

# Geometric data of cross-sections of beams

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## Keywords

Engineering mechanics, stress analysis, centroid of an area, second moments of area, section moduli, external forces, displacements of cross-section, normal stresses, shear center, shearing stresses.

## Abstract

This paper describes a program which calculates the geometrical characteristic values of cross-sections of beams with complex shapes (bars or thin-walled profiles with any number of holes in the profiles).

After a short introduction describing the calculations involved for moments of area, we will introduce the capabilities of every component of this program and we will make a survey of the whole system via a flow chart.

At the end of the paper we will summarize the main features of the program and mention some possible extensions.

## Introduction

In many areas in engineering mechanics, geometric characteristic values such as area, centroid of area, and second moments of area, are needed. E.g. in stress analysis to determine and improve the behaviour of a structural member subject to extreme stresses.

The geometrical characteristic values for simple cross-sections (circles, rectangles, etc.) are easily determined using general formulas, or they are available in handbooks for a wide range of standard structural steel shapes (I girder, T girder, etc.).

However, for more complicated cross-sections of beams, a calculation will be required according to the defining

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integration formulas:

$$I_{yy} = \int z^2 dA ; I_{zz} = \int y^2 dA ; I_{yz} = - \int y z dA$$

To simplify the calculation, the area of a beam could be dissected into geometrical simple structures (triangles, rectangles, circles, etc.). From the calculations of the component areas, the value for the whole area is calculated through summation, as follows (figure 1.):

e.g.: • for the area of section:

$$A = \Sigma A_i \quad i = (1, \dots, n)$$

• for the centroid of area:

$$y_s = \frac{\Sigma y_i A_i}{\Sigma A_i} ; z_s = \frac{\Sigma z_i A_i}{\Sigma A_i}$$

• for the second moments of area:

(relating to the initial axes)

$$I_{yy} = \Sigma I_{yy_i} + \Sigma z_i^2 A_i$$

$$I_{zz} = \Sigma I_{zz_i} + \Sigma y_i^2 A_i$$

$$I_{yz} = \Sigma I_{yz_i} - \Sigma y_i z_i A_i$$

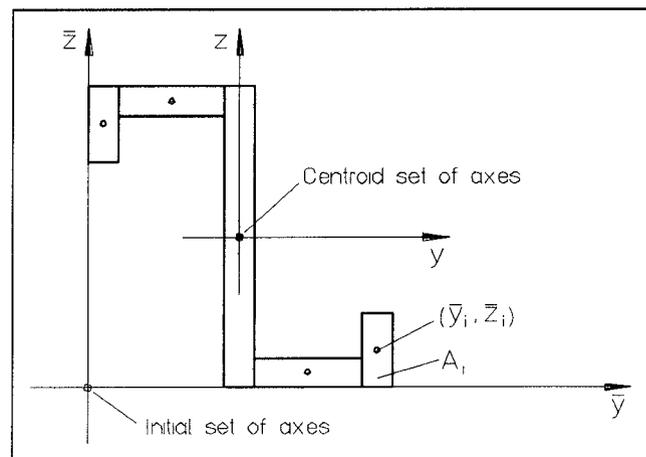


Figure 1. Dissection of cross-section of a beam in geometrical simple structures (rectangles).

In the case of cross-sections of beams with complex outer and inner shapes, this procedure leads to comprehensive calculations. These are processed by this program using the above mentioned formulas and Green's integral theorem in the plane. Moreover, this program offers a user-friendly input for the geometry, as well as a clear representation of the results.

### Input of the beam cross-section

There are 3 possibilities of data input:

- 1) the cross-section is constructed on the screen. The coordinates of the start- and endpoints of the connecting elements and the choice of the type of connection elements between the points are menu-controlled entered through the keyboard. The user can select the coordinate system (i.e. polar or cartesian) for each point entered, and thus can switch between these coordinate systems if needed/desired while entering data points. Permissible connecting element types are straight lines, arcs of circles, ellipses, parabolas, and hyperbolas.
- 2) the data for the cross-section is obtained from a HPGL data file created by a CAD-program.
- 3) the data for the cross-section is obtained from an ASCII-file created with a text editor.

### Calculation of the beam cross-section

The program recognizes outer and inner contours automatically. It calculates the outer contour and the inner contours separately and then subtracts the moment values of the inner contours from the moment values of the outer contour using the formulas mentioned above. To calculate the moment values for a single cross-sectional area the integral taken over the cross-section is transformed to a line integral on the contour by Green's integral theorem in the plane (figure 2.).

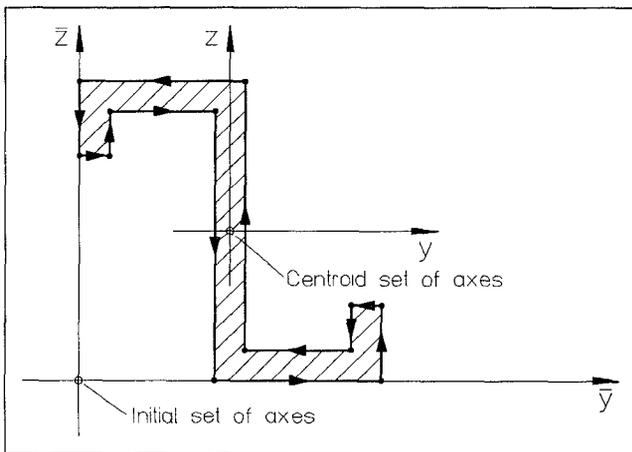


Figure 2. Integration over total cross-sectional area of a beam using Green's integral theorem.

E.g. the defining integration formulas of second moments of area using Green's integral theorem in the plane are:

$$I_{yy} = \int_A z^2 dA = -\oint_{\partial A} \frac{1}{3} z^3 dy = \oint_{\partial A} z^2 y dz$$

$$I_{zz} = \int_A y^2 dA = -\oint_{\partial A} y^2 z dy = \oint_{\partial A} \frac{1}{3} y^3 dz$$

$$I_{yz} = -\int_A y z dA = \oint_{\partial A} \frac{1}{2} z^2 y dy = -\oint_{\partial A} \frac{1}{2} z y^2 dz$$

If the cross-section contains curved boundaries, the program constructs components using arcs of circles, ellipses, parabolas, and/or hyperbolas to represent these curves. Finally the program determines the following geometrical characteristic values of the whole cross-sectional area:

- circumference of area:  $s$
- area of section:  $A$
- centroid of area:  $\bar{y}_s, \bar{z}_s$   
(relating to the initial axes)
- second moments of area:  $I_{yy}, I_{zz}, I_{yz}$   
(relating to the initial axes)
- second moments of area:  $I_{yy}, I_{zz}, I_{yz}$   
(relating to the centroid axes)
- second moments of area:  $I_{\eta}, I_{\zeta}$   
(relating to the principal axes)
- angle of rotation of principal axes:  $\alpha$   
(relating to the centroid axes)
- distance to the extreme outer fibre:  $\bar{y}_{max}, \bar{z}_{max}$   
(relating to the initial axes)
- distance to the extreme outer fibre:  $y_{max}, z_{max}$   
(relating to the centroid axes)
- distance to the extreme outer fibre:  $\eta_{max}, \zeta_{max}$   
(relating to the principal axes)
- section moduli:  $W_{\eta}, W_{\zeta}$   
(relating to the principal axes)
- displacements of cross-section:  $v(x), w(x)$   
(as a result of external forces)
- points of peak bending stresses:  $\sigma(x, y, z)$   
(as a result of external forces)
- shear center:  $y_M, z_M$   
(relating to the centroid axes)
- graph of shearing stresses:  $T(y, z)$   
(as a result of external forces)

## Output of results of the calculation

### 1) Numerical results of calculation:

The calculated geometrical characteristic values are shown in tabular form on the screen. This table of results (figure 3.) may also be printed.

Table of Results								
Initial set of axes			Centroid set of axes			Principal set of axes		
$I_{yy}$	.....	mm <sup>4</sup>	$I_{yy}$	.....	mm <sup>4</sup>	$I_{\eta}$	..	mm <sup>4</sup>
$I_{zz}$	.....	mm <sup>4</sup>	$I_{zz}$	.....	mm <sup>4</sup>	$I_{\zeta}$	..	mm <sup>4</sup>
$I_{yz}$	.....	mm <sup>4</sup>	$I_{yz}$	.....	mm <sup>4</sup>	$\alpha$	.....	°
$\bar{y}_{max}$	.....	mm	$y_{max}$	.....	mm	$\eta_{max}$	.....	mm
$\bar{z}_{max}$	.....	mm	$z_{max}$	.....	mm	$\zeta_{max}$	.....	mm
$\bar{y}_s$	.....	mm	$s$	.....	mm	$W_{\zeta}$	.....	mm <sup>3</sup>
$\bar{z}_s$	.....	mm	$A$	.....	mm <sup>2</sup>	$W_{\eta}$	.....	mm <sup>3</sup>

Figure 3. Tabular representation of the geometrical characteristic values.

### 2) Graphical results of calculation:

The graph is constructed in such a way that the profile is always drawn as large as possible. All sets of axes have a uniform graduation.

The centroid of area of the beam cross-section is represented by the point of intersection of the inscribed centroid axes. The calculated geometrical characteristic values are listed in the right margin of the screen and added to the graph by menu-controlled selections. The graph (figure 4.) may be printed.

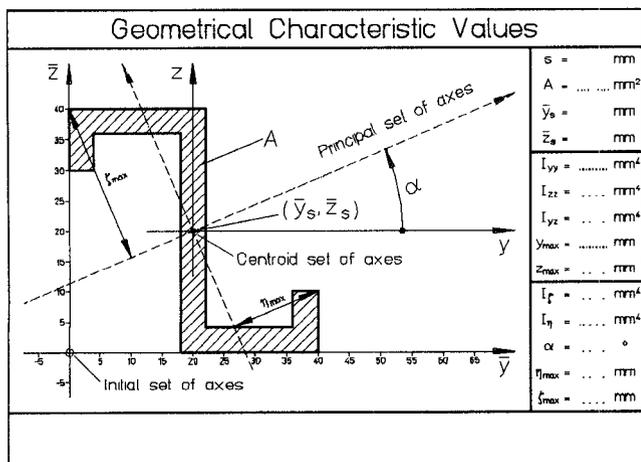


Figure 4. Graphical representation of the geometrical characteristic values.

### 3) Additional graphical results of calculation:

The displacements of the whole cross-sectional area and the graph of the shearing stresses across the beam cross-section are determined for an external force, acting in the y-z-plane.

The external force is defined by the input of magnitude, point of action and angle of action. The respective graphs (figures 5. and 6.) may be printed.

- a) The peak bending stresses of the beam cross-section are calculated only for the points that are a maximum distance from the neutral axis. The results of the angle of inclination of the neutral axis, the peak bending stresses, and the displacements are listed in the right margin of the screen and are added to the graph if required by the user.

For the entire graphical representation of the displacements and the displaced contours, the program chooses a factor such that the profiles are always drawn as large as possible (figure 5.).

The bending stresses of additional points on the contour of the beam are determined by entering their coordinates, and the results are shown in the right margin of the screen.

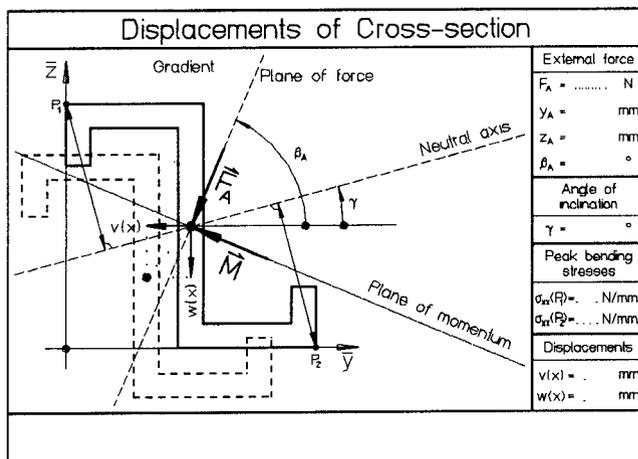


Figure 5. Graphical representation of the displacements of cross-section and the peak bending stresses.

- b) The shearing stresses are calculated for the whole cross-section of a beam. The results of the peak shearing stresses and the coordinates of the shear center are listed in the right margin of the screen and are added to the graph on user request.

For the graphical representation of the shearing stresses, the program chooses a factor such that the maximum value is drawn as large as possible in proportion to the geometry of the beam (figure 6.).

The shearing stresses of additional points of the contour of the beam are determined by entering their coordinates, and the results are shown in the right margin of the screen.

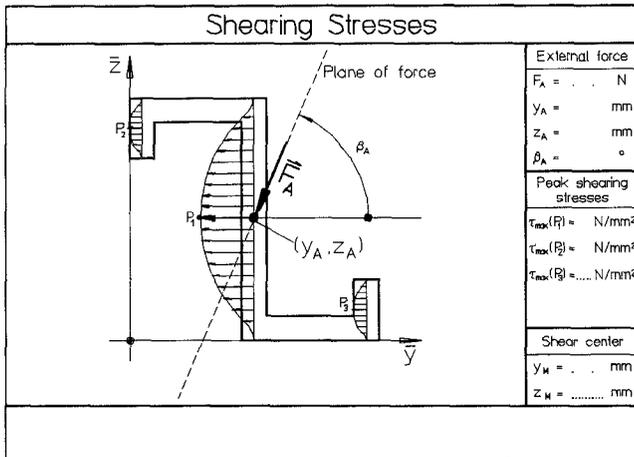


Figure 6. Graphical representation of shearing stresses, peak shearing stresses, and the shear center.

### Program flow chart

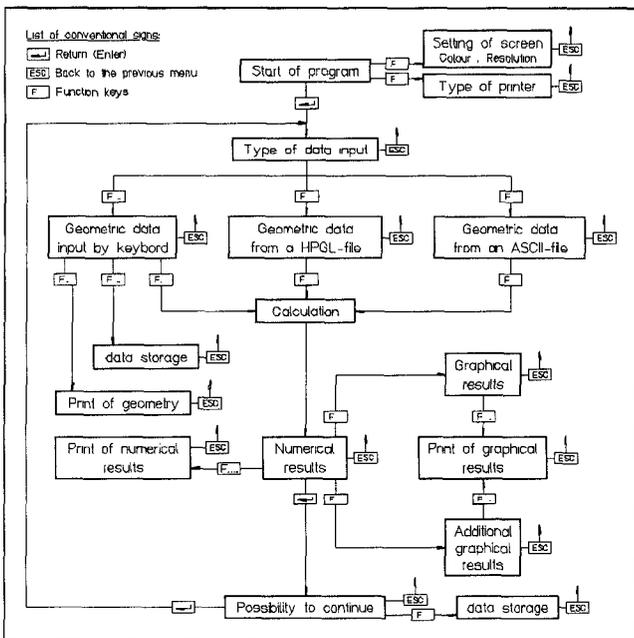


Figure 7. Flow chart of the program.

### Conclusion

The purpose of this program, which has been developed in a thesis, is as follows:

- user-friendly input of the cross-section of a beam by keyboard or directly from a data-file.
- simplified calculation of the geometrical characteristic values of complicated beam cross-sections.

- clear representation of the geometrical characteristic values and their influence on the strength of a structural member.

This program has been developed for IBM PC's and IBM-"compatible" computers with MS-DOS 3.3 (and higher levels) in APL2/PC.

The main reason to use APL for this software development was to use dynamic data structures to avoid restrictions in the number of holes in the cross-section and the number of single contours. APL also simplifies the implementation of matrix calculations.

The appendix contains two routines demonstrating the use of APL.

The program uses the AP207 for graphical display and the AP124 for screen management.

Further extensions and modifications to the program, as listed below, could be considered.

- Additional types of connection elements
- Creating fillets and chamfers by built-in functions within the program
- Input of geometric data from a DXF data file
- Output of graphical results to a DXF data file
- Output of graphical results to a HPGL data file
- Output of numerical results to an ASCII-file, which is readable by the program PAFEC-FE.

### References

- [1] Beitz, W. /Küttner, K.-H. [Hrsg.]: Taschenbuch für den Maschinenbau /Dubbel. 17. Auflage, Springer-Verlag (1990).
- [2] Nash, William A.: Theory and Problems of Strength of Materials 2. Edition, McGraw-Hill Book Co. (1987).
- [3] Schnell, W. /Gross, D. /Hauger, W.: Technische Mechanik, Band 2: Elastostatik. 2. Auflage, Springer-Verlag (1989).
- [4] Szabò, István : Einführung in die Technische Mechanik. 8. Auflage, Springer-Verlag (1984).
- [5] Spiegel, Murray R.: Vektoranalysis, Theorie und Anwendung. Nachdruck, McGraw-Hill Book Co.(1982).

## Appendix 1: Function *Momgfl*

```
mom←s Momgfl e;v;w;⊞FC
A V2.6 20.03.1994
A Calculation of moments of area of a composed area. If a single area
A is to be subtracted the complete input line of this area must be
A multiplied by -1.
A s S 0/1: display table of results ? [DEF.: 0]
A e M per line: input of a partial area (possibly multiplied by -1)
A [;1] area
A [;2 3] first moments of area relating to the initial axis set
A [;4 5] second moments of area relating to the centroid axis set
A [;6] product moment of area relating to the centroid axis set
A mom V moments of area of the whole area
A=
A local variables:
A v M coordinates of centroids of the partial areas relating to the
A initial set of axes
A w M 1. coordinates of centroids of the partial areas relating to
A the centroid set of axes
A 2. summands from parallel-axis theorem, second moments of area
A of the partial areas
⊞FC←'. -' A decimal point, minus sign
A
⊞(2≠⊞NC 's')/'s←0'
mom←+e[;13] A area, first moments of area
w←((ρv)ρmom[3 2]÷mom[1 1])-v+e[;3 2]÷e[;1 1]
mom←mom,(+3+[2]e)+1 1 -1×e[;1]+.×w[;2 1 1]×w[;2 1 2]
→(∼s)/0
' '
-73+ 'Ai xMi yMi Syi Sxi ui=xM-xMi vi=yM-yMi'
-73+ [2]10 3⊞e[;1],v,e[;3 2],w
-73+70ρ' ',8ρ'='
13 3 30 3 10 3⊞mom[1 3 2]
' '
-63+ 'vi2×Ai ui2×Ai -ui×vi×Ai Iuiui Ivivi Iuivi'
-63+ [2]10 3⊞w+(w[;2 1 1]×w[;2 1 2]×e[;1 1 1]×((1+ρe),3)ρ1 1 -1),3+[2]e
-63+60ρ' ',8ρ'='
-63+10 3⊞+fw
```

## Appendix 2: Function *Momhac*

```
r←s Momhac e;d;ms;⊞FC
A V2.6 20.03.1994
A Calculation of moments of area of a composed area. If a single area
A is to be subtracted the complete input line of this area must be
A multiplied by -1.
A s S 0/1: display moments on the screen ?
A e M per line: input of a partial area (possibly multiplied by -1)
A [;1] area
A [;2 3] first moments of area relating to the initial axis set
A [;4 5] second moments of areas relating to the centroid axis set
A [;6] product moment of area relating to the centroid axis set
A V input of total area
```

```

A r V [1] area
A [2 3] coordinate of centroid relating to the initial axis set
A [4,5] second moments of area relating to the principal axis set
A .5*(ms[1]+ms[2]) +- ((ms[1]-ms[2])*2 + ms[3]*2) *.5
A [6] angle from first centroid axis to major principal axis
A .5*arctan (2 ms[3] ÷ (ms[1]-ms[2]) )
A (+ PI/2, if Ixx<Iyy)
A=
A local variables:
A d S .5*(ms[1]-ms[2])
A ms V second moments of area relating to the centroid axis set
AFC+'. - ' A decimal point, minus sign
A
A called routines: Momgfl Momdis
A=====
A(0=NC 's')/'s+0'
A s/'Momdis e'
A →(1=ppr+e)/ENDIF1 A input for total area ?
A r+s Momgfl e
A s/'Momdis r'
A ENDIF1:
A
A d←0.5×- /2†ms←-3†r A second moments of area relating
A to the centroid axis set
A r←(†r),(r[3 2]÷†r),(0.5×+ /2†ms)+(1-1)×(+/(d,ms[3])*2)*0.5
A A r: area, centroid, princ. moments
A axial moments equal ?
A →(0=d)/ELSE A minimum or maximum ?
A r←r,0.5×(3oms[3]÷d)+o0>d ◇ →ENDIF A axial moments equal: angle +- pi/4
A ELSE:r←r,o(×3ms)÷4
A ENDIF:
A
A s/' princ. moments and angle of princ. axes ',10 3*(1,1,180÷o1)×-3†r

```