Automatic Digital Matric Structural Analysis

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List of Symbols

A = cross-sectional area of bar.
A_{max} = maximum cross-sectional area of bar.
A_{min} = minimum cross-sectional area of bar.
A_p = area of panel.
B_{max} = length of longer parallel edge of tapered panel.
B_{min} = length of shorter parallel edge of tapered panel.
$D_x(i)$ = x component of displacement at the joint i in the coordinate system (X, Y, Z). $D_y(i)$ and $D_z(i)$ are defined similarly.
E = Young's modulus.
$F_x(i)$ = x component of the external force applied at the joint i. $F_y(i)$ and $F_z(i)$ are defined similarly.
G = shear modulus.
L_{ij} = length of the bar with end joints i and j.
P_{ij} = load at the joint i on the bar B_{ij}.
q = shear flow.
$\bar{q}$ = average shear flow.
δ_{ij} = the displacement of the joint i in the direction of the bar B_{ij}.
$\Delta x_{ij} = x_i - x_j$. Δy_{ij} and Δz_{ij} are defined similarly.
t_D = thickness of panel at intersection of its diagonals.
i, j, k, m, n = subscripts.

Introduction

THE accelerated pace of computer development has made possible the rapid solution of complex problems. The time necessary to set up problems has begun to surpass greatly machine solution time. Consequently, more emphasis is needed on means of making use of digital machines and allied equipment in setting up problems automatically.

Heretofore, the preparation of certain matric equations appearing in structural analysis has been a tedious task. The procedures have required a large amount of judgment and tiresome hand computation. The chances for errors have been prevalent.

The present paper presents a method whereby the above factors can be minimized or negated. Input data are reduced to a minimum. All logical decisions are carried out completely automatically so as to arrange the matrix automatically. Machine time is found to be very small relative to the time previously needed to set up problems. Therefore, this coded program should prove very useful to structures and allied engineers.

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Some familiarization with Klein[1,2] is helpful but not necessary for the understanding of the development in this paper. The basic concepts are the ones of joint, bar, and panel. The joints connect the bars and the bars border the panels.

Machine Simulation of Elements

The code is built around the basic elements: joints, bars, and panels. The information below noted by asterisk (*) is computed.

1) Joints
 A group of words is assigned to each joint containing the following information:
 a) Its position coordinates x, y, and z
 b) Whether it is fixed
 *c) All the bars attached to the joint.
2) Bars
 A group of words is assigned to each bar with the following data:
 a) Its two end joints
 b) Whether it is tapered
 c) Its cross-sectional area (both maximum and minimum, if tapered)
 *d) The direction cosines x, y, and z
 *e) The length of the bar L_{ij}.
3) Panels
 A group of words is assigned to each panel with information on:
 a) The four corner joints
 *b) The area
 c) The thickness
 *d) All of the bordering bars
 *e) B_{max} and B_{min} (lengths of the parallel sides)
 f) Whether it is tapered.

The number of cells in a group must be multiples of eight to allow the use of multiple index registers. A joint, bar, and panel, grouped together in 40 cells to economize on space, are not necessarily related.

The Code

A) Input
 The information without asterisks, described in the previous section, is read into the proper location. The numbers of the joints appearing in the bars and panels are converted into the two's complement of that joint's address. The joint then can be referred to with an index register.

[1] B. Klein, "A simple method of matric structures analysis," *J. Inst. Aeronaut. Sciences*, vol. 24, pp. 40–46; January, 1957.
[2] B. Klein, "A simple method of metric structural analysis, part II—effects of taper and a consideration of curvature," *J. Inst. Aeronaut. Sciences*, vol. 24, pp. 813–820; November, 1957.

B) Bar Cross Referencing

1) The following is computed for each bar B_{ij}:

$$\text{a) } \Delta x = x_i - x_j, \tag{1}$$

$$\text{b) } \Delta y = y_i - y_j, \tag{2}$$

$$\text{c) } \Delta z = z_i - z_j. \tag{3}$$

2) The bar address complement is stored in the groups of the joints i and j.

3) Every panel is examined. If both the joints i and j are in it, the bar B_{ij} borders it. In this case the two's complement of the bar address is placed in the panel group and the two's complement of the panel address is placed in the bar group.

C) Joint Equations

1) For each joint i the following force equilibrium equations are entered in the matrix

$$\text{a) } \sum_j P_{ij} \left(\frac{\Delta x}{L}\right)_{ij} = F_x(i), \tag{4}$$

$$\text{b) } \sum_j P_{ij} \left(\frac{\Delta y}{L}\right) = F_y(i), \tag{5}$$

$$\text{c) } \sum_j P_{ij} \left(\frac{\Delta z}{L}\right) = F_z(i). \tag{6}$$

D) Panel Area

1) The two parallel bars for each panel are found by comparing the direction cosines. The bars are rearranged so that the longest parallel side is first and the other parallel bar is second.

2) Twice the area A_p is computed by the following (see Fig. 1).

$$2A_p = h(B_{\min} + B_{\max}) \tag{7}$$

where

$$h = L_{ij} \sin \theta_1 \tag{8}$$

$$\sin \theta_1 = \sqrt{1 - \cos^2 \theta_1} \tag{9}$$

$$\cos \theta_1 = \left(\frac{\Delta x}{L}\right)_{ij} \left(\frac{\Delta x}{L}\right)_{jm} + \left(\frac{\Delta y}{L}\right)_{ij} \left(\frac{\Delta y}{L}\right)_{jm} + \left(\frac{\Delta z}{L}\right)_{ij} \left(\frac{\Delta z}{L}\right)_{jm}. \tag{10}$$

The cosine above is equal to the dot product only if both sides are directed away from or toward the joint. If the sides are directed in opposite ways, the dot product must be multiplied by minus one.

E) Panel Equation

1) The shear panel displacement equation for the panel in Fig. 1 is

$$-(2A_p/Gt_D)\bar{q}_{jmki} - B_{\max}(\delta_{ij}+\delta_{jm}) + B_{\min}(\delta_{ki}+\delta_{ik}) - L_{ij}(\delta_{ij}+\delta_{ji}) + L_{km}(\delta_{km}+\delta_{mk}) = 0. \tag{11}$$

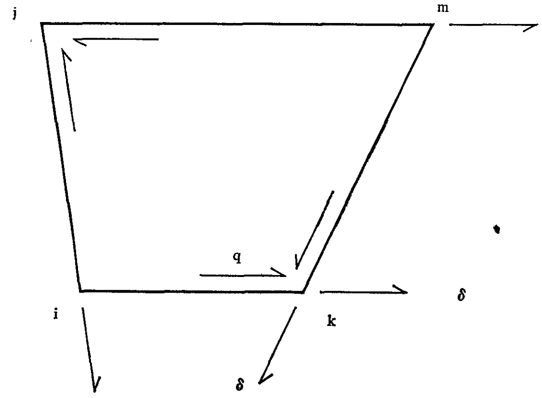


Fig. 1—Shear panel.

2) The arrows inside the panel indicate the direction of shear flow. The arrows outside the panel indicate the direction of displacements.

3) The corresponding panel and bar numbering for Fig. 1 is

panel: P_{jmki}

bars: B_{mj}, B_{ki}, B_{ij}, and B_{km}.

4) The panel numbering determines the positive direction of the shear flows, which are directed toward the joints designated by the first and third subscripts of the panel. (See Fig. 1.) The bar numbering determines the direction of positive displacement of the joint which is toward the joint determined by the first subscript of the bar.

5) The signs of the terms in the equation are:
   a) Shear term. This sign is minus.
   b) Displacement term. If the direction of positive displacement is the same as the direction of positive shear flow along a side, this sign is plus; if they are opposite, the sign is minus.

6) The code determines the signs by examining the joint numbers. If the first bar subscript is equal to the first or third panel subscript, the sign is plus; otherwise, it is minus.

7) The length of the side is the factor outside the parenthesis in the displacement term. Note that the length of the opposite side is used in the case of the parallel sides.

F) Bar Equations

1) The axial element equilibrium equation for each bar is

$$P_{ij} - P_{ji} + \sum_n b_n(\pm \bar{q}_n) = 0. \tag{12}$$

2) When the bar B_{ij} is one of the parallel bars bordering a panel, b_n of (12) is the length of the other parallel bar; otherwise, it is the length of the bar B_{ij}.

3) The sign of a shear term is positive if the positive direction of the shear along an edge of the

panel and the positive direction of the displacement in the bar bordering that edge are opposite; otherwise, the sign of the shear term is negative.

4) The axial force displacement equation for a tapered bar is

$$P_{ij} + L_{ij} \sum_{n} (\pm \bar{q}_n) f(B_{\min}/B_{\max}, A_{\max}/A_{\min}) + (E\bar{A}/L_{ij})(\delta_{ji} - \delta_{ij}) = 0 \quad (13)$$

where P_{ij} is the force at the narrow end of the bar. The sign of the shear term is plus if the shear and displacement directions oppose, and

$$f(B_{\min}/B_{\max}, A_{\max}/A_{\min}) = \frac{(A_{\min}/A_{\max})^{0.175}}{1 + (B_{\min}/B_{\max})^{0.7}} \quad (14)$$

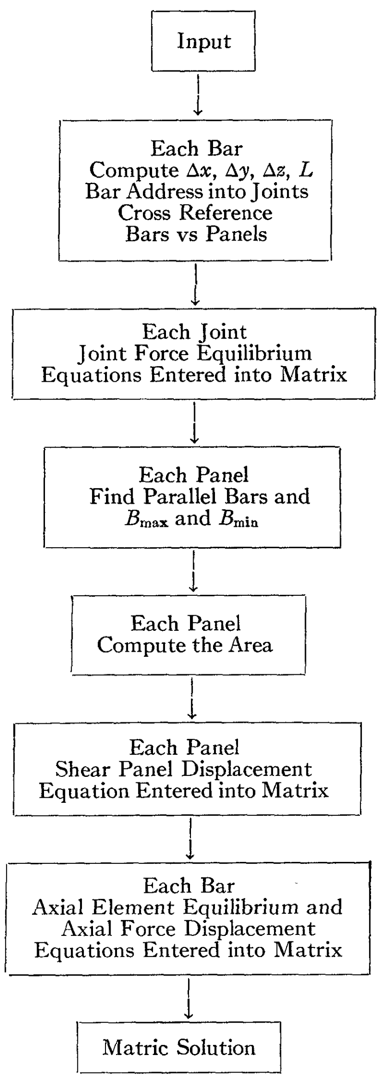
$$A = A_{\min} + \left(\frac{A_{\mathrm{mim}}/A_{\max}}{2}\right)^{0.175} (A_{\max} - A_{\min}) \quad (15)$$

if the taper is linear.

5) For a nontapered bar the equation is

$$P_{ij} + P_{ji} + (2EA/L_{ij})(\delta_{ji} - \delta_{ij}) = 0. \quad (16)$$

G) The Flow Chart follows.



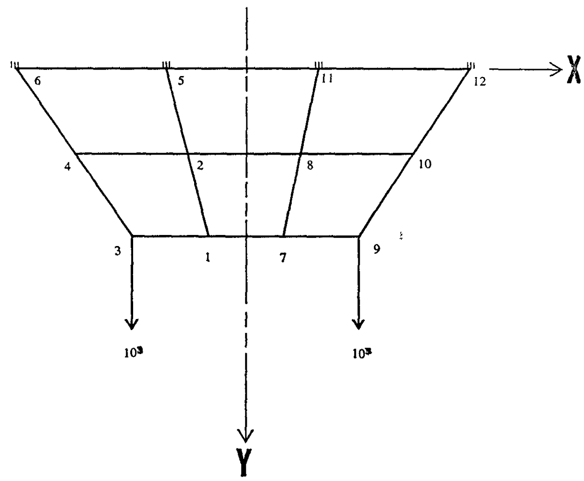


Fig. 2—Example problem.

Example Problem

The structure in Fig. 2 is analyzed by the code. The complete input data describing the structure appears in the next section, which is followed by the matrix generated and the solution with the matrix column numbers. The elements without matrix column numbers are obtained by a simple calculation from matrix computed elements; *e.g.*, Delta 1–7 equals Delta 1–3.

Total computing time, *i.e.*, the time for both matric setup and solution, is about 0.01 hour.

Problems of a much more formidable nature have been arranged and solved by the program, and work is in progress for improving the code. For example, use of instantaneous coordinates may create many more zero elements in the matrix.

INPUT

FORCES

JOINT	*X*	*Y*	*Z*
3	0.0	1000.0	0.0
9	0.0	1000.0	0.0

JOINTS

NUMBER	*X*	*Y*	*Z*	FIXED
1	− 2.5	16.0	0.1	NO
2	− 3.75	8.0	0.1	NO
3	− 7.5	16.0	0.1	NO
4	−11.25	8.0	0.1	NO
5	− 5.0	0.0	0.1	YES
6	−15.0	0.0	0.1	YES
7	2.5	16.0	0.1	NO
8	3.75	8.0	0.1	NO
9	7.5	16.0	0.1	NO
10	11.25	8.0	0.1	NO
11	5.0	0.0	0.1	YES
12	15.0	0.0	0.1	YES

BARS

JOINT 1	JOINT 2	TAPERED	AMAX	AMIN	*E*
3	4	YES	1.0	0.66667	1.0
4	6	YES	1.3333	1.0	1.0
9	10	YES	1.0	0.66667	1.0
10	12	YES	1.3333	1.0	1.0
1	2	YES	1.0	0.66667	1.0
2	5	YES	1.3333	1.0	1.0
7	8	YES	1.0	0.66667	1.0
8	11	YES	1.3333	1.0	1.0
3	1	NO	0.66667	0.66667	1.0
4	2	NO	1.0	1.0	1.0
9	7	NO	0.66667	0.66667	1.0
10	8	NO	1.0	1.0	1.0
1	7	NO	0.66667	0.66667	1.0
2	8	NO	1.0	1.0	1.0

PANELS

NUMBER	JOINT	JOINT	JOINT	JOINT	G-SHEAR	THICKNESS
1	3	4	2	1	0.4	0.07
2	4	6	5	2	0.4	0.1
3	1	2	8	7	0.5	0.07
4	2	5	11	8	0.4	0.1
5	7	8	10	9	0.4	0.07
6	8	11	12	10	0.4	0.1

THE MATRIX

ROW	COL	VALUE
1	1	0.42443388
1	2	−0.42443388
1	3	1.0
2	1	−0.90545894
2	2	0.90545894
3	4	−1.0
3	5	0.90545894
3	6	−0.42443388
4	7	0.15437688
4	8	−0.15437688
4	9	−1.0
4	10	1.0
5	7	−0.98801203
5	8	0.98801203
6	11	−0.15437688
6	12	−1.0
6	13	1.0
7	11	0.98801203
8	14	−0.42443388
8	15	0.42443388
8	16	−1.0
9	14	−0.90545894
9	15	0.90545894
10	17	−1.0
10	18	0.90545894
10	19	0.42443388
11	20	−0.15437688
11	21	0.15437688
11	22	1.0
11	23	−1.0
12	20	−0.98801203
12	21	0.98801203
13	24	0.15437688
13	25	1.0
13	26	−1.0
14	24	0.98801203
15	27	−3499.9999
15	28	−8.8352984
15	29	8.0970674
15	30	−10.0
15	31	−10.0
16	32	−3571.4285
16	28	−8.8352984
16	4	−8.8352984
16	29	8.0970674
16	33	−8.0970674
16	34	−7.5
16	6	−7.5
16	30	5.0
16	31	5.0
17	35	−3499.9999
17	36	8.0970674
17	29	−8.0970674
17	30	−1.0
17	37	1.0
18	38	−3571.4285
18	36	8.0970674
18	39	8.0970674
18	29	−8.0970674
18	33	−8.0970674
18	34	−7.5
18	40	7.5
18	30	5.0
18	37	−5.0
19	41	−3499.9999
19	42	8.8352984
19	36	−8.0970674
19	37	10.0
19	43	10.0
20	44	−3571.4285
20	42	8.8352984
20	17	8.8352984
20	36	−8.0970674
20	39	−8.0970674
20	40	7.5
20	19	−7.5
20	37	−5.0
20	43	−5.0
21	38	5.0
21	35	−1.0
21	10	−1.0
21	23	1.0
22	30	−0.26666667
22	37	−0.26666667
22	10	1.0
22	23	1.0
23	38	−7.5
23	13	−1.0
23	26	1.0
24	34	−0.26666667
24	40	−0.26666667
24	13	1.0
24	26	1.0
25	32	−5.0
25	27	10.0
25	9	−1.0
25	3	1.0
26	30	0.26666667
26	31	−0.26666667
26	9	1.0
26	3	1.0
27	32	7.5
27	12	−1.0
28	34	0.26666667
28	6	−0.26666667
28	12	1.0
29	44	5.0
29	41	−10.0
29	22	−1.0
29	16	1.0
30	37	0.26666667
30	43	−0.26666667
30	22	1.0
30	16	1.0
31	44	−7.5
31	25	−1.0
32	40	0.26666667
32	19	0.26666667
32	25	1.0
33	35	8.0970674
33	27	−8.0970674
33	45	−1.0
33	8	1.0
34	35	4.2360996
34	27	−4.2360996
34	29	−0.1430726
34	8	1.0
35	38	8.0970674
35	32	−8.0970674
35	7	−1.0
35	11	1.0
36	38	4.3028433
36	32	−4.3028433
36	33	−0.10150823
36	29	0.10150823
36	11	1.0
37	41	8.0970674
37	35	−8.0970674
37	46	−1.0
37	21	1.0
38	41	4.2360996
38	35	−4.2360996
38	36	−0.14307260
38	21	1.0
39	44	8.0970674
39	38	−8.0970674
39	20	−1.0
39	24	1.0
40	44	4.3028433
40	38	−4.3028433

ROW	COL	VALUE
40	39	−0.10150823
40	36	0.10150823
40	24	1.0
41	27	8.8352984
41	47	−1.0
41	2	1.0
42	27	4.6223159
42	28	−0.13111821
42	2	1.0
43	32	8.8352984
43	1	−1.0
44	32	4.6951448
44	4	−0.903026737
44	28	0.093026737
45	41	−8.8352984
45	48	−1.0
45	15	1.0
46	41	−4.6223159
46	42	−0.13111821
46	15	1.0
47	44	−8.8352984
47	14	−1.0
48	44	−4.6951448
48	17	−0.093026737
48	42	0.093026737
27	49	−468.75001
28	49	−468.75001
31	49	−468.75001
32	49	−468.75001
43	49	−1104.4123
44	49	−1104.4123
47	49	−1104.4123
48	49	−1104.4123

THE SOLUTION

MATRIX COLUMN NUMBER	ELEMENT			VALUE
24	P	1	2	0.0
25	P	1	3	1631.4448
26	P	1	7	1631.4448
20	P	2	1	3299.3447
22	P	2	4	986.2406
21	P	2	5	3299.3447
23	P	2	8	986.2406
	P	3	1	418.5939
	P	3	4	986.2406
16	P	4	2	0.0
14	P	4	3	7443.9683
15	P	4	6	7443.9683
46	P	5	2	4150.4514
48	P	6	4	6515.2640
13	P	7	1	1631.4448
11	P	7	8	0.0
12	P	7	9	1631.4448
10	P	8	2	986.2406
7	P	8	7	3299.3448
	P	8	10	986.2406
8	P	8	11	3299.3448
	P	9	7	418.5939
9	P	9	10	986.2406
3	P	10	8	0.0
1	P	10	9	7743.9684
2	P	10	12	7743.9684
45	P	11	8	4150.4514
47	P	12	10	6515.2640
44	Q	1		407.4740
41	Q	2		105.1129
38	Q	3		0.0
35	Q	4		0.0
32	Q	5		−407.4740
27	Q	6		−105.1129
39	DELTA	1	2	43445.283
40	DELTA	1	3	6117.8879
	DELTA	1	7	6117.8879
36	DELTA	2	1	26172.822
37	DELTA	2	4	3698.4027
	DELTA	2	8	3898.4027
	DELTA	2	5	26172.822
	DELTA	3	1	29813.814
17	DELTA	3	4	151221.70
43	DELTA	4	2	7396.8052
42	DELTA	4	3	53067.405
	DELTA	4	6	53067.405
	DELTA	7	1	6117.8879
33	DELTA	7	8	43445.282
34	DELTA	7	9	6117.8875
	DELTA	8	2	3698.4027
29	DELTA	8	7	26172.822
30	DELTA	8	10	3698.4022
	DELTA	8	11	26172.822
	DELTA	9	7	−29813.814
4	DELTA	9	10	151221.70
31	DELTA	10	8	7396.8045
28	DELTA	10	9	53067.405
	DELTA	10	12	53067.405
19	DX	3		−29813.814
18	DY	3		180986.33
6	DX	9		29813.813
5	DY	9		180986.33