Art, computers and mathematics

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The computer

The computer is having an implosive effect upon the way we deal with a variety of problems. As an extension of man's senses, computer technology can provide an exciting new potential for the creation of art. The frontiers of knowledge in computer research suggest a new approach to problem solving in the arts. With a computer the artist can now deal with different variables in his decision making process than with conventional methods. For example, it is possible to put into the memory of the computer a color representation of a landscape. This landscape can be simulated on a graphic console. Then with computer programs which implement mathematical functions, the artist can watch the effects of wind velocity, temperature and factors which involve the amount of daylight upon his landscape. He can also observe data which are generally unavailable such as the effects of molecular structure, weight, mass and time upon the landscape. In his decision making process, the artist can rely on non-visual cues as well as visual cues. He can modify many more parameters in the total landscape environment to create a work of art than by conventional methods.

The artist and the modern environment

The frontiers of knowledge in computer research offer a glimpse into the future role of the artist. At M.I.T. and Stanford University considerable research is in progress which attempts to deal with artificial intelligence programs. Some researchers suggest that once we provide computer programs with sufficiently good learning techniques, these will improve to the point where they will become more intelligent than humans. Suppose we have a machine which has stored in it, a knowledge of art history, theories of philosophy and aesthetics, in fact, the intellectual history of man. Every known technique about painting, sculpture and the computer graphics will also be stored in the computer, not to mention an ability to make judgments more logically than man. What happens to questions about art when there is a dialogue between man and such a computer program? What becomes the problem? Who is the artist? What are the implications for man? It is both terrifying and exciting at the same time.

The emergence of new forms and media in contemporary art indicates the artist's deep involvement in twentieth century technology. Robert Rauschenberg creates forms which combine objects such as automobile doors and heating ducts with sophisticated electronic devices that create sounds and smells. Artists Tom Wesselman and George Segal utilize plastics and neon lights as well as radios and TV sets. Kinetic art has produced a vast array of objects which move or vibrate. Current artists have used practically every product in our society to make art. In the tradition of the craftsman, these artists seem to be more concerned about materials and technical processes than any underlying scientific concepts which produced these products.

The computer which handles fantastic amounts of data for processing brings the artist close to the scientist. Both can now use the same disciplines and knowledge in different ways. For the first time, the artist is in a position to deal directly with the basic scientific concepts of the twentieth century. He can now enter the world of the scientist and examine those laws which describe a physical reality. The artist can enter a microuniverse of science and alter parameters to create a different kind of artistic world. In a highly systematic and disciplined manner he can deal with fantasy and imagination. One example of the use of a scientific concept for artistic purposes is the well-known Lorentz transformation. It is a theory of special relativity which is a scientific explanation of the apparent distortion of a form as it approaches the speed of light. It would be interesting to see what happens graphically to a drawing of a turtle or a hummingbird, which can be used as input to the computer, as it approaches the speed of light. The artist may be interested in the absurdity of such an idea and it may give him a different kind of form. He may enjoy the contradiction of a turtle traveling near the speed of light.

The artist need not necessarily stop at the parameters defined by a transformation in relativity. He can arbitrarily declare that objects will move at a speed which is five times that of light (provided the mathematical equations do not degenerate). In fact, he can, with the computer, take a broad variety of well-known equations which describe our physical universe and change the parameters. He can create his own personal fiction.

It is quite apparent that the computer artist will have contact with the scientist. The computer and the mathematical disciplines required to solve problems, artistic and scientific, make for a common ground. Those fields will be brought closely together, and both will benefit by the dialogue made possible through computer science.

The artist, when involved in the creative process, feels free to deal with experience in any terms which can express his conception. He is not restricted to the rules required of the scientist to express a reality. In a sense, one might say that he takes the many parameters of ordinary experience and changes them to express his imagination. He creates a new universe. Since his purpose is to make art, the artist is not bound by the laws which account for the physical world. On the other hand, the scientist is also interested in realities. He explains the behavior of physical phenomena and usually verifies it in mathematical terms. The famous mathematician and writer Jacob Bronowski summarizes the similarities and the differences between artist and scientist in the following statement:

"The creative act is alike in art and in science; but it cannot be identical in the two; there must be a difference as well as a likeness. For example, the artist in his creation surely has open to him a dimension of freedom which is closed to the scientist. I have insisted that the scientist does not merely record the facts, but he must conform to the facts. The sanction of truth is an exact boundary which encloses him, in a way which it does not constrain the poet or the painter. . . ."*

Mathematics and the arts

Some artists throughout history have been intrigued with the possibility of making use of mathematics in their art. The renaissance idea of virtual space (the imitation of 3-D space) was generated by mathematical formulae for linear perspective. Modern artists such as Paul Klee, Moholy Nagy, Naum Gabo, and Antoine Pevsner have used simple mathematical systems to analyze and develop forms. In the past, mathematics has been given limited application as a tool for the discovery of aesthetic form because the techniques employed were slow and extremely time-consuming. Traditional ways of solving problems of measurement and plotting were too awkward and mechanical for artistic application. As a consequence, the artist's concept of structure was limited by what he was able to design or draw by hand. If an artist was able to understand or use the traditional methods of mathematical analysis, he was quickly discouraged because of the many repetitive steps in the computation of a problem. What he could visualize was limited by traditional methods.

An artist can now make use of complex mathematical functions. With the advent of computers the artist has at his disposal the computational power to apply many mathematical transformations to a variety of images. By implementing mathematical functions through computer programs, and generating the results on a mechanical plotter or CRT display, the artist can pursue an orderly in-depth inquiry of visual form. He can examine closely related or widely divergent functions. The results can be evaluated artistically against the background of the mathematical functions involved. Above all, there are new form possibilities that can be generated by the computer. Artists are faced with the prospect

*From J. Bronowski *Science and Human Values*, 1956.

of a new medium to enable the exploration of visual ideas.

The interest here is in the use of mathematical functions to modify form in a variety of ways. There is no intention to define a particular mathematical theory of art, but simply to say that mathematics is a useful means to create many kinds of art. One artist may prefer to work with realistic images while another one may prefer abstract images. An artist may select visual information in such a way as to communicate qualities of surrealism or expressionism, and even a tragic or lyrical view of reality. Questions about artistic content involve artistic decisions. Mathematics per se does not make the art. One brings artistic criteria to the object and makes value judgments about it. This criteria must deal with the nature of aesthetic experience and those aspects which make it art. Mathematics and the computer only provide us with another means to make artistic inquiry.

In a broad sense certain modern educational theories, and the way in which mathematical strategies can be applied, are also relevant. Currently there is considerable discussion about "psychological sets" in learning. This concept of sets suggests that the artist is often the victim of his own "set producing tendencies." At times, he has great difficulty in breaking down his own biases in order to solve an artistic problem in a creative way. Because of his biases, the artist usually gets only slight variations on a basic structural theme. A mathematical orientation toward visual problem solving can enable the artist both to break down his biases and to express another range of solutions. Essentially this would depend upon the types of strategies which the artist employed to effect his original data. To say this another way, the computer's capabilities, if exploited by the artist, would be made to yield a greater number of alternatives more rapidily and efficiently. Within a short period of time a computer-cathode ray tube-plotter or graphic console capability can give hundreds of variations of a form.

It is conceivable that any given transformation can be of use in computer art. Special consideration seems appropriate for transformations with properties which enable an artist to visualize the approximate result of the transformation and to transformations which are easily adapted to machine computation.

Mathematical functions are found in almost every branch of mathematics and they can produce an unlimited number of variations for the discrete set of coordinate points associated with a line drawing. Successive applications of the same or different functions can produce interesting results. In the example of SINE CURVE MAN there was a successive application of a sine curve function in which the amplitude was increased each time. Numerous possibilities exist if one uses a multiplicity of functions to achieve a final transformation. Some of the applications include a sequence of functions in which many functions are used. The example PLUS SIGNS, includes functions which deal with orientation, size and distribution. The motion picture HUMMINGBIRD, which is in time, utilizes several functions simultaneously, each varying independently as in the "three bird scramble" sequence. Each bird was on a different mathematically defined path, while their size was changed sinusoidally—each at its own frequency. Also each image moved from an abstract picture to a realistic bird at an independent and non-uniform rate.

Figure 1—SINE CURVE MAN, 1967. A digitized line drawing of a man was used as the input figure to a computer program which applied a mathematical function. The X value remained constant and a sine curve function was placed upon the Y value. Given the X and Y coordinates for each point, the figure was plotted by the computer from $X' = X$, $Y' = Y + C^* \text{SIN}(X)$ where C is increased for each successive image.

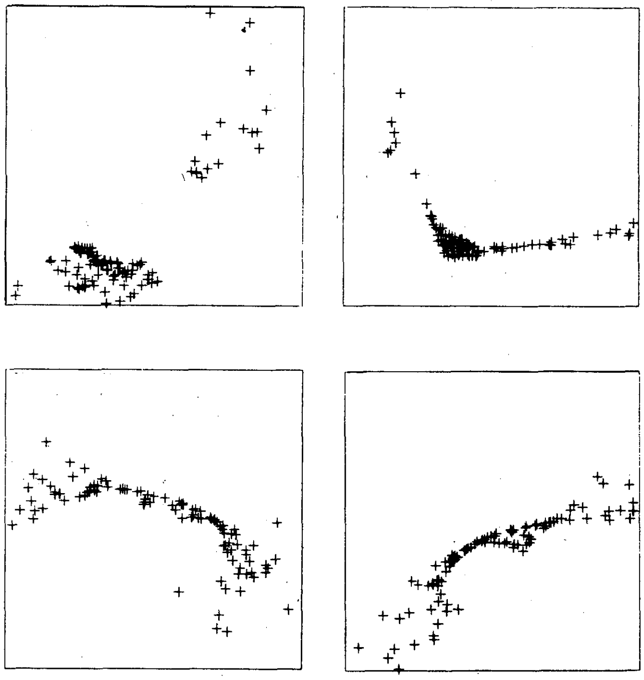
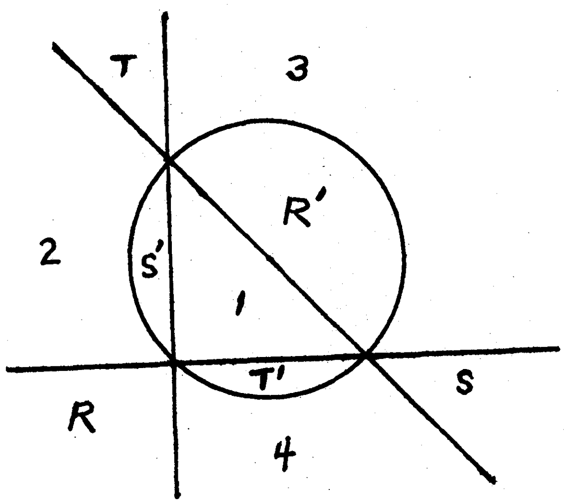


Figure 2—In figure 2 the equations $x' = y^* (x + y - 1)/(x^2 + y^2 - x - y)$ $y' = x^* (x + y - 1)/(x^2 + y^2 - x - y)$ cause the lettered primed and unprimed areas to interchange while the numbered areas map into themselves. To produce the accompanying figures rectangular areas were defined in which random coordinates were generated. After applying the transformation equations those coordinates which fell inside a second rectangle were accepted as the location for a fly or plus sign.



Applicable types of mathematical transformations

Transformations can be classified in many ways. One approach is to group transformations ac-

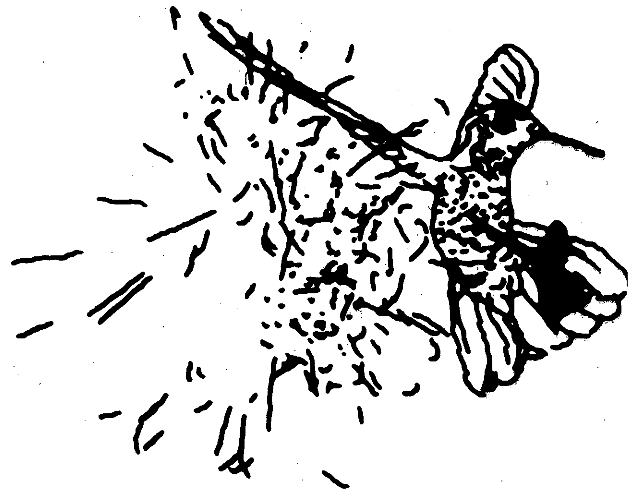


Figure 3—Computer film HUMMINGBIRD, 1967. One still frame from a computer animated movie.

cording to properties such as: those which preserve size and shape; those which preserve lines; those which preserve angles of intersection; those which preserve continuity; and so forth. If transformations are given by equations for coordinates of image points, the transformations can be classified by the form of the equations. Perhaps one might even try to develop a mathematical theory of art, and include a heirarchy of functions based upon ideas about periodicity, harmony, permutation, ratio, progression, and so forth. This would seem to presuppose a theory of artistic structure and one may find many problems in such a mathematical theory of art. There is no clear cut criteria on which to make a judgment about classification. What is important, however, is that the artist have a broad range of functions and computer programs available to him.

It is quite true that an artist can probably imitate any image the computer can generate. Some images would be difficult, but not impossible. However, the question is not one of hand skills versus machine skills. The problem is one of conception, wherein the mathematical transformations made possible by the computer present a new dimension to art.

Research

During the past two years we have produced several hundred pictures by computer. Our work has involved color as well as black and white. We have developed many programs which make use of functions in trigonometry, coordinate systems, conformal mapping, n-dimensional geometry and randomness. In addition we have developed several programs which deal with special problems of graphic display by computer.

The technique we use is as follows: (1) An artistic drawing is made with line segments of points of the subject matter to be used. (2) The drawing is digitized line by line with the resultant coordinates punched into cards. (3) Decisions are made about the type of form modification and the mathematical steps required to accomplish it. (4) The mathematical algorithm is programmed for a computer which then generates the plotter commands. (5) At this point another decision is made about the color and line width for the transformation. (6) The transformed image is plotted on a CalComp 563 plotter.

Using a variation of the photo-nylon screen technique we have developed a method to transfer the plotter output to canvas and plastic sheets. This allows us to use a permanent non-fading paint to represent the image.

We have also completed a ten minute computer animated motion picture entitled HUMMINGBIRD.* The subject was a line drawing of a hummingbird for which a sequence of movements appropriate to the bird were outlined. Over 30,000 images comprising some 25 motion sequences were generated by the computer. For these, selected sequences were used for the film. A microfilm plotter recorded the images directly on film.

To facilitate control over the motion of some sequences the programs were written to read all the controlling parameters from cards, one card for each frame. Curve fit or other data generating programs were used to punch the parameter decks. We also built a windowing option into our plot subroutine.

Our most recent project is sculpture using a 3-axis, continuous path, numerically controlled milling machine. Mathematically generated surfaces such as the Bessel function were our first works. We have developed our own supporting

*Awarded a prize at the 4th International Experimental Film Competition, Brussels, Belgium, 1967.

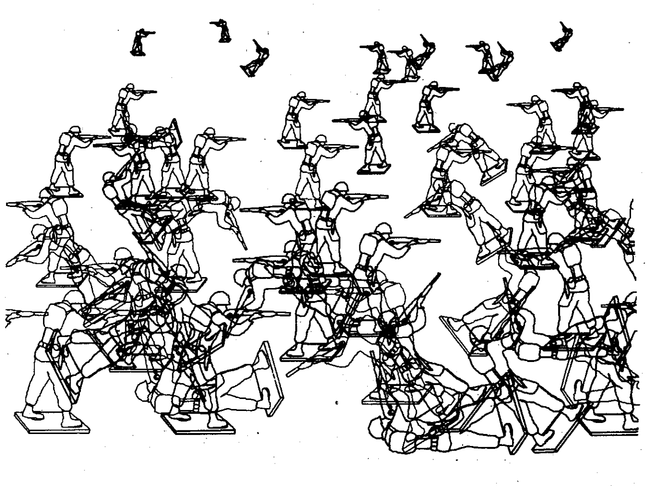


Figure 4—RANDOM WAR, 1967. A drawing was made of one toy soldier and this became the data deck. A computer program which generates random numbers is called a pseudo-random number generator. Such a program determined the distribution and position of 400 soldiers on the battlefield. One side is called the "Red" and the other one the "Black", and the names of real people were given to the program. Another computer program assigned military ranks and army serial numbers at random. The random number generator decided the following information and the computer made this picture with the casualty list. (1) Dead (2) Wounded (3) Missing (4) Survivors (5) One Hero for each side (6) Medals for Valor (7) Good Conduct (8) Efficiency Medals.

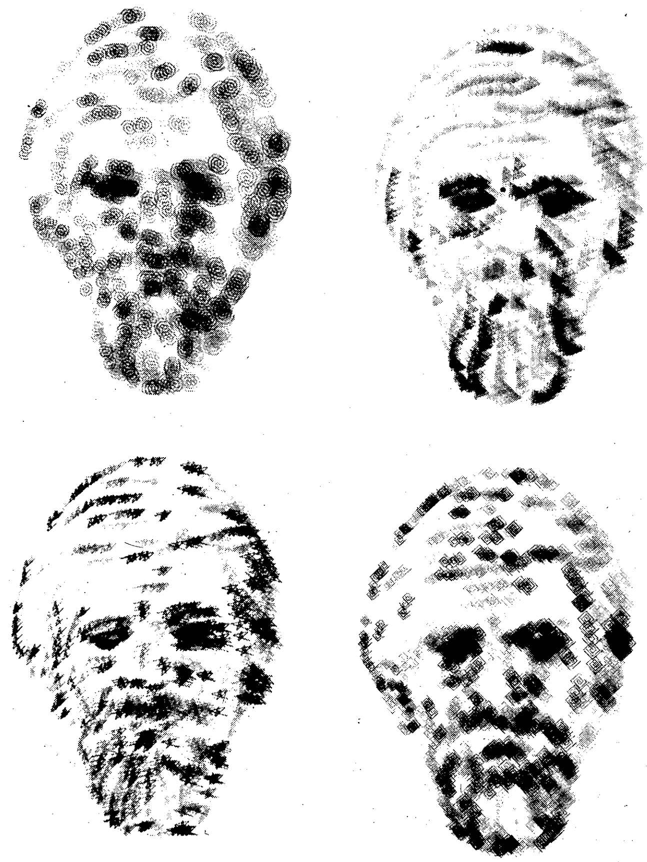


Figure 5—RANDOM COLOR DISTRIBUTION, 1967. A realistic line drawing of an old man was used as the data deck. A line drawing was transformed mathematically into a shaded image. A spiral, rectangle, triangle and star were the character symbols used to replace each line segment. The position of the four colors was determined by the random number generator. The size of each character symbol is a function of distance from a reference point outside the picture space.

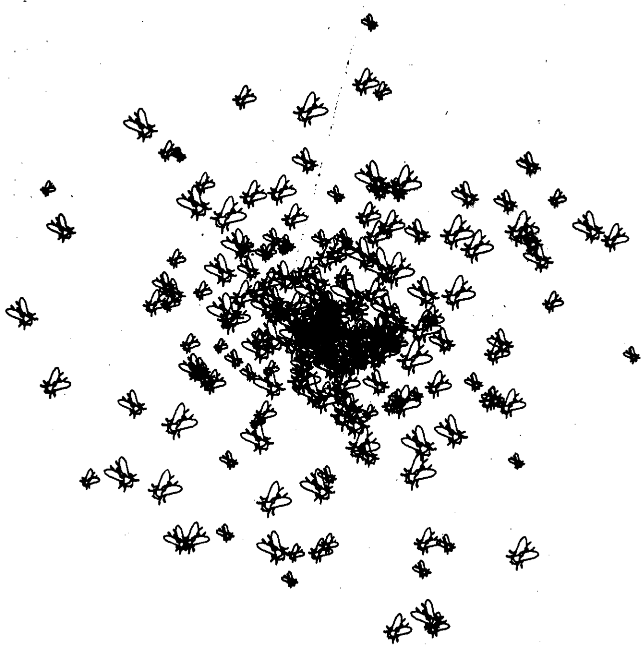


Figure 6—FLIES IN A CIRCLE, 1966. A computer program generates random numbers which determine the distribution of a specific number of flies in a series of 1″ concentric rings. Within predetermined limits the random number generator also decides the orientation and the size of each fly.

routines rather than using one of the special languages such as APT. The rigid program—part correspondence as in APT was not suited for our purposes. This problem lead us to develop a method which provides a more flexible input capability for general artistic use.

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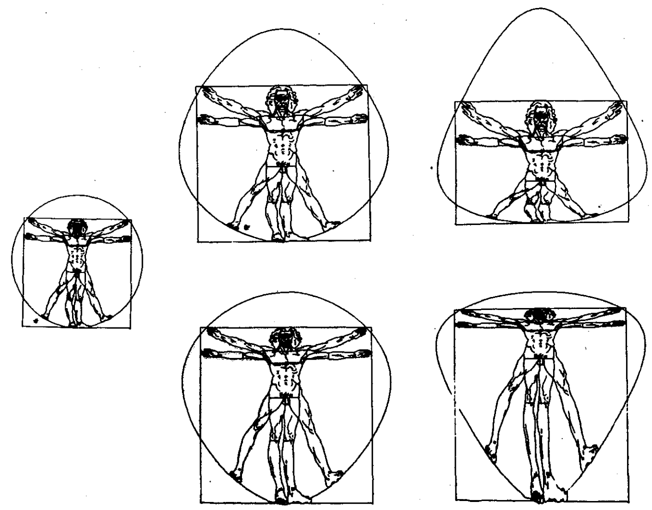

After Leonardo's HUMAN PROPORTIONS

Figure 7—The program finds Y_{min} and Y_{max} (or X_{min} and X_{max}) for the input image. Then from λ_{min} and λ_{max}, read as input parameters, λ_i can be calculated for each Y_i. The mapping function is $Y'_i = K^* \tan(\lambda_i)$ where K is a constant calculated to yield a final figure at the desired height.

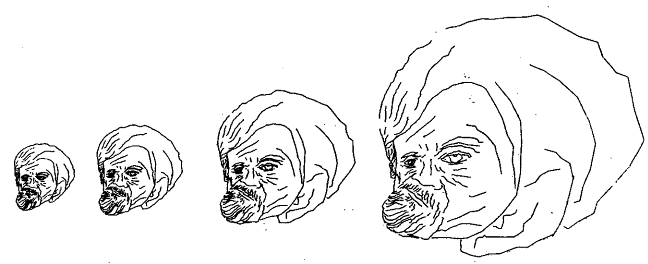

Figure 8—CIRCLE INVERSION, 1967. The input image is scaled to fit inside a circle of unit radius. Working in polar coordinates the transformed image is $\theta' = \theta$, $r' = 1/r$. A point at the unit circle's center transforms to a point at infinity while a point on the circle's circumference is invariant.