

Simplified Ordering for Fixed-complexity Sphere Decoder

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ABSTRACT

This paper proposes a simplified ordering algorithm for the fixed-complexity sphere decoder (FSD). The new algorithm is developed from the analysis of the ordering for FSD from a geometrical point of view. Computer simulation is used to assess the improvements in bit-error rate (BER) performances of MIMO systems using the FSD with the original and the simplified ordering. Simulation results show that the new ordering method can achieve nearly the same BER as the original ordering method but with much less complexity.

Categories and Subject Descriptors

C.2 [Computer-Communication Networks]: Wireless Communication.

General Terms

Algorithms, Design.

Keywords

geometrical analysis, fixed-complexity sphere decoder, ordering, MIMO.

1. INTRODUCTION

Multiple-input-multiple-output (MIMO) system has attracted more and more attention in wireless communications due to its potentially high data rate and low bit-error rate (BER). However, detection has been a big issue that makes MIMO systems difficult to implement in practice. It is well known that maximum-likelihood (ML) detection achieves the best block-error rate (BLER) performance, but its complexity is too high because it has to search the solution in the whole signal space of the transmitted signals. Linear detection algorithms such as zero-forcing (ZF) and minimum mean-square-error (MMSE) are much simpler, but the BER performance is not satisfactory because they do not fully explore the diversity advantages of the MIMO channel. Sphere decoding (SD) [1]-[3] can achieve the BER performance of ML detection with much less complexity by searching within only a subset of the whole signal space. Geometric decoding algorithms [4]-[6] have also been proposed to reduce the search signal space to a small subset. However, these algorithms share a common disadvantage that the detection complexity varies with the signal-to-noise ratio (SNR), making these algorithms difficult for hardware implementation.

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Recently, a fixed-complexity sphere decoder (FSD) has been proposed to solve the variable complexity problem of SD [7]. The algorithm used an ordering process to determine the order of candidates for each symbol. Then it calculated the Euclidean distances between the received signal and all the vector combinations of these candidates and chose the vector with the smallest Euclidean distance as the detected symbol. Since the FSD searches for the candidates in a processive way rather than a recursive way as in SD, the detection complexity is fixed for all SNRs.

The error performance of FSD was analyzed in [8] algebraically. In this paper, we look at FSD from a geometrical point of view. We show how the ordering criterion in FSD works in the lattice of the receive signal space. Then we adopt a simplified ordering criterion to approximate the original ordering criterion for FSD. Simulation results show that, in a 4×4 MIMO system used for study, the simplified ordering achieves nearly the same BER performance as the original ordering for FSD with much less complexity.

The rest of this paper is organized as follows: Section II describes the system model and the FSD algorithm in [7]. Section III explains the geometrical analysis of the ordering used in FSD. Section IV presents the simplified ordering algorithm based on the analysis in Section III. Simulation results are presented in Section V. Section VI concludes the paper.

2. SYSTEM MODEL AND FIXED-COMPLEXITY SPHERE DECODER

2.1 System Model

The system model used for the study is an $N_t \times N_r$ MIMO system with N_t transmit antennas and N_r receive antennas, over a fast Rayleigh fading channel which changes at each transmitted symbol vector. The received signal vector \mathbf{r} with length N_r is given by:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \quad (1)$$

where the entries of $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ represent the channel coefficients which are assumed to be perfectly known at the receiver but not at the transmitter, $\mathbf{s} \in \mathbb{C}^{N_t}$ is the transmitted uncoded signal vector, and $\mathbf{w} \in \mathbb{C}^{N_r}$ represents the complex additive white Gaussian noise (AWGN) matrix with elements being independently and identically distributed (iid) and following the normal distribution $N_{\mathbb{C}}(0, N_0)$.

The detection process aims to find the signal vector with the smallest Euclidean distance from the received vector:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \mathbb{S}} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \quad (2)$$

where \mathbb{S} is the signal space containing all the possible combinations of transmitted symbols.

2.2 FSD algorithm

In [7], the FSD algorithm tried to solve (2) in two steps. First, an ordering was used to determine the order of symbols to be detected. For the first p symbols, all or some of the possible values were enumerated (called *full* or *partial expansion*). An SD-like detection algorithm based on the Cholesky decomposition of the channel matrix was then used to detect the rest of the symbols. Finally, the Euclidean distances between all the enumerated symbol vectors and the received signal \mathbf{r} were calculated and compared. The symbol vector with the smallest Euclidean distance from \mathbf{r} was taken as the detected vector. These two steps can be described in details as follows.

a. Ordering

The channel matrix ordering of [7] can be described as follows.

In the $(i+1)^{\text{th}}$ iteration, for $i=0, \dots, N_t-1$:

1) The matrix $\mathbf{Q}_i = (\mathbf{H}_{N_t-i}^H \mathbf{H}_{N_t-i})^{-1}$ is calculated, where \mathbf{H}_{N_t-i} is the channel matrix with the i columns selected in the previous iterations removed and $\mathbf{H}_{N_t-i}^H$ denotes the conjugate transposition of \mathbf{H}_{N_t-i} .

2) The k^{th} column of \mathbf{H}_{N_t-i} is selected according to

$$k = \begin{cases} \arg \max_j [\mathbf{Q}_i]_{jj}, & \text{for full expansion,} \\ \arg \min_j [\mathbf{Q}_i]_{jj}, & \text{for single expansion,} \end{cases} \quad (3)$$

where $[\mathbf{Q}_i]_{jj}$ is the j^{th} -diagonal element of \mathbf{Q}_i . In [8], $[\mathbf{Q}_i]_{jj}$ was interpreted as the post-processing noise amplification of the j^{th} symbol in the $(i+1)^{\text{th}}$ iteration. In the next section, we show what $[\mathbf{Q}_i]_{jj}$ is in the geometrical expression of FSD detection.

b. Detection

After ordering, expansions and detections are used. As an example, consider a system transmitting four 16QAM symbols (i.e. $N_t=4$) at a time. To detect the first symbol (e.g. s_2 after ordering), full expansion is used (i.e. $p=1$). For each of the 16 possible values of s_2 , the contribution of s_2 is removed from the received signal \mathbf{r} and hard decision is applied to detect the next symbol by minimizing the *partial Euclidean distance* (PED) [9]. Interference in the detected symbol is then removed from \mathbf{r} and similarly, the PED of the next level is minimized to detect the next symbol. The detection process goes like this until all the 16 symbol-vector candidates are obtained for each value of s_2 . The Euclidean distances of these 16 candidates to the received signal \mathbf{r} are compared. The candidate with the smallest Euclidean distance is taken as the detected vector.

3. GEOMETRICAL ANALYSIS OF FSD ORDERING

In this section, we illustrate the ordering for FSD from a geometrical point of view.

Consider an example of a 3-dimension real received signal space (i.e. $N_r = N_t = 3$) for 4PAM as shown in Fig. 1, where the symbol \star denotes the received signal vector \mathbf{r} . The generation matrix of the lattice is \mathbf{H} , having column vectors \mathbf{h}_1 , \mathbf{h}_2 and \mathbf{h}_3 as the directions of s_1 , s_2 and s_3 , respectively. The detector aims to find the lattice point closest to \mathbf{r} . In Fig. 1, the dots of the same color denote the lattice points with the same values of s_2 . We call the hyperplane formed by the dots with the same value of s_2 a *layer* of s_2 . For example, the layers with slashes are layers of s_3 .

In Section II, it is described that, in each iteration of the ordering process, one column of \mathbf{H} is removed. Therefore, in the $(i+1)^{\text{th}}$ iteration, for $i=0, 1, \dots, N_t-1$, the space is spanned by only the columns of \mathbf{H}_{N_t-i} and the lattice is generated by \mathbf{H}_{N_t-i} . We denote the index of the symbol corresponding to the j^{th} column of \mathbf{H}_{N_t-i} by $I(i, j)$, and the distance between the layers of $s_{I(i, j)}$ in the $(i+1)^{\text{th}}$ iteration by l_j^i (for $j=1, 2, \dots, N_t$). Then in the 1st iteration ($i=0$) we have the following equation.

$$l_j^0 = \frac{2}{\sqrt{[\mathbf{Q}_0]_{jj}}} \quad (4)$$

Proof: In this proof, we use the 3-dimensional example of Fig. 1. The extension to an N_t -dimensional space is straightforward.

Denoting the matrix $(\mathbf{H}_0^H \mathbf{H}_0)^{-1} \mathbf{H}_0^H$ as $\mathbf{M}_{\mathcal{F}}$, we have

$$\mathbf{M}_{\mathcal{F}} \mathbf{H}_0 = \mathbf{I} \quad (5)$$

Then denoting the j^{th} row of $\mathbf{M}_{\mathcal{F}}$ as \mathbf{m}_j^H , (5) implies \mathbf{m}_j is orthogonal to all but the j^{th} column vectors of \mathbf{H}_0 . Also note that

$$\mathbf{m}_j = \mathbf{H}_0 \mathbf{q}_j \quad (6)$$

where \mathbf{q}_j^H is the j^{th} row of \mathbf{Q}_0 . (6) means that \mathbf{m}_j is within the vector space spanned by the columns of \mathbf{H}_0 . From (5) and (6), we can see that \mathbf{m}_j is orthogonal to the layers of s_j . The length of the line segment of \mathbf{m}_j between adjacent layers of s_j is equal to the distance between adjacent layers of s_j , which is l_j^0 . We denote \mathbf{h}_j^0 as the j^{th} column of \mathbf{H}_0 and $\|\cdot\|$ as the Euclidean norm of a vector. Then, from Fig. 1, we can see that the length of the line segment of \mathbf{h}_j^0 between adjacent layers of s_j is equal to $2\|\mathbf{h}_j^0\|$. Since the layers of s_j are parallel to each other, we have:

$$l_j^0 = 2\|\mathbf{h}_j^0\| \cos \theta \quad (7)$$

where θ is the angle between \mathbf{m}_j and \mathbf{h}_j^0 .

From (5) we can write:

$$\mathbf{m}_j^H \mathbf{h}_j^0 = |\mathbf{m}_j| |\mathbf{h}_j^0| \cos \theta = 1 \quad (8)$$

Combining (7) and (8) yields:

$$l_j^0 = \frac{2}{|\mathbf{m}_j|} \quad (9)$$

From (6) we know

$$|\mathbf{m}_j|^2 = \mathbf{q}_j^H \mathbf{H}_0^H \mathbf{H}_0 \mathbf{q}_j \quad (10)$$

Since \mathbf{q}_j^H is the j^{th} row of \mathbf{Q}_0 and $\mathbf{Q}_0 \mathbf{H}_0^H \mathbf{H}_0 = \mathbf{I}$, $\mathbf{q}_j^H \mathbf{H}_0^H \mathbf{H}_0 \mathbf{q}_j$ is equal to the j^{th} element of \mathbf{q}_j , which is $[\mathbf{Q}_0]_{jj}$. Substituting $\mathbf{q}_j^H \mathbf{H}_0^H \mathbf{H}_0 \mathbf{q}_j$ into (10) and then to (9) yields

$$l_j^0 = \frac{2}{\sqrt{[\mathbf{Q}_0]_{jj}}} \quad (11)$$

Similarly, it can be proved that in the $(i+1)^{\text{th}}$ iteration:

$$l_j^i = \frac{2}{\sqrt{[\mathbf{Q}_i]_{jj}}} \quad (12)$$

From the previous analysis, we can see that the ordering for FSD actually depends on the distance between the layers of the symbols. The p symbols with the smallest layer distances are fully expanded. In single expansion, the symbol with a larger layer distance is detected first.

To understand the reason for this ordering used in FSD, we can look at it from a geometrical way. Consider the same lattice shown in Fig.1. The layer distance of s_2 is obviously smaller than those of s_1 and s_3 , so the lattice point closest to the received point \mathbf{r} (the \star) is equally likely to be in one of the four layers of s_2 . Thus all the four layers of s_2 need to be searched and the full expansion process is used in FSD. In single expansion, the 2nd column of \mathbf{H} is removed and the space becomes a 2-dimensional space spanned by \mathbf{h}_1 and \mathbf{h}_3 , as depicted in Fig. 2. Now the symbols are detected iteratively and only one candidate of each symbol is taken. If the layer distance of a symbol is large, it is likely that the lattice point closest to \mathbf{r} is on the layer closest to \mathbf{r} . In the example of Fig. 2, the distance between the layers of s_3 is large, so the point closest to \mathbf{r} is more likely to be on the layer where $s_3=-1$, which is the closest layer to \mathbf{r} . This means that the decision of $s_3=-1$ has a higher probability of being correct, so detecting s_3 first will reduce the error propagation in single expansion detection. This is the reason why the symbols corresponding to small $[\mathbf{Q}_i]_{jj}$ are detected first in single expansion detection in FSD.

4. SIMPLIFIED ORDERING

As we can see from section II, the ordering for FSD requires at least N_t-1 matrix inversion operations, which contributes to much of decoding complexity. This problem is even worse for a fast fading channel where the channel matrix changes at each transmitted symbol vector. In this section, we propose a simplified ordering criterion based on the geometrical analysis to

approximate the original ordering criterion for FSD. This new ordering requires only one matrix inversion operation and thus reduces the complexity of the ordering process for FSD. The simplified ordering is described as follows:

The k^{th} symbol is selected according to

$$k = \begin{cases} \arg \max_j [\mathbf{Q}_0]_{jj}, & \text{for full expansion,} \\ \arg \min_j [\mathbf{Q}_0]_{jj}, & \text{for single expansion,} \end{cases} \quad (13)$$

where $[\mathbf{Q}_0]_{jj}$ is the j^{th} -diagonal element of \mathbf{Q}_0 and $\mathbf{Q}_0 = (\mathbf{H}^H \mathbf{H})^{-1}$.

In the example of Fig. 1 and Fig. 2, the simplified ordering is equivalent to using l_3^0 to approximate l_2^1 . This approximation is more clearly depicted in Fig. 3. The extension to an N_t -dimensional system is straightforward. Recall the definition of $I(i, j)$ in the last section. It is easy to see that in the i^{th} iteration (for $i=0,1,\dots,N_t-1$) of ordering in an N_t -dimensional system, the symbol corresponding to l_j^i is $s_{I(i,j)}$ and $I(0, j) = j$. Thus a simplified method is to use $l_{I(i,j)}^0$ to approximate l_j^i . As we can see from the example of Fig. 3, this approximation uses the layer distance in a 3-dimensional space to approximate the layer distances in a 2-dimensional space. Generally, it uses an N_t -dimensional layer distance to replace all the layer distances in the lower dimensional spaces. Although errors may occur in approximating the distances, the ordering is less affected. Moreover, the simplified ordering of (13) only requires one matrix inversion operation, which substantially reduces the complexity of the ordering, especially in the fast fading channels.

5. SIMULATION RESULTS

Computer simulation has been used to evaluate the effects of ordering and simplified ordering on the BER performance and the complexity of a MIMO system. A 4×4 MIMO system transmitting 4 uncoded 16QAM symbols at a time interval has been used for studies. The channel is a fast fading channel which varies from each transmitted symbol vector to another one.

The BER performances of SD, original FSD, FSD without using the ordering and FSD with the simplified ordering are shown in Fig. 4. It can be seen that the BER performance of the FSD algorithm is very close to that of ML detection. The BER performance of FSD without using the ordering is also shown in Fig. 4. This algorithm is the same as the FSD algorithm except that the order for symbol detection is random. Comparing the BER performance with that of FSD shows that significant improvements in BER performance can be achieved by using the ordering. The BER of the FSD with our proposed simplified ordering is also shown in the same figure. It can be seen that the BER curve is nearly identical to that of the original FSD.

Figure 5 compares the complexity of FSD using the original and simplified orderings at different SNRs. The solid lines correspond to the average number of multiplication operations while the dashed lines correspond to the average number of addition operations. It can be seen that the simplified ordering reduces the complexity of FSD by more than 20%.

6. CONCLUSIONS

In this paper, we have analyzed the ordering of FSD from a geometrical point of view. Based on the analysis, we proposed a new ordering criterion to approximate the original ordering criterion for FSD. Computer simulation results have shown that the simplified ordering algorithm does not cause obvious degradation to the BER performance of the FSD, but reduces the complexity by more than 20%.

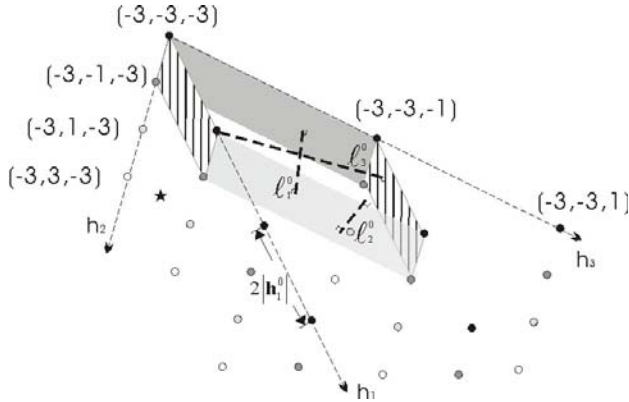


Fig. 1. Lattice in received signal space.

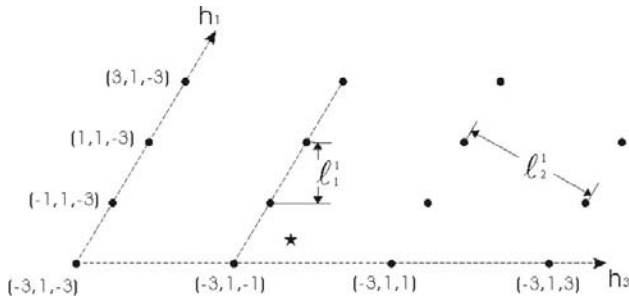


Fig. 2. Space spanned by h_1 and h_3 .

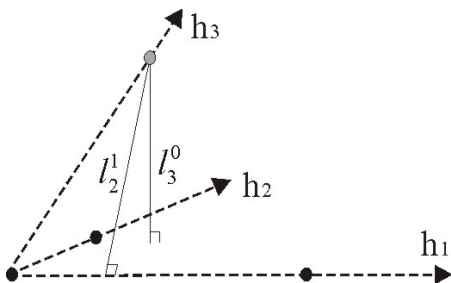


Fig. 3. Approximation of simplified ordering.

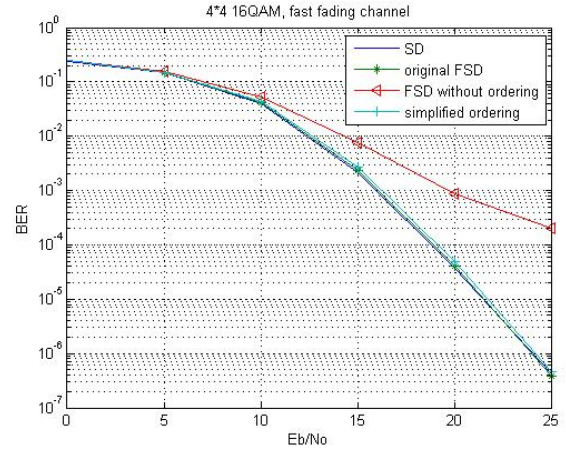


Fig. 4. Bit-error rate of FSD with and without simplified ordering.

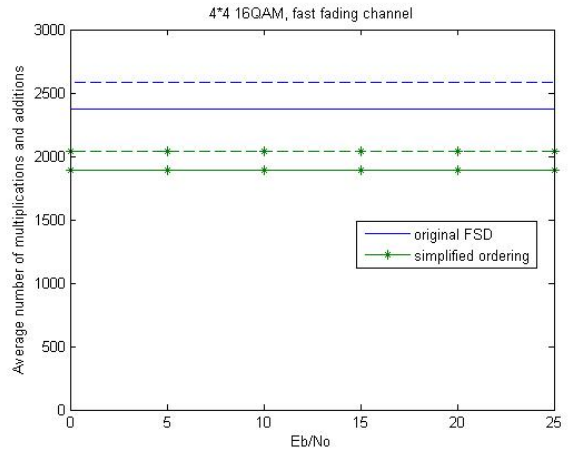


Fig. 5. Complexity of FSD with original and simplified ordering.

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