TECHNICAL REPORT
Sensor Activation and Radius Adaptation (SARA) in Heterogeneous Sensor Networks

NOVELLA BARTOLINI, TIZIANA CALAMONERI
"Sapienza" University of Rome, Italy
TOM LA PORTA
Pennsylvania State University, USA
and
CHIARA PETRIOLI, SIMONE SILVESTRI
"Sapienza" University of Rome, Italy

In this paper we address the problem of prolonging the lifetime of wireless sensor networks (WSNs) deployed to monitor an area of interest. In this scenario, a helpful approach is to reduce coverage redundancy and therefore the energy expenditure due to coverage.

We introduce the first algorithm which reduces coverage redundancy by means of Sensor Activation and sensing Radius Adaptation (SARA) in a general applicative scenario with two classes of devices: sensors that can adapt their sensing range (adjustable sensors) and sensors that cannot (fixed sensors). In particular, SARA activates only a subset of all the available sensors and reduces the sensing range of the adjustable sensors that have been activated. In doing so, SARA also takes possible heterogeneous coverage capabilities of sensors belonging to the same class into account. It specifically addresses device heterogeneity by modeling the coverage problem in the Laguerre geometry through Voronoi-Laguerre diagrams.

SARA executes quickly and is guaranteed to terminate. It provides a configuration of the active set of sensors that meets lifetime and coverage requirements of demanding WSN applications, not met by current solutions.

By means of extensive simulations we show that SARA achieves a network lifetime that is significantly superior to that obtained by previous algorithms in all the considered scenarios.

Categories and Subject Descriptors: C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Distributed networks; network topology*

General Terms: Algorithms, Design, Performance

Additional Key Words and Phrases: Area coverage, wireless sensor networks, heterogeneous devices, variable radii

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Author's address: Novella Bartolini, Tiziana Calamoneri, Chiara Petrioli and Simone Silvestri, Department of Computer Science, "Sapienza" University of Rome, Via Salaria 113, 198 Rome, Italy . E-mail: {novella, calamo, petrioli, silvestris}@di.uniroma1.it
Tom La Porta, Pennsylvania State University, University Park, PA, USA. E-mail:tlp@cse.psu.edu
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1. INTRODUCTION

As large collections of networked, inexpensive devices, Wireless Sensor Networks (WSNs) are the technology of choice for applications requiring seamless and pervasive coverage of geographic areas, buildings and public or private spaces and structures. Critical applications such as access control and intrusion/hazard detection as well as less critical tasks of which wildlife monitoring and precision agriculture are typical examples, are best served by the infrastructure-less and unobtrusive nature of WSNs.

Since sensor nodes typically have limited battery power, meeting coverage requirements with minimal energy expenditure is a primary issue. For years this problem has been tackled by designing protocol stacks that are energy efficient, implicitly assuming that the culprit of most of the energy consumption of a node is the communication circuitry. As a consequence, solutions that enhance network performance (lifetime, capacity, etc.) have been proposed that are based on methods that reduce communication costs: Data fusion and filtering techniques (for limiting the number of transmissions) [Nakamura et al. 2007], new and advanced forms of energy provisioning [Sharma et al. 2010; Moser et al. 2010; Kansal et al. 2007], clever exploitation of the mobility of network components [Basagni et al. 2008] as well as optimized protocol design [Yick et al. 2008]. However, the level of improvement that energy-efficient techniques for communication can produce is starting to plateau because of the inevitable trade-offs that they impose (e.g., energy conservation versus latency). At the same time, the sensing devices mounted on the wireless node have become more numerous and more sophisticated. Along with the cheap sensors, e.g., those for temperature and humidity, it is now common to endow even small nodes with cameras and active sensors such as radars and sonars, which demand non-negligible energy from the node. Therefore, for providing critical enhancement to network performance, it is no longer possible to focus only on reducing communication costs. Careful consideration must be also given to the sensory component of the node. We also note that, unlike"on-off" sensors, like those for temperature, light, and humidity, more sophisticated sensors consume energy depending on their sensing range. Therefore, similar to communication power control, *sensing coverage control* becomes an important element in the overall WSN performance optimization process. In particular, *sensor activation and radius adaptation*, the ability of selecting which sensor to activate[1]

and to what level of coverage, are necessary new ingredients for the design of durable and reliable WSNs.

In this paper we present a new solution for the joint problem of dynamically scheduling the activation of different subsets of sensor nodes and of tuning their sensing radii (if their technology allows) for prolonging the network lifetime while ensuring the coverage of the given Area of Interest (AoI)[2]. Sensor activation as a research area has received considerable attention in the recent past. In particular, two selective activation algorithms have been proposed that have been shown to outperform other solutions for the problem: The Distributed Lifetime Maximization (`DLM`) scheme [Kasbekar et al. 2009], and the Variable Radii Connected Sensor Cover (`VRCSC`) [Zou et al. 2009].

In this paper we propose an algorithm called `SARA`, standing for *Sensor Activation and*

[1] With *sensor activation* we indicate the turning on of the sensing and communication units of a node. When this happens, the sensor is *awake*. A sensor goes to *sleep* by turning off (or by switching to low power mode) both its sensing and communication units.
[2] As usually done in the literature, we assume that the AoI is a convex region.

Radius Adaptation that, to the best of the authors' knowledge, is the first algorithm working in the general scenario of heterogeneous networks. Our algorithm follows an original approach to solve the coverage problem, as it makes use of the Voronoi diagrams in the Laguerre geometry to determine the coverage responsibility of each node.

SARA achieves the following desirable properties (theoretically and experimentally proven in the following).

- It ensures maximum sensing coverage at all times, i.e., activated nodes are able to cover the same area that would be covered if all nodes that are still operational were activated with their maximum transmission range.
- It accommodates WSNs comprised of heterogeneous nodes, endowed with active and passive sensors with fixed or adjustable sensing radius.
- It is Pareto optimal, unlike DLM and VRCSC. (This property constitutes a necessary requirement for a sensor activation and radius adaptation policy to be optimal.)
- It is robust with respect to different definitions of coverage requirements and network lifetime.

The performance of SARA has been evaluated by means of simulation experiments on WSNs with heterogeneous nodes. The results of our experiments show that SARA is able to quickly configure the network in a way that ensures low energy consumption and long lifetime. We also conducted a comparative performance evaluation of SARA with DLM and VRCSC, which revealed the superiority of SARA in terms of coverage extension and network lifetime in a wide range of operative settings, including the ones for which those previous solutions were specifically designed.

The paper is organized as follows. Section 2 introduces the problem of radius adaptation and sensor activation. Section 3 motivates the use of the Voronoi-Laguerre measure to address device heterogeneity and provides the notions of computational geometry needed to fully understand the proposed solution. In Sections 4 and 5 we describe SARA and prove its Pareto optimality, convergence and termination. Section 6 briefly describes the protocols selected as benchmarks: DLM and VRCSC. A thorough performance evaluation of SARA is then provided in Section 7, including a comparison between SARA, DLM and VRCSC performance. Finally, Section 9 surveys the literature on related topics, while Section 10 concludes the paper.

2. PROBLEM FORMULATION

In this paper we consider heterogeneous WSNs, where the nodes are endowed with several kinds of sensing technologies. In particular, we focus on the use of two types of sensors, namely, those with adjustable sensing radius and those with fixed radius. The capability to adjust the sensing range is typical of devices based on active sensing technologies, such as those equipped with radars and sonars. The power consumption of this kind of sensor depends on the extent of the sensing radius. For this type of sensors setting the sensing range to the minimum necessary for coverage decreases energy consumption. Although not all commercial active devices allow radius to be adapted, some sensors with variable sensing ranges are already commercially available [OSIRIS photoelectric sensors 2010; Kompis and Aliwell 2010]. By contrast, for sensors based on passive sensing technologies (e.g., those equipped with piezoelectric sensors or thermometers) the monitoring activity typically consists in taking single point measures. For these devices the sensing range is typically fixed and so is their energy consumption. An exception is the case of low power CMOS cameras, based on a passive sensing approach, where the depth of field can be adjusted to guarantee a given quality of monitoring at certain distances.

We consider a set $\mathcal{S} = \mathcal{S}_{\texttt{adjustable}} \cup \mathcal{S}_{\texttt{fixed}}$ of $|S| = N$ sensors, where $\mathcal{S}_{\texttt{adjustable}}$ contains the nodes with adjustable sensing radius (hereby shortly called *adjustable sensors*) and $\mathcal{S}_{\texttt{fixed}}$ those with a fixed radius (shortly called *fixed sensors*) . If a node s_i belongs to the set $\mathcal{S}_{\texttt{adjustable}}$ its sensing radius r_i can be set to any value from 0 to $r_i^{\texttt{max}}$. For a node $s_j \in \mathcal{S}_{\texttt{fixed}}$ the sensing radius r_j is either 0, meaning that the sensing unit is turned off, or $r_j^{\texttt{fixed}}$, when the sensing unit is turned on. The sensors of the two sets can also have heterogeneous transmission radii $r_i^{\texttt{tx}}$, $i = 1, \ldots, N$. We assume that the transmission radii are such that any two sensors with intersecting or tangential sensing circles are connected to each other. Therefore, complete coverage implies also that the WSN is connected, and no sensor should be kept awake if it is not necessary for coverage.

An exact model of the relationship between the energy consumed by a node for sensing and the extent of its sensing radius cannot be given as it is dependent on the sensing technology and electronic circuitry for detection. For the purpose of our work, we refer to a general approximate model also used in [Pattem et al. 2003; Zou et al. 2009] according to which if sensor s_i has sensing radius r_i the energy consumption per time unit is given by

$$E_{\texttt{sensing}}(r_i) = a \cdot r_i^c + b. \tag{1}$$

The parameters a and b are device specific constants. The parameter c is related to the sensing technology in use and typically varies in the range $[2, 4]$ in case of sensors adopting an active sensing technology.

The energy consumption due to communications is also dependent on the specific type of device being considered. It is typically an increasing function of the transmission radius, which takes into account all the energy consuming activities related to radio communications, namely transmissions, receptions and idle listening to the radio channel. In this paper we consider the energy cost model of Telos nodes [Polastre et al. 2005].

The problem addressed in this paper is the following: *Given a WSNs each sensor $s_i \in S$ has to decide whether to activate itself or not at any given time and, if active, how to set its sensing radius r_i at that time. The objective is guaranteeing maximum sensing coverage while prolonging the network lifetime as much as possible.*

Here we define the network lifetime as the time during which the network is able to guarantee the coverage of a given percentage p of the AoI. For instance, if $p = 100\%$ the network lifetime is the time at which the first coverage hole appears. If $p = x\%$ the network lifetime is the first time at which less than $x\%$ of the AoI is covered [3].

3. PRELIMINARIES ON VORONOI LAGUERRE DIAGRAMS AND ON THEIR USE TO DETERMINE AND REDUCE COVERAGE REDUNDANCY

Prior works on sensor networks very often rely on the use of Voronoi diagrams to model coverage, such as in [Wang et al. 2006] for mobile sensors, in [Ammari and Das 2008] for energy aware routing, or in [Zou et al. 2009] for selective activation. Voronoi diagrams can be used to model the coverage problem only in the case of sensors endowed with equal

[3] Definitions of lifetime based on the percentage of alive nodes [Bough and Santi 2002] can be adopted as well. Although more commonly used in the literature, these different notions of lifetime are less suitable than our when the applicative task is coverage of an AoI.

sensing radii as discussed in [Bartolini et al. 2009]. In order to address the problem of coverage in the presence of heterogeneous devices, namely devices with different sensing ranges and different capability to adapt their setting, in this section we introduce the notion of Voronoi diagrams in *Laguerre geometry*. We also discuss how these diagrams can be exploited to decrease coverage redundancy (and thus the energy consumption due to sensing) while preserving network coverage and connectivity.

In a Voronoi diagram, we call the axis generated by two sensors which is equidistant from them and perpendicular to their connecting segment the *Vor line* . This line divides the plane into two halves. In the case of sensors with the same sensing radius the Vor line properly delimits the responsibility regions of the two sensors as it is the symmetry axis between the two. If the sensors have heterogeneous radii, the Vor line may not determine the responsibility region correctly, as depicted in Fig. 1. Indeed, according to a Voronoi-based partition of coverage responsibilities, the sensor positioned in C_1 has the responsibility to sense anything to the left of the Vor line, and the sensor positioned in C_2 should sense anything to the right. In particular, the grey areas in the figure would incorrectly be assigned to the sensor in C_1, whereas they are covered only by the sensor in C_2. The line which correctly delimits the responsibility regions of the two sensors is the one that is equidistant from C_1 and C_2 in Laguerre geometry. In Figure 1 this line is called *VorLag*.

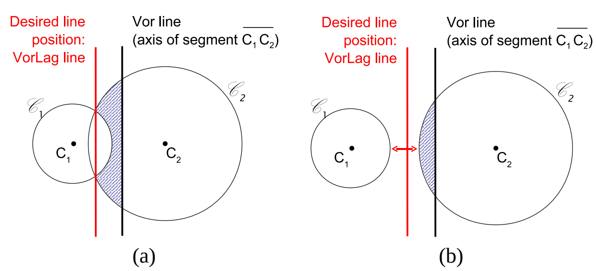


Fig. 1. Different positions of the line equidistant from C_1 and C_2 according to the Euclidean (Vor) and to the Laguerre (VorLag) distance in the case of intersecting (a) and non-intersecting circles (b).

Formally, given a circle $\mathscr{C}$ with center $\mathrm{C} = (x_\mathrm{C}, y_\mathrm{C})$ and radius r_C, and a point $\mathrm{P} = (x_\mathrm{P}, y_\mathrm{P})$ in the plane $\Re^2$, the Laguerre distance $d_L(\mathscr{C}, \mathrm{P})$ between $\mathscr{C}$ and P is defined as follows:

$$d_L^2(\mathscr{C}, \mathrm{P}) = d_E^2(\mathrm{C}, \mathrm{P}) - r_\mathrm{C}^2, \tag{2}$$

where $d_E(\mathrm{C}, \mathrm{P})$ is the Euclidean distance between the points C and P. In Laguerre geometry, given two circles with distinct centers and possibly different radii, the locus of the points equally distant from them is a line, called *VorLag line*, that is perpendicular to the segment connecting the centers. If the two circles intersect each other, their VorLag line crosses their intersection points, as in Fig. 1 (a) [Imai et al. 1985].

Given N circles $\mathscr{C}_i$ with centers $\mathrm{C}_i = (x_i, y_i)$ and radii r_i, $i = 1, \ldots, N$, the *Voronoi-Laguerre polygons* $V(\mathscr{C}_i)$ for the circles $\mathscr{C}_i$ are defined as

$$V(\mathscr{C}_i) = \{\mathrm{P} \in \Re^2 | d_L^2(\mathscr{C}_i, \mathrm{P}) \leq d_L^2(\mathscr{C}_j, \mathrm{P}), \forall j \neq i\}, i = 1, \ldots, N.$$

A Voronoi-Laguerre polygon is always convex. A tessellation of the plane into Voronoi-Laguerre polygons is called a *Voronoi-Laguerre diagram*. Obviously, if $r_i = r_j$ for all $i, j = 1, \ldots, N$, the Voronoi-Laguerre diagram reduces to the ordinary Voronoi diagram. Notice that it may happen that the Voronoi-Laguerre polygon $V(\mathscr{C}_i)$ does not contain any point of the plane. This happens when the half-planes generated by the VorLag lines formed by $\mathscr{C}_i$ and its nearby circles have no overlap. In this case, $V(\mathscr{C}_i)$ is called a *null polygon*. The occurrence of null polygons is specific of Voronoi-Laguerre diagrams and reflects a situation of complete redundancy that is not captured by traditional Voronoi diagrams for which the generated polygons are always not null.

In the following the sensor s_i whose sensing circle $\mathscr{C}_i$ generates the polygon $V(\mathscr{C}_i)$ is called the *generator* of $V(\mathscr{C}_i)$; the vertices of the same polygon are hereby shortly referred to as *Voronoi-Laguerre vertices*.

Two sensors are *Voronoi-Laguerre neighbors* if their polygons have one edge in common. Given a sensor $s_i \in S$, the set of its Voronoi-Laguerre neighbors is hereafter referred to as $\mathcal{N}_S(s_i)$. Furthermore, we refer to $\mathcal{N}_S^{\emptyset}(s_i)$ as the set of sensors with null polygons which have a sensing overlap with the sensor s_i:

$$\mathcal{N}_S^{\emptyset}(s_i) = \{s_j \in S : d_E(s_i, s_j) \leq (r_i + r_j) \wedge V(\mathscr{C}_j) = \emptyset\}.$$

The reason why Voronoi Laguerre diagrams perfectly model the coverage problem in the case of heterogeneous sensors is their capability to partition the area of interest into polygonal regions which in fact represent the responsibility regions of the deployed sensors. Indeed, a fundamental property of the Voronoi diagrams in the Laguerre geometry is the following:

THEOREM 3.1. *([Bartolini et al. 2009]) Let us consider N circles $\mathscr{C}_i$, with centers $C_i = (x_i, y_i)$ and radii r_i, $i = 1, \ldots, N$, and let $V(\mathscr{C}_i)$ be the Voronoi-Laguerre polygon of the circle $\mathscr{C}_i$. For all $k, j = 1, 2, \ldots, N$, $V(\mathscr{C}_k) \cap \mathscr{C}_j \subseteq \mathscr{C}_k$.*

Less formally, if a point P of the area of interest is covered by at least one sensor, it is certainly covered also by the sensor s_i that generates the Voronoi-Laguerre polygon $V(\mathscr{C}_i)$ that includes P.

3.1 Characterization of coverage redundancy

We define as *redundant* any sensor $s_i \in \mathcal{S}$ such that the sensing circle $\mathscr{C}_i$ is completely covered by other sensors, namely $\mathscr{C}_i \subseteq \cup_{s_j \in \mathcal{S}, j \neq i} \mathscr{C}_j$. The following corollaries 3.1, 3.2 and 3.3 of Theorem 3.1 show the criteria to decide whether s_i is redundant.

COROLLARY 3.1. *If a sensor s_i does not cover any point of its Voronoi-Laguerre polygon $V(\mathscr{C}_i)$, then its sensing circle $\mathscr{C}_i$ is completely covered by other sensors in $\mathcal{S}$. Therefore s_i is redundant.*

PROOF. Since by hypothesis $V(\mathscr{C}_i) \cap \mathscr{C}_i = \emptyset$, $\mathscr{C}_i$ contains only points that are external to its polygon. Therefore, if $P \in \mathscr{C}_i$ then P is covered by the generating sensor of the polygon to which it belongs (for Theorem 3.1). □

Corollary 3.1 affirms that if s_i does not cover its polygon, it can be turned off without affecting coverage.

COROLLARY 3.2. *Given a sensor s_i which covers only a portion of its polygon $V(\mathscr{C}_i)$, let ℓ be a circular segment on the intersection between the boundary of $\mathscr{C}_i$ with the polygon $V(\mathscr{C}_i)$. All the points on ℓ which are not on edges of $V(\mathscr{C}_i)$ are covered only by s_i.*

PROOF. By hypothesis, the region $V(\mathscr{C}_i) \setminus \mathscr{C}_i$ is not covered by the generating sensor of the polygon to which it belongs (that is the sensor s_i). Therefore, due to theorem 3.1, it is not covered by any sensor. Consider any circular segment ℓ on the boundary of $\mathscr{C}_i$ and inside $V(\mathscr{C}_i)$ (see Fig. 2 in which ℓ is the arc $\widehat{DF}$) and a point P on ℓ but not on the edges of $V(\mathscr{C}_i)$. We want to show that s_i is the only sensor which covers P. Since P is not on the edges of the polygon, it is possible to find a value of ϵ arbitrarily small, such that the ϵ-surrounding of P is internal to $V(\mathscr{C}_i)$. The intersection of this ϵ-surrounding with the region $V(\mathscr{C}_i) \setminus \mathscr{C}_i$ (that in Fig. 2 is delimited by the segments $\overline{EF}$, $\overline{DE}$ and by the arc $\widehat{DF}$) is obviously uncovered.

We now proceed by contradiction. Let us assume that there is another sensor $s_j \in \mathcal{S}$ such that P is also covered by s_j. Since, by construction, any ϵ-surrounding of P contains an uncovered region, the circle $\mathscr{C}_i$ can cover P only with its boundary. Furthermore, since s_j cannot cover points of $V(\mathscr{C}_i) \setminus \mathscr{C}_i$, then s_j must be tangential to $\mathscr{C}_i$ in P, and must have a lower sensing radius $r_j < r_i$. However this implies that P would be crossed by the Voronoi-Laguerre edge formed by s_i and s_j, and the portion of $V(\mathscr{C}_i)$ on the opposite side of this edge with respect to $\mathscr{C}_i$ could not belong to $V(\mathscr{C}_i)$, contradicting our construction. ∎

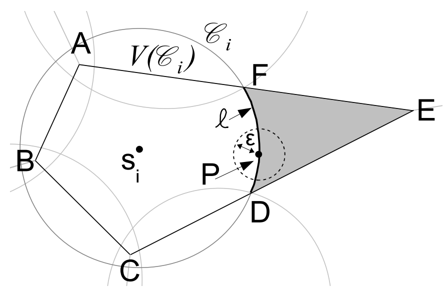


Fig. 2. Voronoi-Laguerre polygon partially covered by its generating sensor.

Corollary 3.2 states that if s_i only partially covers its polygon, it cannot reduce its sensing radius without affecting coverage.

COROLLARY 3.3. *Let us consider a sensor s_i, with sensing circle $\mathscr{C}_i$ and Voronoi-Laguerre polygon $V(\mathscr{C}_i)$. Let P be a point that is covered by s_i and is internal to its polygon, that is $P \in V(\mathscr{C}_i) \cap \mathscr{C}_i$. If P is covered by a sensor in $\mathcal{S}$ other than s_i, then there exists a sensor $s_k \in \mathcal{N}_{\mathcal{S}}(s_i) \cup \mathcal{N}_{\mathcal{S}}^{\emptyset}(s_i)$ such that P is also covered by s_k. In other words, any point of $V(\mathscr{C}_i)$ that is covered by more than one sensor, is certainly covered at least by the generating sensor s_i and by one of its Voronoi-Laguerre neighbors or a sensor with null polygon.*

PROOF. Let $\mathscr{D}$ be the Voronoi-Laguerre diagram generated by $\mathcal{S}$ and $\mathscr{D}'$ be the diagram generated by $\mathcal{S}' = \mathcal{S} \setminus \{s_i\}$. In the diagram $\mathscr{D}$, $P \in V(\mathscr{C}_i)$. By contrast, in the diagram $\mathscr{D}'$, the sensor s_i is not present.

Since by the hypothesis, P is covered by a sensor in $\mathcal{S}'$, thanks to Theorem 3.1 we can affirm that P is also covered by the generating sensor s_k of the polygon, such that $P \in V'(\mathscr{C}_k)$ defined in $\mathscr{D}'$. Obviously, $V'(\mathscr{C}_k) \neq V(\mathscr{C}_k)$. Let us assume, for sake of contradiction, that $s_k \notin N_{\mathcal{S}}(s_i) \cup \mathcal{N}^{\emptyset}_{\mathcal{S}}(s_i)$ If the sensor s_k is not a Voronoi-Laguerre neighbor of s_i and it has not a null polygon in $\mathscr{D}$, its polygon in $\mathscr{D}'$ would be the same as in $\mathscr{D}$, because it would be delimited by edges formed by sensors other than s_i. Therefore it would be $V'(\mathscr{C}_k) = V(\mathscr{C}_k)$, which is a contradiction.

□

Corollary 3.3 states that in order to decide whether s_i can reduce its radius or be turned off it is sufficient to evaluate the coverage of the sensors in $\mathcal{N}_{\mathcal{S}}(s_i) \cup \mathcal{N}^{\emptyset}_{\mathcal{S}}(s_i)$.

3.2 Reducing the redundancy of sensors with adjustable sensing radius

The corollaries 3.1, 3.2 and 3.3 let us determine whether an adjustable sensor s_i can reduce its sensing radius or turn itself off. In particular: (1) if the sensor s_i does not cover any point of its polygon, s_i can be turned off (in consequence of Corollary 3.1); (2) if s_i covers its polygon only partially, s_i must stay awake and work with its current radius (in consequence of Corollary 3.2); (3) if s_i covers its polygon completely, it may reduce its sensing radius of an extent that can be determined on the basis of the coverage of its neighbors (in consequence of Corollary 3.3).

We now address the third situation more in detail. If s_i covers its polygon completely, it can shrink its sensing radius to the distance between s_i and the farthest vertex $f(V(\mathscr{C}_i))$ of its polygon, without compromising maximum sensing coverage.

As an example of sensing radius reduction, let us consider the sensor s_1 in Figure 3. In Figure 3(a) the farthest vertex of $V(\mathscr{C}_1)$ is at a distance from s_1 which is smaller than its radius. Because of Theorem 3.1 we can assert that all the points that are internal to $\mathscr{C}_1$ but do not belong to $V(\mathscr{C}_1)$ are covered by the sensors generating the Voronoi-Laguerre polygon to which they belong. Therefore s_1 redundantly covers the region within its circle that is external to its polygon and it can reduce its radius to cover no farther than $f(V(\mathscr{C}_1))$, maintaining full coverage of its responsibility region. Such a reduction of the sensing radius of s_1 is shown in Figure 3(b). Changing the sensing radius of s_1 requires the Voronoi Laguerre polygons of s_1 and its Laguerre neighbors to be recomputed, as shown in Fig. 3(c). This reduction step can be repeated until the radius of the sensor s_1 is such that the farthest vertex of the polygon $V(\mathscr{C}_1)$ is on the circle $\mathscr{C}_1$ and the radius cannot be reduced any more (see Figure 3(d)). A convergence proof is given in Section 5, Theorem 5.2.

This repeated reduction of the sensing radius is at the basis of `SARA`, where sensing radii of adjustable sensors are reduced until even a single radius reduction would leave a coverage hole. Note that this process may even lead some sensors to shrink their sensing range to zero (in case of redundant sensors), which means that such sensors are deactivated.

3.2.1 *On a characterization of boundary farthest vertices: Loose and strict farthest vertices.* `SARA` typically considers the distance to the farthest vertex of a Voronoi-Laguerre polygon as a lower bound for the reduction of the sensing radius of the generating sensor. If the radius is reduced below this threshold, there is a loss of coverage in almost all cases. Nevertheless in some extremely rare configurations[4] it is possible to reduce the radius below this distance without any coverage loss, by enforcing an ordering in the radius reduction of neighbor sensors.

Given a sensor s_i, let $f(V(\mathscr{C}_i))$ be the farthest vertex of its Voronoi-Laguerre polygon

[4] In the experiments we obtained such a situation only by construction.

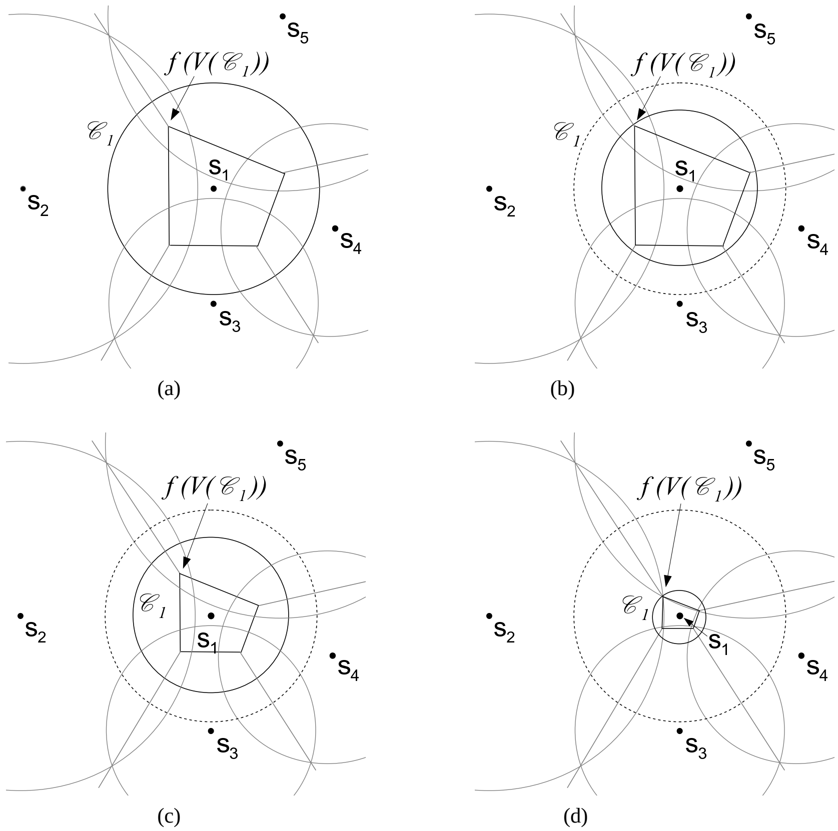


Fig. 3. Iterative reduction of the sensing radius of sensor s_1 to the farthest vertex of its Voronoi-Laguerre polygon.

$V(\mathscr{C}_i)$. We call that the sensor s_i is called the *generating sensor* of the farthest vertex $f(V(\mathscr{C}_i))$ and we call $f(V(\mathscr{C}_i))$ a *boundary farthest* if it lies on the boundary of $\mathscr{C}_i$.

A boundary vertex is the intersection point of three circles and of their three Voronoi-Laguerre axes, and therefore is a boundary vertex for at least three sensors. In the following we say that the boundary farthest vertex of a sensor s_i is a *strict farthest* if the radius of s_i cannot be reduced without leaving a coverage hole. Otherwise such a vertex is called a *loose farthest*. An example of strict and loose boundary farthest vertex is given in Fig.4 (a) and (b), respectively. In the example all sensor nodes have reduced their radius to their farthest vertex which is therefore a boundary farthest vertex. This is when it makes the difference whether a farthest vertex is loose or strict. Let us focus on point F which is a common boundary farthest vertex for the three generating sensors s, s_l and s_k. As Fig.4 (b) shows, F is a loose boundary farthest for sensor s, in fact, s can significantly reduce its sensing radius without compromising coverage. However, a common farthest that is loose for a generating sensor is not necessarily loose for the others. Point F is a strict farthest for the three other sensors s_i, s_l and s_k which cannot reduce their radius.

In general, if s is the only generating sensor for which a boundary farthest is loose, it can reduce its radius without creating any coverage hole: The other generating sensors cannot perform any concurrent reduction since their farthest vertex is strict. In this case, in order to calculate its new radius, s has to subtract from its responsibility region $V(\mathscr{C})$ all the

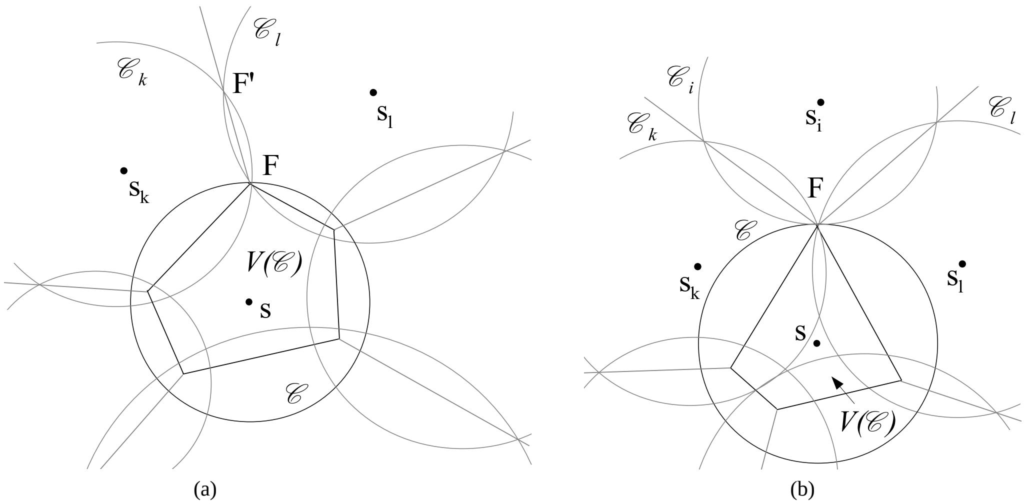


Fig. 4. Strict (a) and loose (b) farthest vertices

areas covered by the other generating sensors and guarantee to cover the farthest point of the remaining region $\overline{V}(\mathscr{C}) \triangleq V(\mathscr{C}) \setminus (\mathscr{C}_k \cup \mathscr{C}_l)$. Fig. 5 (a) shows how the sensor s seen in Figure 4 (b) can reduce its radius to the minimum needed to cover the farthest point B of the region ABCD $=\overline{V}(\mathscr{C})$, shaded in the figure. After this radius reduction, s needs to recalculate its Voronoi-Laguerre polygon and possibly perform a further radius reduction, as in Fig. 5 (b).

Although it is very unlikely to occur, it is theoretically possible for a boundary farthest vertex to be loose for two or more generating sensors. In such a case, a concurrent radius reduction of the two or more sensors having a loose farthest vertex might result in a coverage hole. For this reason we introduce a simple *decision serialization scheme for loose farthest vertices*. This can be easily implemented by means of either a back-off policy or a leader election and a leader arbitrated sensor nodes radius reduction. As there are many well established techniques to solve the problem of serializing decisions in a distributed computing setting, for the sake of simplicity and brevity, we do not address this aspect in the presentation of the algorithm. We refer the reader to the Appendix of this paper for the details of the simple geometrical rules sensors adopt to determine if their boundary farthest vertex is strict or loose.

3.3 Turning off sensors with fixed sensing radius

Not having the capability of tuning the extent of its sensing radius, the only way that a node with fixed radius has to save energy is to go to sleep when it is *redundant*. Therefore, the approach we take for selecting which node with fixed radius should deactivate is based on a greedy algorithm run by each node s that, after a local exchange of information, determines if neighboring nodes can completely cover for s, and if s is the “best” node for deactivation, i.e., the node that allows us to obtain the most energy conservation.

The extent of information needed by a node for deciding whether or not to deactivate can be kept significantly low by exploiting the Voronoi-Laguerre tessellation, in agreement

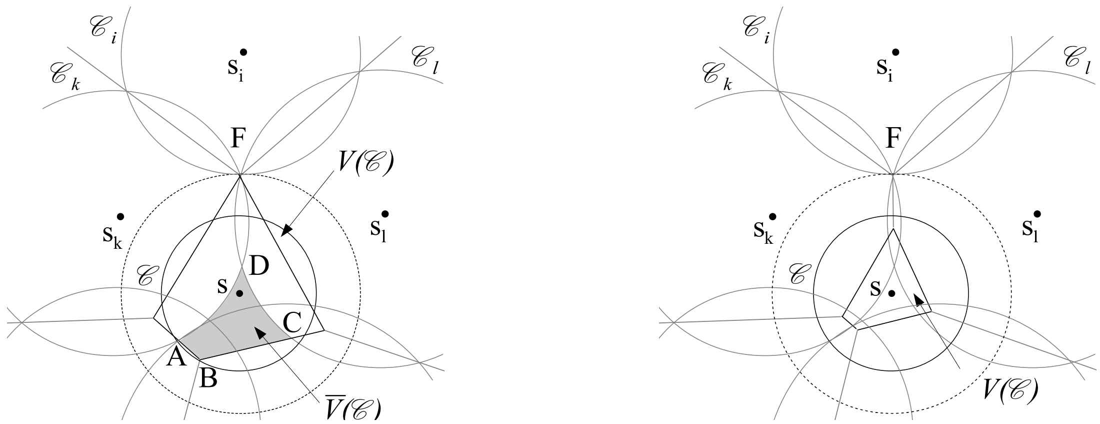


Fig. 5. Reduction of the sensing radius in a situation of loose boundary farthest vertex.

with Corollaries 3.1, 3.2 and 3.3. Three cases may occur: (1) the sensing circle $\mathscr{C}$ of s does not cover any point of its Voronoi-Laguerre polygon $V(\mathscr{C})$, (2) the sensing circle $\mathscr{C}$ only partially covers $V(\mathscr{C})$, (3) the sensing circle $\mathscr{C}$ completely covers the polygon $V(\mathscr{C})$.

In case (1), Corollary 3.1 states that s is certainly redundant. In case (2), Corollary 3.2 states that sensor s cannot be turned off. In case (3) the sensor s must evaluate the coverage of its Voronoi-Laguerre neighbors and of the sensors intersecting $V(\mathscr{C})$ which have null polygons, and determine its redundancy on the basis of Corollary 3.3. The mentioned corollaries set the limit to the number of nodes with which a node s needs to exchange information in order to decide whether to deactivate or not.

4. THE ALGORITHM SARA

SARA is executed in parallel by all the sensors of the network. Its execution results in the selection of a subset of sensors to be kept awake while the others go to sleep, i.e., they are put in a low energy modality or turned off. SARA also allows a node with adjustable radius that is awake to set its sensing range. The obtained sensor activation and radius adjustment is used for a time, called *operative time interval*, that lasts until SARA is re-executed. The operative time interval is not necessarily fixed since SARA execution can be event-driven.[5]

Each sensor makes the decision about whether to (de)activate and about reducing its radius (if possible) iteratively. In order to do so, at each iteration k each node determines its own Voronoi-Laguerre polygon. This requires the node to be aware of its one-hop neighbors (nodes it can communicate with directly), their location[6] and their sensing radius. The iteration is then composed by two phases. During the first phase nodes with fixed

[5] An event-driven reconfiguration requires that sensors operating in low power mode can be contacted by the sink by means of an interest dissemination. Deactivated nodes equipped with a wake-up radio[Gu and Stankovic 2004] can be woken up upon need and can therefore safely turn off their radio for the whole duration of the operative time interval. If such extra HW is not available nodes in low power mode must periodically wake up according to a very low duty cycle so that changes in the mode of operation of the network can be signaled.

[6] This information may be obtained through extra hardware such as GPS, if available, or through one of the many localization schemes recently proposed.

radius decide whether to go to sleep or not. In the second phase, the nodes with adjustable radius perform their radius reduction. Each node s_i bases its decision on a parameter $\alpha_i^{(k)} \in (0, 1]$, which depends on the energy gain that the sensor will achieve by either going to sleep or by reducing its sensing radius. This parameter is used differently depending on whether a node has a fixed or an adjustable radius. Specifically, a node s_i with fixed radius will go to sleep with probability $\alpha_i^{(k)}$ provided that there are neighboring nodes that are awake and redundantly cover its sensing circle. If s_i has an adjustable radius it will reduce it by the fraction $\alpha_i^{(k)}$ of the maximum radius reduction that does not alter the coverage of its responsibility region. As we will prove in Section 5, the iterative execution of the two phases leads to a network configuration in which there is no redundant fixed sensor and it is not possible to further reduce the radius of any adjustable sensor without creating new coverage holes.

4.1 SARA in details

4.1.1 *Initialization.* SARA is described by Algorithms 1 and 3, for nodes with fixed and adjustable radius, respectively. At the start of SARA operations, each sensor sets the iteration counter k and the value of its sensing radius (the maximum value in the case of sensors with adjustable radius). The flag `decision_made` is set to `false` indicating that the node is undecided. The node remains awake and undecided until in one of the iterations it makes a final decision on the value of its sensing radius to be used till a new SARA execution.

Initialization also includes the setting of a timer needed for protocol operations.

4.1.2 *Computing $\alpha_i^{(k)}$.* Consider the k-th iteration of SARA. Let $S^{(k)}_{\texttt{undecided}} \subseteq S^{(k)}_A$ be the set of sensors that have not made their final configuration decision, where $S^{(k)}_A$ is the set of sensors that are still awake. Similarly, $S^{(k)}_{\texttt{decided}} = S^{(k)}_A \setminus S^{(k)}_{\texttt{undecided}}$ is the set of sensors that are still awake and have already made their configuration decision.

Consider $s_i \in S^{(k)}_{\texttt{undecided}}$. Let $\mathscr{L}^{(k)}(s_i)$ be the subset of $S^{(k)}_{\texttt{undecided}}$ including s_i and all the undecided sensors that are either Voronoi-Laguerre neighbors of s_i or have a null polygon and their sensing circle intersects $\mathscr{C}_i$: $\mathscr{L}^{(k)}(s_i) = S^{(k)}_{\texttt{undecided}} \cap (\mathcal{N}_{S_A^{(k)}}(s_i) \cup \mathcal{N}^{\emptyset}_{S_A^{(k)}}(s_i) \cup \{s_i\})$.

Let also $\mathscr{D}^{(k)}(s_i) = \mathcal{S}^{(k)}_{\texttt{decided}} \cap (\mathcal{N}_{S_A^{(k)}}(s_i) \cup \mathcal{N}^{\emptyset}_{S_A^{(k)}}(s_i))$ be the subset of the sensors that have already made their decision and are either Voronoi-Laguerre neighbors of s_i or have a null polygon and overlap the sensing circle $\mathscr{C}_i$.

The computation of the parameter $\alpha_i^{(k)}$ depends on the comparison between s_i and the nodes in $\mathscr{L}^{(k)}(s_i)$ with respect to the decrease in energy consumption that is achievable through sensing radius reduction while ensuring coverage. The comparison is motivated by the fact that these nodes are those that still have the chance to reduce their sensing radius and consequently their energy expenditure. The value of $\alpha_i^{(k)}$ should be higher for a node s_i when choosing it for sensing radius reduction or for going to sleep leads to a better performance gain than choosing the other nodes in the neighborhood.

The criterion we propose to compute $\alpha_i^{(k)}$ is based on the *energy gain*, defined as the amount of energy that a sensor can save by reducing its sensing radius to the farthest point of the responsibility region (in case of sensors with adjustable radius) or by going to sleep (case of sensors with fixed sensing radius).

We recall that $E_{\texttt{sensing}}()$ is the energy expenditure per unit time due to sensing, defined in Equation 1. The energy gain of sensor s_i in the k-th iteration is defined as $\Delta E_i^{(k)} = E_{\texttt{sensing}}(r_i^{\texttt{fixed}})$ for sensors with fixed sensing radius. For sensors with adjustable sensing radius, it is either $\Delta E_i^{(k)} = E_{\texttt{sensing}}(r_i^{(k-1)})$ for sensors which have a null or an uncovered polygon, or $\Delta E_i^{(k)} = E_{\texttt{sensing}}(r_i^{(k-1)}) - E_{\texttt{sensing}}(d_E(s_i, f(\overline{V}^{(k)}(\mathscr{C}_i))))$, otherwise. Here $\overline{V}^{(k)}(\mathscr{C}_i) = V^{(k)}(\mathscr{C}_i) \setminus \cup_{s_j \in \mathscr{D}^{(k)}(s_i)} \mathscr{C}_j$.

The energy gain criterion sets the value of $\alpha_i^{(k)}$ as follows:

$$\alpha_i^{(k)} = \max\left\{ \frac{\Delta E_i^{(k)} - \Delta E_i^{\texttt{min}\ (k)}}{\Delta E_i^{\texttt{max}\ (k)} - \Delta E_i^{\texttt{min}\ (k)}}, \alpha_{\texttt{min}} \right\}, \tag{3}$$

where the parameter $\alpha_{\texttt{min}}$ is an arbitrarily small constant, such that $0 < \alpha_{\texttt{min}} \ll 1$, $\Delta E_i^{\texttt{max}\ (k)} = \max_{s_j \in \mathscr{L}^{(k)}(s_i)} \Delta E_j^{(k)}$ is the maximum achievable gain in the neighborhood of s_i and $\Delta E_i^{\texttt{min}\ (k)} = \min_{s_j \in \mathscr{L}^{(k)}(s_i)} \Delta E_j^{(k)}$ is its minimum value. If $\Delta E_i^{\texttt{max}\ (k)} = \Delta E_i^{\texttt{min}\ (k)}$ we consider $\alpha_i^{(k)} = 1$. According to Eq. 3, the more a node s_i allows energy savings the higher is the probability that it is selected for going to sleep if s_i is a fixed sensor, or the higher is the reduction of sensing radius that is allowed if s_i is an adjustable sensor. This setting of $\alpha_{\texttt{min}}$ ensures that even the sensor with smallest potential energy gain can make a decision that improves its energy expenditure.

The energy gain criterion has been compared by means of extensive simulations with several others, including one based on the node residual energy and one based on an estimate of the node expected lifetime. In all the scenarios the energy gain criterion showed superior performance. Therefore, we will focus only on such a criterion for the remainder of the paper.

4.1.3 SARA *for sensors with fixed sensing radius.* At the beginning of SARA operations, all the sensors with fixed radius are awake and undecided. Let us consider the k-th iterative step of SARA (k-th execution of the `while` cycle in Algorithm 1). The set of sensors that are still awake at the k-th iteration is referred to as $S_A^{(k)} = S_{\texttt{fixed}}^{(k)} \cup S_{\texttt{adjustable}}^{(k)}$.

Each undecided sensor $s_i \in S_{\texttt{fixed}}^{(k)}$ performs an information exchange with its neighbors that are still undecided to gather information regarding their radius and position[7]. With this information, s_i is able to construct its Voronoi-Laguerre polygon $V(\mathscr{C}_i^{(k)})$ and to determine the set $\mathcal{N}_{S_A^{(k)}}(s_i)$.

Node s_i then informs its neighbors with which it has a sensing overlap about the nullity of its polygon. This information allows its neighbors to compute their sets $\mathcal{N}^{\emptyset}_{S_A^{(k)}}$. Each node then evaluates its redundancy status (according to Corollary 3.3).

If s_i is not redundant at the k-th iteration, it cannot become redundant in any of the successive iterations because SARA in each iteration can only reduce the number of sensors that can cover an area. Therefore, in the case of non redundancy, s_i communicates this to the neighbors with a sensing overlap (sending a `turn-on` message), ends the decision

[7] It is not necessary to exchange information with the sensors that have already made their configuration decisions. Also, the node location is communicated at the start of each SARA execution and only if the node location has changed.

phase (setting the `decision_made` flag to `true`), and stays awake.

If sensor s_i is redundant it communicates its potential energy gain to the nodes in $\mathcal{N}_{S_A^{(k)}}(s_i) \cup \mathcal{N}^{\emptyset}_{S_A^{(k)}}(s_i)$.

Nodes with a null polygon also send their potential energy gain to all their neighbors with sensing overlap. Each node is then able to construct the set $\mathscr{L}^{(k)}(s_i)$ and compute $\alpha_i^{(k)}$. The calculus of $\alpha_i^{(k)}$ is executed by running the function `get_alpha` described in Algorithm 2.

Algorithm 1: Algorithm SARA for fixed sensors

```
Algorithm SARA executed by node s_i
    Initialization:
    k = 0;
    Back-off interval = [0, t_max^backoff];
    r_i^(k) = r_i^fixed;
    decision_made=false;
    Exchange position information with neighbors;

    Iterative Voronoi-Laguerre diagram construction:
    while !decision_made do
        Exchange info on radius with neighbors;
        Construct the VorLag polygon V^(k)(C_i);
        Exchange info on null polygons;
        Evaluate redundancy and energy gain;
        if s_i is not redundant then
            // Case of fixed sensors that need to stay awake
            Send turn-on message;
            decision_made=true;
            Stay awake;
        else
            // Case of redundant fixed sensor
            Exchange info on energy gain;
        Build set L^(k)(s_i);
        alpha_i^(k) =get_alpha(L^(k)(s_i));
        Choose a random instant t_i^* ∈ [0, t_max^backoff];
        while t < t_i^* do
            Listen to update messages from the neighborhood;
        if s_i is not redundant anymore then
            Send turn-on message;
            decision_made=true;
            Stay awake;
        else
            With probability alpha_i^(k)
                Send turn-off message;
                decision_made=true;
                Go to sleep;
        k = k + 1;
```

Since more than one sensor may decide to turn themselves off at the same iteration, possibly leaving coverage holes, we introduce a simple back-off scheme to avoid conflicting

Algorithm 2: Function to compute parameter α_i

Function `get_alpha` ($\mathscr{L}^{(k)}(s_i)$)
  Set $\Delta E_i^{\mathtt{max}\ (k)} = \max_{s_j \in \mathscr{L}^{(k)}(s_i)} \Delta E_j^{(k)}$ and
   $\Delta E_i^{\mathtt{min}\ (k)} = \min_{s_j \in \mathscr{L}^{(k)}(s_i)} \Delta E_j^{(k)}$;
  $\alpha_i^{(k)} = \max \left\{ \frac{\Delta E_i^{(k)} - \Delta E_i^{\mathtt{min}\ (k)}(n)}{\Delta E_i^{\mathtt{max}\ (k)} - \Delta E_i^{\mathtt{min}\ (k)}}, \alpha_{\mathtt{min}} \right\}$;
  return $\alpha_i^{(k)}$;

decisions. More precisely, given a back-off interval $t_{\mathtt{max}}^{\mathtt{backoff}}$, each sensor s_i chooses a random instant $t_i^* \in [0, t_{\mathtt{max}}^{\mathtt{backoff}}]$, hereafter called *backoff timeout*. It then waits for a time t_i^*, during which it considers all the messages received from the nodes in radio proximity that may make s_i redundant.

After the expiration of the backoff timeout t_i^*, the sensor s_i verifies if it is still redundant or not. If it is not redundant anymore, s_i decides to stay awake and sets the `decision_made` flag to `true`. It then communicates this decision to its neighbors by sending them a `turn-on` message.

If instead s_i is still redundant, it goes to sleep with probability $\alpha_i^{(k)}$. In the case the node goes to sleep, it sets the `decision_made` flag to `true` and communicates its decision by sending a `turn-off` message.

Notice that a redundant sensor with fixed sensing radius does not necessarily go to sleep at the first iteration. Therefore, the execution of a single iteration of the algorithm does not eliminate the existing redundancy completely. Nevertheless, at each iteration the sensors with higher priority are the ones that more likely will go to sleep. The other redundant sensors will eventually either go to sleep or become non-redundant in one of the subsequent iterations depending on the decisions of their neighbors.

4.1.4 `SARA` *for sensors with adjustable sensing radius.* All sensors with adjustable radius start executing `SARA` by setting their radii to their maximum value. They are also all undecided. As before, we consider the generic k-th iteration of `SARA` (k-th execution of the `while` cycle in Algorithm 3). At each algorithm iteration, the radius reduction decision at node $s_i \in S_{\mathtt{adjustable}}^{(k)}$ is made after the back-off phase of its neighbors in $S_{\mathtt{fixed}}^{(k)}$. At the end of such a phase, every sensor $s_i \in S_{\mathtt{adjustable}}^{(k)}$ updates its Voronoi-Laguerre polygon $V^{(k)}(\mathscr{C}_i)$, updates its information for computing $\alpha_i^{(k)}$ if any of its fixed radius neighbor turned itself off during the back-off, and determines the sets $\mathscr{L}^{(k)}(s_i)$ and $\mathscr{D}^{(k)}(s_i)$. In this way, the sensor s_i has the necessary information to calculate the maximum radius reduction that does not create coverage holes. Notice that in this calculus, the sensors belonging to the two sets $S_{\mathtt{decided}}^{(k)}$ and $S_{\mathtt{undecided}}^{(k)}$ play a different role since the sensors in $S_{\mathtt{decided}}^{(k)}$ will no longer change their configuration for the current execution of `SARA`, therefore their sensing circles can be considered definitely covered and can be subtracted from the responsibility region of those sensors that still have to make their configuration decision. This is the reason why the maximum radius reduction for s_i is computed as the one that does not alter the coverage of the region $\overline{V}^{(k)}(\mathscr{C}_i) = V^{(k)}(\mathscr{C}_i) \setminus \cup_{s_j \in \mathscr{D}^{(k)}(s_i)} \mathscr{C}_j$.

The minimum extent of s_i sensing radius reduction $\overline{d}_i^{(k)}$ is based on Corollaries 3.1, 3.2, and 3.3. If s_i is not able to cover any point of $\overline{V}^{(k)}(\mathscr{C}_i)$ then $\overline{d}_i^{(k)} = 0$ (due to Corollary 3.1). If s_i only partially covers its polygon ($d_E(s_i, c(\overline{V}^{(k)}(\mathscr{C}_i))) < r_i^{(k)} < d_E(s_i, f(\overline{V}^{(k)}(\mathscr{C}_i)))$ where $c(\overline{V}^{(k)}(\mathscr{C}_i))$ is the closest point of $\overline{V}^{(k)}(\mathscr{C}_i)$ from s_i) then $\overline{d}_i^{(k)} = r_i^{(k)}$ (the radius does not change, as determined by Corollary 3.2). Finally, if s_i completely covers its polygon, $\overline{d}_i^{(k)}$ is set to $d_E(s_i, f(\overline{V}^{(k)}(\mathscr{C}_i)))$, that is the Euclidean distance between s_i and the farthest point of $\overline{V}^{(k)}(\mathscr{C}_i)$.

The sensor s_i, whose radius at the k-th iteration is $r_i^{(k)}$, will then reduce its radius to an intermediate value in the range $[\overline{d}_i^{(k)}, r_i^{(k)}]$, whose position is determined by the priority value $\alpha_i^{(k)}$. Therefore s_i calculates the new value of its radius $r_i^{(k+1)}$ as $r_i^{(k+1)} = r_i^{(k)} - \alpha_i^{(k)} \cdot (r_i^{(k)} - \overline{d}_i^{(k)})$.

Each sensor belonging to $S_{\texttt{adjustable}}$ that reduces its radius affects the potential decisions of its Voronoi-Laguerre neighbors, so the process is iterated until no further reductions are possible, because either a strict farthest vertex is on the boundary of the sensing circle, or the Voronoi-Laguerre polygon of the sensor gradually became null, and the sensor is put to sleep.

5. PROPERTIES OF SARA

The execution of SARA on a set of sensors $\mathcal{S}$ leads to a final configuration that will be hereby called *cover set*. In the following we will shortly denote with S_{SARA} such a cover set, where S_{SARA} is a set of awake sensors with their radius configuration decided by SARA.

The following theorem shows that the cover set calculated by SARA provides the same coverage as the starting configuration (the one where all sensors are active at maximum radius).

THEOREM 5.1. **(Coverage equivalence)** *Consider a set of adjustable and fixed sensors $\mathcal{S} = \mathcal{S}_{adjustable} \cup \mathcal{S}_{fixed}$. Let $\mathscr{A} \subseteq AoI$ be the area that the sensors in $\mathcal{S}$ are able to cover if they are all active and the adjustable sensors work at their maximum radius. Let S_{SARA} be the cover set calculated by* SARA. *The coverage extension of S_{SARA} is equal to $\mathscr{A}$.*

PROOF. Let us denote with $S_{\text{SARA}}^{(k)}$ the cover set determined by SARA at the k-th iteration, with $S_{\text{SARA}}^{(0)} = S$. Let us also denote with $\mathscr{A}^{(k)} \subseteq AoI$ the portion of the AoI that is covered by $S_{\text{SARA}}^{(k)}$, therefore

$$\mathscr{A}^{(k)} = \cup_{s_j \in S_{\text{SARA}}^{(k)}} \mathscr{C}_j^{(k)}.$$

The Voronoi-Laguerre diagram of $S_{\text{SARA}}^{(k)}$ creates a partition of the AoI. Therefore, in order to prove that the coverage extension does not decrease after the algorithm execution, it is enough to prove that, at each iteration, the coverage of each polygon is preserved, that is:

$$V(\mathscr{C}_i^{(k)}) \cap \mathscr{A}^{(k)} \subseteq \mathscr{A}^{(k+1)}, \forall s_i \in S_{\text{SARA}}^{(k)}. \quad (4)$$

Regarding fixed sensors, SARA allows them to go to sleep one at a time and only if their polygon is already covered by other sensors, so if one of them decides to go to sleep the

Algorithm 3: Algorithm SARA for adjustable sensors

Algorithm SARA executed by node s_i

// before starting the next operative time interval, the sensor s_i works with the radius

// determined at the previous execution of SARA

Initialization:

$k = 0$;

Back-off interval = $[0, t^{\mathrm{backoff}}_{\mathrm{max}}]$;

$r_i^{(k)} = r_i^{\mathrm{max}}$;

decision_made=false;

Exchange position information with neighbors;

Iterative Voronoi-Laguerre diagram construction:

while !*decision_made* **do**

  Exchange info on radius with neighbors;

  Construct the VorLag polygon $V^{(k)}(\mathscr{C}_i)$;

  Exchange redundancy/polygon nullity information messages and potential energy gain;

  while $t < t^{backoff}_{max}$ **do**

    listen to update messages from the fixed nodes in the neighborhood;

  Update the VorLag polygon $V^{(k)}(\mathscr{C}_i)$;

  Build sets $\mathscr{L}^{(k)}(s_i)$ and $\mathscr{D}^{(k)}(s_i)$;

  Let $\overline{V}^{(k)}(\mathscr{C}_i) = V(\mathscr{C}_i) \setminus \cup_{s_j \in \mathscr{D}^{(k)}(s_i)} \mathscr{C}_j$;

  Let $f(\overline{V}^{(k)}(\mathscr{C}_i))$ be the farthest point of $\overline{V}^{(k)}(\mathscr{C}_i)$ from s_i;

  Let $c(\overline{V}^{(k)}(\mathscr{C}_i))$ be the closest point of $\overline{V}^{(k)}(\mathscr{C}_i)$ from s_i;

  if $(d_E(s_i, c(\overline{V}^{(k)}(\mathscr{C}_i))) < r_i^{(k)} < d_E(s_i, f(\overline{V}^{(k)}(\mathscr{C}_i)))) \vee$ $(f(\overline{V}^{(k)}(\mathscr{C}_i))$ *is a strict farthest)* $\vee (r_i^{(k)} = 0))$ **then**

    // reached minimum radius

    decision_made $=$ true;

  else

    if $r_i^{(k)} < d_E(s_i, c(\overline{V}^{(k)}(\mathscr{C}_i)))$ **then**

      // completely uncovered polygon

      $\overline{d}_i^{(k)} = 0$;

    else

      $\overline{d}_i^{(k)} = d_E(s_i, f(\overline{V}^{(k)}(\mathscr{C}_i)))$;

    $\alpha_i = get\_alpha(\mathscr{L}(s_i))$;

    $r_i^{(k+1)} = r_i^{(k)} - \alpha_i^{(k)}(r_i^{(k)} - \overline{d}_i^{(k)})$;

    $k = k + 1$;

if $r_i^{(k)} = 0$ **then**

  // null or completely uncovered polygon

  go to sleep;

else

  Adjust the sensing radius to $r_i^{(k)}$;

coverage of its polygon does not decrease, thus guaranteeing that Eq. 4 is trivially verified for fixed sensors.

For what concerns the case of adjustable sensors, let us consider any sensor s_i still active in the k-th iteration. Theorem 3.1 affirms that the covered area of $V(\mathscr{C}_i^{(k)})$ is all covered by s_i. This means that, for any $s_i \in S_{\texttt{SARA}}^{(k)}$ and for any iteration k:

$$V(\mathscr{C}_i^{(k)}) \cap \mathscr{A}^{(k)} \stackrel{Th.3.1}{=} V(\mathscr{C}_i^{(k)}) \cap \mathscr{C}_i^{(k)}.$$

Therefore in order to prove Eq. 4, it is sufficient to prove that

$$V(\mathscr{C}_i^{(k)}) \cap \mathscr{C}_i^{(k)} \subseteq \mathscr{A}^{(k+1)}, \forall s_i \in S_{\texttt{SARA}}^{(k)} \cap S_{\texttt{adjustable}}. \tag{5}$$

SARA reduces the radius of an adjustable sensor s_i to a value such that the coverage of the region $\overline{V}(\mathscr{C}_i^{(k)}) = V(\mathscr{C}_i^{(k)}) \setminus (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k)})$ is not altered.

By the definition of $\mathscr{D}_i^{(k)}$, sensors belonging to $\mathscr{D}_i^{(k)}$ are such that their sensing circles do not change in the following iterations, therefore if $s_j \in \mathscr{D}_i^{(k)}$ then $\mathscr{C}_j^{(k)} = \mathscr{C}_j^{(k+1)}$.

Let us consider a further partition of $V(\mathscr{C}_i^{(k)})$ in the following two subsets:

$V_{i,k}^1 \triangleq \overline{V}(\mathscr{C}_i^{(k)}) = V(\mathscr{C}_i^{(k)}) \setminus (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k)})$, and $V_{i,k}^2 \triangleq V(\mathscr{C}_i^{(k)}) \setminus \overline{V}(\mathscr{C}_i^{(k)}) = V(\mathscr{C}_i^{(k)}) \cap (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k)})$.

We will now prove that Eq. 5 is verified by separately considering the two subsets $V_{i,k}^1$ and $V_{i,k}^2$. Let us first consider $V_{i,k}^1$.

$$V_{i,k}^1 \cap \mathscr{C}_i^{(k)} \stackrel{\texttt{SARA}}{=} V_{i,k}^1 \cap \mathscr{C}_i^{(k+1)} \subseteq \mathscr{C}_i^{(k+1)} \subseteq \mathscr{A}^{(k+1)}.$$

We now show that the same property holds for $V_{i,k}^2$.

$$V_{i,k}^2 \cap \mathscr{C}_i^{(k)} \subseteq V_{i,k}^2 = V(\mathscr{C}_i^{(k)}) \cap (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k)}) \stackrel{\texttt{Def of } \mathscr{D}_i^{(k)}}{=}$$

$$= V(\mathscr{C}_i^{(k)}) \cap (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k+1)}) \subseteq \cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k+1)} \subseteq \mathscr{A}^{(k+1)}.$$

Since $V(\mathscr{C}_i^{(k)}) \cap \mathscr{C}_i^{(k)} = (V_{i,k}^1 \cup V_{i,k}^2) \cap \mathscr{C}_i^{(k)} \subseteq \mathscr{A}^{(k+1)}$, Eq. 4 is verified.

$\square$

THEOREM 5.2. **(Convergence in the case of adjustable sensors)** *Given a set $\mathcal{S} = \mathcal{S}_{adjustable}$ of only adjustable sensors, under the execution of* SARA*, each sensor will converge to a final configuration decision.*

PROOF. Consider the adjustable sensor $s_i \in \mathcal{S}$, positioned in C_i. Let $r_i^{(k)}$ be its sensing radius at the k-th iteration of SARA, and let $\mathscr{C}_i^{(k)}$ and $V(\mathscr{C}_i^{(k)})$ be its sensing circle and its Voronoi-Laguerre polygon, respectively. We distinguish three cases: (1) $V(\mathscr{C}_i^{(k)})$ is completely covered (notice that this case includes the situation of null polygons which can be considered a degeneration of non null polygons), (2) $V(\mathscr{C}_i^{(k)})$ is only partially covered and (3) $V(\mathscr{C}_i^{(k)})$ is not covered (neither by s_i nor by any other sensor, due to Theorem 3.1).

Convergence in case (1). Theorem 3.1 ensures that $V(\mathscr{C}_i^{(k)})$ is completely covered by s_i. Since `SARA` preserves coverage (for Theorem 5.1), the new polygon and its farthest point will also be covered by s_i at any successive iteration of `SARA`. We recall that $\overline{V}(\mathscr{C}_i^{(k)}) = V(\mathscr{C}_i^{(k)}) \setminus (\cup_{s_j \in \mathscr{D}_i^{(k)}} \mathscr{C}_j^{(k)})$. We define $\overline{d}_i^{(k)} = d_E(s_i, f(\overline{V}(\mathscr{C}_i^{(k)})))$ and $d_i^{(k)} = d_E(s_i, f(V(\mathscr{C}_i^{(k)})))$. As $\overline{V}(\mathscr{C}_i^{(k)}) \subseteq V(\mathscr{C}_i^{(k)}) \subseteq \mathscr{C}_i^{(k)}$ the following holds:

$$0 \leq \overline{d}_i^{(k)} \leq d_i^{(k)} \leq r_i^{(k)}. \tag{6}$$

Since $r_i^{(k)}$ is strictly decreasing and non-negative, when $k \to \infty$, it converges to a value $R_i \geq 0$. `SARA` sets the radius of s_i for the next iteration as: $r_i^{(k+1)} = r_i^{(k)} - \alpha_i^{(k)} \cdot (r_i^{(k)} - \overline{d}_i^{(k)})$, where $\alpha_i^k \in (0,1]$. It follows that $R_i = R_i - \lim_{k\to\infty} \alpha_i^k \cdot (R_i - \lim_{k\to\infty} \overline{d}_i^{(k)})$. As $\alpha_i^k > \alpha_{\mathtt{min}}$ is strictly positive and lower than 1, then $\lim_{k\to\infty} \overline{d}_i^{(k)} = R_i$.

The convergence of $\lim_{k\to\infty} d_i^{(k)}$ follows, due to Equation 6, by applying the comparison criterion. This means that the radius of s_i converges to the minimum value to cover the farthest vertex of its polygon, which is a *boundary farthest configuration*.

If such a boundary farthest vertex is strict, then s_i terminates its execution of `SARA`.

Otherwise, the adoption of the *serialization scheme for loose farthest vertices* discussed in Section 3.2.1 ensures that all the sensors with loose vertices will perform their additional radius reduction one at a time. After this radius reduction, s_i will never generate again a loose farthest with the same neighbors (as this would require an increase in the sensing range of at least one sensor, which is not allowed by `SARA`). Since there is a finite number of neighbor sensors that can generate a loose farthest with s_i, then s_i will eventually reach a strict farthest situation and will exit.

Convergence in case (2). In this case, as the coverage of the polygon is only partial, the sensor cannot reduce its radius (due to Corollary 3.2) and `SARA` immediately terminates.

Convergence in case (3). Consider $k = 0$. In case (3) $V(\mathscr{C}_i^{(0)}) \cap \mathscr{C}_i^{(0)} = \emptyset$. At the successive iterations, the polygon of s_i can only be altered by the radius reductions performed by the neighbors of s_i and s_i itself.

As the polygon $V(\mathscr{C}_i^{(0)})$ is not covered, the polygons of the Voronoi-Laguerre neighbors of s_i are either partially covered or completely uncovered, because they share an edge with $V(\mathscr{C}_i^{(0)})$. A radius reduction of a neighbor with completely uncovered polygon may result in an extension of the polygon of s_i with new uncovered zones. By contrast, the neighbors of s_i which partially cover their polygons will not change their radius. Therefore, for any iteration $k > 0$, $V(\mathscr{C}_i^{(k)}) \cap \mathscr{C}_i^{(k)} = \emptyset$, that is a polygon which is initially uncovered will remain uncovered.

Hence, for a sensor s_i being in case (3), $\overline{d}_i^{(k)} = 0$, $\forall k \geq 0$. This implies that $r_i^{(k+1)} = (1 - \alpha_i^{(k)}) \cdot r_i^{(k)} \leq (1 - \alpha_{\mathtt{min}}) \cdot r_i^{(k)}$, $\forall k \geq 0$. Therefore $\lim_{k\to\infty} r_i^{(k)} \leq \lim_{k\to\infty}(1 - \alpha_{\mathtt{min}})^k \cdot r_i^{(0)} = 0$, proving that the sensor s_i converges to a final configuration in which it will be switched off[8].

□

[8]Notice that, although s_i knows from the beginning that its decision will be to turn off, it still executes the algorithm iteratively in order not to alter the decision priority established by the energy gain criterion.

THEOREM 5.3. **(Termination in the case of fixed sensors)** *Given a set $\mathcal{S} = \mathcal{S}_{fixed}$ of only fixed,* SARA *puts to sleep all the redundant sensors in a finite time.*

PROOF. At the k-th iteration of SARA, every fixed sensor determines whether it is redundant or not. If it is not redundant it immediately ends its execution with the decision to stay awake. If instead it is redundant it turns itself off with probability α_i (see Algorithm 1). At every iteration k of the algorithm, there is at least one sensor s_i (namely the one with maximum value of ΔE_i) whose value of $\alpha_i^{(k)}$ is equal to 1 and therefore has probability 1 to go to sleep. It follows that at each iteration at least one redundant sensor turns itself off (although in practice many sensors go to sleep at each iteration, as shown in Section 7). Hence, in a finite number of steps all redundant fixed sensors will go to sleep. ∎

THEOREM 5.4. *(***Convergence of** SARA **in the general scenario***) Given a set $\mathcal{S} = \mathcal{S}_{adjustable} \cup \mathcal{S}_{fixed}$ of both adjustable and fixed sensors, under the execution of* SARA*, each sensor converge to a final configuration decision.*

PROOF. The convergence of SARA easily descends from the Theorems 5.2 and 5.3.

It has to be noted that although the presence of fixed sensors does not alter the convergence property of the adjustable sensors, the opposite is not true. In fact, the presence of adjustable sensors in the mix alters the behavior of the fixed sensors as it is no longer guaranteed that at every iteration k there will be a redundant fixed sensor that will turn itself off. Although it is still true that there will be at least one sensor $s_i^{(k)}$ in $\mathcal{S}$ with $\alpha_i^{(k)} = 1$, this sensor may belong to the adjustable class. Therefore, the convergence speed of the fixed class is slowed down by the presence of the adjustable sensors[9] . For this reason this theorem only affirms the convergence and not the termination of SARA in the mixed scenario, as in the case of only adjustable sensors. ∎

Theorem 5.4 states the convergence of SARA in the mixed scenario. The question is how to ensure that convergence does not take too long: The adjustable sensors might reduce their radius of an infinitesimal step at each iteration. In order to ensure the theoretical termination of the algorithm in a finite number of steps we can set an upper limit K on the number of iterations (*faster termination condition*). Despite convergence might theoretically take quite long time we have observed that no more than 20 iterations are sufficient to achieve termination of the 95% of sensors. Setting a value of K as low as 20 has a negligible impact on the performance of SARA, but has the advantage to ensure a very fast termination of the algorithm execution.

The following Lemma 5.5 analyzes the property of the cover set obtained after the execution of SARA focusing in particular on the polygons generated by the adjustable sensors.

LEMMA 5.5. **(Properties of the cover set)** *Consider a mixed set of adjustable and fixed sensors $\mathcal{S} = \mathcal{S}_{adjustable} \cup \mathcal{S}_{fixed}$. Consider the cover set S_{SARA} obtained after the execution of* SARA *on $\mathcal{S}$. If $s_i \in S_{SARA} \cap \mathcal{S}_{adjustable}$ either s_i partially covers its polygon $V(\mathscr{C}_i)$, or its farthest vertex $f(V(\mathscr{C}_i))$ is a strict boundary farthest vertex.*

[9]This is because we want the two classes of sensors to reduce their radius in parallel without favoring a given class. If, due to a particular operative setting, one of the two classes should have a higher priority in making configuration decisions, this can be handled by redefining accordingly the priority parameter $\alpha_i^{(k)}$.

PROOF. Let s_i exit SARA at time T_{s_i} with its radius set to $r_i^k > 0$[10]. According to Algorithm 3 s_i terminated SARA execution either because its polygon is not completely covered or because it has reached a strict boundary farthest configuration. We now show that changes in the sensing coverage of other nodes s_j which occur at a time $T > T_{s_i}$ cannot change this property. As this is obvious for sensors which partially cover their polygons, let us consider the case of s_i completely covering its polygon.

Two types of events can occur after T_{s_i} which affect sensor s_i responsibility region: 1) other adjustable sensors s_j reduce their radius, 2) fixed or adjustable sensors are turned off. Both these events may result in an increase of sensor s_i responsibility region. However, since s_i radius cannot change (s_i has exited), since the reduction of other nodes radius preserves coverage (Theorem 5.1) and since if a point P is covered it is covered by the node to which responsibility region it belongs (theorem 3.1) it derives that s_i responsibility region stays within the circle centered in s_i and with radius equal to r_i^k. Therefore, each boundary farthest point at time T_{s_i} is still a boundary farthest at the end of SARA execution. ∎

According to SARA, each sensor pursues an individual utility that is to reduce its power consumption and at the same time to do its best to cover the AoI. In terms of this utility function, the cover set S_{SARA} obtained by SARA starting from $\mathcal{S}$, is Pareto optimal. In fact, it is not possible to increase the utility of a single sensor (i.e., by reducing the sensing range of an adjustable sensor or turning off a fixed one) without decreasing the utility (i.e., increasing the sensing range of an adjustable sensor or turning on a fixed one that was previously sleeping) of at least another device in the network.

THEOREM 5.6. **(Pareto optimality)** *Given a set $\mathcal{S} = \mathcal{S}_{adjustable} \cup \mathcal{S}_{fixed}$ of sensors, after the execution of* SARA *(without the faster termination condition), the produced cover set S_{SARA} is Pareto optimal.*

PROOF. In order to prove the Pareto optimality of SARA we need to show that there is no action that could improve the utility of a single sensor, i.e. a sensing radius reduction or the deactivation of a device, without reducing the coverage achieved by S_{SARA}.

This property is true for fixed sensors, since all redundant fixed sensors will eventually turn themselves off according to the back-off scheme provided by SARA. This trivially derives from Theorems 5.3 and 5.4.

In the case of adjustable sensors, consider $s_i \in \mathcal{S}_{\text{adjustable}}$, Theorem 5.2 states that under the execution of SARA s_i will eventually reach a final configuration decision, while Lemma 5.5 gives a characterization of the final solution, affirming that if s_i completely covers its polygon, s_i is in a strict boundary farthest vertex configuration whereas if s_i covers its polygon only partially, Corollary 3.2 proves that in this case s_i cannot reduce its radius without affecting coverage. ∎

Pareto optimality is a *necessary condition for global optimality*. Unfortunately, the Pareto optimality of the cover set does not have implications in terms of quality of the solution to the lifetime problem, as there are infinite Pareto optimal solutions. Nevertheless, by adopting an energy-aware policy, SARA is able to choose a cover set among all the

[10]Notice that the case $r_i^k = 0$ is exluded because s_i belongs to the cover set S_{SARA}

possible Pareto-optimal ones, which reduces the energy consumption of the network and prolongs its lifetime, as experimentally shown in Section 7.

6. TWO RECENTLY PROPOSED SELECTIVE ACTIVATION AND RADIUS ADAPTATION ALGORITHMS

To the best of our knowledge there is no prior work in the literature that addresses the problem of selective activation and sensing radius adaptation in a general applicative scenario combining fixed sensors and sensors endowed with variable sensing capabilities. Moreover, previous works rarely consider device heterogeneity. For these reasons, we compare SARA to the Distributed Lifetime Maximization (DLM) algorithm [Kasbekar et al. 2009] which is designed to work with fixed radius sensor and to the Variable Radii Connected Sensor Cover (VRCSC) algorithm [Zou et al. 2009] which is designed to work only with devices that can adjust their sensing radius. The choice of these two algorithms is motivated by the performance analysis carried out by the same authors which shows that DLM and VRCSC achieve better performance to previous schemes proposed in the same class.

In this section we give a short description of DLM and VRCSC and of our extensions to adapt them for a general scenario. We also discuss why they do not provide Pareto optimal solutions.

DLM addresses the problem of activating a subset of sensors so that each point of the AoI is monitored by at least k sensors [11]. DLM considers the case of heterogeneous sensors with fixed sensing radii. The authors call *intersection point* any point where two sensing circles intersect with each other and observe that if each intersection point is k-covered, then the whole AoI is k-covered. DLM is a round based algorithm. At each round, maximum coverage is obtained by iteratively waking up sensors according to an ordered list of nodes that are in radio proximity. The list is sorted on the basis of the energy consumed by the nodes and of the number of intersection points that they can cover. Such a list provides the priority order for the iterative waking up of the sensors in a neighborhood. At each iteration, the sensors whose sensing range is already k-covered by other already awake sensors are removed from the list (they will not wake up themselves). We refer to [Kasbekar et al. 2009] for the details of the algorithm.

We extend DLM to the case of sensors with adjustable sensing radii by considering the devices with variable radii as if they were fixed. This means that each sensor, independently of the class to which it belongs, will either wake up (i.e., operate at maximum transmission radius) or go to sleep. As DLM is not designed to deal with variable radii devices, this variant is introduced only to show that to apply DLM to a more general setting requires non trivial changes.

VRCSC explicitly addresses the problem of k-covering the AoI with sensors with adjustable radii (both transmission and sensing radii).

VRCSC makes use of Voronoi diagrams to determine which sensors are completely redundant. It then reduces the radius of each sensor to the minimum necessary to cover the farthest point of its Voronoi polygon. For each redundant sensor s, VRCSC calculates the energy benefit obtained by putting it to sleep. This benefit is compared to the additional energy expenditure that the neighbors of s would incur to enlarge their radius with respect

[11] For the sake of simplicity, in this paper we do not address the problem of k-coverage. Hence in all our experiments, detailed in Section 7, we assume that all the algorithms work with $k = 1$.

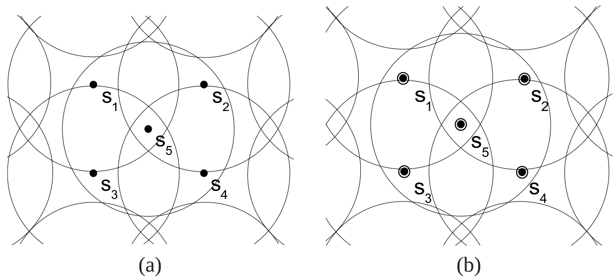


Fig. 6. About Pareto optimality. Initial configuration (a). Selective activation with DLM (b) and SARA (c). The nodes with double circle are awake, while the other ones are sleeping.

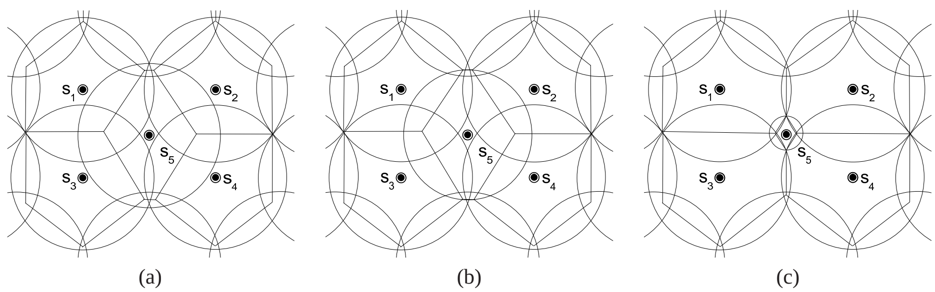


Fig. 7. About Pareto optimality. Initial configuration (a). Selective activation with VRCSC (b) and SARA (c).

to their minimum setting (i.e. the one needed to cover their Voronoi polygon) so as to cover the Voronoi polygon of s on its behalf. We refer the reader to [Zou et al. 2009] for more details on VRCSC.

We extend the use of VRCSC to the case of sensors with fixed radii. In the case of fixed sensors VRCSC only operates the waking up/putting to sleep decisions, while the rules to reduce sensor radius are disabled. The purpose of this variant is to show how trivial extensions of VRCSC perform in a more general scenario than the one for which it is designed.

Unlike our approach, both DLM and VRCSC do not meet the necessary condition for optimality discussed in Section 5. This is explained in Figures 6 and 7.

Figure 6(a) represents an initial configuration with fixed sensors. Observe that sensors s_1, s_2, s_3 and s_4 must be awake to ensure a complete coverage of the AoI, as they cover portions of the AoI that cannot be covered by any other sensor in the network. According to DLM, if the energy available to sensor s_5 is sufficiently high, s_5 can be the first sensor to be woken up in its neighborhood. Once awake, it stays awake despite the waking up of the other four sensors makes s_5 unnecessary (see Figure 6(b)).

Under the same initial setting SARA would not activate s_5, as the backoff policy ensures the sleeping of all redundant sensors. This is shown in Figure 6(c).

Figure 7 displays a scenario with adjustable sensors. Figure 7(a) shows the initial configuration where all sensors are awake and work at their maximum radius. The figure also highlights the Voronoi diagram of the considered sensors. In this example all sensors (s_1, s_2, s_3, s_4 and s_5) are needed to achieve full coverege. Sensors s_1, s_2, s_3 and s_4 cannot reduce their radius as their uniquely covered zone reaches the boundary of their sensing circle. Sensor s_5, instead, can significanlty reduce its radius without affecting coverage.

According to VRCSC each sensor sets its radius to the distance from it to the farthest vertex of its Voronoi polygon. Therefore, s_5 reduces its radius as shown in Figure 7(b). Since no sensor can be put to sleep, this is the final configuration achieved by VRCSC. Nevertheless, sensor s_5 can still significantly reduce its radius. By iteratively adjusting the radius of s_5, SARA reaches a Pareto optimal configuration, where the radius of s_5 is set to the minimum value that does not leave a coverage hole, as shown in Figure 7(c).

We conclude this subsection by underlying that if DLM and VRCSC are not properly extended as discussed above, VRCSC cannot be used in the case of non adjustable radii and, vice-versa, DLM cannot be applied to the case of variable radii. Our algorithm, instead, works in both the operative settings. Moreover, our algorithm is also able to work in a mixed scenario characterized by both sensors with adjustable and fixed radii, even in the presence of heterogeneous devices, showing an impressive versatility. In the performance evaluation section we will also show that SARA achieves significant performance improvements over the other two schemes in all operative settings.

We summarize the features of the three schemes in Table I.

	Fixed type		Adjustable type		Both types	
	Hom.	Het.	Hom.	Het.	Hom.	Het.
DLM	Y	Y	N	N	N	N
VRCSC	N	N	Y	N	N	N
SARA	Y	Y	Y	Y	Y	Y

Table I. Scenarios where the considered algorithms are applicable.

To give a fair performance comparison, in Section 7 we compare SARA to DLM and VRCSC in their restrictive operative settings and then we extend their use to the general applicative scenario where devices belong to both the two classes of sensors with fixed and adjustable radii and are heterogeneous in their sensing capabilities.

7. EXPERIMENTAL RESULTS

7.1 Experimental setting

In all the experiments we use the following setting. The AoI is a square shaped region of 80m $\times$ 80m. We adopt the Telos [Polastre et al. 2005] communication cost model. Concerning the sensing model of sensors with adjustable radius we consider the cost of six Maxbotix sonar devices [MAXBOTIX sonar datasheets 2010] with different orientations, working at 2Hz. We adopted the cubic law of energy cost ($c = 3$ in Equation 1) with respect to the sensing radius.

According to these models, each sensor has a transmission range of 30m. The battery capacity is 1840 mAh and sensors are endowed with an initial energy that is uniformly distributed in the interval (0, 1840mAh]. The length of the operative time interval between two successive executions of the algorithm SARA and DLM is set to 24h which is equal to 1.5% of the total time a sensor can remain awake. Notice that the algorithm VRCSC, as defined in [Zou et al. 2009], reconfigures the network every time a sensor has exhausted its available energy.

Regarding the setting of the sensing radius, it varies from one applicative scenario to the other. We consider all the applicative scenarios described in Table I.

The algorithms were implemented by using the Wireless module of the OPNET modeler software [OPNET Technologies].

7.2 Choice of the reduction criterion

Before we give the comparative performance evaluation between SARA and other previous works, we show an extract of the many experiments that motivated our choice in the formulation of the priority decision parameter α described in Subsection 4.1.2.

Notice that all the properties of SARA that we demonstrated in Section 5 hold no matter which is the formulation of the parameter $\alpha_i^{(k)}$.

In particular, we proved that, independently of the particular choice for the setting of $\alpha_i^{(k)}$, the algorithm SARA guarantees that the solution will be Pareto-optimal, therefore no sensor will be able to decrease its energy consumption by turning itself off or reducing its sensing radius, without requiring other sensors to increase their energy expenditure to compensate the coverage loss deriving from the decision of the first sensor.

The order in which the sensors operate their decisions is determined by the particular choice for the formulation of the decision priority $\alpha_i^{(k)}$ and has a direct impact on the solution, i.e. different (all Pareto-optimal) solutions are obtained executing the algorithm by giving sensors different priorities. Nevertheless, it is clear that the network lifetime of different Pareto optimal solutions can vary significantly.

Since the setting of $\alpha_i^{(k)}$ influences the policy decisions at a local level only, it is not completely intuitive to determine the effects of such local decisions on a global performance metric such as the network lifetime. Therefore we considered several possible formulation of the decision priority parameter.

First we can consider the *residual energy* of the devices, and gave higher priority to sensors with lower residual energy in making turning-off or radius reduction decisions, therefore $\alpha_i^{(k)\texttt{residual\_energy}} = \frac{\max_{j\in\mathscr{L}^{(k)}(s_i)} E^{(n)}_{\texttt{available}}(s_j) - E^{(n)}_{\texttt{available}}(s_i)}{\max_{j\in\mathscr{L}^{(k)}(s_i)} E^{(n)}_{\texttt{available}}(s_j) - \min_{j\in\mathscr{L}^{(k)}(s_i)} E^{(n)}_{\texttt{available}}(s_j)}$.

Nevertheless this criterion revealed that by turning off the sensors with lower energy (or significantly reducing their responsibility region) would cause other sensors (those with large residual energy) to consume an arbitrary large amount of energy, possibly making the alive sensors end up with much lower energy than the turned off sensor would have had if it were kept awake. This is the reason why this criterion performs worse than others as we show in Figure 8.

Then, we can consider a formulation of the parameter $\alpha_i^{(k)}$ which results in a priority setting based on the *expected residual lifetime* of individual sensors.

The expected residual lifetime (in number of equally sized operative time intervals) $\hat{L}_i(k)$ of the sensor s_i at the k-th iteration of the algorithm is calculated as the ratio between the currently available energy $E_{\texttt{available}}(s_i)$ and the energy consumption per operative time interval with the currently calculated radius, i.e. $\hat{L}_i(k) = \frac{E_{\texttt{available}}(s_i)}{E_{\texttt{active\_sensing}}(r_i^{(k)})}$.

The residual lifetime criterion consists therefore in setting the value of $\alpha_i^{(k)}$ as

$$\alpha_i^{(k)\texttt{residual\_lifetime}} = \frac{L^{(k)}_{\texttt{max}}(n) - \hat{L}^{(k)}_i(n)}{L^{(k)}_{\texttt{max}}(n) - L^{(k)}_{\texttt{min}}(n)}$$

Although this setting of $\alpha_i^{(k)}$ is superior to the previous ones in all the considered scenarios, it still tends to favor the improvement of the lifetime of single sensors with respect to the utility of the global network. In particular there are still some situations in which the algorithm gives higher priority to turning off some sensors with smaller residual lifetime at the expense of sensors with larger residual lifetime that in this way are forced to work longer, spending more energy than all the others in the neighborhood.

We experimented other formulations of $\alpha_i^{(k)}$ that we do not discuss in this paper for the sake of brevity. We experimentally obtained the best results by setting $\alpha_i^{(k)}$ as we described in Subsection 4.1.2, giving higher priority to the decisions that lead to a better energy gain.

In Figures 8(a) and (b) we considered a scenario with 900 homogeneous sensors with adjustable radius, whose sensing range was allowed to vary in the interval $[2m, 6m]$. Figure 8 (a) shows a comparison of the residual energy obtained by the three criteria, while Figure 8(b) shows how the criterion that we chose guarantees a longer lifetime than the other ones.

7.3 Impact of the faster termination condition

In this subsection we analyze the impact of the *faster termination condition* introduced in Section 5, namely of the configuration of the maximum number of iterations K that SARA is allowed to execute at the beginning of each operative time interval. We recall that the algorithm SARA is guaranteed to converge, but theoretically it may do so in an infinite number of smaller and smaller steps. Although in the experiments we never encountered a scenario where SARA did not terminate in a finite number of steps, we introduced the condition for faster termination, by imposing an upper bound K on the number of algorithm iterations at each operative time interval.

The experiments shown in Figure 9 (a-d) are made in a scenario with 900 sensors with adjustable sensing radii ranging from 2m to 6m. These experiments highlight that even by setting K to a small value (e.g. 20) the algorithm SARA shows the same performance of the unbounded case in terms of active, sleeping, and dead nodes (Figure 9 (a-b-c)) and of network coverage (Figure 9(d)).

7.4 Adjustable sensors: Homogeneous setting

This section is devoted to a comparative analysis of the performance of SARA, DLM and VRCSC in a scenario with only sensors with equal capabilities to adjust their sensing range. As in the experiments of the previous sections, we considered 900 sensors whose range varied in the interval $[2m, 6m]$. It should be noted that this scenario is not the most general, but it is the one for which VRCSC was specifically designed. Therefore, it is reasonable to expect that VRCSC show a good performance. Nevertheless, the experiments highlight that even in this case, the algorithm SARA performs better. Indeed thanks to the device homogeneity, the algorithm VRCSC is able to work in this scenario without creating coverage holes, but it is not able to fully exploit the adaptability of the sensing range as SARA does, thanks to the use of Voronoi diagrams in the Laguerre geometry.

In the following experiments we consider the modified version of DLM described in Section 6 in order to apply it to the scenario with adjustable sensing radii. As we have already argued, this modified version is introduced in these experiments to highlight that this algorithm cannot be trivially extended to the general scenario without a significant loss in performance.

Figure 10(a) shows how the coverage achieved by the three algorithms decreases with

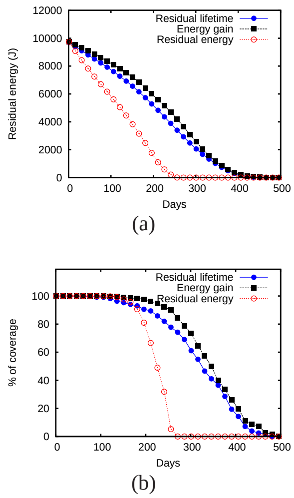


Fig. 8. Average residual energy (a) and coverage of `SARA` under different formulations of the decision priority parameter.

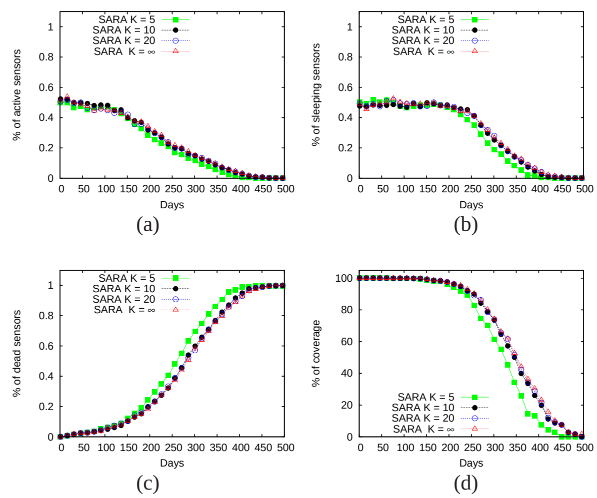


Fig. 9. Performance of `SARA` under different settings of the maximum number of iterations K (faster termination condition). Percentage of active sensors (a), sleeping sensors (b), dead sensors (c), percentage of coverage of the AoI (d).

time. The loss in coverage with `DLM` is much faster than with `VRCSC` and `SARA`, evidencing its inapplicability to this operative scenario. In this case `SARA` performs better than `VRCSC`. For instance, in correspondence to day 350, `SARA` is still capable to cover about twice the extension of the area covered by `VRCSC`. This evidences the capability of `SARA` to prolong the network lifetime, when this is formulated as the time within which the network is still capable to cover a given percentage x of the AoI, independently of the value of x.

Figure 10(b) and (c) show how the percentage of active and sleeping sensors varies with time. These percentages are calculated with respect to the whole set of available sensors. It should be noted that although `DLM` activates a very small percentage of the available sensors, it is penalized by the fact that the radius of the active sensors cannot be modulated by the algorithm (Figure 10(e)). Thence the energy consumption per sensor is very high, as demonstrated by Figure 10(f) which shows how small is the residual energy under `DLM` after few operative time intervals, resulting in a very high percentage of dead sensors [12] Notice that under `DLM` the number of active sensors shows a peak after about 50 operative intervals for a twofold reason. On the one hand the greedy nature of the algorithm `DLM` results in the activation of redundant sensors, as it gives higher priority to the activation of sensors which have consumed less energy in the previous operative intervals. On the other hand, while at the first intervals, the algorithm is able to select the best suitable sensors to cover the AoI, after the death of several sensors some regions can only be covered by sensors which cause larger overlaps.

The algorithms `SARA` and `VRCSC` are able to modulate the sensing radius of the active sensors so as to reduce the coverage overlaps and save energy. Therefore, with respect to `DLM`, more sensors are activated (Figure 10(b)) working with lower sensing radius (Figure

[12] Hereby we refer to *dead sensor* as to devices which have exhausted their available energy.

10(e)). This permits to the algorithms VRCSC and SARA to preserve more energy than DLM (10(f)). Notice that, SARA activates a higher number of sensors with smaller radius than VRCSC, thus being able to prolong the network lifetime by reducing the amount of consumed energy.

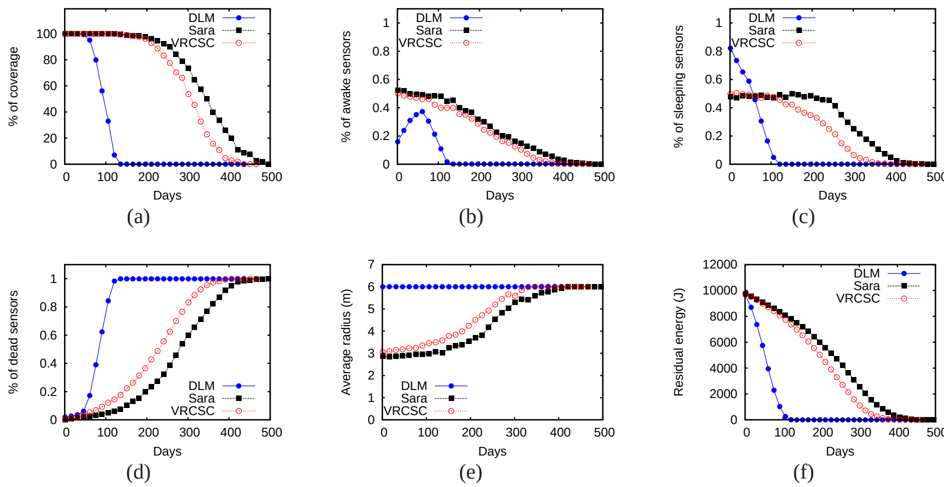


Fig. 10. Adjustable sensors: homogeneous setting. Comparative analysis of SARA, DLM and VRCSC. Percentage of coverage (a), active sensors (b), sleeping sensors (c), sensors with no residual energy (d). Average radius of the awake sensors (e) and average residual energy per sensor (f). Scenario with 900 equally equipped sensors.

We now evaluate the benefits of the three algorithms in terms of lifetime improvements. Figure 11(a) shows the time when the algorithms are no longer able to achieve a coverage higher than the 80% of the area of interest by varying the number of available sensors.

Figure 11(b) shows the distribution of the sensors over the AoI in the operative scenario with 300 sensors. This figure evidences that, despite the general redundancy, a significant portion of the area of interest is either uncovered or covered by only few sensors. These sensors deplete their energy faster than others no matter which algorithm is in use. This implies that with a tolerance of only 20% (due to the definition of lifetime as the time at

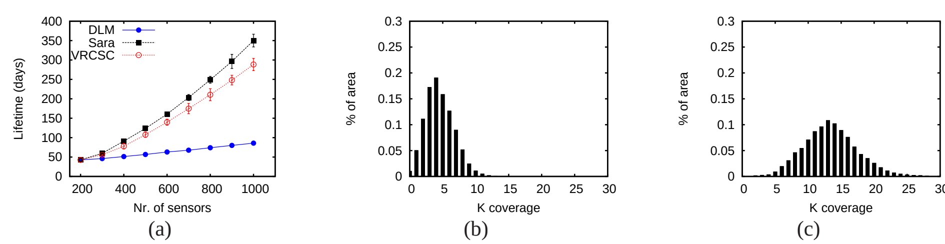


Fig. 11. Adjustable sensors: homogeneous setting. Lifetime (a) and distribution of the sensors over an area of interest of 80m x 80m with a random deployment of 300 sensors (b) and 900 equally equipped sensors (c).

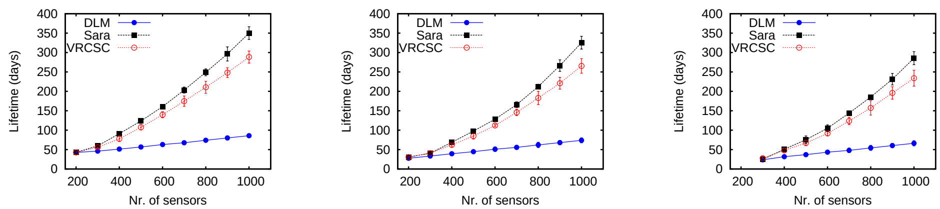


Fig. 12. Adjustable sensors: homogeneous setting. Lifetime achieved by the three algorithms expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

which more than 20% of coverage is lost) the three algorithms cannot do much to improve the network lifetime.

For this reason, in this applicative scenario, the lifetime of DLM, VRCSC and SARA is about the same, as seen in Figure 11(a) when the number of available sensors is 300. By contrast, Figure 11(c) shows the distribution of the sensors over the AoI in the operative scenario with 900 sensors. It shows that due to this higher density, there is more room for the algorithms to improve the network lifetime by means of selective activation and radius reduction, as also evidence by Figure 11(a) when the number of available sensors is 900. The algorithm SARA outperforms the other two by achieving a longer lifetime being able to perform a more efficient activation policy. In particular, although this scenario is the most favorable to the algorithm VRCSC, SARA is able to always achieve a longer lifetime. For instance, when the number of sensors is 1000, the algorithm SARA achieves and increase of 20% in the network lifetime with respect to VRCSC (350 days for SARA versus 290 days for VRCSC).

In Figure 12 we compare the algorithms in terms of network lifetime by increasing the number of deployed sensors. We consider the time at which the coverage of the AoI goes below the 80% (a), 90% (b) and 95%(c). Notice that, even if our algorithm does not specifically target a particular notion of lifetime, it outperforms the other two also under other possible definitions of lifetime.

7.5 Adjustable sensors: heterogeneous setting

In this section we analyze a scenario with only sensors with adjustable radius. Differently from the setting of the previous experiments, we now consider the case of sensors unequally equipped. In particular the 50% of the available sensors is capable to adjust its radius up to a value of 6m, whereas the remaining 50% can only reach a sensing radius 3m long.

In Figure 13(a) we show the coverage achieved by the set of active sensors after the configuration obtained by running the algorithms. Notice that in this case we selected the very first execution of the algorithms, hence the number of dead sensors is zero for all of them. Despite the high energy availability of all the sensors, the algorithm VRCSC is not able to guarantee the complete coverage of the AoI at any time, even though it activates a very high percentage of sensors as shown in Figure 13(b). This is due to the way it governs the radius configuration decisions on the basis of Voronoi diagrams. As we have already mentioned the use of Voronoi diagrams is correct only in the case of homogeneous sensing

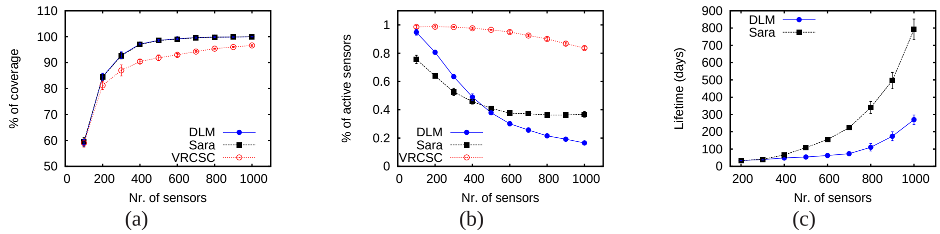


Fig. 13. Adjustable sensors: heteoregeneous setting. Coverage (a), percentage of active sensors (b), and lifetime of the network (c) in the operative scenario with **heterogeneous** sensors with **adjustable** sensing radius.

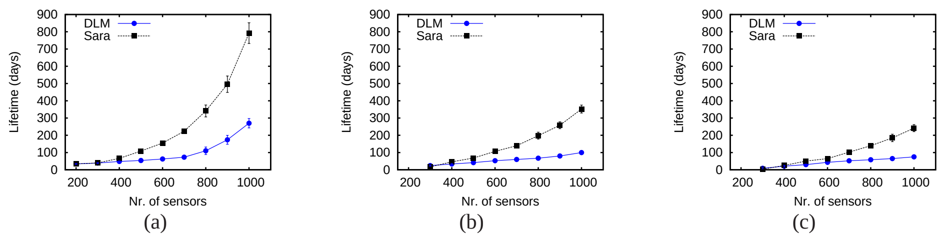


Fig. 14. Adjustable sensors: heterogeneous setting. Lifetime achieved by SARA and DLM expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

radii, while in the case of heterogeneous setting it is necessary to model the coverage responsibility regions of the devices in the Laguerre metric space. On the contrary both SARA and DLM are able to achieve the maximum coverage extent with a significantly lower percentage of active sensors.

Figure 13(b) highlights that, when working with a low number of available sensors, DLM activates a large fraction of redundant sensors having a small radius. In fact, being DLM based on a priority criterion which takes into account the number of intersection points when making activation decisions, when the number of sensors is so small, it is not able to give more priority to the sensors which contribute a better coverage. When the initial density of the available sensors is higher, the number of intersection points is more significant in reflecting the coverage that each sensor is capable to contribute to, resulting in a higher number of active sensors with larger sensing range.

Figure 13(c) shows the network lifetime achieved by DLM and SARA varying the number of available sensors in the heterogeneous setting. Although DLM is able to work in a heterogeneous scenario with sensors having different sensing ranges, it cannot exploit the device capability to adjust their radius. Therefore the lifetime under DLM is much shorter than under SARA. In particular, when the number of sensors is 900, the lifetime of SARA is almost twice the lifetime of DLM.

In Figure 14 we compare the algorithms in terms of network lifetime by increasing the number of deployed sensors. We consider the time at which the coverage of the AoI goes below the 80% (a), 90% (b) and 95%(c). Even under other possible definition of network lifetime, SARA outperforms DLM.

Notice that the network lifetime has a non linear dependence on the number of available sensors, as it increases more than linearly. This is due to the non-linear dependence of the energy consumption with respect to the sensing range. Indeed, the more sensors can be activated at small radius, the lower is the energy consumption and the longer is the lifetime.

7.6 Fixed sensors: heterogeneous setting

In this section we consider a scenario where sensors have a fixed sensing radius. We focus on the case where sensors have heterogeneous sensing capabilities: Half of the sensors have a sensing radius of 3m while the other half have a sensing radius of 6m. This is the scenario for which DLM was specifically designed. In this setting VRCSC is not able to guarantee maximum coverage in case of sensor heterogeneity. Therefore we will display only results for DLM and SARA.

The experiments show that SARA outperforms DLM in terms of percentage of the AoI covered over time (Figure 15(a)) and results into a lower number of dead sensors over time (Figure 15(c)). The percentage of awake sensors, displayed in Figure 15(b), shows a similar trend (for the same reason) than that discussed in Section 7.4. DLM experiences a higher number of awake sensors than SARA during the first 120 days. As a consequence, the number of sensors which are put to sleep (obtained as a complement to 1 of the sum of awake and dead sensors) will be much lower than in SARA. When time increases the reduced number of awake sensors in DLM reflects the high number of dead nodes, and consequently the poor coverage performance. These observations explain the fact that SARA experiences longer network lifetimes than DLM. This improvement is as high as twofold (Figure 15(f)).

Figure 15(d) and (e) shows the percentage of awake sensors with large and small radius under the execution of DLM and SARA, respectively. It is interesting to note that initially, under DLM, the majority of awake sensors have large radius. Nevertheless, after very few operative time intervals, nodes with large radius quickly deplete their energy, and after day 100, DLM can only work with sensors having small radius. On the contrary, SARA is able to successfully exploit device heterogeneity from the beginning, by activating sensors with large and small radius in different percentages, on the basis of coverage requirements. As a consequence, only at about day 200 SARA works with only sensors having small radius. For this reason the peak in Figure 15(b) in the number of active sensors is located on the right with respect to the one of DLM.

In Figure 16 we compare the algorithms SARA and DLM in terms of network lifetime by increasing the number of deployed sensors. We consider the time at which the coverage of the AoI goes below the 80% (a), 90% (b) and 95%(c). Notice that, in Figure 16(c) the point corresponding to the deployment of 300 sensors is missing, because even if all the sensors were kept awake, this amount of sensors would not be sufficient to cover the 95% of the AoI. Even in this case, although our algorithm does not specifically address a particular notion of lifetime, it outperforms DLM also under other possible lifetime requirements.

7.7 Mixed sensors: homogeneous setting

We consider the most general applicative scenario, with sensors belonging to both classes of fixed and adjustable sensing radius, we refer to a scenario with 900 uniformly deployed sensors. The set of available sensors is composed by 50% of fixed sensors with sensing radius equal to 6m and 50% of adjustable sensors with a sensing radius which varies in the interval [2m,6m]. Notices that this scenario is considered homogeneous because all

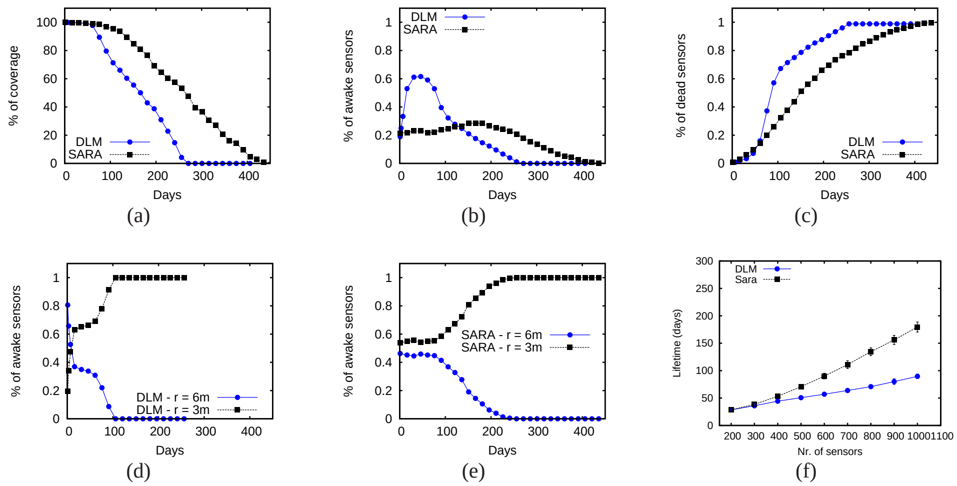


Fig. 15. Fixed sensors, heterogeneous setting. Scenario with 900 sensors. Comparative analysis of SARA and DLM. Percentage of AoI covered (a), percentage of awake sensors (b), percentage of dead sensors (c). Composition of the set of awake sensors under DLM (d) and SARA (e). Network lifetime (f).

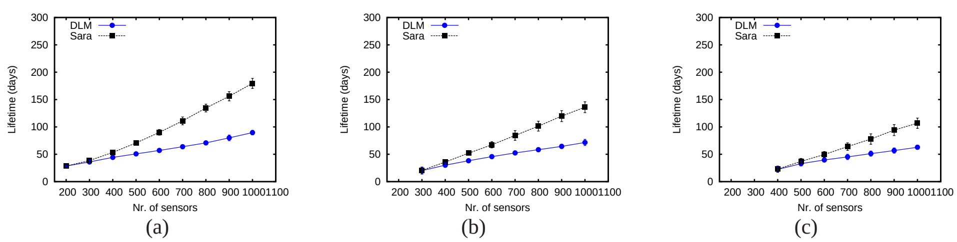


Fig. 16. Fixed sensors: heterogeneous setting. Lifetime achieved by the three algorithms expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

the sensors are able to reach the same maximum extension of the sensing range, no matter which class they belong to.

Figure 17(a) shows the percentage of the AoI that the algorithm SARA is able to cover as the time increases. The figure also shows the percentage of the AoI that is covered by the only sensors with adjustable radius, and by those with fixed radius separately. It is worth noting that at the first operative time intervals SARA privileges the sensors with adjustable range in the active set, as also detailed in Figure 17(b). This is due to the higher flexibility of the solution that can be obtained using this class of devices. As time progresses, the adjustable sensors that have been used extensively in the previous intervals begin to deplete their energy, thence SARA requires more fixed sensors to be included in the active set. It should be noted also that, see Figure 17(a), the percentage of dead sensors is about the same for the sensors of the two classes. This is due to the fact that, in this homogeneous setting, as long as a fixed sensor is activated, it consumes energy at the same rate of an adjustable sensor working at maximum sensing range. While this behavior was expected in the case of the algorithms DLM and VRCSC, as they do not distinguish the two classes, this has to be considered a nice property for the algorithm SARA as it evidences its capability to do the best with the two classes, exploiting their energy when possible in an equal manner. The Figure 17(d) shows the composition of the set of sleeping sensors, which is a complement to the values of figure (b) and (c).

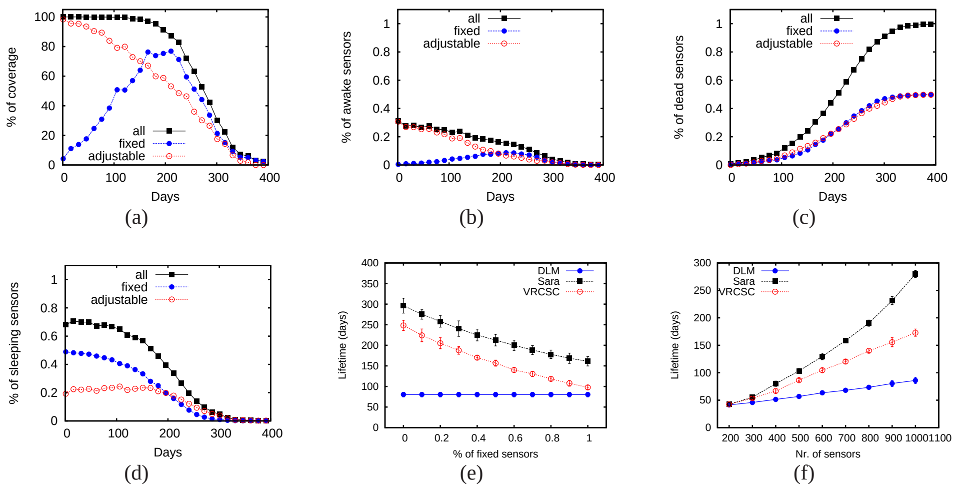


Fig. 17. Mixed sensors: homogeneous scenario. The maximum radius of adjustable sensors and the radius of fixed sensors are 6m. Case of 900 sensors, 50 % fixed and 50 % adjustable range: coverage (a), active (b), dead (c), and sleeping (d) sensors. Lifetime of the network by varying the percentage of fixed sensors with respect to total (e). Lifetime of the network by varying the number of available sensors (f).

We now comparatively analize the behavior of the algorithms VRCSC and DLM with respect to SARA.

Thanks to the device homogeneity, the algorithm VRCSC in its modified version introduced in Section 6, is able to work in this scenario without creating coverage holes. Nevertheless, the presence of fixed sensors in the available set, compromises the capability of VRCSC to correctly determine the maximum extent of the radius reduction to be adopted by sensors with adjustable range.

The algorithm DLM, in its modified version instead does not find more difficulties when dealing with this scenario than those encountered in addressing the case of only adjustable sensors, as it treats every sensor as if it were fixed.

The Figure 17(e) shows the lifetime of the network when the percentage of fixed sensors in the available set increases. Not surprisingly the performance of DLM is not affected by this increase as it treats the two classes alike. Both VRCSC and SARA show a decreasing behavior of the network lifetime due to the decreasing flexibility of the network. Indeed it is intuitive that by increasing the percentage of fixed sensors, in the homogeneous case, we are significantly reducing the set of possible solutions that can be reached by any algorithm. Nevertheless SARA is less affected by this phenomenon as it can exploit the capability of the two classes of sensors with more specifically tailored decisions.

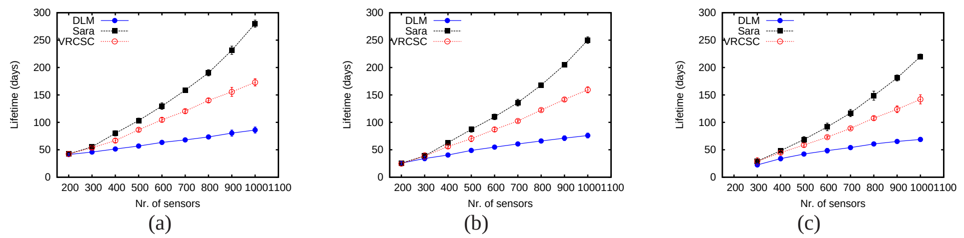


Fig. 18. Mixed sensors: homogeneous setting. Lifetime achieved by the three algorithms expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

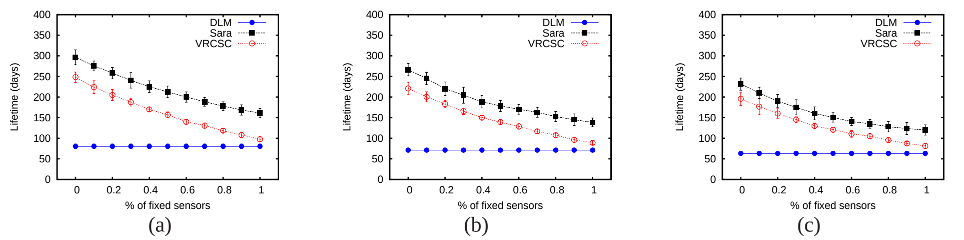


Fig. 19. Mixed sensors: homogeneous setting. Lifetime of the network by varying the percentage of fixed sensors with respect to total. Lifetime achieved by the three algorithms expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

All the above considerations justify the significant improvement in terms of lifetime achieved by `SARA` with respect to `DLM` and `VRCSC`. To highlight this difference we now consider an experiment conducted by varying the number of available sensors, with a set composed of 50% of sensors with adjustable radius, and 50% with fixed radius. The Figure 17(e) illustrates the behavior of the three algorithms in this setting. For instance, when the number of sensors is 1000, `SARA` achieves a lifetime of 280 days, whereas `VRCSC` reaches 170 days, and `DLM` only 80 days.

In Figure 18 we compare the algorithms `SARA`, `VRCSC` and `DLM` in terms of network lifetime by increasing the number of deployed sensors. We consider the time at which the coverage of the AoI goes below the 80% (a), 90% (b) and 95%(c). Even in this case, although `SARA` does not specifically address a particular notion of lifetime, it outperforms the other two also under other possible lifetime requirements.

Figure 19 shows the performance of the three algorithms by varying the percentage of fixed sensors in the available set when 900 sensors are deployed. The figure highlights that even under other possible definitions of lifetime `SARA` outperforms `DLM` and `VRCSC`.

7.8 Mixed sensors: heterogeneous setting

In this latter subsection, we consider the most general applicative scenario, where sensors belong to both classes and where the two classes have heterogeneous sensing capabilities. In particular, the radius of fixed sensors is 3m long, while the radius of adjustable sensors varies in the interval [2m, 6m].

The qualitative analysis of the results shown in Figures 20(a-f) is analogous to the case of the homogeneous setting. Nevertheless, Figure 20(c) highlights that, in this case, the set of dead sensors is composed by a higher fraction of adjustable sensors with respect to the homogeneous case. This is due to the fact that we are considering fixed sensors with lower range, that implies for this class a lower energy consumption rate with respect to the homogeneous case, resulting in a higher residual energy for the fixed sensors, as shown in Figure 20(e). Notice that, as in all the heterogeneous cases treated in this paper, we did not analyze the behavior of VRCSC in this scenario, as the comparison cannot be fair, because VRCSC does not succeed in completing the coverage of the AoI. Finally, in Figure 20(f) we show that, as expected, the lifetime achieved by SARA is significantly longer than under DLM. For instance, when the number of sensors is 1000, SARA achieves a lifetime of about 750 days, while DLM is only capable to last no more than 270 days.

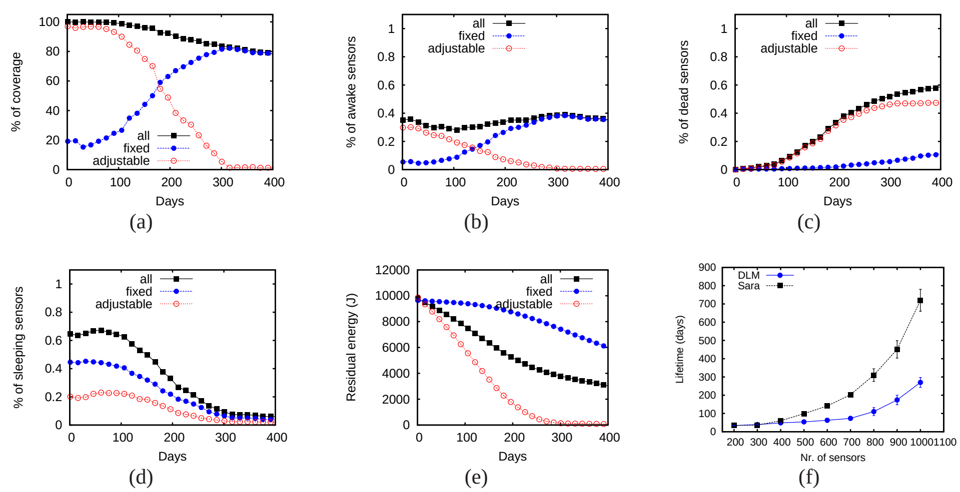
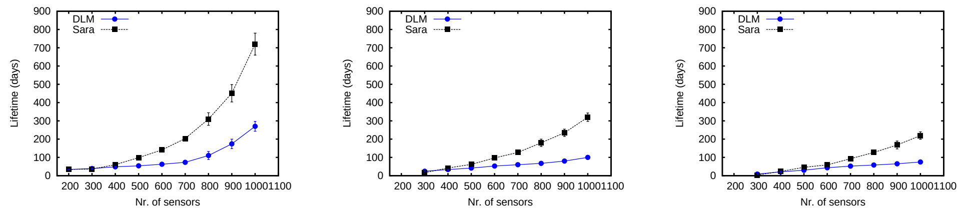


Fig. 20. Percentage of coverage (a), active (b), dead (c), sleeping (d) sensors and residual energy (e) in a scenario with 900 **heterogeneously** equipped sensors of **both classes** of devices (50 % with fixed and 50 % with adjustable sensing range). Lifetime of the network when varying the number of sensors (50 % of each class).

Notice that in this heterogeneous setting, it does not make sense to analyze the performance of the algorithms when the percentage of the two classes of sensors varies. This is because the fixed sensors have different sensing capabilities than the maximum for adjustable sensors. Therefore, by varying the composition of the mix we would alter the coverage capability of the network.

In Figure 21 we compare the algorithms SARA and DLM in terms of network lifetime by increasing the number of deployed sensors. We consider the time at which the coverage of the AoI goes below the 80% (a), 90% (b) and 95%(c). Even in this case, although SARA does not specifically address a particular notion of lifetime, it outperforms DLM also under other possible lifetime requirements.



(a) (b) (c)

Fig. 21. Mixed sensors: heterogeneous setting. Lifetime achieved by the three algorithms expressed as the time after which the algorithm is no longer capable to cover more that 80% (a), 90% (b) and 95% (c) of the AoI.

8. RELATED WORKS

The problem of exploiting network redundancies to prolong the network lifetime has been largely investigated in the literature so far. Depending on the application requirements, the approach to the problem may vary significantly. For example, some works only aim at guaranteeing network connectivity, the SPAN [Chen et al. 2002] and ASCENT [Cerpa and Estrin 2004] protocols just to mention the most acknowledged, without considering coverage issues. Due to space limitations, in this section we only consider the works dealing with the problem of completely covering an area of interest and we refer the reader to the work [Rowaihy et al. 2007] from Rawaihy et al. for a survey of sensor scheduling policies in several other applicative scenarios.

The PEAS protocol proposed by Ye et al. in [Ye et al. 2003] was designed to address both coverage and connectivity at the same time. According to this protocol only a subset of nodes stay awake while the others are put to sleep. A sleeping node occasionally wakes up to determine the presence of coverage holes in its proximity and make activation decisions accordingly. This approach does not ensure complete coverage, as coverage holes cannot be discovered until a nearby sleeping sensor wakes up. Another randomized algorithm is proposed by Xiao et al. in [Xiao et al. 2010]. Different sets of sensors work alternatively according to a probabilistic scheduling. The authors study the performance of the proposed approach in terms of coverage extension and detection delay. Differently from the works in [Ye et al. 2003; Xiao et al. 2010], our approach aims at ensuring the coverage completeness as long as the available sensors have enough energy.

Xing et al. [Xing et al. 2005] propose the protocol CPP to achieve k-coverage of an area of interest while maintaining the network connectivity.. They address both coverage and connectivity, and in particular they define an operative setting in which the the former implies the latter, namely when the transmission radius is at least twice the sensing range. They also provide necessary and sufficient conditions for an area to be k-covered. The authors point out that the network lifetime achieved by their algorithm does not linearly scale with the number of sensing nodes, due to the higher energy consumption related to periodic beacon messages. Our work addresses the same operative setting (with $k = 1$) with a more aggressive scheduling policy by resorting to the Laguerre metric space rather than to the Euclidean one, thus allowing a better scalability. The geometric analysis made in [Xing et al. 2005] is at the basis of several subsequent works, such as the one from Kaskebar et al. [Kasbekar et al. 2009] that we study in more detail in section 6.

In the work [Cardei and Du 2005] by Cardei, Du et al., the sensor nodes are divided into disjoint sets, such that at a specific time only one sensor set is responsible for sensing the targets, while the sensors of the other sets are kept in a low power mode. The sets are scheduled in a round robin manner and operate for equal time intervals. The authors prove that finding the maximum number of disjoint sets is an NP-complete problem. For this reason they propose the use of a heuristic approach to calculate the set covers on the basis of a mixed integer programming model. The main drawback of this approach is that it is centralized, which is not desirable in a sensor network environment. The constraint of having disjoint set covers operating for equal time intervals is relaxed in the work [Cardei et al. 2005] by Cardei, Thai et al., and two heuristics are proposed, one using linear programming and the other using a greedy approach.

In [Funke et al. 2007], Funke et al. consider the problem of selecting a set of awake sensors of minimum cardinality so that sensing coverage and network connectivity are maintained. The authors analyze the performance of a greedy solution for complete coverage showing that it achieves an approximation factor no better than $\Omega(\log n)$, where n is the number of sensor nodes. For this reason, the authors also present algorithms that provide approximate coverage while the number of nodes selected is a constant factor far from the optimal solution.

The same problem is addressed by Tian et al. in [Tian and Georganas 2002] and by Bulut et al. in [Bulut et al. 2008]. These works considered the coverage problem aiming at activating only a minimal number of sensors and letting the others conserve their energy in a low power mode. Each sensor periodically evaluates its sensing area to determine whether it is also covered by other sensors. Once a sensor has determined its redundancy, it can deactivate itself. Since several sensors may determine that they can go to sleep at the same time, a back-off based policy is proposed to prevent collisions and impose a unique order of deactivation. These proposals are similar to the way our algorithm eliminates the redundancies in the case of sensors endowed with fixed sensing capabilities. Nevertheless, the way we give priority to sensors having higher overlaps is completely different, as it is based on a more refined evaluation of the coverage diagram of the network deployment.

None of the aforementioned works addresses the problem of continuously covering an area of interest with some or all sensors being able to modulate their sensing ranges as we do in this paper. This operative setting, but with discrete coverage targets, is analyzed by Cardei, Wu et al. in [Cardei et al. 2006]. The proposed solution is based on non-disjoint set cover scheduling. The approach is centralized and the problem is proved to be NP-complete. For this reason the authors provide two heuristics, both centralized and distributed.

A limitation of all the above mentioned methods, is that they cannot be dynamically reconfigured to accommodate different density requirements, being them time-varying or position dependent. Nevertheless, our approach proves to be very versatile in this operative scenario, and is also robust to network heterogeneity and diverse energy availability and harvesting capacity.

An approach that is able to take account of event dynamics is the one proposed in [He et al. 2009] by He et al., where the scheduling policy is based on a probabilistic technique. Nevertheless, this work assumes a Boolean sensing model and does not address the case of non uniform device capabilities and energy availability. Furthermore it does not address the case of sensors endowed with adjustable sensing ranges.

Two recent works by Kaskebar et al. [Kasbekar et al. 2009], and by Zou et al. [Zou et al. 2009], propose the algorithms `DLM` and `VRCSC`, respectively. These algorithms are described in deeper details in Section 6 where we also make performance comparisons with our proposal.

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A policy based on the setting of a back-off period for putting redundant devices to sleep was proposed in [Tian and Georganas 2002; Bulut et al. 2008]. Unlike these previous proposals, we are able to set the sleep priority of the individual devices on the basis of the parameter α which can be defined according to specific application goals. Furthermore, the mentioned proposals do not deal with the case of heterogeneous networks with the contemporary presence of sensors with fixed and adjustable sensing capabilities.

10. CONCLUSIONS

We proposed a new algorithm for prolonging the lifetime of a heterogeneous wireless sensor network (WSN) through selective Sensor Activation and sensing Radius Adaptation (SARA). Our approach to joint sensor activation and radio adaptation is very general, and is the first to be applicable to scenarios with devices with adjustable and fixed sensing ranges (heterogeneous WSNs). In particular we focus on networks where some devices are able to adjust their sensing range so as to decrease the energy consumption. The proposed algorithm is based on a model of the coverage problem which uses Voronoi-Laguerre diagrams. This model allows to explicitly take account of device heterogeneity. We prove the convergence, termination and the Pareto-optimality of our approach. The proposed algorithm achieves longer lifetime and higher coverage than previous solutions in all the considered scenarios.

11. APPENDIX

Details about loose boundary farthest vertices

To complete our geometrical analysis, we now detail the general methodology according to which a sensor s can determine whether a boundary farthest vertex $F = f(V(\mathscr{C}))$ of its polygon is strict or loose.

In order to illustrate the methodology, let us consider the example of Fig. 4(b). The sensors s, s_i, s_k and s_l generate a common boundary farthest vertex $F = f(V(\mathscr{C}))$. This vertex is a strict boundary farthest for all the generating sensors with the exception of s. Indeed, as also shown in Fig. 5, the sensor s can still reduce its sensing radius without leaving a coverage hole.

The edges of the polygon $V(\mathscr{C})$ that intersect in F are generated by intersecting the circles $\mathscr{C}$ and $\mathscr{C}_l$ and the circles $\mathscr{C}$ and $\mathscr{C}_k$, where s_l and s_k are the Voronoi-Laguerre neighbors that with s generate the common farthest F. The circles $\mathscr{C}_l$ and $\mathscr{C}_k$ also intersect each other in the point F' that we call *opposite farthest with respect to* s.

Notice that if F is a loose farthest vertex, then F' must be internal to the angle formed by the VorLag axes generating the boundary farthest F and on the side of $V(\mathscr{C})$. Indeed, if $f(V(\mathscr{C}))$ is loose, then it exists a finite value δ such that every point at distance less than δ from F and internal to $V(\mathscr{C})$ is covered by at least another sensor. As a consequence of Theorem 3.3, the points of the δ-surrounding of F internal to $V(\mathscr{C})$ are also covered by a Voronoi-Laguerre neighbor [13].

Therefore let us consider the only Voronoi-Laguerre neighbors of s. The neighbors that will be able to cover an area arbitrarily close to F are therefore s_l and s_k. The only way for these sensor to avoid leaving any point 1-covered in the surrounding of F is to intersect each other in a point (the opposite farthest F') that is internal to the axes generating F and on the side of $V(\mathscr{C})$.

Observe that also the opposite implication holds as, if F' is included in the angle formed by the VorLag axes generating $f(V(\mathscr{C}))$, then $\mathscr{C}_l \cup \mathscr{C}_k$ cover both F and any other point

[13] The theorem also mentions sensors with null polygons, but the points around a boundary farthest cannot be covered by a null polygon because any sensor added in a position where it overlaps a boundary farthest generated by other sensors, would generate its own polygon, containing the point that was a boundary farthest before its addition

at distance less than δ from it, for some finite value of δ, and hence F is a loose farthest. We can summarize the previous reasonings in the following:

Characterization of loose farthest vertices. *Consider a sensor s, and let the farthest vertex $f(V(\mathscr{C}))$ of its Voronoi-Laguerre cell from s be determined as the intersection point of the edges of $V(\mathscr{C})$ lying on the axes formed by s and s_l and by s and s_k. $f(V(\mathscr{C}))$ is loose for s if and only if the opposite farthest F' with respect to s lies inside the angle formed by the VorLag axes generating $f(V(\mathscr{C}))$ which contains s.*

Figure 4 evidences some examples of positions of the circles $\mathscr{C}$, $\mathscr{C}_l$ and $\mathscr{C}_k$ that generate a strict farthest (in (a)), and a loose one (in (b)). The figure highlights the position of the farthest vertex F and of the opposite farthest F'. Notice that the opposite farthest F' can be external to $V(\mathscr{C})$ (on the opposite side of the polygon with respect to F) and still determine a loose farthest vertex situation.

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