Analytically Modelling the Performance of Piggybacking on Beacons in VANETs

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ABSTRACT

Piggybacking on beacons is a forwarding technique in vehicular ad-hoc networks (VANET) as a means to disseminate data. With this technique data is attached to and transmitted along with scheduled beacons. Nodes are assumed to beacon asynchronously.

In this paper we present a first version of an analytical model that is able to accurately capture the performance of a piggybacking protocol inside a VANET in a number of closed-form expressions, assuming some simplifications. For a given forwarding distance, transmission range, node density, and beacon frequency, the model is able to give the stochastic distribution of the end-to-end delay. The model also provides the distribution of the per-hop delay, the hop length, and the position of the i^{th} forwarder. We have verified our analytical model using a simulation study.

The most relevant assumptions in our model are a fixed inter-node distance and a fixed deterministic transmission range. Having completed this stage of our work, our next goal is to extend our model so that we can drop these assumptions.

Categories and Subject Descriptors

C.2.2 [**Computer-Communication Networks**]: Network Protocols—*Routing protocols*; C.4 [**Performance of Systems**]: Modeling techniques

General Terms

Performance, Verification

Keywords

Beaconing, Piggybacking, Vehicular Ad-Hoc Network, VANET

1. INTRODUCTION

Piggybacking is a forwarding technique in which data is not forwarded independently, but is attached to (and transmitted along with) packets that have already been scheduled for transmission in the near future. Often these latter packets will be network- or transport level control packets, i.e., packets containing information that must be transmitted for the proper functioning of the protocol.

The piggybacking protocol that is considered here operates in the context of a vehicular ad-hoc network (VANET), and attaches higher-level data to network-level *beacons*. Beacons have been defined by ETSI as short status messages that every node in a VANET regularly broadcasts [9]; they play a critical role both at the network level (for routing) and the application level (to create mutual awareness). At the link level beacons are transmitted as IEEE 802.11p [4] broadcast messages. The IEEE 802.11p amendment to the IEEE 802.11 standard has been specifically designed to support wireless access in vehicular environments (WAVE).

Piggybacking is a forwarding technique that is regularly used in VANETs, see Section 2. Previous research [10] [11] has shown that the performance of piggybacking depends on a limited set of network parameters, such as the dissemination distance, the number of nodes within transmission range, and the frequency with which beacons are broadcasted. The main research question of this paper is whether we can analytically model the performance of a piggybacking protocol as a function of these network parameters.

The main contribution of this paper is an analytical model that expresses the performance of a piggybacking protocol inside a VANET in analytical and often even closed-form expressions. Verification of the model showed that our model results are accurate within a few percent. The model takes the dissemination distance, the node density, the transmission range, and the beacon frequency into account, and gives (closed-form) expressions of the following performance metrics:

1. the distribution of the end-to-end delay;
2. the distribution of the required number of hops to reach the sink;
3. the distribution of the per-hop delay, and;
4. the distribution of the position of the i^{th} forwarder.

The model contains some assumptions, mainly regarding the topology of the network and transmission reception probabilities.

The outline of this document is as follows. We start by discussing some related work on VANETs, beaconing, and piggybacking in Section 2. Then we introduce our model of a piggybacking system in Section 3. In this section the considered piggybacking protocol is specified and our assumptions are listed. We consider two types of beaconing: one in which inter-beacon times are deterministic, and one in which inter-beacon times are exponentially distributed. In Section 4 we present a model analysis when inter-beacon times are distributed exponentially. In Section 5 we present a model analysis when inter-beacon times are deterministic. We have performed a simulation study in order to verify the correctness of our model analysis. The simulation set-up is described in Section 6, while we discuss the verification results in Section 7. In Section 7 we also discuss the validity of the assumptions of our piggybacking model. Finally, in Section 8, we conclude our work and give a preview on our next steps.

2. RELATED WORK

In VANET research *beaconing* refers to the periodical 1-hop broadcast of network-level status messages. Because of their function they have been defined by ETSI in [9] as Cooperative Awareness Messages (CAM). These messages contain information that is both relevant for application-level safety applications and network-level routing: almost all co-operative road safety applications defined by ETSI are based on beaconing, and received beacons are used to create location tables that are used by geo-networking routing protocols [5].

As the node density in a network increases it becomes a challenging task to create a scalable beaconing approach[19]. In general the more advanced beaconing solutions propose to adapt in a distributed fashion the frequency and the used transmission power with which beacons are sent. Their goal is to have nodes beacon at the highest possible frequency without congesting the wireless medium, while sharing bandwidth fairly [15] [18] [16].

Using beacons to piggyback data inside a VANET is a common method that has been applied in a number of dissemination schemes [10] [16] [20]. The schemes in [10] and [16] can be considered pure piggybacking approaches. The approach in [20] suffers from the scalability problems mentioned in [19], causing the network to become congested. Although all of the works mentioned here have evaluated the performance of their respective beaconing-based solutions, their main performance metric has been the delivery ratio, and none has made an attempt at expressing delay as a function of network parameters.

Although there is a plethora of studies on multi-hop forwarding protocols in VANETs (see above), practically all of these studies are simulation based, which is the preferred method of study in VANET research. Analytical studies are mainly restricted to single-hop situations in which nodes all share the same collision domain, and in general focus on the scalability of beaconing, see e.g. [7]. In the area of sensor networks analytical models are more often used to evaluate the performance of an entire network (e.g., [8]), but the context in which these networks work (a static network topology with low rate data collection), their communication methods (synchronized beaconing and forwarding), and their performance metrics of interest (usually the network lifetime) do not apply to our application.

The main performance metric of our model is the end-to-end delay. In [11] an extensive simulation study shows how

Figure 1: An example of a network where $d_D = 300$ m, $d_{IN} = 25$ m, and $R = 100$ m. The lower row of numbers denotes the positions of the nodes.

the end-to-end delay of a piggybacking protocol depends for the most part on the average inter-node distance (i.e., the node density), the transmission range, and the beacon frequency. The paper does not quantify the impact these parameters have on the delay; it only shows trends for different simulation scenarios. In this paper we do model these dependencies and give not only the average delay, but also the end-to-end delay distribution.

3. THE SYSTEM MODEL

In this section our model of a piggybacking system is described. We include the considered network topology, the packet reception probabilities, beacon timing distributions, and of course the protocol itself. Relevant assumptions contained in the model are listed and discussed at the end.

Nodes are equidistantly spaced every d_{IN} m ('inter-node distance') over a straight line, with the source at 0 m and the sink at d_D m (the 'dissemination distance'), see Fig. 1. For ease of explanation we number the nodes in ascending order from source to sink: node j (denoted n_j) is located at position $j \cdot d_{IN}, j = 0, 1, \ldots, d_D/d_{IN}$. Nodes are static: they remain at their position. The node that forwards the message for the i^{th} time after the source's original transmission is referred to as the i^{th} forwarder. Every node has the same deterministic transmission range R: all nodes within this range are assumed to successfully receive a transmitted signal without suffering from fading or from interfering signals. Finally, all delays related to transmitting and processing a signal (i.e., transmission delay, switching times, etc.) are set to zero.

Nodes beacon independently of each other. Every time a node has beaconed it will draw a new i.i.d. inter-beacon time from the inter-beacon time distribution T (in s). T is distributed either deterministically or exponentially, both with average inter-beacon time T_b. Although congestion-avoidance mechanisms may regulate beacon frequencies over time, for a given beaconing rate the inter-beacon times are generally assumed to be deterministic.

The piggybacking protocol works as follows. All data forwarding is done by means of piggybacking. At time t_{start} the source node piggybacks the application message on top of a beacon. The message has a geographically defined destination address which is the position of the sink. Nodes are assumed to know their own position and include it in their beacons. A receiver of a beacon thus knows the position of the sender of the beacon. All nodes apply the following forwarding rule: when a node receives a piggybacked message at time t, and it has scheduled to beacon at time t_b ($t_b > t$), then it will itself piggyback the message at t_b *if and only if* by that time it has not received the message from any node positioned closer to the sink than itself. This effectively makes sure that each new forwarder will be closer to the sink than the previous forwarder.

In the next two sections we analyse our model when inter-beacon times are distributed exponentially and when they are distributed deterministically. The difference between both methods lies in the fact that the exponential distribution is memoryless, while the deterministic distribution is not. With exponentially distributed inter-beacon times the behaviour of the model only depends on its current state, whereas with deterministically distributed inter-beacon times the previous states must also be taken into account.

4. ANALYSING THE SYSTEM USING EXPONENTIALLY DISTRIBUTED INTER-BEACON TIMES

In this section we focus on analysing our piggybacking model, as presented in Section 3, when inter-beacon times are distributed exponentially. We calculate the hop time (H) in Section 4.1 and the hop length (L) in Section 4.2. Using the latter result we calculate the distribution of the required number of hops to reach the sink node (N) in Section 4.4. Finally we combine the distributions of H and N to determine the distribution of the delay (D) in Section 4.5.

4.1 Hop Time

Let H_i be the hop-time for hop i: the time between the moment that the message is forwarded for the $(i-1)^{th}$ and the i^{th} time. Let $t_{start} = 0$ be the moment that the source piggybacks the message. H_1 is the time between t_{start} and the first time it is retransmitted. In this section we will focus on determining H_i using the *residual inter-beacon time* $\dot{T}$.

We first define any node that is within R m of the most recent forwarder and positioned closer to the sink than the most recent forwarder, to be a *candidate forwarder*. The set of candidate forwarders is by definition of size $\frac{R}{d_{IN}}$, except if the distance between the last forwarder and the sink is less than R. We ignore this effect however, since in that case the sink has already received the message.

Let the random variable $\dot{T}$ then describe the residual inter-beacon time of a candidate forwarder. For a candidate forwarder that has its next beacon moment at t_b, its residual inter-beacon time $\dot{t}$ at time t_{now} ($t_{now} \leq t_b$) is given by

$$\dot{t} = t_b - t_{now}. \tag{1}$$

When the source has transmitted the message at t_{start} there are $\frac{R}{d_{IN}}$ candidate forwarders (in Figure 1 nodes n_1, ..., n_4 can all act as the first forwarder). The node that acts as the first forwarder is the node that has the smallest residual inter-beacon time at t_{start}. H_1 is thus distributed as the minimum value of the $\frac{R}{d_{IN}}$ residual inter-beacon times of the candidate forwarders, which are independent and distributed as $\dot{T}$. Since T is distributed exponentially, $\dot{T}$ is also exponentially distributed with the same mean T_b, due to the memoryless property of the exponential distribution. The minimum of $\frac{R}{d_{IN}}$ residual inter-beacon times with mean T_b is also exponentially distributed, with mean $\frac{T_b}{R/d_{IN}}$. The CDF of H_1 is thus given by

$$F_{H_1}(t) = \begin{cases} 1 - \exp^{\frac{R}{d_{IN}} \cdot \frac{-t}{T_b}} & t \geq 0, \\ 0 & t < 0. \end{cases} \tag{2}$$

To calculate H_2 we are again interested in the moment when one of $\frac{R}{d_{IN}}$ nodes beacons for the first time. It is important to note here that some of the candidate nodes may have received the message twice (from the source node and from the first forwarder), while the other nodes have only received it once (from the first forwarder). A part of the former group's residual inter-beacon time has thus already passed. However, again due to the memorylessness of the exponential distribution, this does not affect the distribution of the residual inter-beacon time of nodes in the former group. The distribution of H_2 is therefore equal to the distribution of H_1. Since this argument applies for each hop we can state that all successive hop times H_2, H_3, ... have the same distribution as H_1 and are mutually independent.

4.2 Hop Length

Let X_i be the position of the i^{th} forwarder. Let L_i be the length of a hop, defined as

$$L_i = X_i - X_{i-1}. \tag{3}$$

The source is positioned at $X_0 = 0$ m, so $L_1 = X_1$. When the source transmits the message at t_{start} there are $\frac{R}{d_{IN}}$ first hop candidate forwarders, positioned every d_{IN} m:

$$X_1 \in \{d_{IN}, 2d_{IN}, ..., R\}. \tag{4}$$

The node that acts as the first forwarder is the candidate node that has the smallest residual inter-beacon time $\dot{T}$. Since all candidate forwarders have the same exponential rate $\frac{1}{T_b}$ for $\dot{T}_1$, each candidate forwarder has equal probability of becoming the next forwarder. This holds for all hops (see Section 4.1), so L_i is given by

$$L_i \sim U(d_{IN}, 2d_{IN}, \ldots, R) \ \forall\, i > 0, \tag{5}$$

where $U(\cdot)$ is a discrete uniform distribution.

4.3 Position of the Forwarder

Because the distribution of the hop length is i.i.d. (see Eq. (5)), the distribution of the position of the i^{th} forwarder is equal to the i^{th} convolution of L_i, i.e.,

$$X_i = \sum_{j=1}^{i} L_j, \quad \forall\, i > 0. \tag{6}$$

In [12] the following explicit expression has been given for the i^{th} convolution of a discrete, uniformly distributed random variable

$$P(X_i = x_i) = \frac{1}{l^i} \sum_{j=0}^{\lfloor k/l \rfloor} (-1)^j \binom{i}{j} \binom{i+k-lj-1}{i-1}, \tag{7}$$

with $l = R/d_{IN}$, $k = x_i/d_{IN}$.

4.4 Required Number of Hops

Let N_x be the distribution of the number of hops required to have the message piggybacked by a node at position x or beyond. To have the sink at position d_D receive the message, a node that is positioned on or beyond $d_D - R$ must transmit the message. We are therefore actually interested in the distribution of $N_{d_D - R}$. In our discussion we use the more general term N_x however.

The position of the i^{th} forwarder is a convolution of i hop lengths, see Eq. (6). For arbitrary values of $x > 0$ we can thus state

$$P(N_x \geq i) = P(X_i \leq x_i) = P(L_1 + \cdots + L_i \leq x). \tag{8}$$

To calculate the latter term we can again use Eq. (7):

$$P(X_i \leq x_i) = \sum_{x=0}^{x=x_i} P(X_i = x) \tag{9}$$

For large values of x we can also use the following method. We know that N_x is distributed as the largest integer $i > 0$ for which holds that $X_i \leq x$ for $x > 0$, and that X_i is a convolution of i.i.d. hop lengths, see Eq. (6). According to [17] (page 5), as x goes to infinity, N_x then has the following asymptotically normal distribution:

$$N_x \sim N(\frac{x}{\mu_{L_i}}, \sigma^2_{L_i} \frac{x}{(\mu_{L_i})^3}), \tag{10}$$

where $L_i \sim L_1$, and μ_{L_i} and $\sigma^2_{L_i}$ are the mean and variance of L_i, which can easily be calculated.

When we calculate N_x in our discussion in Section 7 we use Eq. (9) when $x \leq 2R$, else we use Eq. (10).

4.5 Delay

Let D be the delay to piggyback the message from source to sink. It is a function of the number of hops required to bridge the source-to-sink distance (N_x), and the delay per hop (H_i). Since the delay per hop is exponentially distributed with mean $\frac{T_b}{R/d_{IN}}$, the total delay for k hops, denoted D_k, is given by the Erlang-k distribution, the PDF of which is given by

$$f_{D_k}(t) = \lambda^k \frac{t^{k-1}}{(k-1)!} \exp(-\lambda t), \quad t \geq 0. \tag{11}$$

The minimum number of hops required to deliver the message at the sink is $\lfloor \frac{d_D - R}{R} \rfloor$, the maximum is $\frac{d_D - R}{d_I N}$. If $d_D \leq R$ then the delay is obviously zero. Summing over all possible hop counts n and hop times t, the PDF of D is given by

$$f_D(t) = \sum_{n=\lfloor \frac{d_D - R}{R} \rfloor}^{\frac{d_D}{d_{IN}}} P(N_{d_D - R} = n) \cdot f_{D_n}(t), \quad t \geq 0 \wedge d_D > R. \tag{12}$$

5. ANALYSING THE SYSTEM USING DETERMINISTICALLY DISTRIBUTED INTER-BEACON TIMES

In this section we focus on analysing our model, as presented in Section 3, when inter-beacon times are distributed deterministically. We first introduce our general approach, and then calculate the different performance metrics.

We saw in the previous section that when inter-beacon times are distributed exponentially, the hop length has an identical, discrete uniform distribution for each hop, which allows us to express the required number of hops as a simple convolution of these i.i.d. hop lengths. Similarly, the hop time for each hop has an identical exponential distribution, which allows us to use the Erlang-k distribution to calculate the delay for a given number of hops.

When inter-beacon times are distributed deterministically however, neither the hop length nor the hop time is i.i.d., so we cannot take the same approach. Both distributions depend on the hop time *and* the hop length of *all* the preceding hops, and the distributions must be calculated separately for each hop. As we will show however the distribution of the hop length and of the hop time converge after the first three hops. For the hop length and the hop time of the fourth hop (and following hops) we can therefore assume that they are i.i.d., allowing us to express the number of hops required beyond the third hop as a convolution of i.i.d. hop lengths, and the delay for a given number of hops beyond the third hop as a convolution of i.i.d. hop times.

Our approach in this section is as follows. In Section 5.1 we calculate the distribution of the hop time for the first two hops; we assume that the hop time of following hops is distributed identical to the hop time of the second hop. In Section 5.2 we calculate the distribution of the position of the forwarder for the first three hops. Using this, we calculate the distribution of the hop length for the first three hops in Section 5.3; we assume that the hop length of following hops is distributed identical to the hop length of the third hop. Using this latter assumption we then calculate the distribution of the position of the forwarder for the fourth hop and following in Section 5.4. In Section 5.5 we calculate the required number of hops: for the first three hops we use the distribution of the forwarder that was calculated in Section 5.2, the following hops are expressed as a convolution of i.i.d. hop lengths. Finally in Section 5.6 we calculate the distribution of the delay: for the first hop we use the distribution of the hop time of the first hop, all following hops are expressed as a convolution of i.i.d. hop times.

5.1 Hop Time

The concept of hop time was already explained in Section 4.1. We will again use the residual inter-beacon time as given by Eq. (1). For each hop the hop time is distributed as the minimum value of the residual inter-beacon times of the candidate forwarders. As we will show the residual inter-beacon times for the first hop is i.i.d. for each node. For the following hops however this does no longer hold, and the hop times of the previous hop(s) must be taken into account.

Let the source again transmit the message at $t_{start} = 0$. Since nodes beacon deterministically and t_{start} is chosen randomly, the per-node residual inter-beacon time for all candidate forwarders for the first retransmission, $\dot{T}_1$, is uniformly distributed in $\langle 0, T_b]$. The CDF is given by

$$F_{\dot{T}_1}(t, \dot{T}_{max}) = \frac{t}{\dot{T}_{max}}, \quad 0 < t \leq T_{max}, \tag{13}$$

with the maximum residual inter-beacon time $\dot{T}_{max} = T_b$, $F_{\dot{T}_1}(t, \dot{T}_{max}) = 0$ for $t \leq 0$ and $F_{\dot{T}_1}(t, \dot{T}_{max}) = 1$ for $t > \dot{T}_{max}$. Just as with exponential inter-beacon times, H_1 is distributed as the minimum value of the residual inter-beacon times of the candidate forwarders. The CDF of such a distribution is well known [14], and for k candidate forwarders and a maximum residual inter-beacon time of $\dot{T}_{max}$ is given by

$$F_{H_1}(t, \dot{T}_{max}, k) = 1 - \Big(1 - \frac{t}{\dot{T}_{max}}\Big)^k, \quad 0 \leq t \leq \dot{T}_{max}, \tag{14}$$

with $k = \frac{R}{d_{IN}}$ and $\dot{T}_{max} = T_b$, $F_{H_1}(t, \dot{T}_{max}, k) = 0$ for $t < 0$ and $F_{H_1}(t, \dot{T}_{max}, k) = 1$ for $t > \dot{T}_{max}$. $F_{H_1}(t, \dot{T}_{max}, k)$ gives the probability that one of k nodes will have beaconed in the period $\langle 0, t]$. Because we will have need of it later on we define $\bar{F}_{H_1}(t, \dot{T}_{max}, k)$ as the probability that *none* of k nodes will have beaconed in this period, given by

$$\bar{F}_{H_1}(t, \dot{T}_{max}, k) = 1 - F_{H_1}(t, \dot{T}_{max}, k) \tag{15}$$

When we want to determine the distribution of H_2 we can no longer assume that the per-node residual inter-beacon time distributions of all the candidate nodes are identical. The distribution of a candidate forwarder's residual inter-beacon time depends on whether the forwarder already received the message for the first time in hop 0 (i.e., from the source) or in hop 1. If it received the message first in hop 1 then its distribution is equal to Eq. (13). If it received the message already in hop 0 then the hop time of the first hop must be taken into account: let t_1 be the hop time of hop 1, then the remaining residual inter-beacon time of the candidate forwarder is uniformly distributed in $\langle 0, T_b - t_1]$. We denote the set of candidate forwarders for hop 2 that received the message in hop 0 as $x_{2,0}$ and the set of candidate forwarders for hop 2 that received the message in hop 1 as $x_{2,1}$. The size of both of these sets is defined by the position of the first forwarder. Taking into account all possible positions of the first forwarder and all possible hop-times of the first hop, the CDF of H_2 for $0 \leq t \leq T_b$ can be obtained by the convolution

$$F_{H_2}(t) = \sum_{x_1=d_{IN}}^{R} P(X_1 = x_1) \int_{t_1=0}^{T_b} f_{H_1}(t_1, T_b, R/d_{IN}) \cdot \Big(1 - \bar{F}_{H_1}(t, T_b - t_1, |x_{2,0}|) \cdot \bar{F}_{H_1}(t, T_b, |x_{2,1}|)\Big)\, dt_1, \quad (16)$$

with $F_{H_2}(t) = 0$ for $t < 0$ and $F_{H_2}(t) = 1$ for $t > T_b$. Here $P(X_1 = x_1)$ is the probability that the node positioned at x_1 becomes the first forwarder. How to calculate this is tackled in the next section.

To calculate the distribution of H_3, H_4, etc., it is again necessary to take the distributions of the previous hop times into account, as well as the positions of the previous forwarders. Calculating these distributions becomes resource-intensive for hop 3 and beyond. As we will show in Section 7 however the distribution of H converges and, for the purpose of our model, does not change significantly beyond hop 2. For the remainder of our analysis we therefore state

$$F_{H_i} \sim F_{H_2} \;\forall\, i > 2. \quad (17)$$

5.2 Position of the Forwarder

Let X_i be the position of the i^{th} forwarder. In this section a method is presented to calculate X_i for $i \leq 3$. We first explain the approach of our method and then show the required calculations. Calculating $X_4, X_5, \ldots$ in this manner becomes relatively complex and resource intensive; for this reason an approximate method to calculate X_i for $i > 3$ is given in Section 5.4.

The distribution of X_i depends on two factors. The first factor is the set of positions of all previous forwarders. The general form of this dependency is given by

$$\begin{aligned} &P(X_i = x_i) = \\ &\sum_{x_1=d_{IN}}^{R} P(X_1 = x_1) \cdot \sum_{x_2=2d_{IN}}^{2R} P(X_2 = x_2 \mid X_1 = x_1) \cdots \\ &\sum_{x_{i-1}=(i-1)d_{IN}}^{(i-1)R} P(X_{i-1} = x_{i-1} \mid X_1 = x_1, \ldots, X_{i-1} = x_{i-1}) \cdot \\ &P(X_i = x_i \mid X_{i-1} = x_{i-1}, \ldots, X_1 = x_1) \end{aligned} \quad (18)$$

The second factor is the distribution of the residual inter-beacon times of the candidate forwarders. The last term of Eq. (18) is equal to the probability that the candidate forwarder at position x_i has a smaller residual inter-beacon time than all the other candidate forwarders. We denote the set of *remaining* candidate forwarders (i.e., excluding the node at position x_i) as $\bar{x}_i$. Let $\dot{T}_{x_i}$ be the distribution of the residual inter-beacon time of the node at position x_i, and $\dot{T}_{\bar{x}_i}$ the distribution of the minimum value of the residual inter-beacon times of the nodes in set $\bar{x}_i$. The last term in Eq. (18) can thus be expressed as

$$\begin{aligned} &P(X_i = x_i \mid X_{i-1} = x_{i-1}, \ldots, X_1 = x_1) = \\ &P(\dot{T}_{x_i} < \dot{T}_{\bar{x}_i} \mid X_{i-1} = x_{i-1}, \ldots, X_1 = x_1) \end{aligned} \quad (19)$$

To calculate Eq. (19) we integrate over all possible values of $\dot{T}_{x_i}$ multiplied by the probability that $\dot{T}_{\bar{x}_i}$ has a larger value. Since again the positions of the previous forwarder must be taken into account this is given by

$$\begin{aligned} &P(\dot{T}_{x_i} < \dot{T}_{\bar{x}_i} \mid X_{i-1} = x_{i-1}, \ldots, X_1 = x_1) \\ &= \int_{0}^{T_b} f_{\dot{T}_{x_i}}(t_i, x_1, \ldots, x_{i-1}) \\ &\cdot \Big(1 - F_{\dot{T}_{\bar{x}_i}}(t_i, x_1, \ldots, x_{i-1})\Big)\, dt_i \end{aligned} \quad (20)$$

The distribution of the residual inter-beacon time of a candidate forwarder depends on two factors as well: the hop in which the node first received the message, and the hop times of the hops that have passed since then. Let $\dot{T}_{x_3}$ be the residual inter-beacon time of a candidate forwarder for hop 3 that first received the message in hop 0, and let t_1 and t_2 be the respective hop times of the first two hops. $\dot{T}_{x_3}$ is then uniformly distributed in $\langle 0, T_b - t_1 - t_2]$. $\dot{T}_{\bar{x}_3}$ is distributed as the minimum value of the residual inter-beacon times of all the nodes in the set $\bar{x}_3$, similar to how H_2 was calculated in Eq. (16). For ease of notation we subdivide the set of remaining candidate forwarders $\bar{x}_i$ into different subsets of nodes that received the message in the same hop. We denote the set of remaining candidate forwarders that received the message in hop $i-1$ as $\bar{x}_{i,i-1}$, the set of remaining candidate forwarders that received the message in hop $i-2$ as $\bar{x}_{i,i-2}$, etc.

In the remainder of this section we focus on solving Eq. (20) for $i \leq 3$. For the first hop this is trivial, for the second and third hop we need to calculate the distribution of $\dot{T}_{x_i}$ and $\dot{T}_{\bar{x}_i}$.

After the source has forwarded the message for the first time, all candidate forwarders for the first hop have an identical residual inter-beacon time distribution. The probability that one node beacons first is therefore equal for all nodes, so we can state

$$P(X_1 = x_1) = \frac{d_{IN}}{R}, \quad x_1 \in \{d_{IN}, 2d_{IN}, ..., R\}. \quad (21)$$

The candidate forwarders for the second hop do not have identical residual inter-beacon time distributions, since some nodes will have received the message in hop 0, while other received it in hop 1. The CDF of $\dot{T}_{x_2}$ for $0 \leq t \leq T_b$ is given by

$$F_{\dot{T}_{x_2}}(t) = \begin{cases} F_{\dot{T}_1}(t, T_b) & x_2 > R \\ \int_0^{T_b} f_{H_1}(t_1, T_b, R/d_{IN}) \cdot & \\ F_{\dot{T}_1}(t, T_b - t_1)\, dt_1 & 0 < x_2 \leq R, \end{cases} \tag{22}$$

with $F_{\dot{T}_{x_2}}(t) = 0$ for $t < 0$, and $F_{\dot{T}_{x_2}}(t) = 1$ for $t > T_b$. The CDF of $\dot{T}_{\bar{x}_2}$ for $0 \leq t \leq T_b$ is given by

$$F_{\dot{T}_{\bar{x}_2}}(t, x_1) = \int_0^{T_b} f_{H_1}(t_1, T_b, R/d_{IN}) \cdot \Big(1 - \bar{F}_{H_1}(t, T_b, |\bar{x}_{2,1}|) \cdot \bar{F}_{H_1}(t, T_b - t_1, |\bar{x}_{2,0}|)\Big)\, dt_1, \tag{23}$$

where $|\bar{x}_{2,j}|$ is the amount of candidate forwarders in the set $\bar{x}_2$ that first received the message in hop j, $F_{\dot{T}_{\bar{x}_2}}(t, x_1) = 0$ for $t < 0$, and $F_{\dot{T}_{\bar{x}_2}}(t, x_1) = 1$ for $t > T_b$.

Each candidate forwarders for hop 3 can have received the message in hop 0, hop 1, or hop 2. This is determined by the positions of the previous forwarders. The distributions of $\dot{T}_{x_3}$ and $\dot{T}_{\bar{x}_3}$ must therefore take all possible combinations of previous forwarder into account, as well as all possible hop times of hop 1 and 2. The CDF of $\dot{T}_{x_3}$ is given by

$$F_{\dot{T}_{x_3}}(t, x_1) = \begin{cases} F_{\dot{T}_1}(t, T_b) & x_1 + R < x_3 \\ \int_0^{T_b} f_{H_2}(t_2) \cdot F_{\dot{T}_1}(t, T_b - t_2)\, dt_2 & R < x_3 \leq x_1 + R \\ \int_0^{T_b}\int_0^{T_b} f_{H_1}(t_1, T_b, R/d_{IN}) \cdot f_{H_2}(t_2) & \\ \cdot F_{\dot{T}_1}(t, T_b - t_1 - t_2)\, dt_2\, dt_1 & x_3 \leq R, \end{cases} \tag{24}$$

with $F_{\dot{T}_{x_3}}(t, x_1) = 0$ for $t < 0$ and $F_{\dot{T}_{x_3}}(t, x_1) = 1$ for $t \geq T_b$. The CDF of $\dot{T}_{\bar{x}_3}$ is given by

$$\begin{aligned} &F_{\dot{T}_{\bar{x}_3}}(t, x_1, x_2) \\ &= \int_0^{T_b}\int_0^{T_b} f_{H_1}(t_1, T_b, R/d_{IN}) \cdot f_{H_2}(t_2) \\ &\cdot \Big(1 - \bar{F}_{H_1}(t, T_b, |\bar{x}_{3,2}|) \cdot \bar{F}_{H_1}(t, T_b - t_2, |\bar{x}_{3,1}|) \\ &\cdot \bar{F}_{H_1}(t, T_b - t_1 - t_2, |\bar{x}_{3,0}|)\Big)\, dt_2\, dt_1, \end{aligned} \tag{25}$$

with $F_{\dot{T}_{\bar{x}_3}}(t, x_1, x_2) = 0$ for $t \leq 0$ and $F_{\dot{T}_{\bar{x}_3}}(t, x_1, x_2) = 1$ for $t \geq T_b$.

5.3 Hop Length

The concept of hop lengths was already explained in Section 4.2. Note that Eq. (3) also holds when inter-beacon times are distributed deterministically. We specify the distribution of the first three hop lengths in an exact manner. The hop lengths for beyond hop 3 are approximated.

The general expression for L_i is given by

$$\begin{aligned} P(L_i = l) = &\sum_{x_{i-1}=(i-1)d_{IN}}^{(i-1)R} P(X_{i-1} = x_{i-1}) \cdot \\ &P(X_i = x_{i-1} + l \mid X_{i-1} = x_{i-1}). \end{aligned} \tag{26}$$

The distributions of the positions of the first three forwarders have been given in the previous section. Using these we have an exact expression for the distribution of L_i for $i \leq 3$.

As we will show in Section 7 the distribution of L converges and, for the purpose of our model, does not change significantly beyond hop 3. For the remainder of our analysis we therefore state

$$F_{L_i} \sim F_{L_3}\ \forall\, i \geq 3. \tag{27}$$

We thus assume that L_i for $i > 3$ is distributed identically and independently of the positions of the previous forwarders and the previous hop times.

5.4 Approximated Position of the Forwarder

In this section we approximate the distribution of X_i for $i > 3$. In Section 5.2 an exact expression was given for X_i for $i \leq 3$ and by means of Eq. (27) we assumed that the length of each following hop is i.i.d. – combining these the distribution of the position of every next forwarder is given by

$$\begin{aligned} P(X_i = x_i) = &\sum_{x_{i-1}=(i-1)d_{IN}}^{(i-1)R} P(X_{i-1} = x_{i-1}) \cdot \\ &P(L_i = x_i - x_{i-1}), \quad \forall\, i > 3, \end{aligned} \tag{28}$$

We thus have a recursive approximation of the position of the forwarder for the fourth hop and beyond.

5.5 Required Number of Hops

Let N_x be the number of hops required to have a node at position x or beyond become forwarder. In this section we give two methods to determine N_x. The first method uses the distribution of X_i and can be used regardless of the expected required number of hops. In the second method N_x is modelled as a renewal process; this method should be used if the expected required number of hops is high. Note that to have the sink at position d_D receive the message, a node that is positioned on or beyond $d_D - R$ must transmit the message. We are therefore generally interested in the distribution of $N_{d_D - R}$. In our discussion we use the more general term N_x however.

The first method makes use of the fact that the probability that at most i hops are needed to reach position x is equal to the probability that the i^{th} forwarder is at or beyond position x, i.e.,

$$P(N_x \leq i) = 1 - P(X_i < x), \tag{29}$$

where $P(X_i < x)$ is equal to the summed up probabilities that the forwarder is at position $0 \leq y \leq x - d_{IN}$ as was given by Eq. (28),

$$P(X_n < x) = \sum_{y=0}^{x-d_{IN}} P(X_n = y). \tag{30}$$

In the second method we use the fact that N_x is distributed as the largest integer $i > 0$ for which holds that

$X_i \leq x$ for $x > 0$, and that X_i for $i > 3$ is a convolution of hop i.i.d. hop lengths, see Eq. (28). Let N_{x-x_3} be the number of hops required beyond hop 3. According to [17] (page 5), as x goes to infinity, N_{x-x_3} then has the following asymptotically normal distribution:

$$N_{x-x_3} \sim \mathcal{N}(\frac{x-x_3}{\mu_{L_3}}, \sigma^2_{L_3}\frac{x-x_3}{(\mu_{L_3})^3}), \quad x \geq x_3, \tag{31}$$

where μ_{L_3} and $\sigma^2_{L_3}$ are the mean and variance of the discrete uniform distribution of the hop length L_3, which can easily be calculated. Including the distribution of X_3, N_x is then distributed as

$$P(N_x = n) = \sum_{x_3=3\cdot d_{IN}}^{3R} P(X_3 = x_3) \cdot P(N_{x-x_3} = n-3). \tag{32}$$

5.6 Delay

Let D be the delay to piggyback the message from source to sink. It is a function of the number of hops required to bridge the dissemination distance (N_{d_D-R}), and the delay per hop (H_i). The distribution of the hop time of the first hop is given by Eq. (14), the hop time of subsequent hops is given by Eq. (17). The distribution of D is given by

$$D = \sum_{n=x/R}^{x/d_{IN}} P(N_{d_D-R} = n) \sum_{i=1}^{n} H_i \tag{33}$$

Calculating such a convolution quickly becomes too resource-intensive for practical purposes. We therefore use the central limit theorem, which states that for n i.i.d. variables H_2 with mean μ_{H_2} and variance σ_{H_2}, and with $S_n = \sum_{i=1}^{n} H_2$, S_n can be approximated as $S_n \sim \mathcal{N}(n\cdot\mu_{H_2}, n\cdot\sigma_{H_2})$. Taking the distribution of the first hop time into account the distribution of the delay is then given by

$$P(D = d) = \int_{t=0}^{T_b} f_{H_1}(t) \cdot P(S_{n-1} = d-t). \tag{34}$$

6. THE SIMULATION SET-UP

In this section two simulation experiments are described. The first experiment has been set up in such a way as to resemble the piggybacking model in Section 3, i.e., including all model assumptions. This simulation study is referred to as the *model simulation*. The results of the model simulation are used in Section 7.1 to verify the correctness of our analysis made in Section 4 and Section 5. If our analysis is correct, its results should resemble the results of the model simulation as close as possible.

The second experiment has been set-up in a more realistic manner; it is referred to as the *realistic simulation*. In Section 7.2 we use the results of this simulation study to discuss how the behaviour of the piggybacking system is affected when our model assumptions are dropped.

The specific details of each experimental set-up are addressed in the relevant subsections. The general set-up is as follows. For both set-ups two inter-beacon time distributions were used: the deterministic and the exponential distribution. Both experiments have been performed using the OMNET++ network simulator v4.1 [2] and using a self-modified version of the MiXiM framework v2.1 [1] to model the communication architecture. To model the behaviour of the 802.11p protocol as accurately as possible we have altered the IEEE 802.11 medium access module in such a way that all parameters follow the 802.11p specification [4]. The available 802.11 MiXiM physical layer was adapted to include bit error rates (BER) and packet error rates (PER) for all transmission bit rates used in our experiments. The centre frequency was set to 5.9 MHz and access category (AC) 0 was used.

6.1 The Model Simulation Set-up

Nodes are equidistantly spaced every d_{IN} m from source to sink over a straight line of length d_D. Beaconing and forwarding by means of piggybacking is done as specified in Section 3.

To emulate a deterministic, fixed transmission range, the physical layer was adapted in such a way that any node within R meters of a transmitting node will receive the transmitted signal at a fixed (and relatively high) power level. Outside this range a receiving node receives the signal at zero power. This effectively gives a unit-disc propagation model with reception probabilities one and zero (in case of an isolated transmission). Nodes use the 802.11p MAC as described above. To keep the influence of packet collisions as low as possible however beacon sizes are kept small: 160 bits, regardless whether the beacon has the application message attached to it or not.

6.2 The Realistic Simulation Set-up

Nodes are positioned from source to sink over a straight line of length d_D. The inter-node spacing is exponentially distributed with mean d_{IN}. Work such as [6] suggests that the exponential distribution gives a good description of the inter-vehicle distance in case of free flowing traffic. Beaconing and forwarding by means of piggybacking is done as specified in Section 3.

Due to its ability to model both long-term and short-term fading we use the log-normal shadowing model [13] for signal propagation. The path loss exponent is set to 3.5 and the standard deviation to 6. The transmission power was chosen such that two nodes that are R meters apart, have a packet reception probability of 0.4 if there are no other users on the wireless medium. Beacon sizes are set to 400 bytes, including all headers. An attached application message increases the beacon size with an additional 100 bytes. There is a non-zero probability that the message does not reach the sink due to transmission errors.

7. VERIFICATION & VALIDATION

In this section we verify the correctness of the model analysis in Section 7.1. We also show how the behaviour of the piggybacking system is affected when we drop our model assumptions in Section 7.2. Our main performance metric is the distribution of the end-to-end delay, but we also discuss the hop time and the hop length, as together these two metrics determine the delay. Before we start our discussion however we present our method to compare two distributions.

We use the *Kolmogorov-Smirnov* (K-S) statistic to express the difference between two distributions. The K-S statistic K for two distributions $F_1(x), F_2(x)$ is equal to the largest

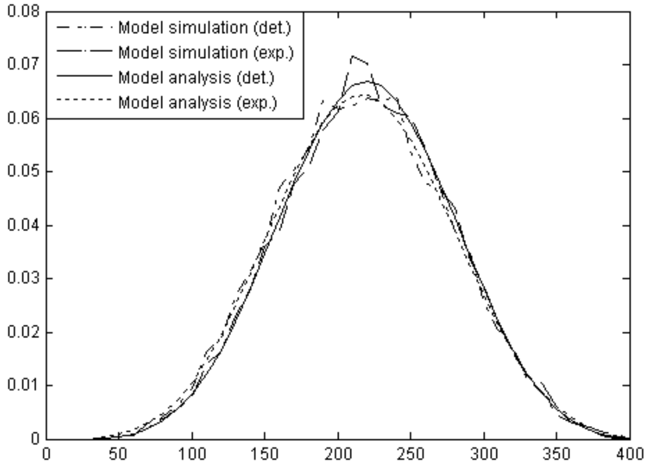

Figure 2: Position of the fourth forwarder when $d_{IN} = 10$ and $R = 100$.

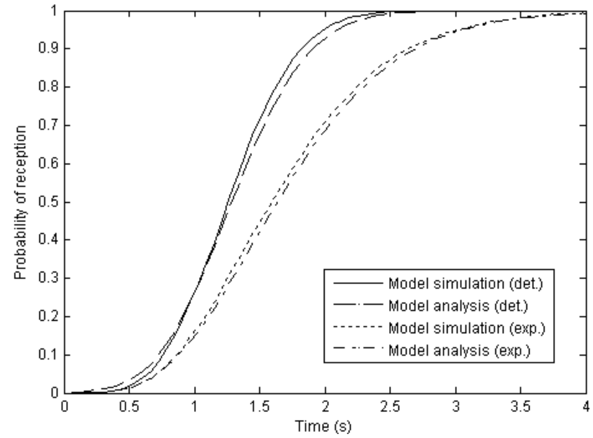

Figure 3: CDF of the end-to-end delay when $d_D = 500$, $d_{IN} = 25$, and $R = 100$.

distance between the CDFs, given by

$$K = \max\{|F_1(x) - F_2(x)|\} \quad \forall\, x. \tag{35}$$

7.1 Verification of the Analysis

In this section we verify the model analysis by comparing its results with the results of the model simulation. We first discuss the results when inter-beacon times are distributed exponentially, then when inter-beacon times are distributed deterministically.

7.1.1 Exponentially distributed inter-beacon times

Although we do not show it here, the distribution of the hop time of the model simulation closely resembles the distribution of the hop time of the model analysis, and the distribution of the hop length of the model simulation closely resembles the distribution of the hop length of the model analysis. The distribution of the position of the forwarder and the distribution of the required number of hops to reach the sink – both of which are a function of the hop length – also closely resemble their respective model analysis distributions. Fig. 2 shows how the distribution of the position of the fourth forwarder of the model analysis closely follows the distribution of the position of the forwarder of the model simulation when $d_{IN} = 10$ and $R = 100$.

The second block of Table 2 shows the K-S statistics of the end-to-end delay distribution of the model analysis and the model simulation, for different dissemination distances and inter-node distances. It can be seen that the two distributions closely follow each other: for all cases the maximum deviation of the two distributions stays within 5%, and it is less than 1% in half of the cases. Fig. 3 shows the CDF of the end-to-end delay for a specific combination of parameters.

7.1.2 Deterministically distributed inter-beacon times

We first discuss the intermediate performance metrics hop time, hop length, and the position of the forwarder, and end with our main performance metric, the end-to-end delay.

As we have discussed in Section 5.1, the distribution of the hop time depends on the hop times and the hop lengths of all the preceding hops. This is most notable when comparing the distribution of the hop time of the first hop with that of the second hop, as can be seen in Fig. 4. In our model however we have assumed that the distribution of the hop time for the third hop and following is identical to the distribution of the hop time of the second hop, see Eq. (17). To test this assumption we have calculated the K-S statistic for a number of hop times, see Table 1. The table shows the K-S statistics when comparing the distributions of the hop times of the model simulation with the distributions of the hop times of the model analysis, for different node densities. It can be seen that our assumption is valid: the difference between the distributions is in almost all cases less than 2%.

Similar to the distribution of the hop time, the distribution of the hop length of a hop also depends on the hop times and the hop lengths of all the preceding hops. The distribution of the hop length differs therefore per hop. This has been visualised in Fig. 3, where the distributions of the hop lengths of hop 1, 2, 3, and 10 are shown when $d_{IN} = 25$ and $R = 100$. The figure also confirms our assumption that after the third hop the distribution of the hop length converges however (see Eq. (27)), since the distribution of the hop length of hop 10 closely resembles that of hop 3.

An interesting observation is that for high node densities the distribution of the hop length can be assumed to have a discrete uniform distribution, similar to how hop lengths are distributed when inter-beacon times are distributed exponentially. Fig. 2 shows for example how the distribution of the position of the forwarder when inter-beacon times are deterministically distributed closely resembles the distribution of the position of the forwarder when inter-beacon times are exponentially distributed, when there are ten nodes inside the transmission range. For high node densities we can thus calculate the distribution of the hop length, the distribution of the position of the forwarder, and the distribution of the

d_{IN} \ Hop	1	2	3	4	5	10
10	0.010	0.006	0.010	0.008	0.013	0.013
25	0.005	0.010	0.007	0.025	0.009	0.011
50	0.012	0.007	0.011	0.013	0.012	0.008

Table 1: K-S statistics of the hop time when $R = 100$ and $T_b = 1$.

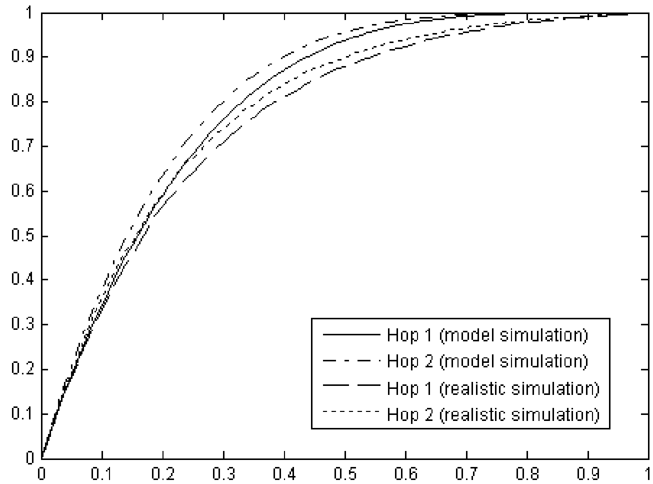

Figure 4: CDFs of the hop times of the model simulation and the realistic simulation when $d_{IN} = 25$, $R = 100$, and $T_b = 1$.

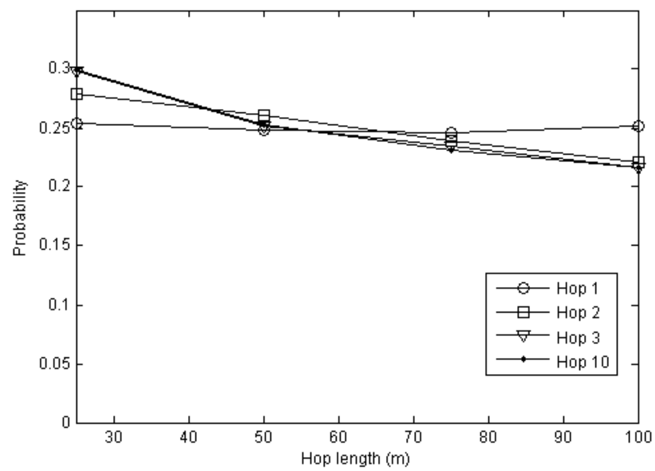

Figure 5: Hop lengths of the model simulation when $d_{IN} = 25$ and $R = 100$.

required number of hops to reach the source, as a convolution of discrete uniform hop lengths, similar to how this is done when inter-beacon times are distributed exponentially. This considerably simplifies our model, as we no longer need to calculate the first three hop lengths separately.

The first block of Table 2 shows the K-S statistics of the end-to-end delay distribution of the model analysis and the model simulation, for different dissemination distances and inter-node distances. It can be seen that the two distributions closely follow each other: for most cases the maximum deviation of the two distributions stays within 5%, and the maximum deviation is 8%. Fig. 3 shows the CDF of the end-to-end delay for a specific combination of parameters.

7.2 Validation of the Model

In this section we compare the results of the model simulation with the results of the realistic simulation, in order to validate how well our model compares to a more realistic situation. Based on this comparison we also judge the feasibility of dropping these assumptions in a next version of our model, in order to improve the model's realism.

A fundamental problem when comparing the results of the two simulations is that the fixed transmission range of the model simulation cannot be directly to any realistic propagation model. We will therefore refrain from directly comparing the results in terms of quantity but instead focus on observed trends. The two main differences between the two simulations are that in the realistic simulation (i) inter-node distances are exponentially distributed (instead of fixed), and (ii) a log-normal shadowing distribution was used to calculate the received signal power (instead of a fixed transmission range). We discuss the distribution of the hop time and the distribution of the hop length.

d_D	200	300	400	500	1000	2500
d_{IN}	det. simulation vs. det. analysis					
10	0.061	0.068	0.041	0.038	0.053	0.083
25	0.051	0.050	0.050	0.042	0.048	0.053
50	0.060	0.040	0.054	0.049	0.033	0.066
d_{IN}	exp. simulation vs. exp. analysis					
10	0.007	0.006	0.010	0.011	0.009	0.000
25	0.008	0.032	0.030	0.026	0.013	0.009
50	0.009	0.004	0.012	0.007	0.040	0.047

Table 2: K-S statistics of the delay when $R = 100$.

Fig. 4 shows the CDF of the hop time for the first two hops, both for the realistic simulation and the model simulation. Similar to the model simulation it can be seen in the realistic simulation that the hop time of the second hop has a larger probability of being shorter than the first hop. Although not shown here, the third hop (and any hop beyond the third hop) has a hop time distribution that is almost identical to that of the second hop. We thus need to find an expression for the hop time distribution of the first two hops only, and can assume the distribution of following hops to be equal to the distribution of the second hop, as we have done in this work.

Although we do not show it here the distribution of the hop length of the realistic simulation is distinctly dissimilar for the first two hops, while the hop length distribution of following hops is similar to that of the second hop. We thus need to find an expression for the hop length distribution of the first two hops only, and can assume the distribution of following hops to be equal to the distribution of the second hop.

8. CONCLUSION

In this paper we have presented an analytical model that is able to express the performance of a piggybacking protocol in analytical and often even closed-form expressions. The model takes the dissemination distance, the node density, the transmission range, and the beacon frequency into account, and gives expressions for (i) the distribution of the end-to-end delay, (ii) the distribution of the required number of hops to reach the sink, (iii) the distribution of the per-hop delay, (iv) the distribution of the length of a hop, and (v) the distribution of the positions of the intermediate forwarders.

Our main performance metric is the end-to-end delay. Verification of our analytical model by means of a simulation study shows that for most cases the delay distribution of the model stays within 6% of the simulated delay distribution.

When inter-beacon times are distributed exponentially the performance of the piggybacking protocol can be captured in a number of simple, closed-form expressions. When inter-beacon times are distributed deterministically and inter-node distances are large, the performance of the first three hops of the piggybacking protocol are given by a number of analytical expressions. The performance of following hops is given in closed form. When inter-beacon times are distributed deterministically and inter-node distances are small, the distribution of the length of a hop is similar to when inter-beacon times are distributed exponentially. For high node densities we can therefore express the distribution of the length of a hop, the distribution of the required number of hops to reach the sink, and the distribution of the positions of the intermediate forwarders using the same closed-form expressions.

The model contains some assumptions, mainly regarding the topology of the network and the transmission reception probabilities. It is our goal to drop these assumptions in future work. To analyse the effect this will have on performance we have also performed a simulation study without these assumptions. Our results suggest that performance (in terms of hop length and per-hop delay) is dissimilar for the first two hops, but that any hop beyond the second hop performs similar to the second hop. We therefore only need to find expressions for the hop length and per-hop delay for the first two hops to be able to model the whole system.

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