Fast Approximation of the Transient Response of Lossy Transmission Line Trees*

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Abstract– An efficient technique for estimating the time-domain response of lossy tree-like interconnect structures is presented. The approach is based on approximating such structures by a tree of RLGC sections, and computing a low order approximation to its transfer function. Unlike the *Asymptotic Waveform Evaluation* approach, the *reciprocal* of the transfer function of the tree is expanded as a polynomial in s. Experimental results are presented which demonstrate the higher accuracy of this approach as compared to AWE. A generalization of the procedure for computation of capacitive crosstalk is also presented.

I. Introduction

Transmission line effects in interchip interconnections on high-performance packages such as multichip modules (MCMs) can cause significant signal integrity problems [1]. Severe ringing caused by reflections and impedance mismatches can cause unexpectedly high delays, and crosstalk between adjacent wires can cause spurious switching. In order to avoid costly design iterations, accurate estimation of these phenomena is essential.

Traditional approaches to lossy transmission-line simulation, suffer from high time complexity (see [2] for references to such approaches). A complex multiterminal net may take several seconds or even minutes for an accurate simulation. Clearly, the simulation step will become a bottleneck in the design process for a high-density MCM with several thousand nets. An alternative approach for rapid interconnect simulation has been proposed in [3], based on the *Asymptotic Waveform Evaluation* (AWE) technique [4]. AWE is a general technique for estimating the behavior of a linear circuit. The first $2N-1$ *moments* of the circuit, which are the coefficients of the Maclaurin expansion of its transfer function, are computed by repeatedly solving a DC circuit consisting of resistors, voltage sources and current sources. The moments are then used to find a N-pole approximation to the transfer function, by a procedure which is equivalent to Padé approximation [5]. When the AWE approach is applied to evaluating interconnects, the tree structure can be exploited to obtain significant savings in computation time.

In this paper, we solve the rapid interconnect evaluation using a new approach which is closely related to the AWE approach. The key difference is that instead of expanding the transfer function $H(s)$, we expand its reciprocal, $1/H(s)$, as a polynomial in s. The coefficients of the polynomial are computed in two steps using efficient recursive algorithms. It is well-known that Padé approximants do not necessarily yield stable approximations to stable systems. Therefore, in a N-pole approximation to $H(s)$, some of the poles may be unstable, and these have to be discarded while computing the time-domain response. We have experimentally found that, using our approach, we are able to extract a greater number of stable poles for a given number of polynomial coefficients. This translates to greater reliability and accuracy in the time-domain waveforms.

Section 2 describes the basic approach and algorithms for computing the reduced-order transfer function of a lossy interconnection tree. Section 3 extends the approach to computation of capacitive crosstalk between adjacent trees. Experimental results and applications are presented in Section 4, and Section 5 concludes the paper.

II. Transfer Function Computation

A lossy transmission line structure can be closely approximated by an RLCG tree (Fig. 1.) Each unit length of the net is replaced by an RLCG section. (Criteria for deciding the number of sections are discussed in [6].) The transfer function of such a tree, from the source node to a particular sink node, is a $2N$-th order function of the form:

$$H(s) = \frac{a_0 + a_1 s + \cdots + a_{2N-1} s^{2N-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_{2N} s^{2N}} \qquad (1)$$

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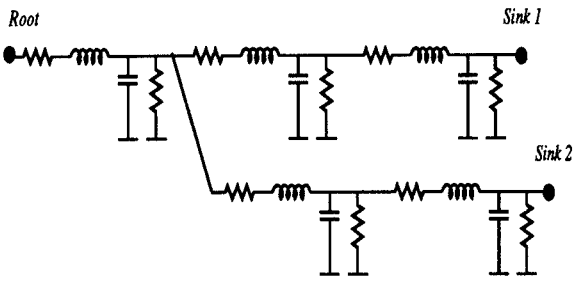

Figure 1: An RLCG interconnection tree

where N is the number of RLCG sections, and the a_i's and b_i's are some functions of the R, L, C and G values and the net topology. Since N may be as high as 100 or more, it is usually more convenient to find a reduced-order approximation for $H(s)$.

Assume that we are interested in finding a n-pole approximation to the transfer function $H(s)$. We divide the computation into two steps, both of which are executed by efficient recursive algorithms:

Step 1: Admittance computation. In this step, for every node v in the tree, the total admittance of the subtree rooted at v is computed, in the form of a polynomial of order $2n-1$ in s.

Step 2: Coefficient computation. In this step, the coefficients of the transfer function from the source node to the desired target node are computed. The target node can be an arbitrary node in the tree.

This approach is a generalization of the approach described in [7]. If the transfer functions to a number of different nodes in the tree are required (for instance, to all the sink nodes in a multiterminal net), step (1) is performed only once, and step (2) is executed once for each transfer function.

A. *Admittance Computation*

Consider the problem of determining the admittance $Y(v)$ of an *RLCG* subtree, "looking in" from its root node v. We shall solve this problem recursively, by assuming that, for each child u of v, the admittance of the subtree rooted at u is known to be $Y(u) = Y_0(u) + Y_1(u)s + \cdots + Y_{2n-1}(u)s^{2n-1}$. From Fig. 2, for a particular child node u, the contribution to the looking in admittance $Y(v)$ is :

$$Y(v) = \frac{Y(u)}{1 + Y(u)(R(u) + sL(u))} \qquad (2)$$

To simplify the algebraic addition of the admittance contributions of multiple children, it is convenient to express

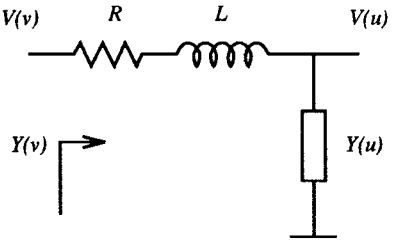

Figure 2: A Single "RLY" Section

the rational function of Eq. 2 as a polynomial of orde $2n-1$:

$$\begin{aligned} Y &= \frac{a_0 + a_1 s + \cdots + a_{2n-1}s^{2n-1}}{1 + b_1 s + \cdots + b_{2n}s^{2n}} \\ &= Y_0 + Y_1 s + \cdots + Y_{2n-1}s^{2n-1} + H.O.T. \quad (3 \end{aligned}$$

The coefficients Y_i can be computed by a simple recursiv formula:

$$Y_k = a_k - \sum_{i=1}^{k} b_i Y_{k-i} \qquad (4$$

The admittances of multiple children of a node ca then be added using straightforward polynomial addi tion. Thus, given the admittance at a node in the form c a $2n-1$-th order polynomial, we can find the admittanc at its parent node in the same form. At any leaf node v $Y_0(v)$ and $Y_1(v)$ are known (the conductance and capaci tance at that node), and the higher-order corfficients ar zero. Thus, using a reverse depth-first traversal of th tree, the looking-in admittance at every node in the tre can be computed as a polynomial of order $2n-1$. Th complexity of this procedure is linear in N, the numbe of sections, and quadratic in n, the order of the approx imation (n is typically less than 10).

If the *RLCG* tree is modified, by attaching a ne branch or deleting an existing branch, the admittanc computation does not need to be repeated for the entir tree, since the admittance values change only for thos nodes lying between the root and the branch node. Th values can be updated by a simple procedure which ir volves a single backtrack from the branch node to th root. This feature is useful for on-line computation c the response during interactive or automatic routing.

B. *Coefficient Computation*

To compute the transfer function from the root to a arbitrary node x in the tree, given the looking-in admi tances at every node, first consider the problem of finc ing the transfer function across a single "RLY" sectio (Fig. 2). In terms of the output voltage $V(u)$ the inpu voltage $V(v)$ can be written as

$$V(v) = V(u)[1 + Y(u)(R(u) + sL(u))] \qquad (5$$

Inductively assume that the node voltage at u is already known, in terms of the voltage at the target node x, as

$$\frac{V(u)}{V(x)} = b_0(u) + b_1(u)s + \cdots + b_{2n-1}s^{2n-1}(u) \quad (6)$$

Then $V(v)$ can be written in terms of $V(x)$ as

$$\frac{V(v)}{V(x)} = b_0(v)+b_1(v)s+\cdots+b_{2n-1}(v)s^{2n-1}+H.O.T. \quad (7)$$

where

$$b_k(v) = (1 + R(u)Y_0(u))b_k(u) + \sum_{i=1}^{k}(L(u)Y_{i-1}(u) + R(u)Y_i(u))\, b_{k-i}(u) \quad (8)$$

Starting with the values $b_0(x) = 1, b_i(x) = 0, i \geq 1$, and backtracking from the target node x to the root node r, the coefficients $b_i(r)$ can be found. We thus have:

$$\begin{aligned} \frac{V(r)}{V(x)} &= b_0(r) + b_1(r)s + \cdots + b_{2n-1}(r)s^{2n-1} \\ &= B(s) \end{aligned} \quad (9)$$

The complexity is linear in N and n.

Unlike the approach of [3], $B(s)$ here represents the moments of the *inverse* of $H(s)$. The polynomial $B(s)$ can be converted to a $[n/(n-1)]$ rational function $D_n(s)/N_{n-1}(s)$ using a Padé approximation [8]. The n-pole approximation for $H(s)$ is then the strictly proper rational function, $N_{n-1}(s)/D_n(s)$.

Computing the step response of the system is then a matter of numerically computing the poles p_i of $sD_n(s)$ and their residues k_i. The time-domain response is then a sum of exponentials and/or damped sinusoids:

$$V_x(t) = \sum_{i=0}^{n} k_i e^{p_i t} \quad (10)$$

The summation in (10) is over all stable poles: if any pole has a positive real part, that pole is ignored.

Although the series impedance in the RLGC sections has been assumed to consist of a resistance in series with an inductance, the same ideas can be used even when the impedance is given by a general polynomial in s. The same is true for the shunt admittance also. This generalization can be useful for modeling frequency dependent skin-effect and dielectric losses [9].

III. Extension to Coupled Trees

Another important phenomenon responsible for signal degradation in high-density interconnects is the *crosstalk* or coupling between long adjacent wires. Simulation of crosstalk waveforms is a difficult task, and is currently not supported by the popular circuit simulation program Spice (Version 3E).

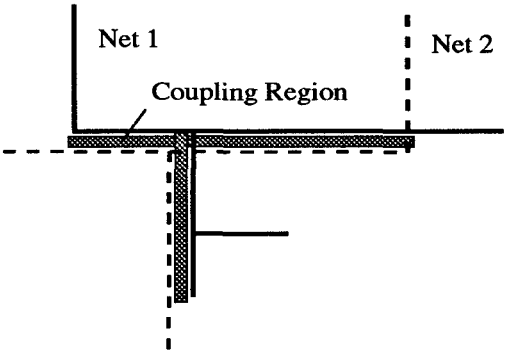

Figure 3: Capacitively Coupled Trees

The procedures described in the previous section for isolated trees can be extended to a pair of capacitively coupled trees, when the region of coupling is itself in the form of a tree (see Fig. 3). Although the coupling introduces loops in the circuit, the simulation algorithms retain their linear complexity.

The crosstalk computation procedure also proceeds in two steps: admittance computation followed by coefficient computation. The result will be a pair of coupled equations of the form:

$$\begin{aligned} V_r(s) &= A(s)V_0(s) + B(s)U_0 s \\ U_r(s) &= C(s)V_0(s) + D(s)U_0(s) \end{aligned} \quad (11)$$

where V_r and U_r are the voltages at the roots of the two trees, V_0 and U_0 are the voltages at target points on the two trees, and A, B, C, and D are polynomials of order $2n-1$ in s. The crosstalk waveform at V_0 due to a pulse at U_r can be computed by setting $V_r = 0$ and solving for V_0:

$$\frac{V_0(s)}{U_r(s)} = \frac{B(s)}{B(s)C(s) - A(s)D(s)} \quad (12)$$

A. Generalization of Admittance Computation

Let the two coupled trees be denoted by T_1 and T_2, and the region in which the coupling occurs be denoted by T_c. We assume that T_c is also a tree. Every edge in T_c corresponds to two RLGC sections, one in T_1 and one in T_2, and every node in T_c corresponds to a pair of coupled nodes, one each in T_1 and T_2. A pair of coupled nodes is connected by a *coupling admittance* γ_K, which is typically a capacitance. For the single tree case, we recursively computed the admittance $Y(v)$ at a node v, which gave us the ratio between the current flowing out of node v and the voltage at node v. For the case of coupled trees, the computation is generalized as follows.

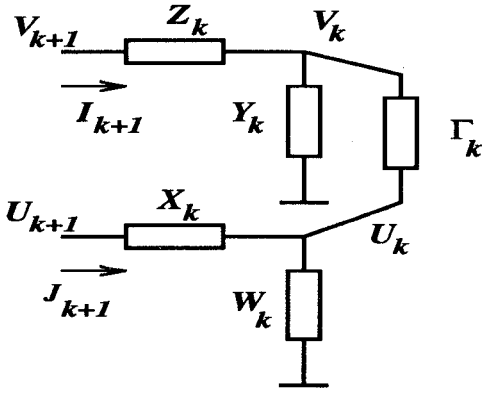

Figure 4: Coupled RLGC Sections

For a pair of coupled nodes, we define three quantities, W_u, Y_v and Γ_{uv}:

$$\begin{aligned} I_v &= Y_v V_v + \Gamma_{uv}(V_v - U_u) \\ J_u &= W_u U_u + \Gamma_{uv}(U_u - V_v) \end{aligned} \quad (13)$$

where U_u, V_v and J_u, I_v are the voltages at, and currents flowing out of, nodes u and v respectively.

Given the admittances at a pair of coupled nodes u_k and v_k, we wish to compute the corresponding quantities at their parent nodes u_{k+1} and v_{k+1} (see Fig. 4, so as to obtain a recursive procedure similar to the single-tree case. Writing equations for the input currents I_{k+1} and J_{k+1}, we have

$$\begin{aligned} I_{k+1}D_k = [&Y_k(1 + (W_k + \Gamma_k)X_k) \\ &+ \Gamma_k(1 + W_k X_k)]V_{k+1} - \Gamma_k U_{k+1} \\ J_{k+1}D_k = [&W_k(1 + (Y_k + \Gamma_k)Z_k) \\ &+ \Gamma_k(1 + Y_k Z_k)]U_{k+1} - \Gamma_k V_{k+1} \end{aligned} \quad (14)$$

where

$$\begin{aligned} D_k = 1 &+ W_k X_k + Z_k Y_k + \Gamma_k(X_k + Z_k) \\ &+ \Gamma_k X_k Z_k(W_k + Y_k) + W_k X_k Y_k Z_k \end{aligned} \quad (15)$$

Comparing (14) with the form of (13), we obtain:

$$Y_{k+1} = \frac{Y_k(1 + (W_k + \Gamma_k)X_k) + \Gamma_k W_k X_k}{D_k} \quad (16)$$

$$W_{k+1} = \frac{W_k(1 + (Y_k + \Gamma_k)Z_k) + \Gamma_k Y_k Z_k}{D_k} \quad (17)$$

$$\Gamma_{k+1} = \frac{\Gamma_k}{D_k} \quad (18)$$

To compute the admittances at all nodes in the two trees, the admittances of those parts of the two trees which extend beyond the leaves of the coupling region are computed first, using the recursion described for the single-tree case. This gives us values for Y and W at all the leaves of the coupling region T_c. The value of Γ at the leaves of T_c is simply the coupling admittance γ. Equations (17–18) are then used to recursively compute the admittances at all nodes of T_c (which are pairs of coupled nodes in T_1 and T_2). The root node of T_c is made to coincide with the root nodes of both trees T_1 and T_2, by conceptually adding "dummy" coupling admittances between uncoupled RLGC sections, and/or adding extra dummy segments to one of the trees, if necessary.

B. *Coupled Coefficient Computation*

The coefficients of the polynomials A, B, C and D in (11) are computed by backtracking from the target nodes of T_1 and T_2. The backtracks on the two trees proceed separately, until they both hit a leaf l of T_c. (In order to ensure that both the backtrack procedures hit the *same* leaf of T_c, only one of the two target nodes can be chosen arbitrarily.) Then, we backtrack from the leaf of T_c to its root (which is also the root of T_1 and T_2), applying the following recursions:

$$V_{k+1} = V_k(1 + Z_k(Y_k + \Gamma_k)) - Z_k \Gamma_k U_k \quad (19)$$

$$U_{k+1} = U_k(1 + X_k(W_k + \Gamma_k)) - X_k \Gamma_k V_k \quad (20)$$

At the leaf l of T_c, the initial separate backtrack computations give us the starting points for the recursion, expressing U_l/U_o and V_l/V_o as polynomials in s. The polynomials are updated at each node in T_c, and at the root, we obtain the final values of the coefficients of A, B, C and D.

IV. Experimental Results

A. *Comparison with AWE and Spice*

The accuracy of the approach described in the previous sections was verified by comparing the computed step response waveforms for a large number of multiterminal nets, with accurate lossy transmission line simulations using Spice (Version 3E). The number of RLGC sections used for the nets in our approach was typically between 50 and 100. The line parameters and driver impedance were all varied over a wide range, and uniformly accurate results were obtained in all cases.

Figs. 5 and 6 show the responses obtained using a 7-pole model for a six-terminal net, with different driver impedances (the transmission line impedance was 50 Ω). Our approach took less than 0.3 seconds for each simulation, as compared to more than 220 seconds for each Spice simulation. The high accuracy of the waveform in these two figures is typical of all the experiments we have conducted.

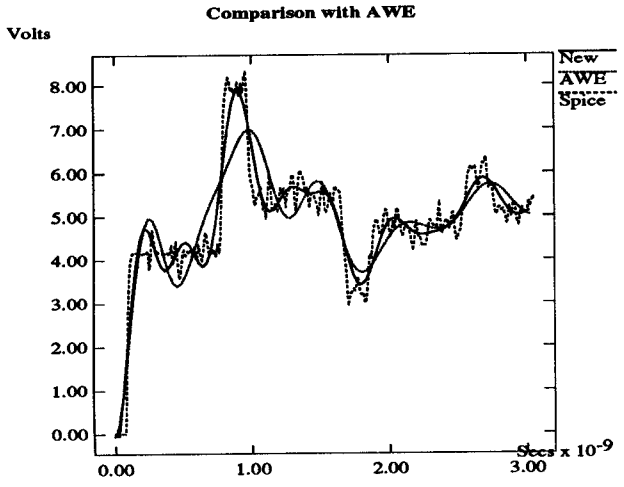

Figure 5: Step response for $R_d = 10\Omega$.

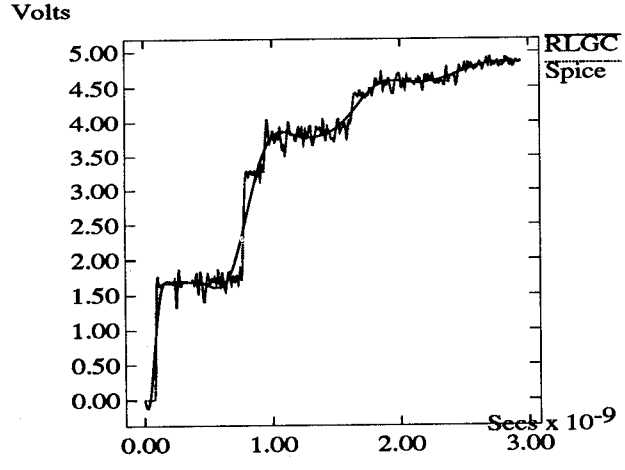

Figure 6: Step response for $R_d = 100\Omega$.

Fig. 5 also shows the waveform obtained using the AWE approach – the waveform does not match the Spice simulation very closely. In general, it was observed that for underdamped situations (when the driver resistance is less than the line impedance), the new approach is more accurate than AWE, and for the matched and overdamped situations, both approaches are equally accurate on the average. This result is significant because it is the underdamped situation which is difficult to simulate accurately, requiring a large number of poles (overdamped lines can be approximated well even by first or second-order models).

Table I shows typical results obtained for a thirteen-pole approximation of ten different nets, with a driver resistance of 10 Ω and a line impedance of 50 Ω. The second and third columns compare the number of unstable poles in the approximation, using AWE and the new approach. The next two columns compare the magnitude of the largest residue associated with an unstable pole. This figure is important since the unstable poles are ignored during the time-domain response computation, and if the residues associated with them are large, then the waveform is likely to have a large error at $t = 0$. From the table, it is evident that the number of unstable poles is less using the new approach in all but three of the ten cases, and the residues associated with these poles are also smaller on the average.

TABLE I
COMPARISON OF AWE AND PROPOSED APPROACH

Ex	*# of Unstable Poles*		*Unstable Residue*	
	AWE	New	AWE	New
1	6	3	10^{-4}	10^{-8}
2	6	4	10^{-1}	10^{-7}
3	5	3	10^{-4}	10^{-3}
4	4	6	10^{-7}	10^{-2}
5	5	3	10^{-1}	10^{-6}
6	6	3	10^{-1}	10^{-8}
7	5	4	10^{-4}	10^{-3}
8	6	3	10^{-2}	10^{-8}
9	6	7	10^{-2}	10^{-2}
10	5	6	10^{-2}	10^{-1}

Fig. 7 shows the crosstalk waveform caused by capacitive coupling between two adjacent lines with 6 RLGC sections each. A 5-pole approximation was used for the computed waveform. The Spice waveform was found by simulating the lumped RLC circuit, since coupled transmission line simulation is not handled by Spice 3E.

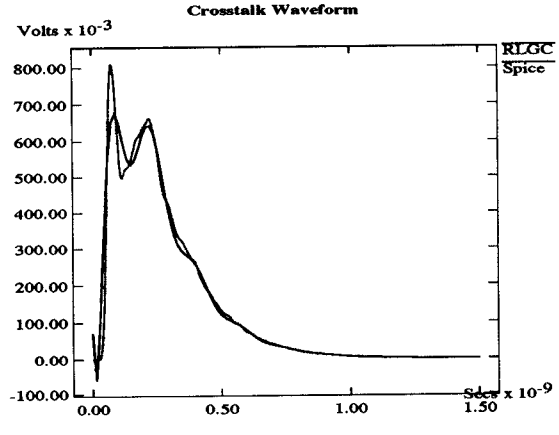

Figure 7: Crosstalk waveform for two coupled lines

B. Application to Global Routing

The speed and accuracy of the simulation algorithm makes it a useful tool for guiding a designer during interactive routing, or for guiding an automatic routing algorithm. As mentioned earlier, a useful feature of the new approach is that changes to the structure of the tree can be handled "on-line". For example, if a tree-construction algorithm adds a new branch to the tree, the admittance values of only those nodes which lie on the path from the root to the branch point need to be updated. This feature has been exploited by a global routing algorithm [10], which begins with the driver node, and constructs a tree by attaching one sink at a time to the partially constructed tree. An optimal branch point on the partial tree is found by performing "trial" constructions, and evaluating their effect on the delay. The algorithm computes a two-pole approximation to the transfer function. The second-order model is superior to a first-order delay model, such as the Elmore model [11], because it is able to predict additional delays introduced by voltage undershoots. At the same time, it is amenable to analytical delay computation, which is not possible with higher-order models.

V. Conclusion

In this paper, we presented a new approach for estimating the time-domain response of lossy coupled interconnects with arbitrary tree structures. Experimental results demonstrate that the approach is efficient and accurate, providing excellent estimates of step responses hundreds of times faster than Spice. The accuracy also compares favorably with the Asymptotic Waveform Evaluation approach, particularly for difficult underdamped lines.

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