the *650* and the five years' history of games was wagered by the computer in accordance with the rules of the strategy. This was done for a number of different values for the cutoff point. It became apparent that a successful strategy was possible for certain values of the cutoff point. Moreover, confidence limits were established in order to assure that there would not be any ruin during the season's play even though the strategy eventually won out.

Statistically the results can be summarized as follows: Let G represent the number of units won or lost after a season's play where $G>0$ if units were won and $G>0$ if units were lost and suppose that the initial investment was 20 units. Then the probability function for G was

$$
\begin{aligned}
&\mathrm{pr}(-20 \leqq G<0)=0.30\\
&\mathrm{pr}(0 \leqq G<20) \quad =0.40\\
&\mathrm{pr}(20 \leqq G<80) \ =0.20\\
&\mathrm{pr}(80 \leqq G) \qquad =0.10
\end{aligned}
$$

Of course, such a probability function indicates a successful strategy. Moreover, the strategy when applied to the 1955 baseball season would have yielded a G equal to 102 units, a return of more than five times the initial investment.

Problem III. Scheduling of Work Assignments of Pilots and Airline Hostesses

Flight crews' salaries and related expenses represent an item of substantial magnitude in the operating costs of an airline. The problem was to determine whether more effiicent utilization of the available manpower and equipment could reduce these costs. This problem consisted of three basic phases.

Phase I. Trip Analysis

Determining the most efficient and economical method to operate flight out of the various bases by taking into consideration the following:

(*a*). The number of crew members at each base, and cost of possibly moving them elsewhere as required.
(*b*). Equipment qualifications of crew members at each base.
(*c*). Route qualifications of crew members at each base.
(*d*). The advantage of combining trips, i.e., combining an outbound trip from a base to a layover station with a possible shuttle to a third layover station, and then combining with a return flight to the base.
(*e*). Expenses incurred by trips requiring crews to remain at layover stations (hotel, meals, transportation).
(*f*). The cost of training personnel to meet requirements prescribed by the schedule, e.g., the costs to train crews who are only qualified to fly *DC-3*'s to fly Convairs.
(*g*). The savings incurred, if any, by mixing equipment in flying schedules.
(*h*). Adequate rest provisions as defined by Civil Aeronautics Regulations and working agreements.

Phase II. Flight Assignments

Constructing the flying schedules for various types of flight categories, i.e., captain, copilots, navigator, flight engineer, stewardess, pursers, subject to the following constraints:

(*a*). There is a maximum number of hours that can be flown in any month.
(*b*). There is a maximum number of hours that can be flown in any week.
(*c*). There is a maximum number of hours that can be flown in any 24-hour period.
(*d*). There is a maximum number of consecutive hours of on-duty time that can be assigned.
(*e*). Flight time schedules are based on a 7-day period.

Phase III. Assignment Improvement

Improve the existing schedules constructed in Phase II taking into account the following:

(*a*). Adjusting schedules and portions of schedules disrupted by leaves, vacations, illnesses, etc.
(*b*). The probability that a percentage of flights will be cancelled in any month because of equipment failure, weather, etc.

Representing the problems in Phases I and II by a mathematical model presented no extraordinary difficulty. The former is, in essence, a linear programming problem to determine the types of flights to be flown in order to reduce the cost for operating these flights. The latter is merely determining the probability distribution function for the various variables involved and producing a table which would indicate, based on a given assumption, what are the revisions in terms of the number and type of flights and flight assignments that have to be eliminated from the assignments already constructed in Phase II.

It is in Phase II where the most difficulty is encountered. The basic problem is simply this: The trip schedules represent the workload *W*, which has to be perfomed in any given month. To accomplish the task of Phase II, *W* has to be broken down into individual work assignments subject to the constraints which were enumerated above; moreover, in doing so, the total amount of layover time had to be minimized and the total number of hours assigned per month up to the maximum hours allowed by constraint (a) had to be minimized. The reason for this latter objective is that a flight crew member's pay is the sum of a monthly base pay and an hourly rate times the number of hours flown in that month.

The solution to this problem was also written and coded for *650* and the final results, particularly in Phase II, have not been obtained. Nevertheless, certain conclusions can be made even at this stage. First, an over-all improvement over the hitherto existing way of accomplishing this task will be of the magnitude of 5 to 10 per cent, a very worthwhile achievement considering the money and costs involved. Secondly, a great deal of this improvement is a consequence of the results obtained in Phase I and to some extent, Phase III. However, there was a decided lack of technique available which would enable the computer to improve in terms of reducing costs, etc., an earlier decomposition of the workload, *W*. More precisely, there was no systematic way to improve an existing decomposition by an iterative procedure which was in the realm of practicality and economically feasible. The only other course of action was to produce another decomposition and select the one which was more efficient than all the previous ones made. Surely, this is nothing more than an elaborate trial and error procedure.

Concluding Remarks

The solutions to the three problems which have been discussed were to a large extent elaborate trial and error schemes. No doubt, the computer serves a very important function in this respect for the basic operations research problem is, to a large extent, initially divorced from a mathematical model. Consequently, any method which assists the analyst in deriving a mathematical model for his problem is a step in the right direction. Nevertheless, it is also true that this primitive approach is relied upon to such a large extent because of the state of the art. The problems that remain as a challenge to the computer user are much more complex than those which are presently being solved. They will require a more efficient utilization of the capabilities and capacities of the computer in use today.

On the other hand, the management of large businesses and industrial companies should realize that, in less than a decade, the computer specialist has gone from deriving mathematical tables and doing standard accounting work to solving operational and managerial problems of enormous magnitude and complexity. If management is to derive a greater utilization from its computing facilities, it must broaden its perspective and give to the computer specialist the challenge that the latter is willing to accept.

Reference

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A Progress Report on Computer Applications in Computer Design

S. R. CRAY   R. N. KISCH

THE subject of computers designing other computers has been a popular one for several years. This subject generally brings to mind Boolean algebra reduction or generation of design logic. This is a difficult problem which the authors of this paper have investigated only superficially, and is not the subject of this paper. Another aspect of computer development work, however, lends itself to mechanization and represents the greatest portion of the time, money, and manpower consuming business of developing a new computing system. This paper summarizes the progress which has been made to date in writing, debugging, and placing in production a general purpose computer program (ERA *1103*) for handling this portion of the development work. This is a program for processing the logical design engineer's work through simulated operation of the proposed equipment to the production of detailed wiring tabulations for manufacturing purposes.

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The mechanization program described in this paper necessarily is based on a particular computer building block and particular type of cabinet design. It is independent, however, of any specific computer and any logical design can be processed which uses the selected building blocks and cabinet structure. The program takes into account all of the physical as well as electrical factors in planning component placement and in computing wire lengths and cable paths in tabulations for manufacture.

The Building Blocks

The particular building block chosen for the design program was a 1-microsecond magnetic switch developed at the St. Paul laboratories of Sperry Rand Corporation. This element performs 3-level "and-or-not" logic and provides one bit of temporary storage in each package.