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Second Edition

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Mathematical Basics of Motion and Deformation in Computer Graphics

Second Edition

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SYNTHESIS LECTURES ON VISUAL COMPUTING: COMPUTER GRAPHICS, ANIMATION, COMPUTATIONAL PHOTOGRAPHY, AND IMAGING #27

ABSTRACT

This synthesis lecture presents an intuitive introduction to the mathematics of motion and deformation in computer graphics. Starting with familiar concepts in graphics, such as Euler angles, quaternions, and affine transformations, we illustrate that a mathematical theory behind these concepts enables us to develop the techniques for efficient/effective creation of computer animation.

This book, therefore, serves as a good guidepost to mathematics (differential geometry and Lie theory) for students of geometric modeling and animation in computer graphics. Experienced developers and researchers will also benefit from this book, since it gives a comprehensive overview of mathematical approaches that are particularly useful in character modeling, deformation, and animation.

KEYWORDS

motion, deformation, quaternion, Lie group, Lie algebra

Contents

Preface

In the computer graphics community, many technical terms, such as Euler angle, quaternion, and affine transformation, are fundamental and quite familiar words, and have a pure mathematical background. While we usually do not have to care about the deep mathematics, the graphical meaning of such basic terminology is sometimes slightly different from the original mathematical entities. This might cause misunderstanding or misuse of the mathematical techniques. Or, if we have just a bit more curiosity about pure mathematics relevant to computer graphics, it should be easier for us to explore a new possibility of mathematics in developing a new graphics technique or tool.

This volume thus presents an intuitive introduction to several mathematical basics that are quite useful for various aspects of computer graphics, focusing on the fundamental procedures for deformation and animation of geometric objects, and curve/surface editing. The objective of this book, then, is to fill the gap between the original mathematical concepts and the practical meanings in computer graphics without assuming any prior knowledge of pure mathematics. We then restrict ourselves to the mathematics for matrices, while we know there are so many other mathematical approaches far beyond matrices in our graphics community. Though this book limits the topics to matrices, we hope you can easily understand and realize the power of mathematical approaches. In addition, this book demonstrates our ongoing work, which benefits from the mathematical formulation we develop in this book.

This book is an extension of our early work that was given as SIGGRAPH Asia 2013 and SIGGRAPH 2014 courses. The exposition developed in this book has greatly benefited from the advice, discussions, and feedback of a lot of people. The authors are very much grateful to Shizuo Kaji at Yamaguchi University and J.P. Lewis at Victoria University of Wellington, who read a draft of this book and gave many invaluable ideas. The discussions and feedback from the audience at the SIGGRAPH courses are also very much appreciated. Many thanks also go to Gengdai Liu and Alexandre Derouet-Jourdan at OLM Digital for their help in making several animation examples included in this book.

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The authors wish to thank Ayumi Kimura for the constructive comments and suggestions made during the writing of this volume. Thanks also go to Yume Kurihara for the cute illustrations. Last, but not least, the authors are immensely grateful to Brian Barsky, the editor of the Synthesis

Lectures on Compute Graphics and Animarion series, and Mike Morgan at Morgan & Claypool Publishers for giving the authors such an invaluable chance to publish this book in the series.

Ken Anjyo and Hiroyuki Ochiai
October 2014

Preface to the Second Edition

In this edition, we added an appendix where we derive several formulas for 3D rotation and deformation. We also incorporated a number of references, particularly relating to our SIGGRAPH 2016 course and its accompanying video. These additions will help readers to better understand the basic ideas developed in this book. We also resolved the mathematical notation inconsistency from the first edition, which makes this book more easily accessible.

We would like to thank the graduate students at Kyushu University who carefully went through the book with the second author in his seminar. Finally, we are very grateful to Ayumi Kimura who worked with us for the SIGGRAPH 2016 course and helped significantly with the editing of the second edition.

Ken Anjyo and Hiroyuki Ochiai
April 2017

Symbols and Notations

T_b	translation, 5
R_θ	2D rotation matrix, 6
$SO(2)$	2D rotation group (special orthogonal group), 7
$M(n, \mathbb{R})$	the set of square matrices of size n with real entries, 7
I, I_n	the identity matrix of size n, 7, 11, 30,
A^T	transpose of a matrix A, 7
$SE(2)$	2D motion group (the set of non-flip rigid transformations), 8
$O(2)$	2D orthogonal group, 9
$SO(n)$	special orthogonal group, 10
$O(n)$	orthogonal group, 10
$SO(3)$	3D rotation group, 11
$R_x(\theta)$	3D axis rotations, 11
$\mathbb{H}$	the set of quaternions, 3, 13
$\overline{q}$	conjugate of a quaternion, 13
$\mathrm{Re}(q)$	real part of a quaternion, 14
$\mathrm{Im}(q)$	imaginary part of a quaternion, 14
$\mathrm{Im}\,\mathbb{H}$	the set of imaginary quaternions, 14
$\lvert q \rvert$	the absolute value of a quaternion, 14
$\mathbb{S}^3$	the set of unit quaternions, 15
exp	exponential map, 16, 29
$\mathrm{slerp}(q_0, q_1, t)$	spherical linear interpolation, 16
ε	dual number, 16
$M(2, \mathbb{H})$	the set of square matrices of size 2 with entries in $\mathbb{H}$, 16
$\check{\mathbb{C}}$	the set of anti-commutative dual complex numbers (DCN), 18
$E(n)$	rigid transformation group, 19
$SE(n)$	n-dimensional motion group, 20
$GL(n)$	general linear group, 23
$\mathrm{Aff}(n)$	affine transformations group, 23
$GL^+(n)$	general linear group with positive determinants, 23
$\mathrm{Aff}^+(n)$	the set of orientation-preserving affine transformations, 23
$\ltimes$	semi-direct product, 25
$\mathrm{Sym}^+(n)$	the set of positive definite symmetric matrices, 26
$\mathrm{Diag}^+(n)$	the set of diagonal matrices with positive diagonal entries, 27