# Alternating Automata on Data Trees and XPath Satisfiability 

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#### Abstract

A data tree is an unranked ordered tree whose every node is labelled by a letter from a finite alphabet and an element ("datum") from an infinite set, where the latter can only be compared for equality. The article considers alternating automata on data trees that can move downward and rightward, and have one register for storing data. The main results are that nonemptiness over finite data trees is decidable but not primitive recursive, and that nonemptiness of safety automata is decidable but not elementary. The proofs use nondeterministic tree automata with faulty counters. Allowing upward moves, leftward moves, or two registers, each causes undecidability. As corollaries, decidability is obtained for two data-sensitive fragments of the XPath query language. Categories and Subject Descriptors: F.4.1 [Mathematical Logic and Formal Languages]: Formal Languages-Decision problems; F.1.1 [Computation by Abstract Devices]: Models of Computation-Automata; H.2.3 [Database Management]: Languages-Query languages


General Terms: Algorithms, Verification

## 1. INTRODUCTION

Context. Logics and automata for words and trees over finite alphabets are relatively well-understood. Motivated partly by the search for automated reasoning techniques for XML and the need for formal verification and synthesis of infinitestate systems, there is an active and broad research programme on logics and automata for words and trees which have richer structure.

Initial progress made on reasoning about data words and data trees is summarised in the survey by Segoufin [2006]. A data word is a word over $\Sigma \times \mathcal{D}$, where $\Sigma$ is a finite alphabet, and $\mathcal{D}$ is an infinite set ("domain") whose elements ("data") can only be compared for equality. Similarly, a data tree is a tree (countable, unranked and ordered) whose every node is labelled by a pair in $\Sigma \times \mathcal{D}$.

First-order logic for data words was considered by Bojańczyk et al. [2006], and related automata were studied further by Björklund and Schwentick [2007]. The logic has variables which range over word positions $(\{0, \ldots, l-1\}$ or $\mathbb{N})$, a unary predicate for each letter from the finite alphabet, and a binary predicate $x \sim y$ which denotes equality of data labels. $\mathrm{FO}^{2}(+1,<, \sim)$ denotes such a logic with two variables and binary predicates $x+1=y$ and $x<y$. Over finite and over infinite

[^0]data words, satisfiability for $\mathrm{FO}^{2}(+1,<, \sim)$ was shown decidable and at least as hard as nonemptiness of vector addition automata. Whether the latter problem is elementary has been open for many years. Extending the logic by one more variable causes undecidability.

Over data trees, $\mathrm{FO}^{2}(+1,<, \sim)$ denotes a similar first-order logic with two variables. The variables range over tree nodes, +1 stands for two predicates "child" and "next sibling", and < stands for two predicates "descendant" and "younger sibling". Complexity of satisfiability over finite data trees was studied by Bojańczyk et al. [2009]. For $\mathrm{FO}^{2}(+1, \sim)$, it was shown to be in 3NExpTime, but for $\mathrm{FO}^{2}(+1,<, \sim)$, to be at least as hard as nonemptiness of vector addition tree automata. Decidability of the latter is an open question, and it is equivalent to decidability of multiplicative exponential linear logic [deGroote et al. 2004]. However, Björklund and Bojańczyk [2007] showed that $\mathrm{FO}^{2}(+1,<, \sim)$ over finite data trees of bounded depth is decidable.

XPath [Clark and DeRose 1999] is a prominent query language for XML documents [Bray et al. 1998]. The most basic static analysis problem for XPath, with a variety of applications, is satisfiability in the presence of DTDs. In the two extensive articles on its complexity [Benedikt et al. 2008; Geerts and Fan 2005], the only decidability result that allows negation and data (i.e., equality comparisons between attribute values) does not allow axes which are recursive (such as "self or descendant") or between siblings. By representing XML documents as data trees and translating from XPath to $\mathrm{FO}^{2}(+1, \sim)$, Bojańczyk et al. [2009] obtained a decidable fragment with negation, data and all nonrecursive axes. Another fragment of XPath was considered by Hallé et al. [2006], but it lacks concatenation, recursive axes and sibling axes. A recent advance of Figueira [2009] shows Exp-Time-completeness for full downward XPath, but with restricted DTDs.

An alternative approach to reasoning about data words is based on automata with registers [Kaminski and Francez 1994]. A register is used for storing a datum for later equality comparisons. Nonemptiness of one-way nondeterministic register automata over finite data words has relatively low complexity: NP-complete [Sakamoto and Ikeda 2000] or PSPACE-complete [Demri and Lazić 2009], depending on technical details of their definition. Unfortunately, such automata fail to provide a satisfactory notion of regular language of finite data words, as they are not closed under complement [Kaminski and Francez 1994] and their nonuniversality is undecidable [Neven et al. 2004]. To overcome those limitations, one-way alternating automata with 1 register were proposed by Demri and Lazić [2009]: they are closed under Boolean operations, their nonemptiness over finite data words is decidable, and future-time fragments of temporal logics such as LTL or the modal $\mu$-calculus extended by 1 register are easily translatable to such automata. However, the nonemptiness problem over finite data words turned out to be not primitive recursive. Moreover, already with weak acceptance [Muller et al. 1986] and thus also with Büchi or co-Büchi acceptance, nonemptiness over infinite data words is undecidable (more precisely, co-r.e.-hard). When the automata are restricted to those which recognise safety properties [Alpern and Schneider 1987] over infinite data words, nonemptiness was shown to be ExpSpace-complete, and inclusion to be decidable but not primitive recursive [Lazić 2006].

Contribution. This article addresses one of the research directions proposed by Segoufin [2006]: investigating modal logics and automata with registers on data trees. Nondeterministic automata with registers which can be nondeterministically reassigned on finite binary data trees were recently studied by Kaminski and Tan [2008]: top-down and bottom-up variants recognise the same languages, and nonemptiness is decidable. However, they inherit the drawbacks of one-way nondeterministic register automata on data words: lack of closure under complement and undecidability of nonuniversality.

We consider alternating automata that have 1 register and are forward, i.e., can move downward and rightward over tree nodes: for short, ATRA $A_{1}$. They are closed under Boolean operations, and we show that their nonemptiness over finite data trees is decidable. Moreover, forward fragments of CTL and the modal $\mu$-calculus with 1 register are easily translatable to ATRA $A_{1}$ [Jurdziński and Lazić 2007]. The expressiveness of $\operatorname{ATRA}_{1}$ is incomparable to those of $\mathrm{FO}^{2}(+1, \sim)$ and the automata of Kaminski and Tan [2008]: for example, the latter two formalisms but not ATRA $A_{1}$ can check whether some two leaves have equal data, and the opposite is true of checking whether each node's datum is fresh, i.e., does not appear at any ancestor node. By lower-bound results for register automata on data words in [Neven et al. 2004; David 2004; Demri and Lazić 2009], we have that ATRA $A_{1}$ nonemptiness is not primitive recursive, and that it becomes undecidable (more precisely, r.e.-hard) if any of the following is added: upward moves, leftward moves, or one more register.

Motivated partly by applications to XML streams (cf., e.g., [Olteanu et al. 2004]), we consider both finite and countably infinite data trees, where horizontal as well as vertical infinity is allowed. For ATRA $_{1}$ with the weak acceptance mechanism, the undecidability result over infinite data words [Demri and Lazić 2009] carries over. However, we show that, for safety ATRA $_{1}$, which are closed under intersection and union but not complement, inclusion is decidable and not primitive recursive. When a data tree is rejected by an automaton with the safety acceptance mechanism, there exists an initial segment whose every extension is rejected. We also obtain that nonemptiness of safety ATRA $_{1}$ is not elementary. The latter is the most surprising result in the article: it means that the techniques in the proof that nonemptiness over infinite data words of safety one-way alternating automata with 1 register is in ExpSpace cannot be lifted to trees to obtain a 2ExpTime upper bound.

The proofs of decidability involve translating from ATRA $_{1}$ to forward nondeterministic tree automata with faulty counters. The counters are faulty in the sense that they are subject to incrementing errors, i.e., can spontaneously increase at any time. That makes the transition relations downwards compatible with a well-quasiordering (cf. [Finkel and Schnoebelen 2001]), which leads to lower complexities of some verification problems than with error-free counters.

We define forward XPath to be the largest downward and rightward fragment in which, whenever two attribute values are compared for equality, one of them must be at the current node. By translating from forward XPath to ATRA $_{1}$, we obtain decidability of satisfiability over finite documents and decidability of satisfiability for a safety subfragment, both in the presence of DTDs. In contrast to the decidable fragments of XPath mentioned previously, forward XPath has sibling axes, recursive axes, concatenation, negation, and data comparisons.

## 2. PRELIMINARIES

After fixing notations for trees and data trees, we define two kinds of forward automata and look at some of their basic properties: alternating automata with 1 register on data trees, and nondeterministic automata with counters with incrementing errors on trees.

### 2.1 Trees and Data Trees

For technical simplicity, we shall work with binary trees instead of unranked ordered trees. Firstly, as e.g. Björklund and Bojańczyk [2007], we adopt the insignificant generalisation of considering unranked ordered forests, in which the roots are regarded as siblings with no parent. Secondly, the following is a standard and trivial one-to-one correspondence between unranked ordered forests $t$ and binary trees $\mathrm{bt}(t)$ : the nodes of $\mathrm{bt}(t)$ are the same as the nodes of $t$, and the children of each node $n$ in $\operatorname{bt}(t)$ are the first child and next sibling of $n$ in $t$. The correspondence works for finite as well as infinite unranked ordered forests. In the latter, there may be infinite (of type $\omega$ ) branches or siblinghoods or both.

Without loss of generality, each node will either have both children or be a leaf, only nonleaf nodes will be labelled, and the root node will be nonleaf. Formally, a tree is a tuple $\langle N, \Sigma, \Lambda\rangle$, where:

- $N$ is a prefix-closed subset of $\{0,1\}^{*}$ such that $|N|>1$ and, for each $n \in N$, either $n \cdot 0 \in N$ and $n \cdot 1 \in N$, or $n \cdot 0 \notin N$ and $n \cdot 1 \notin N$;
$-\Sigma$ is a finite alphabet;
$-\Lambda$ is a mapping from the nonleaf elements of $N$ to $\Sigma$.
A data tree is a tree as above together with a mapping $\Delta$ from the nonleaf nodes to a fixed infinite set $\mathcal{D}$. For a data tree $\tau$, let tree $(\tau)$ denote the underlying tree.

For a data tree $\tau$ and $l>0$, let the $l$-prefix of $\tau$ be the data tree obtained by restricting $\tau$ to nodes of length at most $l$. For each $\Sigma$, the set of all data trees with alphabet $\Sigma$ is a complete metric space with the following notion of distance: for distinct $\tau$ and $\tau^{\prime}$, let $d\left(\tau, \tau^{\prime}\right)=1 / l$ where $l$ is least such that $\tau$ and $\tau^{\prime}$ have distinct $l$-prefixes.

### 2.2 Alternating Tree Register Automata

Automata. A run of a forward alternating automaton with 1 register on a data tree will consist of a configuration for each tree node. Each configuration will be a finite set of threads, which are pairs of an automaton state and a register value, where the latter is a datum from $\mathcal{D}$.

Following Brzozowski and Leiss [1980], transitions will be specified by positive Boolean formulae. For a set of states $Q$, let $\mathcal{B}^{+}(Q)$ consist of all formulae given by the following grammar, where $q \in Q$ :

$$
\varphi::=q(0, \downarrow)|q(0, \npreceq)| q(1, \downarrow)|q(1, \not \subset)| \top|\perp| \varphi \wedge \varphi \mid \varphi \vee \varphi
$$

Given a configuration $G$ at a nonleaf tree node $n$, for each thread $\langle q, D\rangle$ in $G$, the automaton transition function provides a formula $\varphi$ in $\mathcal{B}^{+}(Q)$, which depends on $q$, on the letter labelling $n$, and on whether $D=E$, where $E$ is the datum labelling $n$. In $\varphi$, an atom $r(d, \downarrow)$ requires that thread $\langle r, E\rangle$ be in the configuration for node ACM Transactions on Computational Logic, Vol. V, No. N, Month $20 Y Y$.
$n \cdot d$ (i.e., the register value is replaced by the datum at $n$ ), and an atom $r(d, \not \subset)$ requires the same for thread $\langle r, D\rangle$ (i.e., the register value is not replaced).

Formally, a forward alternating tree 1 -register automaton (shortly, $A T R A_{1}$ ) $\mathcal{A}$ is a tuple $\left\langle\Sigma, Q, q_{I}, F, \delta\right\rangle$ such that:
$-\Sigma$ is a finite alphabet and $Q$ is a finite set of states;
$-q_{I} \in Q$ is the initial state and $F \subseteq Q$ are the final states;
$-\delta: Q \times \Sigma \times\{t t, f f\} \rightarrow \mathcal{B}^{+}(Q)$ is a transition function.
Runs and Languages. The semantics of the positive Boolean formulae can be given by defining when a quadruple $R_{0}^{\downarrow}, R_{0}^{\downarrow}, R_{1}^{\downarrow}, R_{1}^{\nless}$ of subsets of $Q$ satisfies a formula $\varphi$ in $\mathcal{B}^{+}(Q)$, by structural recursion. The cases for the Boolean atoms and operators are standard, and for the remaining atoms we have:

$$
R_{0}^{\downarrow}, R_{0}^{\npreceq}, R_{1}^{\downarrow}, R_{1}^{\npreceq} \models r(d, ?) \stackrel{\text { def }}{\Leftrightarrow} r \in R_{d}^{?}
$$

We can now define the transition relation of $\mathcal{A}$, which is between configurations and pairs of configurations, and relative to a letter and a datum. We write $G \rightarrow_{a}^{E}$ $H_{0}, H_{1}$ iff, for each thread $\langle q, D\rangle \in G$, there exist $R_{0}^{\downarrow}, R_{0}^{\not 又}, R_{1}^{\downarrow}, R_{1}^{\not 又} \models \delta(q, a, D=E)$ such that, for both $d \in\{0,1\}$ :

$$
\left\{\langle r, E\rangle: r \in R_{d}^{\downarrow}\right\} \cup\left\{\langle r, D\rangle: r \in R_{d}^{\nless}\right\} \subseteq H_{d}
$$

A run of $\mathcal{A}$ on a data tree $\langle N, \Sigma, \Lambda, \Delta\rangle$ is a mapping $n \mapsto G_{n}$ from the nodes to configurations such that:
-the initial thread is in the configuration at the root, i.e. $\left\langle q_{I}, \Delta(\varepsilon)\right\rangle \in G_{\varepsilon}$;
-for each nonleaf $n$, the transition relation is observed, i.e. $G_{n} \rightarrow_{\Lambda(n)}^{\Delta(n)} G_{n \cdot 0}, G_{n \cdot 1}$.
We say that the run is:
-final iff, for each leaf $n$, only final states occur in $G_{n}$;
-finite iff there exists $l$ such that, for each $n$ of length at least $l, G_{n}$ is empty.
We may regard $\mathcal{A}$ as an automaton on finite data trees, a safety automaton, or a co-safety automaton. We say that:

- $\mathcal{A}$ accepts a finite data tree $\tau$ iff $\mathcal{A}$ has a final run on $\tau$;
- $\mathcal{A}$ safety-accepts a data tree $\tau$ iff $\mathcal{A}$ has a final run on $\tau$;
- $\mathcal{A}$ co-safety-accepts a data tree $\tau$ iff $\mathcal{A}$ has a final finite run on $\tau$.

Observe that, for finite data trees, the three modes of $\mathcal{A}$ coincide.
Let $\mathrm{L}^{\text {fin }}(\mathcal{A})$ denote the set of all finite data trees with alphabet $\Sigma$ that $\mathcal{A}$ accepts, and $\mathrm{L}^{s a f}(\mathcal{A})$ (resp., $\left.\mathrm{L}^{\cos }(\mathcal{A})\right)$ denote the set of all data trees with alphabet $\Sigma$ that $\mathcal{A}$ safety-accepts (resp., co-safety-accepts).

Remark 2.1. The valid initial and successor configurations in runs were defined in terms of lower bounds on sets. In other words, while running on any data tree, at each node the automaton is free to introduce arbitrary "junk" threads. However, final and finite runs were defined in terms of upper bounds on sets, so junk threads can only make it harder to complete a partial run into an accepting one. This will play an important role in the proof of decidability in Theorem 3.1.

Boolean Operations. Given an $\operatorname{ATRA}_{1} \mathcal{A}$, let $\overline{\mathcal{A}}$ denote its dual: the automaton obtained by replacing the set of final states with its complement and replacing, in each transition formula $\delta(q, a, p)$, every $\top$ with $\perp$, every $\wedge$ with $\vee$, and vice versa. Observe that $\overline{\overline{\mathcal{A}}}=\mathcal{A}$. Considering $\mathcal{A}$ (resp., $\overline{\mathcal{A}}$ ) as a weak alternating automaton whose every state is of even (resp., odd) parity, we have by [Löding and Thomas 2000, Theorem 1] that $L^{\cos }(\overline{\mathcal{A}})$ is the complement of $\mathrm{L}^{\text {saf }}(\mathcal{A})$. Hence, we also have that $\mathrm{L}^{\text {saf }}(\overline{\mathcal{A}})$ is the complement of $\mathrm{L}^{\cos }(\mathcal{A})$, and that $\mathrm{L}^{\text {fin }}(\overline{\mathcal{A}})$ is the complement of $\mathrm{L}^{\text {fin }}(\mathcal{A})$.

For each m of $f i n$, saf, cos, given $\operatorname{ATRA}_{1} \mathcal{A}_{1}$ and $\mathcal{A}_{2}$ with alphabet $\Sigma$, an automaton whose language in mode m is $\mathrm{L}^{\mathrm{m}}\left(\mathcal{A}_{1}\right) \cap \mathrm{L}^{\mathrm{m}}\left(\mathcal{A}_{2}\right)$ (resp., $\mathrm{L}^{\mathrm{m}}\left(\mathcal{A}_{1}\right) \cup \mathrm{L}^{\mathrm{m}}\left(\mathcal{A}_{2}\right)$ ) is constructible easily. It suffices to form a disjoint union of $\mathcal{A}_{1}$ and $\mathcal{A}_{2}$, and add a new initial state $q_{I}$ such that $\delta\left(q_{I}, a, t t\right)=\delta\left(q_{I}^{1}, a, t t\right) \wedge \delta\left(q_{I}^{2}, a, t t\right)$ (resp., $\left.\delta\left(q_{I}, a, t t\right)=\delta\left(q_{I}^{1}, a, t t\right) \vee \delta\left(q_{I}^{2}, a, t t\right)\right)$ for each $a \in \Sigma$, where $q_{I}^{1}$ and $q_{I}^{2}$ are the initial states of $\mathcal{A}_{1}$ and $\mathcal{A}_{2}$. (Since the initial thread's register value is always the root node's datum, the formulae $\delta\left(q_{I}, a, f f\right)$ are irrelevant.)

We therefore obtain:
Proposition 2.2. (a) $A T R A_{1}$ on finite data trees are closed under complement, intersection and union.
(b) Safety $A T R A_{1}$ and co-safety $A T R A_{1}$ are dual, and each is closed under intersection and union.
In each case, a required automaton is computable in logarithmic space.
Safety Languages. A set $L$ of data trees with alphabet $\Sigma$ is called safety [Alpern and Schneider 1987] iff it is closed with respect to the metric defined in Section 2.1, i.e. for each data tree $\tau$, if for all $l>0$ there exists $\tau_{l}^{\prime} \in L$ such that the $l$-prefixes of $\tau$ and $\tau_{l}^{\prime}$ are equal, then $\tau \in L$. The complements of safety languages, i.e. the open sets of data trees, are called co-safety.

Proposition 2.3. For each $\operatorname{ATR}_{1} \mathcal{A}$, we have that $\mathrm{L}^{\text {saf }}(\mathcal{A})$ is safety and $\mathrm{L}^{\cos }(\mathcal{A})$ is co-safety.

Proof. By Proposition 2.2(b), it suffices to show that $\mathrm{L}^{s a f}(\mathcal{A})$ is safety. Suppose for all $l>0$ there exists $\tau_{l}^{\prime} \in \mathrm{L}^{\text {saf }}(\mathcal{A})$ such that the $l$-prefixes of $\tau$ and $\tau_{l}^{\prime}$ are equal.

For each $l>0$, let us fix a final run $n \mapsto G_{l, n}^{\prime}$ of $\mathcal{A}$ on $\tau_{l}^{\prime}$. For each $0 \leq k \leq l$, let $\mathcal{G}_{l, k}$ denote the restriction of the run $n \mapsto G_{l, n}^{\prime}$ to nodes $n$ of length $k$.

Consider the tree consisting of the empty sequence and all sequences $\mathcal{G}_{l, 0} \cdot \mathcal{G}_{l, 1}$. $\cdots \mathcal{G}_{l, k}$ for $l>0$ and $0 \leq k \leq l$. Without loss of generality, each register value in each $G_{l, n}^{\prime}$ labels some node of $\tau_{l}^{\prime}$ on the path from the root to $n$, so the tree is finitely branching. By König's Lemma, it has an infinite path $\mathcal{H}_{0} \cdot \mathcal{H}_{1} \cdots$. For each $0 \leq k, \mathcal{H}_{k}$ is a mapping from the nodes of $\tau$ of length $k$ to configurations of $\mathcal{A}$. It remains to observe that $n \mapsto \mathcal{H}_{|n|}(n)$ is a final run of $\mathcal{A}$ on $\tau$.

Example 2.4. By recursion on $k \geq 1$, we shall define ATRA $_{1} \mathcal{B}_{k}$ with alphabet $\left\{b_{1}, \ldots, b_{k}, *\right\}$. As well as being interesting examples of ATRA $_{1}$, the $\mathcal{B}_{k}$ will be used in the nonelementarity part of the proof of Theorem 4.1.

Let $\mathcal{B}_{1}$ be the automaton depicted in Figure 1. It has three states, where $q$ is initial, and $q^{\prime \prime}$ is final. We have $\delta\left(q, b_{1}, p\right)=q^{\prime}(0, \npreceq \sim) \wedge q^{\prime \prime}(1, \npreceq)$ and $\delta\left(q^{\prime}, b_{1}, p\right)=$ $q^{\prime \prime}(0, \not \not \subset) \wedge q^{\prime \prime}(1, \not \not \subset)$ for both $p \in\{t t, f f\}$, and the transition function gives $\perp$ in all


Fig. 1. Defining $\mathcal{B}_{1}$
other cases. (Recalling that the initial thread's register value is the root node's datum, the formula $\delta\left(q, b_{1}, f f\right)$ is in fact irrelevant.) Observe that $\mathcal{B}_{1}$ safety-accepts exactly data trees that have two nonleaf nodes, the root and its left-hand child, and both are labelled by letter $b_{1}$.

For each $k \geq 1, \mathcal{B}_{k+1}$ is defined so that it safety-accepts a data tree over $\left\{b_{1}, \ldots, b_{k+1}, *\right\}$ iff:
(i) the root node is labelled by $b_{k+1}$, its left-hand child is labelled by $b_{k+1}$, and its right-hand child is a leaf;
(ii) for each node $n$ labelled by $b_{k+1}$, which is not the root, the left-hand child of $n$ is labelled by $*$ and its both children are labelled by $b_{k+1}$, and the right-hand subtree at $n$ is safety-accepted by $\mathcal{B}_{k}$;
(iii) whenever a node $n$, which is not the root, and a descendant $n^{\prime}$ of $n$ are labelled by $b_{k+1}$, we have that their data labels are distinct, and that the datum at $n$ equals the datum at some node which is labelled by $b_{k}$ and which is in the righthand subtree at $n^{\prime}$.
By Proposition 2.2(b), it suffices to define automata for (i)-(iii) separately. Expressing (i) and (ii) is straightforward, and an automaton for (iii) is depicted in Figure 2. It has four states, where $q_{0}$ is initial, and $q_{1}$ and $q_{2}$ are final. For all letters $a$ and Booleans $p$, we have $\delta\left(q_{0}, a, p\right)=q_{1}(0, \not \chi)$, so initially the automaton moves to the left-hand child of the root and changes the state to $q_{1}$. From $q_{1}$, if the current node is labelled by $*$, the automaton moves to both children: $\delta\left(q_{1}, *, p\right)=q_{1}(0, \not \subset) \wedge q_{1}(1, \not \subset)$ for both $p$. Also from $q_{1}$, if the current node $n$ is labelled by $b_{k+1}$, the automaton both moves to the left-hand child without changing the state, and moves to the left-hand child with storing the datum at $n$ in the register and changing the state to $q_{2}: \delta\left(q_{1}, b_{k+1}, p\right)=q_{1}(0, \npreceq) \wedge q_{2}(0, \downarrow)$ for both $p$. From $q_{2}$, the behaviour for $*$ is analogous to that from $q_{1}$, but if the current node's letter is $b_{k+1}$ and its datum is distinct from the datum in the register, the automaton both moves to the left-hand child without changing the state and moves to the right-hand child with changing the state to $q_{3}: \delta\left(q_{2}, b_{k+1}, f f\right)=q_{2}(0, \not \not \subset) \wedge q_{3}(1, \not 又 \not)$. The remainder of Figure 2 is interpreted similarly, and in cases not depicted, the transition function gives $\perp$. Since the mode of acceptance is safety, the automaton in fact expresses:
(iii') whenever a node $n$, which is not the root, and a descendant $n^{\prime}$ of $n$ are labelled by $b_{k+1}$, we have that their data labels are distinct, and that either the datum at $n$ equals the datum at some node which is labelled by $b_{k}$ and which is in the right-hand subtree at $n^{\prime}$, or that subtree is infinite.
Let $2 \Uparrow 0=1$, and $2 \Uparrow k=2^{2 \Uparrow(k-1)}$ for $k \geq 1$. By induction on $k \geq 1$, the safety language of $\mathcal{B}_{k}$ has the following two properties. In particular, in the presence of (i) and (ii), we have that (iii) and (iii') are equivalent.


Fig. 2. Defining $\mathcal{B}_{k+1}$
-for every $\tau$ safety-accepted by $\mathcal{B}_{k}$, every downward sequence which is from the left-hand child of the root and which consists of nodes labelled by $b_{k}$ is of length at most $2 \Uparrow(k-1)$, so $\tau$ is finite and has at most $2 \Uparrow k$ nodes labelled by $b_{k}$;
-for some $\tau$ safety-accepted by $\mathcal{B}_{k}$, the nodes labelled by $b_{k}$ other than the root form a full binary tree of height $2 \Uparrow(k-1)$ (after removing the nodes labelled by $*$ ), so there are $2 \Uparrow k$ nodes labelled by $b_{k}$, and moreover the data at those nodes are mutually distinct.
Finally, we observe that for computing $\mathcal{B}_{k}$, space logarithmic in $k$ suffices.

### 2.3 Faulty Tree Counter Automata

In Section 3, we shall establish decidability of nonemptiness of forward alternating tree 1-register automata over finite data trees, by translating them to automata which have natural-valued counters with increments, decrements and zero-tests. The translation will eliminate conjunctive branchings, by having configurations of the former automata (which are finite sets of threads) correspond to pairs of states and counter valuations, so the latter automata will be only nondeterministic. Also, data will be abstracted in the translation, so the counter automata will run on finite trees (without data).

The feature that will make nonemptiness of the counter automata decidable (on finite trees) is that they will be faulty, in the sense that one or more counters can erroneously increase at any time. The key insight is that such faults do not affect the translation's preservation of nonemptiness: they in fact correspond to introductions of "junk" threads in runs of ATRA $_{1}$ (cf. Remark 2.1).

For clarity of the correspondence between the finitary languages of $\operatorname{ATRA}_{1}$ and the languages of their translations, the counter automata will have $\varepsilon$-transitions.

We now define the counter automata, and show their nonemptiness decidable.
Automata. An incrementing tree counter automaton (shortly, ITCA) $\mathcal{C}$, which is forward and with $\varepsilon$-transitions, is a tuple $\left\langle\Sigma, Q, q_{I}, F, k, \delta\right\rangle$ such that:
$-\Sigma$ is a finite alphabet and $Q$ is a finite set of states;
$-q_{I} \in Q$ is the initial state and $F \subseteq Q$ are the final states;
$-k \in \mathbb{N}$ is the number of counters;
$-\delta \subseteq(Q \times \Sigma \times L \times Q \times Q) \cup(Q \times\{\varepsilon\} \times L \times Q)$ is a transition relation, where $L=\{$ inc, $\operatorname{dec}$, ifz $\} \times\{1, \ldots, k\}$ is the instruction set.

Runs and Languages. A counter valuation is a mapping from $\{1, \ldots, k\}$ to $\mathbb{N}$. For counter valuations $v$ and $v^{\prime}$, we write:

$$
\begin{array}{rll}
v \leq v^{\prime} & \text { iff } & v(c) \leq v^{\prime}(c) \text { for all } c \\
v \xrightarrow{\langle\text { inc }, c\rangle} \sqrt{ } v^{\prime} & \text { iff } & v^{\prime}=v[c \mapsto v(c)+1] \\
v \xrightarrow{\langle\text { dec }, c\rangle} \sqrt{ } v^{\prime} & \text { iff } & v^{\prime}=v[c \mapsto v(c)-1] \\
v \xrightarrow{\langle\text { ifz }, c\rangle} \\
\sqrt{ } v^{\prime} & \text { iff } & v(c)=0 \text { and } v^{\prime}=v \\
v \xrightarrow{l} v^{\prime} & \text { iff } & v \leq v_{\sqrt{ }} \xrightarrow{l} \sqrt{ } v_{\sqrt{ }}^{\prime} \leq v^{\prime} \text { for some } v_{\sqrt{ }}, v_{\sqrt{ }}^{\prime}
\end{array}
$$

A configuration of $\mathcal{C}$ is a pair $\langle q, v\rangle$, where $q$ is a state and $v$ is a counter valuation. To define runs, we first specify that a block is a nonempty finite sequence of configurations obtainable by performing $\varepsilon$-transitions, i.e. for every two adjacent configurations $\left\langle q_{i}, v_{i}\right\rangle$ and $\left\langle q_{i+1}, v_{i+1}\right\rangle$ in a block, there exists $l$ with $\left\langle q_{i}, \varepsilon, l, q_{i+1}\right\rangle \in \delta$ and $v_{i} \xrightarrow{l} v_{i+1}$.

Now, a run of $\mathcal{C}$ on a finite tree $\langle N, \Sigma, \Lambda\rangle$ is a mapping $n \mapsto B_{n}$ from the nodes to blocks such that:
$-\left\langle q_{I}, \mathbf{0}\right\rangle$ is the first configuration in $B_{\varepsilon} ;$
-for each nonleaf $n$, there exists $l$ with $\left\langle q, \Lambda(n), l, r_{0}, r_{1}\right\rangle \in \delta, v \xrightarrow{l} w_{0}$ and $v \xrightarrow{l} w_{1}$, where $\langle q, v\rangle$ is the last configuration in $B_{n}$, and $\left\langle r_{0}, w_{0}\right\rangle$ and $\left\langle r_{1}, w_{1}\right\rangle$ are the first configurations in $B_{n \cdot 0}$ and $B_{n \cdot 1}$ (respectively).

We regard such a run accepting iff, for each leaf $n$, the state of the last configuration in $B_{n}$ is final. The language $\mathrm{L}(\mathcal{C})$ is the set of all finite trees with alphabet $\Sigma$ on which $\mathcal{C}$ has an accepting run.

Decidability of Nonemptiness. We remark that, since nonemptiness of incrementing counter automata over words is not primitive recursive [Demri and Lazić 2009, Theorem 2.9(b)], the same is true of nonemptiness of ITCA.

Theorem 2.5. Nonemptiness of ITCA is decidable.
Proof. Consider an ITCA $\mathcal{C}=\left\langle\Sigma, Q, q_{I}, F, k, \delta\right\rangle$.
For counter valuations $v$ and $v^{\prime}$, and an instruction $l$, we say that $v$ under $l$ yields $v^{\prime}$ lazily and write $v \xrightarrow{l}_{b} v^{\prime}$ iff either $v \xrightarrow{l} \sqrt{ } v^{\prime}$ (i.e., there are no incrementing errors), or $l$ is of the form $\langle\mathrm{dec}, c\rangle, v(c)=0$ and $v^{\prime}=v$ (i.e., 0 is erroneously decremented to 0 ). Observe that:
$(*)$ Whenever $v \leq w$ and $w \xrightarrow{l} w^{\prime}$, there exists $v^{\prime}$ such that $v \xrightarrow{l}_{b} v^{\prime}$ and $v^{\prime} \leq w^{\prime}$.
To reduce the nonemptiness problem for $\mathcal{C}$ to a reachability problem, let a level of $\mathcal{C}$ be a finite set of configurations. For levels $\mathcal{G}$ and $\mathcal{G}^{\prime}$ of $\mathcal{C}$, let us write $\mathcal{G} \rightarrow \mathcal{G}^{\prime}$ iff $\mathcal{G}^{\prime}$ can be obtained from $\mathcal{G}$ as follows:

- each $\langle q, v\rangle \in \mathcal{G}$ with $q \notin F$ is replaced either by the two configurations that some firable transition $\left\langle q, a, l, r_{0}, r_{1}\right\rangle$ yields lazily, or by the one configuration that some firable transition $\langle q, \varepsilon, l, r\rangle$ yields lazily;
- each $\langle q, v\rangle \in \mathcal{G}$ with $q \in F$ is removed.

Performing transitions of $\mathcal{C}$ lazily ensures that, for every level $\mathcal{G}$, the set $\left\{\mathcal{G}^{\prime}\right.$ : $\left.\mathcal{G} \rightarrow \mathcal{G}^{\prime}\right\}$ of all its successors is finite. The latter set is also computable. By the definition of accepting runs and $\left(^{*}\right)$, we have that $\mathcal{C}$ is nonempty iff the empty level is reachable from the initial level $\left\{\left\langle q_{I}, \mathbf{0}\right\rangle\right\}$.

For configurations $\langle q, v\rangle$ and $\langle r, w\rangle$, let $\langle q, v\rangle \leq\langle r, w\rangle$ iff $q=r$ and $v \leq w$. Now, let $\preceq$ be the quasi-ordering obtained by lifting $\leq$ to levels: $\mathcal{G} \preceq \mathcal{H}$ iff, for each $\langle q, v\rangle \in \mathcal{G}$, there exists $\langle r, w\rangle \in \mathcal{H}$ such that $\langle q, v\rangle \leq\langle r, w\rangle$. By Higman's Lemma [Higman 1952], $\preceq$ is a well-quasi-ordering, i.e., for every infinite sequence $\mathcal{G}_{0}, \mathcal{G}_{1}, \ldots$, there exist $i<j$ such that $\mathcal{G}_{i} \preceq \mathcal{G}_{j}$. Observe that, in the terminology of Finkel and Schnoebelen [2001], $\preceq$ is strongly downward-compatible with $\rightarrow$ : whenever $\mathcal{G} \preceq \mathcal{H}$ and $\mathcal{H} \rightarrow \mathcal{H}^{\prime}$, there exists $\mathcal{G}^{\prime}$ such that $\mathcal{G} \rightarrow \mathcal{G}^{\prime}$ and $\mathcal{G}^{\prime} \preceq \mathcal{H}^{\prime}$. Also, $\preceq$ is decidable.

Since $\mathcal{G} \preceq \emptyset$ iff $\mathcal{G}=\emptyset$, we have reduced nonemptiness of $\mathcal{C}$ to the subcovering problem for downward well-structured transition systems with reflexive (which is weaker than strong) compatibility, computable successor sets and decidable ordering. The latter is decidable by [Finkel and Schnoebelen 2001, Theorem 5.5].

## 3. DECIDABILITY OVER FINITE DATA TREES

ThEOREM 3.1. Nonemptiness of $A T R A_{1}$ over finite data trees is decidable and not primitive recursive.

Proof. By considering data words as data trees (e.g., by using only left-hand children starting from the root), the lower bound follows from non-primitive recursiveness of nonemptiness of one-way co-nondeterministic (i.e., with only conjunctive branching) automata with 1 register over finite data words [Demri and Lazić 2009, Theorem 5.2].

We shall establish decidability by reducing to nonemptiness of ITCA, which is decidable by Theorem 2.5. More specifically, by extending to trees the translation in the proof of [Demri and Lazić 2009, Theorem 4.4], which is from one-way alternating automata with 1 register on finite data words to incrementing counter automata on finite words, we shall show that, for each $\operatorname{ATRA}_{1} \mathcal{A}$, an ITCA $\mathcal{C}_{\mathcal{A}}$ with the same alphabet and such that $\mathrm{L}\left(\mathcal{C}_{\mathcal{A}}\right)=\left\{\operatorname{tree}(\tau): \tau \in \mathrm{L}^{\text {fin }}(\mathcal{A})\right\}$, is computable (in polynomial space).

Let $\mathcal{A}=\left\langle\Sigma, Q, q_{I}, F, \delta\right\rangle$. For a configuration $G$ of $\mathcal{A}$ and a datum $D$, let the bundle of $D$ in $G$ be the set of all states that are paired with $D$, i.e. $\{q:\langle q, D\rangle \in G\}$. The computation of $\mathcal{C}_{\mathcal{A}}$ with the properties above is based on the following abstraction of configurations of $\mathcal{A}$ by mappings from $\mathcal{P}(Q) \backslash\{\emptyset\}$ to $\mathbb{N}$. The abstract configuration $\bar{G}$ counts, for each nonempty $S \subseteq Q$, the number of data whose bundles equal $S$ :

$$
\bar{G}(S)=|\{D:\{q:\langle q, D\rangle \in G\}=S\}|
$$

Thus, two configurations have the same abstraction iff they are equal up to a bijective renaming of data. For $1 \leq i \leq \bar{G}(S)$ and $q \in S$, we shall call pairs $\langle S, i\rangle$ abstract data and triples $\langle q, S, i\rangle$ abstract threads.

For abstract configurations $v, w_{0}$ and $w_{1}$, letters $a$, and sets of states $Q=$ with either $v\left(Q_{=}\right)>0$ or $Q_{=}=\emptyset$, we shall define transitions $v \rightarrow{ }_{a}^{Q=} w_{0}, w_{1}$, and show that they are bisimilar to transitions $G \rightarrow_{a}^{E} H_{0}, H_{1}$ such that $v=\bar{G}, w_{0}=\overline{H_{0}}$, $w_{1}=\overline{H_{1}}$ and $Q==\{q:\langle q, E\rangle \in G\}$. The sets $Q=$ can hence be thought of as abstractions of the data $E$. The abstract transitions will then give us a notion of
abstract run of $\mathcal{A}$ on a finite tree (without data), where the sets $Q=$ are guessed at every step. By the bisimilarity, we shall have that:
(I) $\mathcal{A}$ has an accepting abstract run on a finite tree $t$ with alphabet $\Sigma$ iff it has an accepting run on some data tree $\tau$ such that $t=\operatorname{tree}(\tau)$.
In other words, we shall have reduced the question of whether $\mathrm{L}^{\text {fin }}(\mathcal{A})$ is nonempty, i.e. whether there exists a finite tree with alphabet $\Sigma$, a data labelling of its nonleaf nodes, and an accepting run of $\mathcal{A}$ on the resulting data tree, to whether there exists a finite tree and an accepting abstract run of $\mathcal{A}$ on it. It will then remain to show how to compute (in polynomial space) an ITCA $\mathcal{C}_{\mathcal{A}}$ which guesses and checks accepting abstract runs of $\mathcal{A}$, so that:
(II) $\mathcal{C}_{\mathcal{A}}$ has an accepting run on a finite tree $t$ with alphabet $\Sigma$ iff $\mathcal{A}$ has an accepting abstract run on $t$.

To begin delivering our promises, we now define transitions from abstract configurations $v$ for letters $a$ and sets of states $Q=$ with either $v\left(Q_{=}\right)>0$ or $Q_{=}=\emptyset$ to abstract configurations $w_{0}$ and $w_{1}$, essentially by reformulating the definition of concrete transitions (cf. Section 2.2) in terms of abstract threads. For each abstract datum $\langle S, i\rangle$ of $v$ and both $d \in\{0,1\}$, the abstract threads whose abstract datum is $\langle S, i\rangle$ will contribute two sets of states to such a transition: $R^{\prime}(S, i)_{d}^{\downarrow}$, for which the automaton's register is updated, and $R^{\prime}(S, i)_{d}^{k}$, for which the automaton's register is not updated. If $Q_{=}$is nonempty, we take $\left\langle Q_{=}, 1\right\rangle$ to represent the datum abstracted by $Q^{=}$, i.e. with which the register is updated, so states in the union of the set $R^{\prime}\left(Q_{=}, 1\right)_{d}^{\mathcal{A}}$ and all the sets $R^{\prime}(S, i)_{d}^{\downarrow}$ will be associated to the same abstract datum of $w_{d}$. Formally, let $v \rightarrow_{a}^{Q=} w_{0}, w_{1}$ mean that, for each abstract datum $\langle S, i\rangle$ of $v$, there exist sets of states $R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{1}^{\downarrow}, R^{\prime}(S, i)_{1}^{\not /}$ such that:
(i) for each abstract thread $\langle q, S, i\rangle$ of $v$, there exist

$$
R_{0}^{\downarrow}, R_{0}^{\not 又}, R_{1}^{\downarrow}, R_{1}^{\npreceq} \models \delta\left(q, a,\langle S, i\rangle=\left\langle Q_{=}, 1\right\rangle\right)
$$

which satisfy $R_{d}^{?} \subseteq R^{\prime}(S, i)_{d}^{?}$ for both $d \in\{0,1\}$ and $? \in\{\downarrow, \not \downarrow\}$;
(ii) for both $d \in\{0,1\}$ and each nonempty $S^{\prime} \subseteq Q$, we have

$$
\left|\left\{\langle S, i\rangle:\langle S, i\rangle \neq\left\langle Q_{=}, 1\right\rangle \wedge R^{\prime}(S, i)_{d}^{\not x}=S^{\prime}\right\}\right|+\left\{\begin{array}{l}
1, \text { if } R_{d}^{=}=S^{\prime} \\
0, \text { otherwise }
\end{array}\right\} \leq w_{d}\left(S^{\prime}\right)
$$

for some $R_{d}^{=} \supseteq R^{\prime}(Q=, 1)_{d}^{\nsim} \cup \bigcup_{1 \leq i \leq v(S)} R^{\prime}(S, i)_{d}^{\downarrow}$.
It is straightforward to check the following two-part correspondence between the abstract transitions just defined and concrete transitions:
(IIIa) Whenever $G \rightarrow_{a}^{E} H_{0}, H_{1}$, we have $v \rightarrow_{a}^{Q=} w_{0}$, $w_{1}$, where $v=\bar{G}, w_{0}=\overline{H_{0}}$, $w_{1}=\overline{H_{1}}$ and $Q_{=}=\{q:\langle q, E\rangle \in G\}$.
(IIIb) Whenever $\bar{G}=v$ and $v \rightarrow_{a}^{Q=} w_{0}, w_{1}$, there exist $E, H_{0}$ and $H_{1}$ such that $G \rightarrow{ }_{a}^{E} H_{0}, H_{1}, w_{0}=\overline{H_{0}}, w_{1}=\overline{H_{1}}$ and $Q==\{q:\langle q, E\rangle \in G\}$.
Let $\alpha$ be a bijection between the abstract data of $v$ and the data that occur in $G$, which is bundle preserving (i.e., whenever $\alpha\langle S, i\rangle=D$, we have that $S$ is the bundle of $D$ in $G$ ), and if $Q_{=}$is nonempty then $\alpha\left\langle Q_{=, 1}\right\rangle=E$.
-To show (IIIa), for each abstract datum $\langle S, i\rangle$ of $v$ and both $d \in\{0,1\}$, take $R^{\prime}(S, i)_{d}^{\downarrow}$ and $R_{d}^{=}$to be the bundle of $E$ in $H_{d}$, and take $R^{\prime}(S, i)_{d}^{\not 又}$ to be the bundle of $\alpha\langle S, i\rangle$ in $H_{d}$.
-For (IIIb), if $Q_{=}$is empty then take $E$ to be an arbitrary datum which does not occur in $G$, pick the same quadruples for the threads in $G$ as for the corresponding (via $\alpha$ ) abstract threads of $v$, and for both $d \in\{0,1\}$, obtain $H_{d}$ from $w_{d}$ by replacing each set of abstract data $\left\langle S^{\prime}, 1\right\rangle, \ldots,\left\langle S^{\prime}, w_{d}\left(S^{\prime}\right)\right\rangle$ with: the data $\alpha\langle S, i\rangle$ such that $\langle S, i\rangle \neq\left\langle Q_{=}, 1\right\rangle$ and $R^{\prime}(S, i)_{d}^{\neq}=S^{\prime}$, the datum $E$ if $R_{d}^{=}=S^{\prime}$, and fresh further data if the inequality in (ii) is strict.

Composing abstract transitions gives us abstract runs of $\mathcal{A}$. Such a run on a finite tree $\langle N, \Sigma, \Lambda\rangle$ is a mapping $n \mapsto v_{n}$ from the nodes to abstract configurations such that, for each nonleaf $n$, there exists $Q=$ with $v_{n} \rightarrow_{\Lambda(n)}^{Q=} v_{n \cdot 0}, v_{n \cdot 1}$, and if $n$ is the root then $q_{I} \in Q=$. Defining the run to be accepting iff $v_{n}(S)=0$ for all leaves $n$ and all $S \nsubseteq F$, we have (I) above by (IIIa) and (IIIb).

We are now ready to define $\mathcal{C}_{\mathcal{A}}$, as an ITCA that performs the steps (1)-(9) below. States of $\mathcal{C}_{\mathcal{A}}$ are used for control and for storing $a, Q_{=}$, root (initially $t t$ ), $S, R_{0}^{\downarrow}, R_{0}^{\prime \downarrow}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \downarrow}, q, R_{0}^{\downarrow}, R_{0}^{\downarrow}, R_{1}^{\downarrow}, R_{1}^{\nless}, d, ?$ and $R_{d}^{=}$. There are $2^{|Q|}-1$ counters in the array $c$, and $2^{|Q|^{4}}$ counters in the array $c^{\prime}$. The steps are implemented by $\varepsilon$-transitions, except for the $a$-transition in (4). The choices are nondeterministic. If a choice in (3.2) is impossible, or a check in (2), (3.2) or (5) fails, then $\mathcal{C}_{\mathcal{A}}$ blocks.

The steps (1)-(9) guess and check an accepting abstract run of $\mathcal{A}$ on a finite tree. The counter array $c$ is used to store abstract configurations, and the counter array $c^{\prime}$ is auxiliary. The initial condition in the definition of abstract runs is checked in (2), the final condition in (8), and steps (3)-(7) are essentially a reformulation of the definition of abstract transitions. This particular reformulation is tailored for a development in the proof of Theorem 4.1, and is based on observing that the quadruples of sets $R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{1}^{\downarrow}, R^{\prime}(S, i)_{1}^{\downarrow}$ for abstract data $\langle S, i\rangle \neq$ $\left\langle Q_{=}, 1\right\rangle$ do not need to be stored simultaneously, i.e. that it suffices to store numbers of such identical quadruples, which is done using the counter array $c^{\prime}$.
(1) Choose $a \in \Sigma$, and $Q=$ with either $c[Q=]>0$ or $Q==\emptyset$.
(2) If root $=t t$, then check that $q_{I} \in Q=$ and set root $:=f f$.
(3) For each nonempty $S \subseteq Q$, while $c[S]>0$ do:
(3.1) choose $R_{0}^{\prime \downarrow}, R_{0}^{\prime \star}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \star} \subseteq Q$;
(3.2) for each $q \in S$, choose $R_{0}^{\downarrow}, R_{0}^{\nless}, R_{1}^{\downarrow}, R_{1}^{\downarrow} \models \delta\left(q, a,\langle S, c[S]\rangle=\left\langle Q_{=}, 1\right\rangle\right.$ ), and check that $R_{d}^{?} \subseteq R^{\prime ?}{ }_{d}$ for both $d \in\{0,1\}$ and $? \in\{\downarrow, \not \downarrow\}$;
(3.3) decrement $c[S]$, and if $\langle S, c[S]\rangle=\left\langle Q_{=, 0}\right.$, , then choose $R_{d}^{=} \supseteq R_{d}^{\prime \downarrow} \cup R_{d}^{\prime \prime}$ for both $d \in\{0,1\}$, else increment $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \not}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \not}\right]$.
(4) Perform an $a$-transition, forking with $d:=0$ and $d:=1$.
(5) Check that $R_{d}^{=} \supseteq \bigcup\left\{R_{d}^{\prime \downarrow}: c^{\prime}\left[R_{0}^{\downarrow}, R_{0}^{\prime \downarrow}, R_{1}^{\downarrow}, R_{1}^{\prime \star}\right]>0\right\}$, and increment $c\left[R_{d}^{=}\right]$.
(6) Transfer each $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \not}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \not}\right]$ with nonempty $R_{d}^{\prime \not}$ to $c\left[R_{d}^{\prime \not}\right]$.
(7) Reset (i.e. decrement until 0) each $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \not}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \nless}\right]$ with empty $R_{d}^{\prime \not}$.
(8) If $c[S]=0$ whenever $S \nsubseteq F$, then pass through a final state.
(9) Repeat from (1).

Since $\mathcal{C}_{\mathcal{A}}$ is an ITCA, its runs may contain arbitrary errors that increase one or more counters. Nevertheless, between executions of steps (3)-(7) by $\mathcal{C}_{\mathcal{A}}$ and abstract transitions of $\mathcal{A}$, we have the following two-part correspondence. It shows that the possibly erroneous executions of (3)-(7) match the abstract transitions with the slack allowed by their definition, which in turn match the concrete transitions with their possible introductions of junk threads (cf. (IIIa), (IIIb) and Remark 2.1).
(IVa) Whenever $v \rightarrow{ }_{a}^{Q=} w_{0}, w_{1}$, we have that $\mathcal{C}_{\mathcal{A}}$ can perform steps (3)-(7) beginning with any configuration such that each $c[S]$ has value $v(S)$ and each $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \downarrow}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \not}\right]$ has value 0 , so that for both forks $d \in\{0,1\}$ in (4), the ending configuration is such that each $c[S]$ has value $w_{d}(S)$ and each $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \star}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \star}\right]$ has value 0 .
(IVb) Whenever $\mathcal{C}_{\mathcal{A}}$ can perform steps (3)-(7) beginning with a configuration such that $a$ and $Q_{=}$are as in (1) and each $c[S]$ has value $v(S)$, so that for both forks $d \in\{0,1\}$ in (4), the ending configuration is such that each $c[S]$ has value $w_{d}(S)$, we have $v \rightarrow{ }_{a}^{Q=} w_{0}, w_{1}$.
-In proving (IVa), we can choose where incrementing errors occur. For each iteration of (3.1)-(3.3), let the quadruple chosen in (3.1) be

$$
R^{\prime}(S, c[S])_{0}^{\downarrow}, R^{\prime}(S, c[S])_{0}^{\not 又}, R^{\prime}(S, c[S])_{1}^{\downarrow}, R^{\prime}(S, c[S])_{1}^{\not \perp}
$$

so that (3.2) can succeed by (i) in the definition of abstract transitions. It remains to match by incrementing errors, say at the end of (7), any differences between the two sides of the inequalities in (ii).
—To obtain (IVb), let $R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{0}^{\downarrow}, R^{\prime}(S, i)_{1}^{\downarrow}, R^{\prime}(S, i)_{1}^{\not 又}$ for each abstract datum $\langle S, i\rangle$ of $v$ be the quadruple chosen in the last performance of (3.1) with $i=c[S]$ (due to incrementing errors, there may be more than one). Step (3.2) ensures that (i) is satisfied. Since at the end of (3), each $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \star}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \nless}\right]$ has value at least

$$
\left|\left\{\langle S, i\rangle:\langle S, i\rangle \neq\left\langle Q_{=}, 1\right\rangle \wedge \forall d, ?\left(R^{\prime}(S, i)_{d}^{?}=R_{d}^{\prime ?}\right)\right\}\right|
$$

steps (5) and (6) ensure that (ii) is satisfied.
Now, we have (II) above. The 'if' direction follows by (IVa), and the 'only if' direction by (IVb) once we observe that, without loss of generality, we can consider only runs of $\mathcal{C}_{\mathcal{A}}$ that do not contain incrementing errors on the array $c$ outside of steps (3)-(7) except before the first performance of (1).

To conclude that polynomial space suffices for computing $\mathcal{C}_{\mathcal{A}}$, we observe that each of its state variables is either from a fixed finite set, or an element of $\Sigma$, or an element or subset of $Q$, and that deciding satisfaction of transition formulae $\delta\left(q, a,\langle S, c[S]\rangle=\left\langle Q_{=}, 1\right\rangle\right)$ in step (3.2) amounts to evaluating Boolean formulae.

We remark that, in the opposite direction to the translation in the proof of Theorem 3.1, by extending the proof of [Demri and Lazić 2009, Theorem 5.2] to trees, for each ITCA $\mathcal{C}$, an ATRA $_{1} \mathcal{A}_{\mathcal{C}}$ is computable in logarithmic space such that $\mathrm{L}^{\text {fin }}\left(\mathcal{A}_{\mathcal{C}}\right)$ consists of encodings of accepting runs of $\mathcal{C}$. Moreover, similarly as on words, the two translations can be extended to infinite trees, where ATRA $_{1}$ are equipped with weak acceptance and ITCA with Büchi acceptance. Instead of
decidable and not primitive recursive as on finite trees, nonemptiness for those two classes of automata can then be shown co-r.e.-complete.

## 4. SAFETY AUTOMATA

We now show decidability of nonemptiness of forward alternating tree 1-register automata with safety acceptance over finite or infinite data trees. More precisely, since the class of safety ATRA $_{1}$ is not closed under complement, but is closed under intersection and union (cf. Proposition 2.2(b)), we show decidability of the inclusion problem, which implies decidability of nonemptiness of Boolean combinations of safety ATRA $A_{1}$. However, already for the subproblems of nonemptiness and nonuniversality, we obtain non-elementary and non-primitive recursive lower bounds (respectively).

Theorem 4.1. For safety ATRA $_{1}$, inclusion is decidable, nonemptiness is not elementary, and nonuniversality is not primitive recursive.

Proof. Showing that the inclusion problem is decidable will involve extending:
-the proof of Proposition 2.2 to obtain an intersection of a safety and a co-safety ATRA $_{1}$, which can be seen as a weak parity ATRA $_{1}$ with 2 priorities;
-the proof of Theorem 3.1 to obtain an ITCA with a more powerful set of instructions and no cycles of $\varepsilon$-transitions, which can also be seen as having weak parity acceptance with 2 priorities;
-the proof of Theorem 2.5 to obtain decidability of nonemptiness of such ITCA.
To maintain focus, we shall avoid introducing the extended notions in general, but concentrate on what is necessary for this part of the proof.
Suppose $\mathcal{A}_{1}=\left\langle\Sigma, Q_{1}, q_{I}^{1}, F_{1}, \delta_{1}\right\rangle$ and $\mathcal{A}_{2}=\left\langle\Sigma, Q_{2}, q_{I}^{2}, F_{2}, \delta_{2}\right\rangle$ are ATRA ${ }_{1}$, where we need to determine whether $\mathrm{L}^{\text {saf }}\left(\mathcal{A}_{1}\right)$ is a subset of $\mathrm{L}^{\text {saf }}\left(\mathcal{A}_{2}\right)$. By the proof of Proposition 2.2(b), that amounts to emptiness of the intersection of $\mathrm{L}^{\text {saf }}\left(\mathcal{A}_{1}\right)$ and $\mathrm{L}^{\cos }\left(\overline{\mathcal{A}_{2}}\right)$, where $\overline{\mathcal{A}_{2}}=\left\langle\Sigma, Q_{2}, q_{I}^{2}, \overline{F_{2}}, \overline{\delta_{2}}\right\rangle$ is the dual automaton to $\mathcal{A}_{2}$. Assuming that $Q_{1}$ and $Q_{2}$ are disjoint, and do not contain $q_{I}^{\cap}$, let

$$
\mathcal{A}_{\cap}=\left\langle\Sigma,\left\{q_{I}^{\cap}\right\} \cup Q_{1} \cup Q_{2}, q_{I}^{\cap}, F_{1} \cup F_{2}, \delta_{\cap}\right\rangle
$$

be the automaton for the intersection of $\mathcal{A}_{1}$ and $\overline{\mathcal{A}_{2}}$ :

$$
\delta_{\cap}(q, a, p)= \begin{cases}\delta\left(q_{I}^{1}, a, p\right) \wedge \delta\left(q_{I}^{2}, a, p\right), & \text { if } q=q_{I} \\ \delta_{1}(q, a, p), & \text { if } q \in Q_{1} \\ \delta_{2}(q, a, p), & \text { if } q \in Q_{2}\end{cases}
$$

We then have:
$\left(^{*}\right)$ A data tree $\tau$ with alphabet $\Sigma$ is safety-accepted by $\mathcal{A}_{1}$ and co-safety-accepted by $\overline{\mathcal{A}_{2}}$ iff $\mathcal{A}_{\cap}$ has a run on $\tau$ which is final and $Q_{2}$-finite, i.e. there exists $l$ such that the configuration at each node of length at least $l$ contains no threads with states from $Q_{2}$.
Before proceeding, let incrementing tree counter automata with nondeterministic transfers (shortly, ITCANT) be defined as ITCA (cf. Section 2.3), except that $\langle$ ifz,$c\rangle$ instructions are replaced by $\langle\operatorname{transf}, c, C\rangle$ for counters $c$ and sets of counters $C$. Such an instruction is equivalent to a loop which executes while $c$ is nonzero,

[^1]and in each iteration, decrements $c$ and increments some counter in $C$. However, in presence of incrementing errors, the loop may not terminate, whereas $\langle$ transf, $c, C\rangle$ instructions are considered atomic. The effect of $\langle\operatorname{transf}, c, C\rangle$ is therefore to transfer the value of $c$ to the counters in $C$, among which it is split nondeterministically. In particular, $\langle$ ifz, $c\rangle$ instructions can be reintroduced as $\langle$ transf, $c, \emptyset\rangle$.

Now, steps (1)-(9) in the proof of Theorem 3.1 can be implemented by an ITCANT which uses nondeterministic transfers instead of the loops in (3), (6) and (7), and whose transition relation therefore contains no cycles of $\varepsilon$-transitions. More specifically, each reset in (7) can be implemented as a transfer to a new auxiliary counter $c^{\prime \prime},(6)$ already consists of transfers to single counters, and (3) can be replaced by the following two steps:
(3a) If $Q=\neq \emptyset$, then decrement $c\left[Q_{=}\right]$and choose $R_{0}^{=}, R_{1}^{=} \subseteq Q$ such that, for each $q \in Q_{=}$, there exist $R_{0}^{\downarrow}, R_{0}^{\nless}, R_{1}^{\downarrow}, R_{1}^{\nless} \models \delta(q, a, t t)$ with $R_{d}^{=} \supseteq R_{d}^{\downarrow} \cup R_{d}^{\nless}$ for both $d \in\{0,1\}$.
(3b) Transfer each $c[S]$ nondeterministically to the set of all $c^{\prime}\left[R_{0}^{\prime \downarrow}, R_{0}^{\prime \star}, R_{1}^{\prime \downarrow}, R_{1}^{\prime \not}\right]$ such that, for each $q \in S$, there exist $R_{0}^{\downarrow}, R_{0}^{\not 又}, R_{1}^{\downarrow}, R_{1}^{\not 又} \models \delta(q, a$, ff $)$ with $R_{d}^{?} \subseteq$ $R^{\prime}{ }_{d}^{?}$ for both $d \in\{0,1\}$ and $? \in\{\downarrow, \not \subset\}$.

Let $\mathcal{C}_{\cap}$ be such an ITCANT for $\mathcal{A}_{\cap}$, which in addition performs the following step between (7) and (8), where prop is a state variable, initially ff:
( $7 \frac{1}{2}$ ) If $c[S]=0$ whenever $S \cap Q_{2} \neq \emptyset$, then set prop $:=t t$.
As in the proof of Theorem 3.1, we have that $\mathcal{C}_{\cap}$ is computable from $\mathcal{A}_{\cap}$, and therefore from $\mathcal{A}_{1}$ and $\mathcal{A}_{2}$, in polynomial space. Also, $\mathcal{A}_{\cap}$ satisfies (IIIa) and (IIIb), and $\mathcal{A}_{\cap}$ and $\mathcal{C}_{\cap}$ satisfy (IVa) and (IVb). Recalling that $\mathcal{C}_{\cap}$ contains no cycles of $\varepsilon$-transitions, we infer the following from $\left(^{*}\right.$ ) above, where the notion of transitions between levels of $\mathcal{C}_{\cap}$ is as in the proof of Theorem 2.5, and $P$ denotes the set of all states of $\mathcal{C}_{\cap}$ in which prop has value $t t$ :
$\left({ }^{* *}\right) \mathrm{L}^{\text {saf }}\left(\mathcal{A}_{1}\right)$ is a subset of $\mathrm{L}^{\text {saf }}\left(\mathcal{A}_{2}\right)$ iff there does not exist an infinite sequence of transitions $\mathcal{G}_{0} \rightarrow \mathcal{G}_{1} \rightarrow \cdots$ which is from the initial level of $\mathcal{C}_{\cap}$ and such that some $\mathcal{G}_{i}$ contains only states from $P$.

To conclude decidability of inclusion, we show that, given an ITCANT $\mathcal{C}_{\cap}$ and a set $P$ of its states, existence of an infinite sequence of transitions as in $\left({ }^{* *}\right)$ is decidable. For a set $\mathbb{G}$ of levels of $\mathcal{C}_{\cap}$, we write $\uparrow \mathbb{G}$ to denote its upward closure with respect to $\preceq$ : the set of all $\mathcal{H}$ for which there exists $\mathcal{G} \in \mathbb{G}$ with $\mathcal{G} \preceq \mathcal{H}$. We say that $\mathbb{G}$ is upwards closed iff $\mathbb{G}=\uparrow \mathbb{G}$, and we say that $\mathbb{H}$ is a basis for $\mathbb{G}$ iff $\mathbb{G}=\uparrow \mathbb{H}$. As in the proof of Theorem 2.5, we have that successor sets with respect to $\rightarrow$ are computable, $\preceq$ is a well-quasi-ordering, $\preceq$ is strongly (in particular, reflexively) downward-compatible with $\rightarrow$, and $\preceq$ is decidable. Hence, by [Finkel and Schnoebelen 2001, Proposition 5.4], a finite basis $\mathbb{G}_{R}$ of the upward closure of the set of all levels reachable from the initial level is computable. By the strong downward compatibility, the set of all levels from which there exists an infinite sequence of transitions is downwards closed, so its complement is upwards closed. We claim that a finite basis $\mathbb{G}_{T}$ of the latter set is computable. With that assumption, since a finite basis $\mathbb{G}_{N}$ of the set of all levels that contain some state not from $P$ is
certainly computable, we are done because there does not exist an infinite sequence of transitions as in $\left({ }^{* *}\right)$ iff $\uparrow \mathbb{G}_{R}$ is a subset of the union of $\uparrow \mathbb{G}_{T}$ and $\uparrow \mathbb{G}_{N}$.

It remains to establish the claim. For a finite set $\mathbb{G}^{\prime}$ of levels of $\mathcal{C}_{\cap}$, let

$$
K\left(\mathbb{G}^{\prime}\right)=1+\max _{\mathcal{G}^{\prime} \in \mathbb{G}^{\prime}} \max _{\langle q, v\rangle \in \mathcal{G}^{\prime}} \sum_{c \in\{1, \ldots, k\}} v(c)
$$

where $k$ is the number of counters of $\mathcal{C}_{\cap}$. Let also $\operatorname{Pred}_{\forall}\left(\mathbb{G}^{\prime}\right)$ be the upwards-closed set consisting of all $\mathcal{G}$ such that, whenever $\mathcal{G} \rightarrow \mathcal{G}^{\prime}$, we have $\mathcal{G}^{\prime} \in \uparrow \mathbb{G}^{\prime}$. Observe that, whenever $\mathcal{G} \in \operatorname{Pred}_{\forall}\left(\mathbb{G}^{\prime}\right)$, there exists $\mathcal{G}_{\dagger} \in \operatorname{Pred}_{\forall}\left(\mathbb{G}^{\prime}\right)$ such that $\mathcal{G}_{\dagger} \preceq \mathcal{G}$ and, for each $\langle q, v\rangle \in \mathcal{G}_{\dagger}$ and $c \in\{1, \ldots, k\}, v(c) \leq K\left(\mathbb{G}^{\prime}\right)$. Hence, a finite basis of $\operatorname{Pred}_{\forall}\left(\mathbb{G}^{\prime}\right)$ is computable, so the following is an effective procedure:
(i) Begin with $\mathbb{G}_{T}:=\emptyset$.
(ii) Let $\mathbb{H}$ be a finite basis of $\operatorname{Pred}_{\forall}\left(\mathbb{G}_{T}\right)$.
(iii) If $\mathbb{H} \nsubseteq \uparrow \mathbb{G}_{T}$, then set $\mathbb{G}_{T}:=\mathbb{G}_{T} \cup \mathbb{H}$ and repeat from (ii), else terminate.

Since $\preceq$ is a well-quasi-ordering, the procedure terminates and computes a basis of the set of all levels from which every sequence of transitions is finite, as required.

We shall establish that nonemptiness of safety ATRA $_{1}$ is not elementary by a twostage reduction, which separates dealing with the inability of one-way alternating 1-register automata to detect incrementing errors in encodings of computations of counter machines, from ensuring acceptance only of encodings of computations in which counters are bounded by a tower of exponentiations. More precisely, we shall use the following problem as intermediary. The notation $2 \Uparrow m$ is as in Example 2.4.
$\left(^{* * *}\right)$ Given a deterministic counter machine $\mathcal{C}$ and $m \geq 1$ in unary, does $\mathcal{C}$ have a computation which possibly contains incrementing errors, in which every counter value is at most $2 \Uparrow m$, and which is either halting or infinite?

Such a machine is a tuple $\left\langle Q, q_{I}, q_{H}, k, \delta\right\rangle$ where: $Q$ is a finite set of states, $q_{I}$ is the initial state, $q_{H}$ is the halting state, $k \in \mathbb{N}$ is the number of counters, and $\delta: Q \backslash\left\{q_{H}\right\} \rightarrow\{1, \ldots, k\} \times\left(Q \cup Q^{2}\right)$ is a transition function. Thus, from a state $q \neq q_{H}$, either $\delta(q)$ is of the form $\left\langle c, q^{\prime}\right\rangle$, which means that the machine increments $c$ and goes to $q^{\prime}$, or $\delta(q)$ is of the form $\left\langle c, q^{\prime}, q^{\prime \prime}\right\rangle$, which means that, if $c$ is zero, then the machine goes to $q^{\prime}$, else it decrements $c$ and goes to $q^{\prime \prime}$. More precisely, a configuration is a state together with a counter valuation, and we write $\langle q, v\rangle \rightarrow\left\langle q^{\prime}, v^{\prime}\right\rangle$ iff, for some $v_{\sqrt{ }} \geq v$ and $v_{\sqrt{ }}^{\prime} \leq v^{\prime}$,
-either $\delta(q)=\left\langle c, q^{\prime}\right\rangle$ and $v_{\sqrt{ }}^{\prime}=v_{\sqrt{ }}\left[c \mapsto v_{\sqrt{ }}(c)+1\right]$,

—or $\delta(q)=\left\langle c, q^{\prime \prime}, q^{\prime}\right\rangle$ and $v_{\sqrt{ }}^{\prime}=v_{\sqrt{ }}\left[c \mapsto v_{\sqrt{ }}(c)-1\right]$.
We say that the transition is error-free iff the above holds with $v_{\sqrt{ }}=v$ and $v_{\sqrt{ }}=v^{\prime}$. A computation is a sequence $\left\langle q_{0}, v_{0}\right\rangle \rightarrow\left\langle q_{1}, v_{1}\right\rangle \rightarrow \cdots$ such that $q_{0}=q_{I}$ and $v=\mathbf{0}$.

To show that $\left({ }^{* * *}\right)$ is not elementary, we reduce from the problem of whether a deterministic 2-counter machine of size $m$ has an error-free halting computation of length at most $2 \Uparrow \mathrm{~m}$. Given such a machine $\mathcal{C}$ whose counters are $c_{1}$ and $c_{2}$, let $\widehat{\mathcal{C}}$ be a deterministic machine with counters $c_{1}, c_{2}, \overline{c_{1}}, \overline{c_{2}}, c^{\dagger}, c^{\prime}, c^{\prime \prime}$ and $c^{\prime \prime \prime}$, which performs the following and then halts:

```
c
while }\mp@subsup{c}{}{\prime}>
{dec}(\mp@subsup{c}{}{\prime});\mathrm{ while }\overline{\mp@subsup{c}{i}{}}>0{\operatorname{dec}(\overline{\mp@subsup{c}{i}{}});\operatorname{inc}(\mp@subsup{c}{}{\prime\prime})};\operatorname{inc}(\overline{\mp@subsup{c}{i}{}})
    while }\mp@subsup{c}{}{\prime\prime}>
    {\operatorname{dec}(\mp@subsup{c}{}{\prime\prime});\mathrm{ while }\overline{\mp@subsup{c}{i}{}}>0{\operatorname{dec}(\overline{\mp@subsup{c}{i}{}});\operatorname{inc}(\mp@subsup{c}{}{\prime\prime\prime})};
        while }\mp@subsup{c}{}{\prime\prime\prime}>0{\operatorname{dec}(\mp@subsup{c}{}{\prime\prime\prime});\operatorname{inc}(\overline{\mp@subsup{c}{i}{}});\operatorname{inc}(\overline{\mp@subsup{c}{i}{}})}}
```

Fig. 3. Computing $2 \Uparrow m$
(I) For both $i \in\{1,2\}$, set $\overline{c_{i}}$ to $2 \Uparrow m$ by executing the pseudo-code in Figure 3 . The loops over $c^{\prime}, c^{\prime \prime}$ and $c^{\prime \prime \prime}$ implement $\overline{c_{i}}:=2 \Uparrow c^{\prime}, \overline{c_{i}}:=2^{c^{\prime \prime}}$ and $\overline{c_{i}}:=2 \times c^{\prime \prime \prime}$ (respectively).
(II) Simulate $\mathcal{C}$ using $c_{1}$ and $c_{2}$, and after each step:
—increment $c^{\dagger}$;
-if $c_{i}$ has been incremented, then decrement $\overline{c_{i}}$;
-if $c_{i}$ has been decremented, then increment $\overline{c_{i}}$;
-if $\mathcal{C}$ has halted, then go to (III).
(III) For both $i \in\{1,2\}$, transfer $\overline{c_{i}}$ to $c_{i}$.

Observe that $\widehat{\mathcal{C}}$ is computable in space logarithmic in $m$. If $\mathcal{C}$ has an error-free halting computation of length at most $2 \Uparrow m$, running $\widehat{\mathcal{C}}$ without errors indeed halts and does not involve counter values greater than $2 \Uparrow m$. For the converse, suppose $\widehat{\mathcal{C}}$ has a computation which possibly contains incrementing errors, in which every counter value is at most $2 \Uparrow m$, and which is either halting or infinite. By the construction of $\widehat{\mathcal{C}}$ and the boundedness of counter values, the computation cannot be infinite, so it is halting. Since $\overline{c_{1}}$ and $\overline{c_{2}}$ were set to $2 \Uparrow m$ by stage (I), and since stage (III) terminated, the halting computation of $\mathcal{C}$ in stage (II) must have been error-free and it is certainly of length at most $2 \Uparrow m$.

To reduce from $\left({ }^{* * *}\right)$ to nonemptiness of safety ATRA $_{1}$, consider a deterministic counter machine $\mathcal{C}=\left\langle Q, q_{I}, q_{H}, k, \delta\right\rangle$ and $m \geq 1$. We can assume that $q^{\prime} \neq q^{\prime \prime}$ whenever $\delta(q)=\left\langle c, q^{\prime}, q^{\prime \prime}\right\rangle$. By the proof of [Demri and Lazić 2009, Theorem 5.2], which uses essentially the same encoding of computations of counter machines into data words as in the proof of [Bojańczyk et al. 2006, Theorem 14], we have that an ATRA $_{1} \mathcal{A}_{\mathcal{C}}$ with alphabet $Q$ is computable in space logarithmic in $|\mathcal{C}|$, such that it safety-accepts a data tree $\tau$ iff the left-most path in $\tau$ (i.e., the sequence of nodes obtained by starting from the root and repeatedly taking the left-hand child) satisfies the following:

- the letter of the first node is $q_{I}$, and either the letter of the last nonleaf node is $q_{H}$ or the sequence is infinite;
-for all letters $q$ and $q^{\prime}$ of two consecutive nodes $n$ and $n^{\prime}$ (respectively), - either $\delta(q)$ is of the form $\left\langle c, q^{\prime}\right\rangle$ and we say that $n$ is $c$-decrementing, -or $\delta(q)$ is of the form $\left\langle c, q^{\prime}, q^{\prime \prime}\right\rangle$ and we say that $n$ is $c$-zero-testing, -or $\delta(q)$ is of the form $\left\langle c, q^{\prime \prime}, q^{\prime}\right\rangle$ and we say that $n$ is $c$-decrementing;
-for each counter $c$, no two $c$-incrementing nodes are labelled by the same datum, no two $c$-decrementing nodes are labelled by the same datum, and whenever a $c$ incrementing node $n$ is followed by a $c$-zero-testing node $n^{\prime}$, then a $c$-decrementing node with the same datum as $n$ must occur between $n$ and $n^{\prime}$.

Hence, by taking the left-most paths in data trees that are safety-accepted by $\mathcal{A}_{\mathcal{C}}$ and erasing data, we obtain exactly the sequences of states of halting or infinite computations of $\mathcal{C}$ which possibly contain incrementing errors. Assuming that $b_{1}$, $\ldots, b_{m}, *$ are not in $Q$, to restrict further to computations of $\mathcal{C}$ in which every counter value is at most $2 \Uparrow m$, it suffices to strengthen $\mathcal{A}_{\mathcal{C}}$ to obtain a safety ATRA $_{1} \mathcal{A}_{\mathcal{C}}^{2 \Uparrow m}$ with alphabet $Q \cup\left\{b_{1}, \ldots, b_{m}, *\right\}$ which requires that:
-whenever a node $n$ in the left-most path is $c$-incrementing, then the automaton $\mathcal{B}_{m}$ from Example 2.4 safety-accepts the right-hand subtree at $n$;
-whenever a node $n$ in the left-most path is $c$-incrementing, $n^{\prime}$ is either $n$ or a subsequent $c$-incrementing node, and no $c$-decrementing node with the same datum as $n$ occurs between $n$ and $n^{\prime}$, then the right-hand subtree at $n^{\prime}$ contains a node with letter $b_{m}$ and the same datum as $n$.
Finally, that nonuniversality of safety ATRA $_{1}$ is not primitive recursive follows from the same lower bound for nonuniversality of safety one-way alternating automata with 1 register over data words [Lazić 2006].

## 5. XPATH SATISFIABILITY

In this section, we first describe how XML documents and DTDs can be represented by data trees and tree automata. We then introduce a forward fragment of XPath, and a safety subfragment. By translating XPath queries to forward alternating tree 1-register automata, and applying results from Sections 3 and 4, we obtain decidability of satisfiablity for forward XPath on finite documents and for safety forward XPath on finite or infinite documents.

XML Trees. Suppose $\Sigma$ is a finite set of element types, $\Sigma^{\prime}$ is a finite set of attribute names, and $\Sigma$ and $\Sigma^{\prime}$ are disjoint. An XML document [Bray et al. 1998] is an unranked ordered tree whose every node $n$ is labelled by some type $(n) \in \Sigma$ and by a datum for each element of some atts $(n) \subseteq \Sigma^{\prime}$. Motivated by processing of XML streams (cf., e.g., [Olteanu et al. 2004]), we do not restrict our attention to finite XML documents.
Concerning the data in XML documents, we shall consider only the equality predicate between data labels. Equality comparisons with constants are straightforward to encode using additional attribute names. Therefore, similarly as Bojańczyk et al. [2009], we represent an XML document by a data tree with alphabet $\Sigma \cup \Sigma^{\prime}$, where each node $n$ is represented by a sequence of $1+|\operatorname{atts}(n)|$ nodes: the first node is labelled by type $(n)$, the labels of the following nodes enumerate atts $(n)$, the children of the last node represent the first child and the next sibling of $n$ (if any), and for each preceding node in the sequence, its left-hand child is the next node and its right-hand child is a leaf. We say that such a data tree is an XML tree.
Following Benedikt et al. [2008] and Bojańczyk et al. [2009], we assume without loss of generality that document type definitions (DTDs) [Bray et al. 1998] are given as regular tree languages. More precisely, we consider a DTD to be a forward nondeterministic tree automaton $\mathcal{T}$ with alphabet $\Sigma \cup \Sigma^{\prime}$ and without $\varepsilon$-transitions. Such automata can be defined by omitting counters and $\varepsilon$-transitions from ITCA (cf. Section 2.3). Infinite trees are processed in safety mode, i.e. the condition that an infinite run of $\mathcal{T}$ has to satisfy to be accepting is the same as for finite runs: for

$$
\begin{aligned}
& \tau, n, n^{\prime} \models \varepsilon \stackrel{\text { def }}{\Leftrightarrow} n=n^{\prime} \\
& \tau, n, n^{\prime} \models\{\nabla, \Delta, \triangleright, \triangleleft\} \stackrel{\text { def }}{\Leftrightarrow} n\{\widehat{\nabla}, \widehat{\Delta}, \widehat{\triangleright}, \widehat{\triangleleft}\} n^{\prime} \\
& \tau, n, n^{\prime} \models\left\{\nabla^{*}, \Delta^{*}, \triangleright^{*}, \triangleleft^{*}\right\} \stackrel{\text { def }}{\Leftrightarrow} n\left\{\widehat{\nabla}^{*}, \widehat{\Delta}^{*}, \widehat{\triangleright}^{*}, \widehat{\triangleleft}^{*}\right\} n^{\prime} \\
& \tau, n, n^{\prime} \models p_{1} / p_{2} \stackrel{\text { def }}{\Leftrightarrow} \text { there exists } n^{\prime \prime} \text { such that } \\
& \tau, n, n^{\prime \prime} \models p_{1} \text { and } \tau, n^{\prime \prime}, n^{\prime} \models p_{2} \\
& \tau, n, n^{\prime} \models p_{1} \cup p_{2} \stackrel{\text { def }}{\Leftrightarrow} \tau, n, n^{\prime} \models p_{1} \text { or } \tau, n, n^{\prime} \models p_{2} \\
& \tau, n, n^{\prime} \models p[u] \stackrel{\text { def }}{\Leftrightarrow} \tau, n, n^{\prime} \models p \text { and } \tau, n^{\prime} \models u \\
& \tau, n \models p ? \stackrel{\text { def }}{\Leftrightarrow} \text { there exists } n^{\prime} \text { such that } \tau, n, n^{\prime} \models p \\
& \tau, n \models a \stackrel{\text { def }}{\Leftrightarrow} \Lambda(n)=a \\
& \tau, n \models p_{1} / @ a_{1}^{\prime} \bowtie p_{2} / @ a_{2}^{\prime} \stackrel{\text { def }}{\Leftrightarrow} \text { there exist } n_{1}, k_{1}, n_{2}, k_{2} \operatorname{such} \text { that } \\
& \tau, n, n_{1} \models p_{1}, k_{1} \leq\left|\operatorname{atts}\left(n_{1}\right)\right|, \Lambda\left(n_{1} \cdot 0^{k_{1}}\right)=a_{1}^{\prime}, \\
& \tau, n, n_{2} \models p_{2}, k_{2} \leq\left|\operatorname{atts}\left(n_{2}\right)\right|, \Lambda\left(n_{2} \cdot 0^{k_{2}}\right)=a_{2}^{\prime}, \\
& \Delta\left(n_{1} \cdot 0^{k_{1}}\right) \bowtie \Delta\left(n_{2} \cdot 0^{k_{2}}\right)
\end{aligned}
$$

Fig. 4. Semantics of Queries and Qualifiers
each leaf $n$, the state of the configuration at $n$ is final. An XML tree $\tau$ as above is regarded to satisfy $\mathcal{T}$ iff $\mathcal{T}$ accepts tree $(\tau)$.

Fragments of XPath. The fragment of XPath [Clark and DeRose 1999] below contains all operators commonly found in practice and was considered in [Benedikt et al. 2008; Geerts and Fan 2005]. The grammars of queries $p$ and qualifiers $u$ are mutually recursive. The element types $a$ and attribute names $a^{\prime}$ range over $\Sigma$ and $\Sigma^{\prime}$, respectively.

$$
\begin{aligned}
& p::=\varepsilon|\nabla| \Delta|\triangleright| \triangleleft\left|\nabla^{*}\right| \Delta^{*}\left|\triangleright^{*}\right| \triangleleft^{*}|p / p| p \cup p \mid p[u] \\
& u::=\neg u|u \wedge u| p ?|a| p / @ a^{\prime}=p / @ a^{\prime} \mid p / @ a^{\prime} \neq p / @ a^{\prime}
\end{aligned}
$$

We say that a query or qualifier is forward iff:
—it does not contain $\Delta, \triangleleft, \Delta^{*}$ or $\triangleleft^{*}$;
-for every subqualifier of the form $p_{1} / @ a_{1}^{\prime} \bowtie p_{2} / @ a_{2}^{\prime}$, we have that $p_{1}=\varepsilon$ and that $p_{2}$ is of the form $\varepsilon$ or $\nabla / p_{2}^{\prime}$ or $\triangleright / p_{2}^{\prime}$.
A safety (resp., co-safety) query or qualifier is one in which each occurence of $\nabla$, $\nabla^{*}$ or $\triangleright^{*}$ is under an odd (resp., even) number of negations. Since infinite XML documents may contain infinite siblinghoods, $\nabla, \nabla^{*}$ and $\triangleright^{*}$ are exactly the queries that may require existence of a node which can be unboundedly far.

The semantics of queries and qualifiers is standard (cf., e.g., [Geerts and Fan 2005]). We write the satisfaction relations as $\tau, n, n^{\prime} \models p$ and $\tau, n \models u$, where $\tau$ is an XML tree $\left\langle N, \Sigma \cup \Sigma^{\prime}, \Lambda, \Delta\right\rangle$, and $n$ and $n^{\prime}$ are $\Sigma$-labelled nodes. The definition is recursive over the grammars of queries and qualifiers, and can be found in Figure 4. We omit the Boolean cases, and we write $\widehat{\nabla}, \widehat{\Delta}, \widehat{\triangleright}$ and $\widehat{\triangleleft}$ for the relations between $\Sigma$-labelled nodes that correspond to the child, parent, next-sibling and previoussibling relations (respectively) in the document that $\tau$ represents.

We say that $\tau$ satisfies $p$ iff $\tau, \varepsilon, n^{\prime} \models p$ for some $n^{\prime}$.

Example 5.1. Suppose $a_{1}^{\prime}, a_{2}^{\prime} \in \Sigma^{\prime}$. The forward query $p_{a_{1}^{\prime}, a_{2}^{\prime}}=\triangleright^{*} / \nabla^{*}\left[\varepsilon / @ a_{1}^{\prime}=\right.$ $\left(\nabla / \nabla^{*}\right) / @ a_{2}^{\prime}$ ] is satisfied by $\Sigma$-labelled nodes $n_{0}$ and $n_{1}$ iff $n_{0} \widehat{\nabla}^{*} \widehat{\nabla}^{*} n_{1}$ and there exists $n_{2}$ such that $n_{1} \widehat{\nabla}^{+} n_{2}$ and the value of attribute $a_{1}^{\prime}$ at $n_{1}$ is equal to the value of attribute $a_{2}^{\prime}$ at $n_{2}$. Hence, the safety forward query $\varepsilon\left[\neg\left(p_{a_{1}^{\prime}, a_{2}^{\prime}} ?\right)\right]$ is satisfied by an XML tree over $\Sigma$ and $\Sigma^{\prime}$ (whose root may have younger siblings) iff the value of $a_{1}^{\prime}$ at a node is never equal to the value of $a_{2}^{\prime}$ at a descendant.

Suppose a query $p$ and a DTD $\mathcal{T}$ are over the same element types and attribute names. We say that $p$ is satisfiable relative to $\mathcal{T}$ iff there exists an XML tree which satisfies $p$ and $\mathcal{T}$. Finitary satisfiability restricts to finite XML trees.

Complexity of Satisfiability. Let us regard a forward qualifier $u$ over element types $\Sigma$ and attribute names $\Sigma^{\prime}$ as finitely equivalent to an ATRA $\mathcal{A}$ with alphabet $\Sigma \cup \Sigma^{\prime}$ iff, for every finite XML tree $\tau$ over $\Sigma$ and $\Sigma^{\prime}$, and $\Sigma$-labelled node $n$, we have $\tau, n \models u$ iff $\mathcal{A}$ accepts the subtree of $\tau$ rooted at $n$. For safety (resp., co-safety) $u$, safety (resp., co-safety) equivalence is defined by also considering infinite XML trees and safety (resp., co-safety) acceptance by $\mathcal{A}$.

To formalise the corresponding notions for queries, we introduce the following kind of automata "with holes". Query automata are defined in the same way as ATRA $_{1}$ (cf. Section 2.2), except that:
-transition formulae may contain a new atomic formula H ;
-no path in the successor graph from the initial state to a state $q$ such that H occurs in some transition formula at $q$ may contain an update edge.

The vertices of the successor graph are all states, there is an edge from $q$ to $r$ iff $r(0, \downarrow), r(0, \npreceq), r(1, \downarrow)$ or $r(1, \npreceq)$ occurs in some transition formula at $q$, and such an edge is called update iff $r(0, \downarrow)$ or $r(1, \downarrow)$ occurs in some transition formula at $q$.

To define a run of a query automaton on a data tree $\tau$ with the same alphabet and with respect to a set of nodes $N^{\prime}$, we augment the definition of runs of ATRA $A_{1}$ so that whenever a transition formula is evaluated at a node $n$, each occurence of H is treated as $\top$ if $n \in N^{\prime}$, and as $\perp$ if $n \notin N^{\prime}$. Acceptance of a finite data tree, safety acceptance, and co-safety acceptance, all with respect to a set of nodes for interpreting H , are then defined as for ATRA $_{1}$.

For a query automaton $\mathcal{A}$ and an ATRA $_{1}$ or query automaton $\mathcal{A}^{\prime}$ with the same alphabet and initial states $q_{I}$ and $q_{I}^{\prime}$ (respectively), we define the substitution of $\mathcal{A}^{\prime}$ for the hole in $\mathcal{A}$ by forming a disjoint union of $\mathcal{A}$ and $\mathcal{A}^{\prime}$, taking $q_{I}$ as the initial state, and substituting each occurence of H in each transition formula $\delta(q, a, b)$ of $\mathcal{A}$ by $\delta\left(q_{I}^{\prime}, a, b\right)$. Observe that the unreachability in $\mathcal{A}$ of H from $q_{I}$ by a path with an update edge means that the composite automaton transmits initial register values to $\mathcal{A}^{\prime}$ without changes.

Now, we say that a forward query $p$ over element types $\Sigma$ and attribute names $\Sigma^{\prime}$ is finitely equivalent to a query automaton $\mathcal{B}$ with alphabet $\Sigma \cup \Sigma^{\prime}$ iff, for every finite XML tree $\tau$ over $\Sigma$ and $\Sigma^{\prime}$, $\Sigma$-labelled node $n$, and set $N^{\prime}$ of $\Sigma$-labelled nodes, we have $\tau, n, n^{\prime} \models p$ for some $n^{\prime} \in N^{\prime}$ iff $\mathcal{B}$ accepts the subtree of $\tau$ rooted at $n$ with respect to $N^{\prime}$. For safety (resp., co-safety) $p$, safety (resp., co-safety) equivalence is defined by also considering infinite XML trees and safety (resp., cosafety) acceptance by $\mathcal{B}$.


Fig. 5. Defining $\mathcal{A}_{\varepsilon / @ a_{1}^{\prime}=(\nabla / p) / @ a_{2}^{\prime}}^{\Sigma, \Sigma^{\prime}}$
THEOREM 5.2. For each forward query $p$ (resp., forward qualifier u) over $\Sigma$ and $\Sigma^{\prime}$, a finitely equivalent query automaton $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$ (resp., ATR $A_{1} \mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ ) is computable in logarithmic space. If $p$ (resp., $u$ ) is safety, then it is safety equivalent to $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$ (resp., $\mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ ).

Proof. The translations are defined recursively over the grammars of queries and qualifiers:
$-\mathcal{B}_{\varepsilon}^{\Sigma, \Sigma^{\prime}}, \mathcal{B}_{\nabla}^{\Sigma, \Sigma^{\prime}}, \mathcal{B}_{\triangleright}^{\Sigma, \Sigma^{\prime}}, \mathcal{B}_{\nabla * *}^{\Sigma, \Sigma^{\prime}}, \mathcal{B}_{\triangleright *}^{\Sigma, \Sigma^{\prime}}$ and $\mathcal{A}_{a}^{\Sigma, \Sigma^{\prime}}$ are straightforward to define;
$-\mathcal{B}_{p \cup p^{\prime}}^{\Sigma, \Sigma^{\prime}}$ is formed from $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$ and $\mathcal{B}_{p^{\prime}}^{\Sigma, \Sigma^{\prime}}$ by disjunctive disjoint union, $\mathcal{A}_{\neg u}^{\Sigma, \Sigma^{\prime}}$ is formed from $\mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ by dualisation, and $\mathcal{A}_{u \wedge u^{\prime}}^{\Sigma, \Sigma^{\prime}}$ is formed from $\mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ and $\mathcal{A}_{u^{\prime}}^{\Sigma, \Sigma^{\prime}}$ by conjunctive disjoint union (cf. the proof of Proposition 2.2);

- to obtain $\mathcal{B}_{p / p^{\prime}}^{\Sigma, \Sigma^{\prime}}$, we substitute $\mathcal{B}_{p^{\prime}}^{\Sigma, \Sigma^{\prime}}$ for the hole in $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$;
-to obtain $\mathcal{B}_{p[u]}^{\Sigma, \Sigma^{\prime}}$, we substitute a conjunctive disjoint union of $\mathcal{B}_{\varepsilon}^{\Sigma, \Sigma^{\prime}}$ and $\mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ for the hole in $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$;
$-\mathcal{A}_{p ?}^{\Sigma, \Sigma^{\prime}}$ is formed from $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$ by substituting $\top$ for $\mathrm{H} ;$
—an automaton for $\varepsilon / @ a_{1}^{\prime}=(\nabla / p) / @ a_{2}^{\prime}$ is formed by substituting the second automaton depicted in Figure 5 (cf. Example 2.4 for depicting conventions) for the hole in $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$, and substituting the result for the hole in the first automaton depicted in Figure 5;
- the remaining cases in the grammar of qualifiers are handled similarly.

The required equivalences, as well as that if $p$ (resp., $u$ ) is co-safety then it is cosafety equivalent to $\mathcal{B}_{p}^{\Sigma, \Sigma^{\prime}}$ (resp., $\mathcal{A}_{u}^{\Sigma, \Sigma^{\prime}}$ ), are shown simultaneously by induction.

Theorem 5.3. (a) For forward XPath and arbitrary DTDs, satisfiability over finite XML trees is decidable.
(b) For safety forward XPath and arbitrary DTDs, satisfiability over finite or infinite XML trees is decidable.

Proof. Given a forward query $p$ and a DTD $\mathcal{T}$ over element types $\Sigma$ and attribute names $\Sigma^{\prime}$, by Theorem 5.2 , an $\operatorname{ATRA}_{1} \mathcal{A}_{p}^{\Sigma, \Sigma^{\prime}}$ is computable, which is finitely equivalent to the qualifier $p$ ?. We can then compute an ITCA $\mathcal{C}\left(\mathcal{A}_{p \text { ? }}^{\Sigma, \Sigma^{\prime}}\right)$ as in the proof of Theorem 3.1, which recognises exactly trees obtained by erasing data from finite XML trees that satisfy $p$. To conclude (a), we observe that ITCA are closed (in logarithmic space) under intersections with forward nondeterministic tree automata, and apply Theorem 2.5.
For (b), supposing that $p$ is safety, by Theorem 5.2 again, an $\operatorname{ATRA}_{1} \mathcal{A}_{p \text { ? }}^{\Sigma, \Sigma^{\prime}}$ is computable, which is safety equivalent to the qualifier $p$ ?. Applying the proof of Theorem 4.1 to $\mathcal{A}_{p}^{\Sigma, \Sigma^{\prime}}$ and an ATRA ${ }_{1}$ whose safety language is empty, we can compute an ITCANT $\mathcal{C}^{\prime}\left(\mathcal{A}_{p}^{\Sigma, \Sigma^{\prime}}\right)$, which contains no cycles of $\varepsilon$-transitions and recognises exactly trees obtained by erasing data from finite or infinite XML trees that satisfy $p$. It remains to observe that ITCANT with no cycles of $\varepsilon$-transitions are closed (in logarithmic space) under intersections with forward nondeterministic tree automata, and to recall that their nonemptiness was shown decidable also in the proof of Theorem 4.1.

We remark that, by the proof of [Demri and Lazić 2009, Theorem 5.2], finitary satisfiability for forward XPath with DTDs is not primitive recursive, even without sibling axes (i.e., $\triangleright$ and $\triangleright^{*}$ ).

## 6. CONCLUDING REMARKS

It would be interesting to know more about the complexities of nonemptiness for safety ATRA $_{1}$ and satisfiability for safety forward XPath with DTDs. By Theorem 4.1, the former is decidable and not elementary, and by Theorem 5.3(b), the latter is decidable.

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