Non-Markovian Petri Nets

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Abstract

Non-Markovian models allow us to capture a very wide range of circumstances in which it is necessary to model phenomena whose times to occurrence is not exponentially distributed. Events such as timeouts in a protocol, service times at a machine performing the same task on each part, and memory access or instruction execution in a low-level h/w or s/w model, have durations which are constant or with a very low variance. Phase-type distributions can be used to approximate a non-exponential, but they increase the size of the state space.

The analysis of stochastic systems with non-exponential timing is of increasing interest in the literature and requires the development of suitable modeling tools. Recently, some effort has been devoted to generalize the concept of Stochastic Petri Nets *(SPN), by allowing the firing times to be generally distributed.*

A particular case of non-Markovian SPN, *is the class of* Deterministic and SPN (DSPN) *[1]. A* DSPN *is a non-Markovian* SPN *where, in each marking, at most one transition is allowed to have a deterministic firing time with enabling memory policy.*

A new class of stochastic Petri nets has recently been defined [2, 3] by generalizing the deterministic firing times of the DSPN to generally distributed firing times. The underlying stochastic process for these classes of Petri nets is a Markov Regenerative Process *(MRGP). This observation has opened a very fertile line of research aimed at the definition of solvable classes of models whose underlying marking process is an MRGP, and therefore referred to as* Markov Regenerative Stochastic Petri Nets (MRSPN).

Some of the results in this filed will be described in the session. In particular, Ciardo investigates stochastic confusion by defining the selection probability for transitions attempting to fire at the same time. German introduces the "method of supplementary variables" for the derivation of state equations describing the transient behavior of the marking process. Puliafito describes how, under some constraints, concurrent enabling of several generally distributed timed transitions is allowed. Bobbio and Telek discuss how age memory policy can be included to capture preemptive mechanisms of the resume (prs) *type.*

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Two open problems in managing SPN with generally distributed transitions are:

- No efficient algorithm to determine an embedding resulting in numerically computable regeneration intervals is available.
- More than one embedding might be possible, and we do not know how to choose the "best" one.

  In this area, we have shown that the requirement of at most one generally distributed transition enabled in each marking is not necessary , but no alternative necessary condition verifiable at the net-level is known.

The exponential assumption is actually a disadvantage when the effect of contemporary events must be modeled explicitly: a discrete-time model might be more appropriate. But discrete time brings some new problems:

- The semantic of contemporary firings has been discussed only for immediate transitions in GSPNs, while earlier definitions of discrete-time SPNs fail to distinguish this issue from the timing of activities; the definition of preselection and postselection priority for discrete-time Markov SPNs [4] is an improvement, but much work remains to be done.

  In particular, we intend to investigate stochastic confusion, and its resolution at the net level, by defining the selection probability for transitions attempting to fire at the same time.

- If no common timestep exists for the firing times, the process is not Markovian. This case has received little attention.

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We show that the "method of supplementary variables" can be applied for the derivation of state equations describing the transient behavior of the marking process of an MRSPN [5]. The system of state equations consists of partial and ordinary differential equations, and initial and boundary conditions. It turns out that the partial differential equations can be solved in isolation. Inserting the solution into the remaining equations leads to a system of ordinary differential equations combined with initial and boundary conditions. Based on a discretization of the continuous time variables it is then possible to compute the solution iteratively. Several numerical subalgorithms are required, e.g., the randomization technique and Runge-Kutta methods. Furthermore, possible simplifications are discussed for the class of deterministic and stochastic Petri nets (DSPNs), where the general firing times are restricted to deterministic delays.

The software tool TimeNET was developed at the Technische Universität Berlin and provides several components for the analysis and simulation of non-Markovian stochastic Petri nets. Numerical examples using TimeNET will be given in order to illustrate the transient analysis techniques.

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We introduce a class of stochastic Petri nets, indicated as Concurrent Generalized Petri Net (CGPN), which allow simultaneous enabling of several generally distributed timed transitions [6]. Like MRSPN*s, the solution method is based on MRGPs but we extend the field of application, allowing the simultaneous enabling of any number of immediate, exponentially distributed and generally distributed timed transitions, under the hypothesis that the latter are all enabled at the same instant.

The stochastic process underlying a CGPN is shown to be still an MRGP. We evaluate the local (E(t)) and global kernel (K(t)) distributions of the underlying MRGP and define the steps required to generate it automatically. We also deal with the case in which the cumulative distribution functions of the GEN transitions have discontinuities and define the steps required to consider this circumstance for the generation of E(t) and K(t). Steady state as well transient analysis solution techniques are also defined. Finally the use of CGPNs to model and evaluate a system based on a client-server paradigm is described.

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One main limitation of the models discussed in the recent publications is that the generally distributed (or deterministic) transitions must be assigned a firing policy of enabling memory type. The enabling memory policy means that whenever the transition becomes enabled anew, its firing distribution is resampled and the time eventually spent without firing in prior enabling periods is lost. In the language of queueing systems the above mechanism is referred to as (*preemptive repeat different (prd)* policy.

The age memory is able to capture preemptive mechanisms of resume *(prs)* type, where an interrupted activity is recovered by keeping memory of the work already performed, and upon restart, only the residual service needs to be completed. This modeling extension is crucial in connection with fault tolerant and dependable computing systems, where a task, interrupted either during a fault/recovery cycle or for the execution of a higher priority task, must be resumed from the point it was interrupted.

We provide a unified solution for the transient and steady state analysis of a class of *MRSPN* with both age and enabling type general transitions [7].

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