COMBINED ANALOG-DIGITAL SIMULATION

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Introduction

The inherent limitations of the analog computer and the digital computer when each is used exclusively give rise to the need for combined analog-digital techniques. The analog computer, which is ideally suited for dynamic calculations, is at a distinct disadvantage where logical decisions are required, or where demands on accuracy and resolution are stringent. In contrast, digital calculations maintain the required accuracy but can be very time consuming. This is a particular burden when many runs are needed for a statistical analysis of a system.

The calculation of missile trajectories, including missile dynamics, is an area which has been particularly troublesome. The need for high resolution when relatively small, accurate miss distances are sought, prohibits an all-analog simulation. On the other hand the cost of an all-digital simulation can be excessive because of the amount of computer time required to calculate the missile dynamics. The need for a new computing technique is therefore clearly indicated.

A typical missile intercept problem has been simulated using all-analog, all-digital, and combined analog-digital techniques. The results of the hybrid method illustrate the advantages of this type of simulation. This paper will discuss the analog-digital computer system which was employed, the hybrid technique involved, and a comparison of this method with the all-digital and all-analog simulation.

Data-Link

In Figure 1 is shown a block diagram of the Data-Link system used to interconnect an IBM 704 digital computer with a Reeves analog computer (REAC). The system consists of five analog-to-digital (A-to-D) converters, five digital-to-analog (D-to-A) converters, and the necessary control and terminal equipment. It is readily expandable to fifteen channels in either direction. Transistorized circuitry is used throughout with the exception of the cathode followers in the terminal equipment specified by the IBM input-output requirements. The A-to-D converters generate a 12 bit (11 bits plus sign) binary word representing an input voltage in the range of ± 100 volts at a maximum rate of 5 thousand conversions per second. Analog voltages are sampled simultaneously and then read into the IBM 704 sequentially by means of the A-to-D buffer register. In the digital-to-analog direction, the digital words are written sequentially into D-to-A registers, converted to analog voltages, and applied in parallel to the REAC.

The control equipment consists of the logical circuits required to operate the system. Included in this equipment are the channel select counter and a clock pulse generator. The counter selects the channel through which the IBM 704 can supply or receive information. It automatically steps to consecutively numbered channels with each IBM 704 Copy instruction, or may be "jammed" to a specific channel upon appropriate command from the IBM 704. The latter may only be

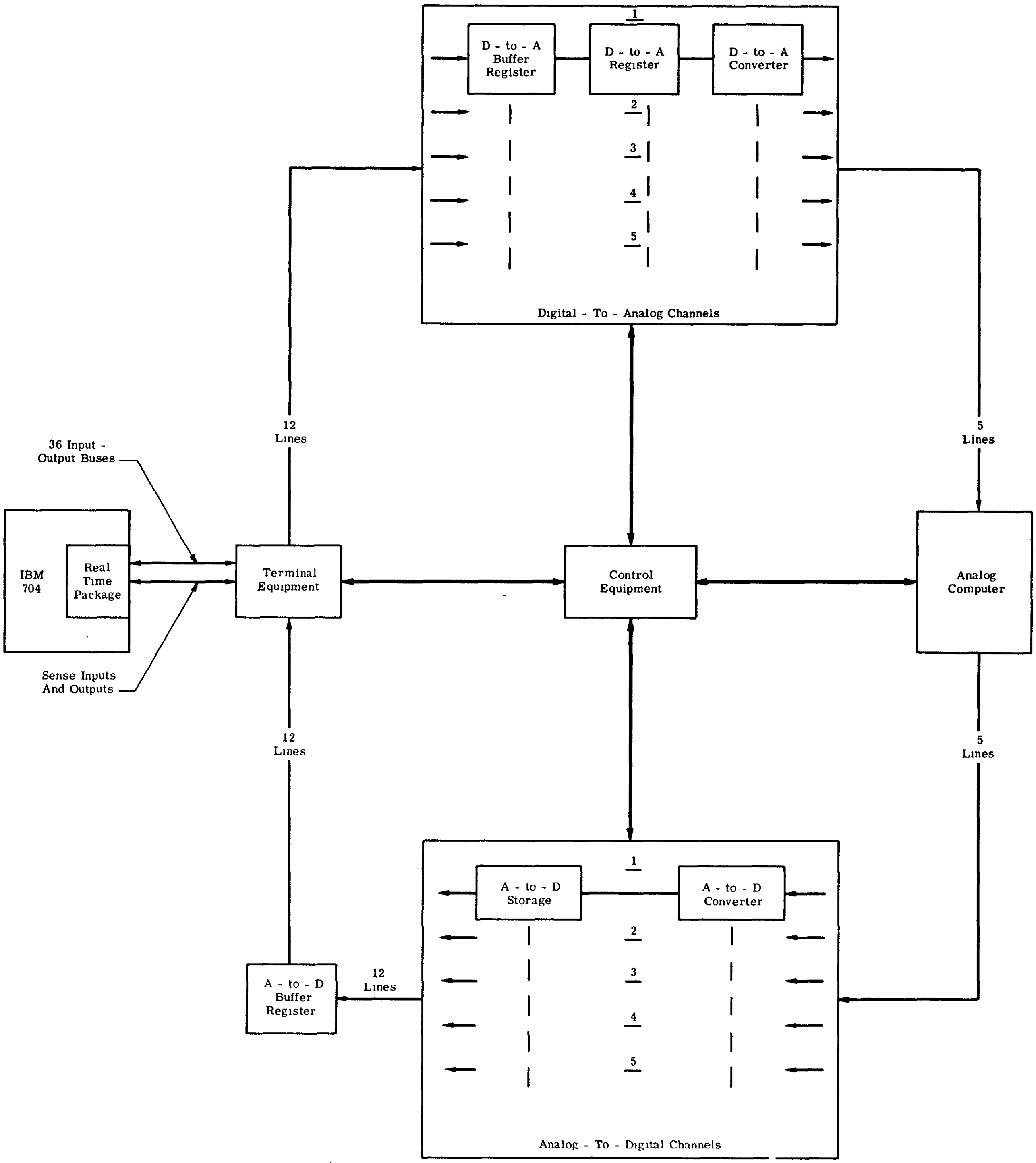


Figure 1. Data-Link System.

accomplished if the IBM 704 has Write Selected the Data-Link. The clock pulse generator, whose rate may be varied from 0.1 cps to 1 KC, is used to initiate a transfer of information between the analog and the digital computer. Analog to digital conversions are triggered by this clock or, in the case of asynchronous operation, by the IBM 704.

In addition to the control equipment terminal equipment is required to make the outputs and inputs of the computer link compatible with the IBM 704. In particular, it translates the IBM 704 logic levels into the computer link logic levels as well as the reverse procedure. Also included in the system is what is known as IBM terminology as a "Real Time Package." This is actually an accessory piece of IBM hardware which provides the necessary connections to the 36 input-output buses and sense inputs and outputs of the IBM 704. Two sense inputs are utilized in the computer link. One indicates whether the REAC is in the "Operate" or "Reset" mode. The second is an overload signal generated when one of the A-to-D converters has an input exceeding ±100 volts. One of the sense outputs is used to complement the state of the REAC; that is send it into "Operate" or "Reset." Another is used to cause analog-to-digital conversion and, if the REAC is in operate, simultaneously cause digital-to-analog conversion. A third causes digital-to-analog conversion exclusively.

Ordinarily 17 of the 36 input-output buses of the IBM 704 are utilized by the link. As shown in Figure 2 these correspond to the 11 information bits (bit positions 25-35), sign (bit position 5), channel address (bit position 1-4), and what is called an "inhibit" bit (bit position 5). Since the channel counter may be "jammed" to a specific channel only when the IBM 704 executes a "writing" sequence with the appropriate channel addressed, the need for this "inhibit" bit arises. In order to select a specific channel for "reading" it is necessary, therefore, to first "write" into the next lower channel so that the counter will be in the correct position at the end of the Copy instruction. Transfer of erroneous information (which would be all zeros) to D-A storage is prevented in this "pseudo-writing" sequence by the inhibit bit. This slightly inefficient method of channel selection for "reading" can be eliminated when an IBM 7090 is used. In this case the additional sense outputs can be used for channel selection prior to either "reading" or "writing."

The same 5 digital-to-analog channels used to transfer information in the "Operate" mode may be used to put initial conditions on the analog integrators while the REAC is in "Reset." Figure 3 shows a simplified block diagram of a digital-to-analog channel and a typical REAC integrator. The initial condition (I.C.) input is only effective while the computer is in "Reset." When it is in "Operate," the integrator integrates the voltage at the input terminal. Designating the input to the integrator at time t = 0 the "initial input," it is possible for the initial conditions to be present on the outputs of the digital-to-analog converters, and for the "initial inputs" to be simultaneously present as a digital number in the buffer register. When a signal from the IBM 704 commands the REAC to go into "Operate" the X and Y relays start dropping out. Relay X is slower in falling out than relay Y. There is a finite length of time, therefore, where the input to the I.C. capacitor and input to the amplifier

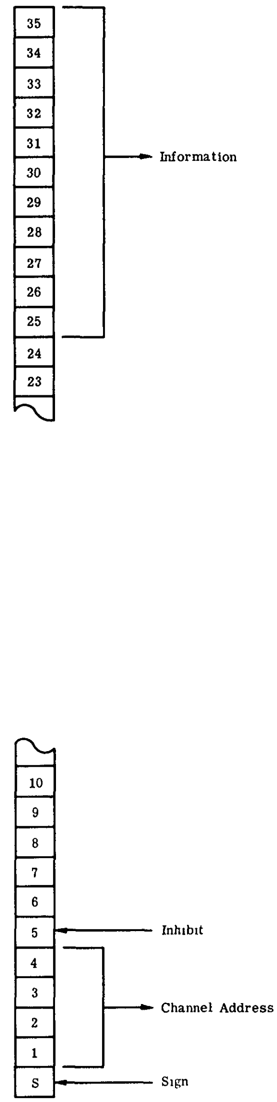


Figure 2. Input-Output Bus Locations Utilized by Data-Link

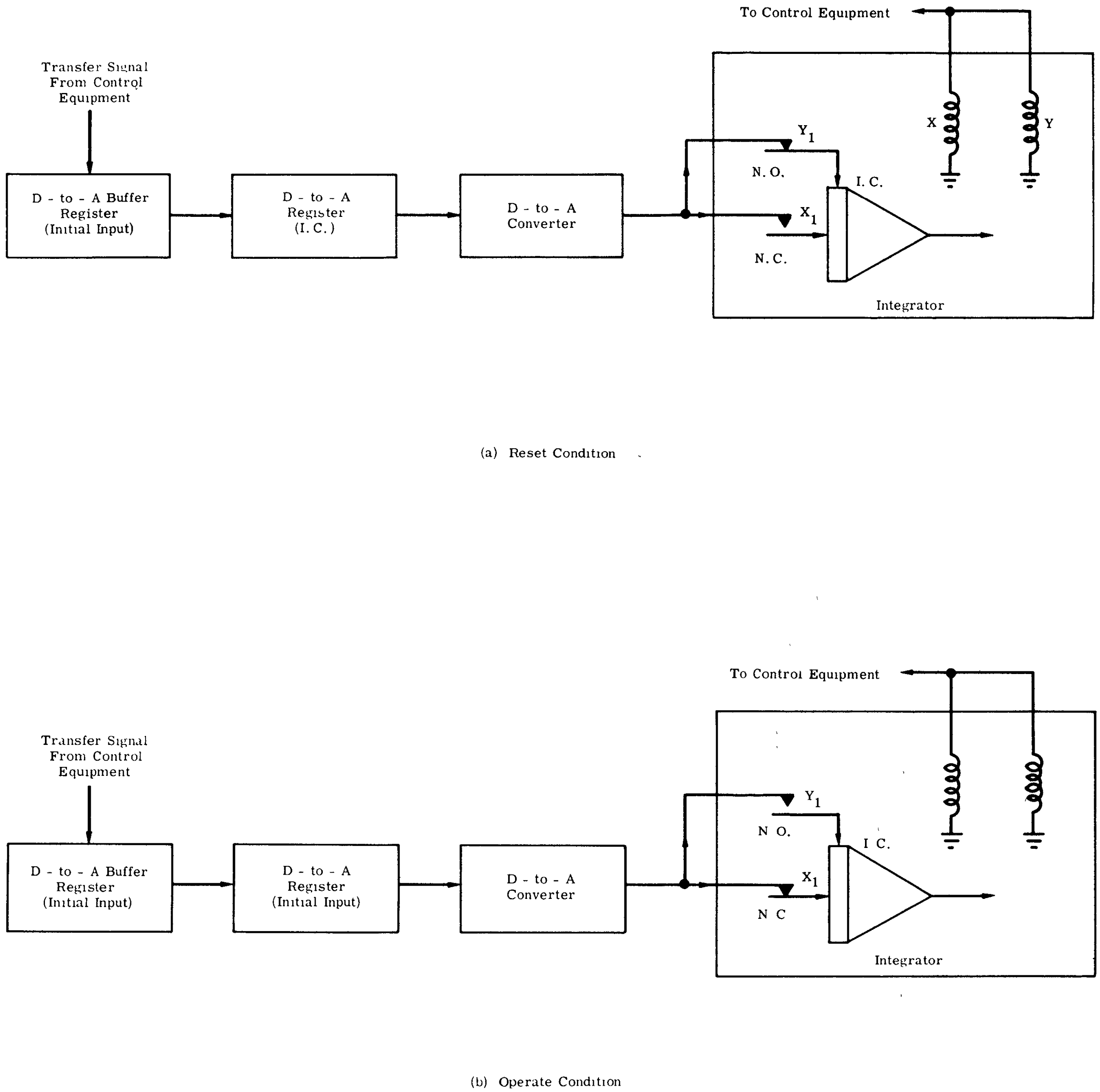


Figure 3. Switching from Reset to Operate Mode.

are both disconnected. It is during this period that the "initial inputs" are automatically transferred from the buffer registers to the digital to analog converters where they are converted to voltages. When relay contact Y_1 has closed the computer is in "Operate," and the "initial inputs" will start being integrated from the initial condition value. Obviously it is not necessary for the output of the D-to-A converters to go to the I.C. and input jacks of the same integrator but may be routed to the inputs of any other amplifier as well.

Missile Intercept Simulation

Figure 4 illustrates the geometry for the terminal phase of a missile intercept problem in two dimensions. The velocity of the missile and target are designated by V_M and V_T respectively, and the range is designated by R. All the angles are defined by the

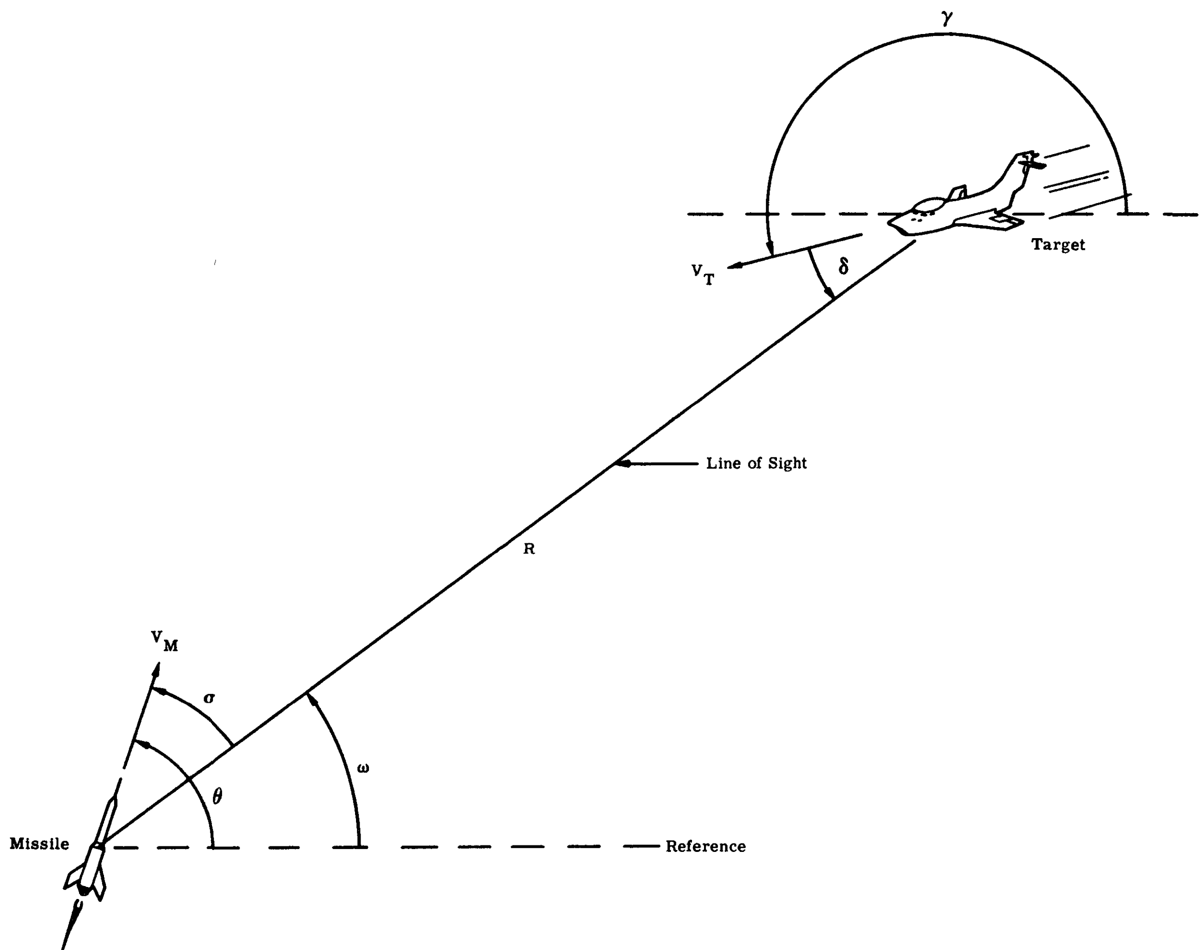


Figure 4. Illustrative Missile Intercept Problem.

geometry. By equating the velocity components normal to the line of sight we obtain,

$$R\dot{\omega} = - V_M \sin \sigma + V_T \sin \delta, \tag{1}$$

and obtain similarly for the velocities transverse to the line of sight,

$$- \dot{R} = V_M \cos \sigma + V_T \cos \delta. \tag{2}$$

From the geometry considerations we also have the angular equations,

$$\delta = \omega - \gamma + \pi, \tag{3}$$

$$\sigma = \theta - \omega. \tag{4}$$

The angle θ_c not shown in Figure 4 is defined as the control surface angle of the missile. For proportional guidance and a second order lag in the missile control system

$$\Theta_c (s) = \frac{N\dot{\Omega}(s)}{(1 + \tau s)^2}, \tag{5}$$

where the capital Greek letters are the Laplace transforms of variables designated by the respective lower case letters, and N is a parameter of the guidance system. For simplicity, the missile transfer function is assumed to be of second order and of the form,

$$\frac{\dot{\Theta}(s)}{\Theta_c(s)} = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1} \tag{6}$$

where K is a gain constant, ξ the relative damping ratio of the missile, and ω_n the natural frequency of the missile.

To include the effects of "g" limiting, the command signal to the missile is limited to $\dot{\omega}_M$. With this constraint Eq. (5) is rewritten in the time domain giving,

$$\tau^2 \ddot{\theta}_c + 2\tau\dot{\theta}_c + \theta_c = N\dot{\omega}^*, \tag{7}$$

where

$$\dot{\omega}^* = \begin{cases} \dot{\omega}_M \text{ when } \dot{\omega} > \omega_M \\ \dot{\omega} \text{ when } -\dot{\omega} < \dot{\omega} < \dot{\omega}_M \\ -\dot{\omega}_M \text{ when } \dot{\omega} < -\dot{\omega}_M. \end{cases} \quad (8)$$

and Eq. (6) is rewritten.

$$\frac{\dddot{\theta}}{\omega_n^2} + \frac{2\xi\ddot{\theta}}{\omega_n} + \dot{\theta} = K\theta_c \quad (9)$$

The equation of target motion is the final equation needed to completely define the problem with the exception of initial conditions. It is assumed in this problem that $\dot{\gamma}$ is constant.

A basic difficulty with an all-analog solution to this intercept problem is the scaling. Range R is chosen to be initially 50,000 feet. Resolution on the order of ±2 feet is desired. To account for the entire variation of R requires solving for R/500 which corresponds to 100 volts for R equal to 50,000 feet. When R is equal to 2 feet, R/500 is 4 millivolts which is well below the probable noise level and practical operation of the analog computer. A second variable which presents a scaling problem is $\dot{\omega}$, the rate of change of the line of sight angle. Initially, when R is large, $\dot{\omega}$ is small. However, when R approaches zero, $\dot{\omega}$ approaches a large value. This does not offer any particular problem in the guidance equation loop since this signal is limited due to the "g" limiting of the missile. On the other hand the true $\dot{\omega}$ is required to compute the kinematics of the problem. Until the missile travels to within a few hundred feet of the target, an all-analog solution is possible. The solution could be obtained to that point in real time. However, the last part of the simulation would have to be run as a separate stage with new scaling to obtain sufficient resolution on R and $\dot{\omega}$.

There is no basic computing difficulty to an all-digital simulation. The disadvantage of this method lies in the relatively longer amount of computing time required for each solution. The total elapsed time is a function of the integration interval used which in turn depends on the accuracy requirements. A 0.025 second time interval, which is sufficient for the dynamics of the problem can be used until the missile is within about 100 feet of the target. A time interval of 0.001 second is necessary from then on due to the ±2 feet resolution requirement. The digital solution, when obtained in this manner consumes about 6-1/2 minutes of IBM 704 time. This is roughly 8 times real time. If a parametric study or statistical analysis of the system is required, the total digital computer time becomes excessive.

The difficulties encountered in the all-analog solution are overcome in the combined analog-digital method by allowing the digital computer, with its eight significant figures to handle the kinematic and guidance equations and, in particular, to solve for R and $\dot{\omega}$. The analog computer is then programmed to compute the higher frequency, lower accuracy portion of the problem represented by the missile dynamics. Thus the equations solved on the REAC are,

$$\ddot{\theta}_c = \frac{N\dot{\omega}^*}{\tau^2} - \frac{2\dot{\theta}_c}{\tau} - \frac{\theta_c}{\tau^2}, \quad (10)$$

and

$$\dddot{\theta} = K\omega_n^2\theta_c - 2\xi\omega_n\ddot{\theta} - \omega_n^2\dot{\theta}, \quad (11)$$

while the equations solved in the IBM 704 are,

$$\gamma = \gamma_0 + \dot{\gamma}t, \quad (12)$$

$$\dot{\omega} = -\frac{(V_M \sin\sigma)}{R} + \frac{(V_T \sin\delta)}{R} \quad (13)$$

$$-\dot{R} = V_M\cos\sigma + V_T\cos\delta, \quad (14)$$

$$\delta = \omega - \gamma + \pi \quad (15)$$

$$\sigma = \theta - \omega \quad (16)$$

$$\dot{\omega}^* = \begin{cases} \dot{\omega}_M \text{ when } \dot{\omega} > \dot{\omega}_M \\ \dot{\omega} \text{ when } -\dot{\omega}_M < \dot{\omega} < \dot{\omega}_M \\ -\dot{\omega}_M \text{ when } \dot{\omega} < -\dot{\omega}_M. \end{cases} \quad (17)$$

A block diagram of the combined analog-digital solution is shown in Figure 5. The digital and analog computations are carried out as shown until the range decreases to 350 feet. At that time, the digital computer is used exclusively for the remainder of the run.

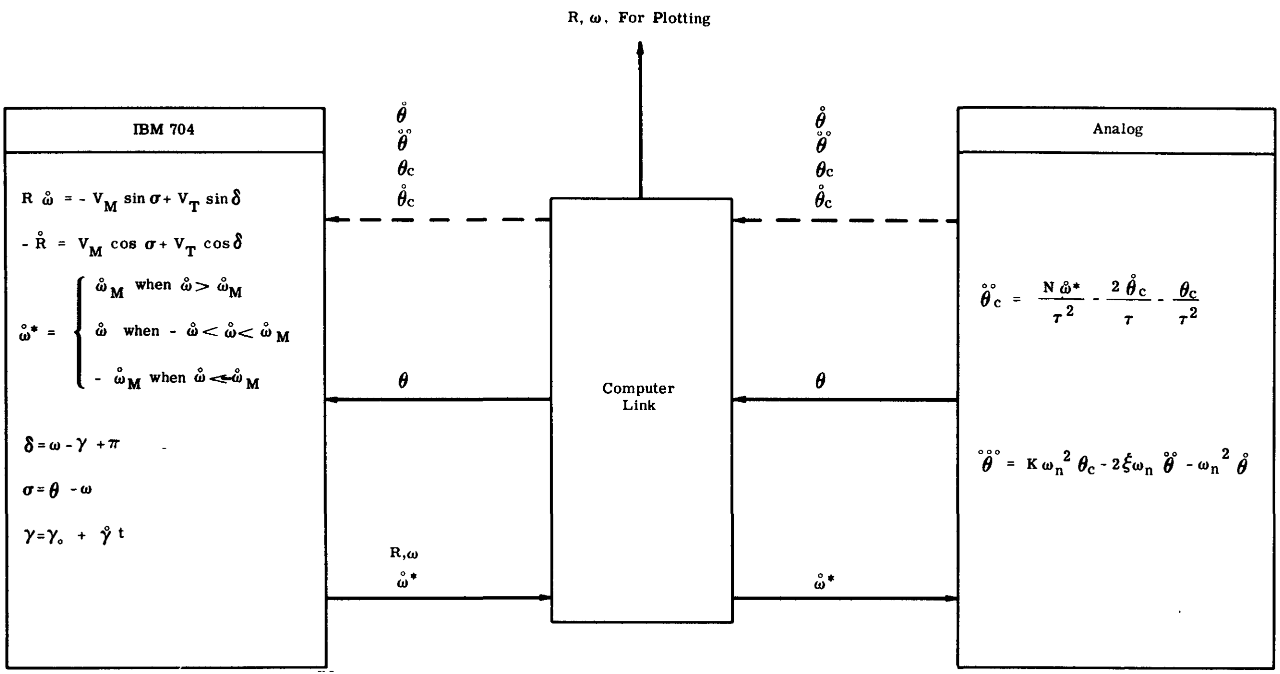


Figure 5. Block Diagram for Combined Analog-Digital Solution.

The flow chart for the digital program is shown in Figure 6. The entire solution is under the control of the IBM 704. After writing the initial condition θ_0 and initial input $\dot{\theta}_0$, it transfers the REAC to "Operate." A clock controlled sampling rate was previously chosen based on dynamic considerations. The IBM 704 is put in the Read Select mode and the variable θ, which upon arrival of the first clock pulse has previously been sampled from the REAC and digitized, is then read into storage. This will be a fixed point number but is converted to floating point to facilitate all floating point arithmetic in the IBM 704. If no overloads have occurred, Eqs. (12) through (17) are then calculated. This is followed by a test on $\dot{R}$ to determine whether it is positive. An affirmative answer indicates a miss and the run would then be terminated. If $\dot{R}$ is negative a test is made on R to determine if the missile is within 350 feet of the target. If it is not, R, $\dot{\omega}^*$, and ω are converted to fixed point numbers and the IBM 704 then goes into the Write Select mode. The command signal in the form of the limited $\dot{\omega}^*$ is sent to the REAC along with R, and ω for plotting purposes. The IBM 704 then awaits the next clock pulse before going to the Read Select mode, and the cycle repeats until the missile comes within 350 feet of the target. When this occurs the IBM 704, in addition to reading θ, reads $\dot{\theta}$, $\ddot{\theta}$, θ_c and $\dot{\theta}_c$. The REAC is sent into "Reset," and the IBM 704 solves the remainder of the run using the previously sampled five variables as initial conditions.

Analysis of Combined Analog-Digital Results

The combined analog-digital solution is obtained in approximately real time with a resolution of ±2 feet. Real time for the problem is 50 seconds while the combined simulation requires 60 seconds. The ten second difference arises due to the all-digital calculation of the last few hundred feet.

A sampling interval of 30 milliseconds corresponded to about 200 samples per cycle for the highest frequency present in the simulation (the natural frequency of the missile). The resulting curves for this run as shown in Figure 7a indicate a negligible deviation from the all-digital and all-analog solutions. The digital print-out of the last portion of the combined simulation revealed that the missile came within 21 feet of the target as compared to 17 feet in the all-digital solution.

At sampling rates of 20 per cycle, the deviation from the all-digital run becomes noticeable as shown in Figure 7b. In this

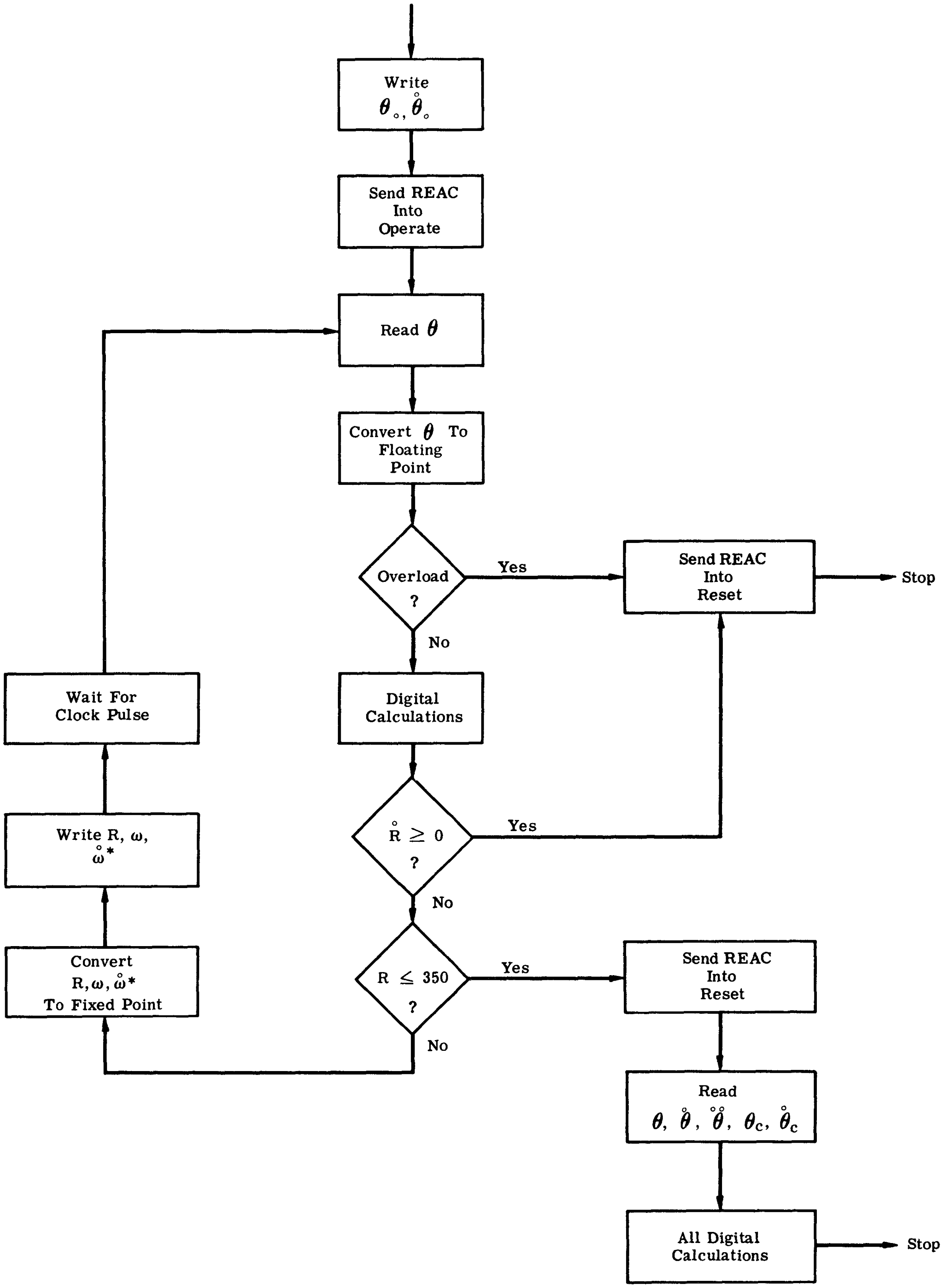
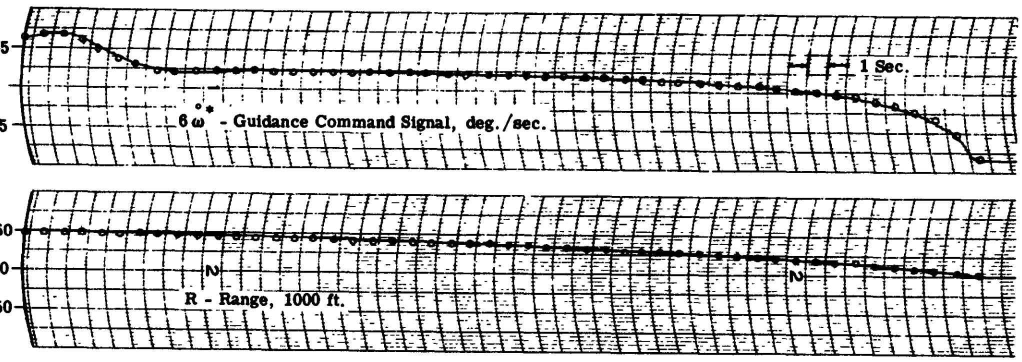
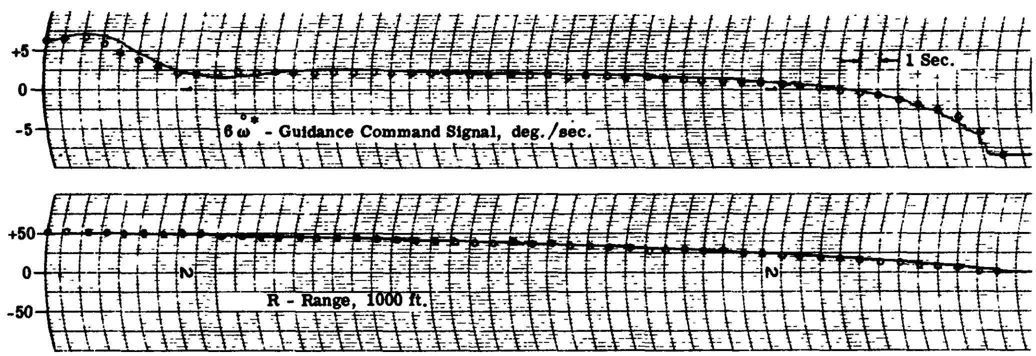


Figure 6. Flow Chart for Combined Analog-Digital Solution.



t ⟶

a. Sampling Interval 30 m. s.



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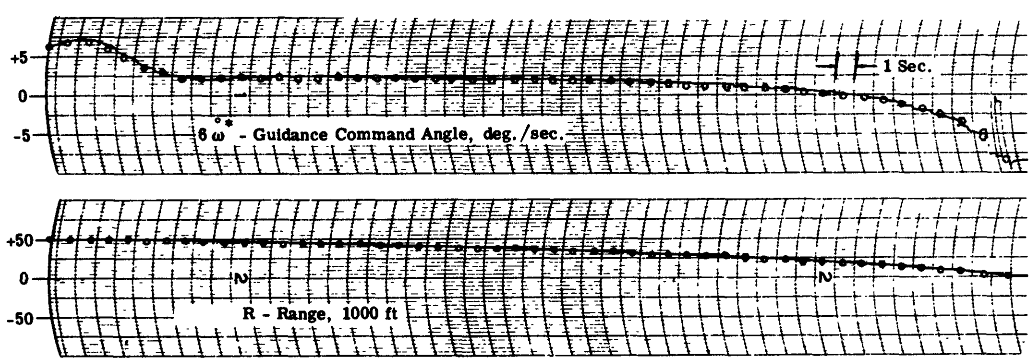
b. Sampling Interval 300 m. s. - No Extrapolation



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c. Sampling Interval 300 m. s. - First Order Extrapolation

—— Combined Analog - Digital
ooooo All - Digital

Figure 7. Missile Intercept Solutions.

case the time delay due to sampling, for which no attempt at compensation has been made, is taking effect. In particular, the guidance command signal $\dot{\omega}^*$ is delayed .3 second. When a simple extrapolation technique is applied on $\dot{\omega}^*$ the variation from the all-digital solution is reduced considerably as shown in Figure 7b. The switch-over to the all-digital portion of the combined analog-digital simulation occurs when the range reaches approximately 180 feet. The final range in this case is 19 feet.

The dynamic equations being solved by the analog computer require no nonlinear elements and the equations being solved by the digital computer have parameters which do not vary rapidly. These facts coupled with the knowledge that a sampling range of 20 per cycle with extrapolation is sufficient, indicate the feasibility of running the problem in one-tenth of real time. The combined analog-digital solution then results in a time improvement of about 65 to 1 over the all-digital solution.

Conclusions

The missile intercept simulation with its associated scaling and expensive computer time problems exemplifies only one area where a combined analog-digital technique is useful. Computer controlled systems that are presently envisioned for future aircraft, missiles, and other space vehicles will require the handling of discrete and continuous information. Hybrid computation is the logical choice for the simulation of these inherently hybrid systems. Since an analog-digital computer link is by nature a sampled-data device it is a valuable tool for research in sampling theory. Stability studies and error analyses are planned using the hardware to simulate the sampled data-system under investigation.

Acknowledgment

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