# Backdoors to Normality for Disjunctive Logic Programs ${ }^{* i}$ 

Johannes Klaus Fichte and Stefan Szeider<br>Vienna University of Technology, Austria<br>fichte@kr.tuwien.ac.at, stefan@szeider.net

September 2, 2018


#### Abstract

Over the last two decades, propositional satisfiability (SAT) has become one of the most successful and widely applied techniques for the solution of NP-complete problems. The aim of this paper is to investigate theoretically how Sat can be utilized for the efficient solution of problems that are harder than NP or co-NP. In particular, we consider the fundamental reasoning problems in propositional disjunctive answer set programming (Asp), Brave Reasoning and Skeptical Reasoning, which ask whether a given atom is contained in at least one or in all answer sets, respectively. Both problems are located at the second level of the Polynomial Hierarchy and thus assumed to be harder than NP or co-NP. One cannot transform these two reasoning problems into Sat in polynomial time, unless the Polynomial Hierarchy collapses.

We show that certain structural aspects of disjunctive logic programs can be utilized to break through this complexity barrier, using new techniques from Parameterized Complexity. In particular, we exhibit transformations from Brave and Skeptical Reasoning to Sat that run in time $O\left(2^{k} n^{2}\right)$ where $k$ is a structural parameter of the instance and $n$ the input size. In other words, the reduction is fixed-parameter tractable for parameter $k$. As the parameter $k$ we take the size of a smallest backdoor with respect to the class of normal (i.e., disjunction-free) programs. Such a backdoor is a set of atoms that when deleted makes the program normal. In consequence, the combinatorial explosion, which is expected when transforming a problem from the second level of the Polynomial Hierarchy to the first level, can now be confined to the parameter $k$, while the running time of the reduction is polynomial in the input size $n$, where the order of the polynomial is independent of $k$. We show that such a transformation is not possible if we consider backdoors with respect to tightness instead of normality.

We think that our approach is applicable to many other hard combinatorial problems that lie beyond NP or co-NP, and thus significantly enlarge the applicability of Sat.


## 1 Introduction

Over the last two decades, propositional satisfiability (SAT) has become one of the most successful and widely applied techniques for the solution of NP-complete problems. Today's Sat-solvers are extremely efficient and robust, instances with hundreds of thousands of variables and clauses can be solved routinely. In fact, due to the success of SAT, NP-complete problems have lost their scariness, as in many cases one can efficiently encode NP-complete problems to SAT and solve them by means of a SAT-solver [Gomes et al., 2008; Biere et al., 2009].

We investigate transformations into SAT for problems that are harder than NP or co-NP. In particular, we consider various search problems that arise in disjunctive answer set programming

[^0](Asp). With Asp one can describe a problem by means of rules that form a disjunctive logic program, whose solutions are answer sets. Many important problems of AI and reasoning can be represented in terms of the search for answer sets [Brewka et al., 2011; Marek and Truszczynski, 1999; Niemelä, 1999]. Two of the most fundamental Asp problems are Brave Reasoning (is a certain atom contained in at least one answer set?) and Skeptical Reasoning (is a certain atom contained in all answer sets?). Both problems are located at the second level of the Polynomial Hierarchy [Eiter and Gottlob, 1995] and thus assumed to be harder than NP or co-NP. It would be desirable to utilize Sat-solvers for these problems. However, we cannot transform these two reasoning problems into Sat in polynomial time, unless the Polynomial Hierarchy collapses, which is believed to be unlikely.

New Contribution In this work we show how to utilize certain structural aspects of disjunctive logic programs to transform the two Asp reasoning problems into SAT. In particular, we exhibit a transformation to SAT that runs in time $O\left(2^{k} n^{2}\right)$ where $k$ is a structural parameter of the instance and $n$ is the input size of the instance. Thus the combinatorial explosion, which is expected when transforming problems from the second level of the Polynomial Hierarchy to the first level, is confined to the parameter $k$, while the running time is polynomial in the input size $n$ and the order of the polynomial is independent of $k$. Such transformations are known as "fpt-transformations" and form the base of the completeness theory of Parameterized Complexity [Downey and Fellows, 1999; Flum and Grohe, 2006]. Our reductions break complexity barriers as they move problems form the second to the first level of the Polynomial Hierarchy.

It is known that the two reasoning problems, when restricted to so-called normal programs, drop to NP and co-NP [Bidoít and Froidevaux, 1991; Marek and Truszczynski, 1991a; Marek and Truszczyński, 1991b], respectively. Hence, it is natural to consider a structural parameter $k$ as the distance of a given program from being normal. We measure the distance in terms of the smallest number of atoms that need to be deleted to make the program normal. Following Williams et al. [2003] we call such a set of deleted atoms a backdoor. We show that in time $O\left(2^{k} n^{2}\right)$ we can solve both of the following two tasks for a given program $P$ of input size $n$ and an atom $a^{*}$ :

Backdoor Detection: Find a backdoor of size at most $k$ of the given program $P$, or decide that a backdoor of size $k$ does not exist.

Backdoor Evaluation: Transform the program $P$ into two propositional formulas $F_{\text {Brave }}\left(a^{*}\right)$ and $F_{\text {Skept }}\left(a^{*}\right)$ such that (i) $F_{\text {Brave }}\left(a^{*}\right)$ is satisfiable if and only if $a^{*}$ is in some answer set of $P$, and (ii) $F_{\text {Skept }}\left(a^{*}\right)$ is unsatisfiable if and only if $a^{*}$ is in all answer sets of $P$.

Tightness is a property of disjunctive logic programs that, similar to normality, lets the complexities of Brave and Skeptical Reasoning drop to NP and co-NP, respectively [Clark, 1978; Fages, 1994]. Consequently, one could also consider backdoors to tightness. We show, however, that the reasoning problems already reach their full complexities (i.e., completeness for the second level of the Polynomial Hierarchy) with programs of distance one from being tight. Hence, an fpt-transformation into Sat for programs of distance $k>0$ from being tight is not possible unless the Polynomial Hierarchy collapses.

Related Work Williams, Gomes, and Selman [2003] introduced the notion of backdoors to explain favorable running times and the heavy-tailed behavior of SAT and CSP solvers on practical instances. The parameterized complexity of finding small backdoors was initiated by Nishimura, Ragde, and Szeider [2004]. For further results regarding the parameterized complexity of problems related to backdoors for Sat, we refer to a recent survey paper [Gaspers and Szeider, 2012]. Fichte and Szeider [2012] formulated a backdoor approach for Asp problems, and obtained complexity results with respect to the target class of Horn programs and various target classes based on acyclicity; some results could be generalized [Fichte, 2012]. Both papers are limited to target classes where we can enumerate the set of all answer sets in polynomial time. The results do not carry over to the present work since here we consider target classes where the problem of determining an answer set is already NP-hard.

Translations from Asp problems to Sat have been explored by several authors; existing research mainly focuses on transforming programs for which the reasoning problems already belong to NP or co-NP. In particular, translations have been considered for head cycle free programs [Ben-Eliyahu and

Dechter, 1994], tight programs [Fages, 1994], and normal programs [Lin and Zhao, 2004; Janhunen, 2006].

Some authors have generalized the above translations to capture programs for which the reasoning problems are outside NP and co-NP. Janhunen et al. [2006] considered programs where the number of disjunctions in the heads of rules is bounded. They provided a translation that allows a Sat encoding of the test whether a candidate set of atoms is indeed an answer set of the input program. Lee and Lifschitz [2003] considered programs with a bounded number of cycles in the positive dependency graph. They suggested a translation that, similar to ours, transforms the input program into an exponentially larger propositional formula whose satisfying assignments correspond to answer sets of the program. As pointed out by Lifschitz and Razborov [2006], this translation produces an exponential blowup already for normal programs (we note that by way of contrast, our translation is in fact quadratic for normal programs).

Over the last few years, several Sat techniques have been integrated into practical Asp solvers. In particular, solvers for normal programs (Cmodels [Giunchiglia et al., 2006], ASSAT [Lin and Zhao, 2004], Clasp [Gebser et al., 2007a]) use certain extensions of Clark's completion and then utilize either black box Sat solvers or integrate conflict analysis, backjumping, and other techniques within the Asp context. ClaspD [Drescher et al., 2008] is a disjunctive AsP-solver that utilizes nogoods based on the logical characterizations of loop formulas [Lee, 2005].

## 2 Preliminaries

Answer set programs We consider a universe of propositional atoms. A disjunctive logic program (or simply a program) $P$ is a set of rules of the form $x_{1} \vee \ldots \vee x_{l} \leftarrow y_{1}, \ldots, y_{n}, \neg z_{1}, \ldots, \neg z_{m}$ where $x_{1}, \ldots, x_{l}$, $y_{1}, \ldots, y_{n}, z_{1}, \ldots, z_{m}$ are atoms and $l, n, m$ are non-negative integers. We write $H(r)=\left\{x_{1}, \ldots, x_{l}\right\}$ (the head of $r$ ), $B^{+}(r)=\left\{y_{1}, \ldots, y_{n}\right\}$ (the positive body of $r$ ), and $B^{-}(r)=\left\{z_{1}, \ldots, z_{m}\right\}$ (the negative body of $r)$. We denote the sets of atoms occurring in a rule $r$ or in a program $P$ by at $(r)=H(r) \cup B^{+}(r) \cup B^{-}(r)$ and $\operatorname{at}(P)=\bigcup_{r \in P}$ at $(r)$, respectively. We abbreviate the number of rules of $P$ by $|P|=|\{r \mid r \in P\}|$. A rule $r$ is negation-free if $B^{-}(r)=\emptyset, r$ is normal if $|H(r)| \leq 1, r$ is a constraint if $|H(r)|=0, r$ is constraint-free if $|H(r)|>0, r$ is Horn if it is negation-free and normal, $r$ is positive if it is Horn and constraint-free, and $r$ is tautological if $B^{+}(r) \cap\left(H(r) \cup B^{-}(r)\right) \neq \emptyset$. We say that a program has a certain property if all its rules have the property. We denote the class of all normal programs by Normal and the class of all Horn programs by Horn. In the following, we restrict ourselves to programs that do not contain any tautological rules. This restriction is not significant as tautological rules can be omitted from a program without changing its answer sets [Brass and Dix, 1998]. Note that we state explicitly the differences regarding tautologies in the proofs.

A set $M$ of atoms satisfies a rule $r$ if $\left(H(r) \cup B^{-}(r)\right) \cap M \neq \emptyset$ or $B^{+}(r) \backslash M \neq \emptyset . M$ is a model of $P$ if it satisfies all rules of $P$. The $G L$ reduct of a program $P$ under a set $M$ of atoms is the program $P^{M}$ obtained from $P$ by first, removing all rules $r$ with $B^{-}(r) \cap M \neq \emptyset$ and second, removing all $\neg z$ where $z \in B^{-}(r)$ from all remaining rules $r$ [Gelfond and Lifschitz, 1991]. $M$ is an answer set (or stable set) of a program $P$ if $M$ is a minimal model of $P^{M}$. The Emden-Kowalski operator of a program $P$ and a subset $A$ of atoms of $P$ is the set $T_{P}(A):=\left\{a \mid a \in H(r), B^{+}(r) \subseteq A, r \in P\right\}$. The least model LM $(P)$ is the least fixed point of $T_{P}(A)$ [Van Emden and Kowalski, 1976]. Note that every positive program $P$ has a unique minimal model which equals the least model $L M(P)$ [Gelfond and Lifschitz, 1988].

Example 1. Consider the program

$$
\left.\begin{array}{rrr}
P=\{a \vee c \leftarrow b ; & b \leftarrow c, \neg g ; & c \leftarrow a ; \\
b \vee c \leftarrow e ; & h \vee i \leftarrow g, \neg c ; & a \vee b ; \\
g & \leftarrow \neg i ; & c
\end{array}\right\} .
$$

The set $A=\{b, c, g\}$ is an answer set of $P$ since $P^{A}=\{a \vee c \leftarrow b ; c \leftarrow a ; b \vee c \leftarrow e ; a \vee b ; g ; c\}$ and the minimal models of $P^{A}$ are $\{b, c, g\}$ and $\{a, c, g\}$.

The main reasoning problems for Asp are Brave Reasoning (given a program $P$ and an atom $a \in \operatorname{at}(P)$, is $a$ contained in some answer set of $P$ ?) and Skeptical Reasoning (given a program $P$ and an atom $a \in \operatorname{at}(P)$, is $a$ contained in all answer sets of $P$ ?). Brave Reasoning is $\Sigma_{2}^{P}$-complete, Skeptical Reasoning is $\Pi_{2}^{P}$-complete [Eiter and Gottlob, 1995].

Parameterized Complexity We give some basic background on parameterized complexity. For more detailed information we refer to other sources [Downey and Fellows, 1999; Flum and Grohe, 2006; Gottlob and Szeider, 2008; Niedermeier, 2006]. A parameterized problem $L$ is a subset of $\Sigma^{*} \times \mathbb{N}$ for some finite alphabet $\Sigma$. For an instance $(I, k) \in \Sigma^{*} \times \mathbb{N}$ we call $I$ the main part and $k$ the parameter. $L$ is fixed-parameter tractable if there exists a computable function $f$ and a constant $c$ such that there exists an algorithm that decides whether $(I, k) \in L$ in time $O\left(f(k)\|I\|^{c}\right)$ where $\|I\|$ denotes the size of $I$. Such an algorithm is called an fpt-algorithm. FPT is the class of all fixed-parameter tractable decision problems.

Let $L \subseteq \Sigma^{*} \times \mathbb{N}$ and $L^{\prime} \subseteq \Sigma^{*} \times \mathbb{N}$ be two parameterized problems for some finite alphabets $\Sigma$ and $\Sigma^{\prime}$. An fpt-reduction $r$ from $L$ to $L^{\prime}$ is a many-to-one reduction from $\Sigma^{*} \times \mathbb{N}$ to $\Sigma^{* *} \times \mathbb{N}$ such that for all $I \in \Sigma^{*}$ we have $(I, k) \in L$ if and only if $r(I, k)=\left(I^{\prime}, k^{\prime}\right) \in L^{\prime}$ such that $k^{\prime} \leq g(k)$ for a fixed computable function $g: \mathbb{N} \rightarrow \mathbb{N}$ and there is a computable function $f$ and a constant $c$ such that $r$ is computable in time $O\left(f(k)\|I\|^{c}\right)$ where $\|I\|$ denotes the size of $I$ [Flum and Grohe, 2006]. Thus, an fpt-reduction is, in particular, an fpt-algorithm. It is easy to see that the class FPT is closed under fpt-reductions. We would like to note that the theory of fixed-parameter intractability is based on fpt-reductions [Downey and Fellows, 1999; Flum and Grohe, 2006].

Propositional satisfiability A truth assignment is a mapping $\tau: X \rightarrow\{0,1\}$ defined for a set $X$ of atoms. For $x \in X$ we put $\tau(\neg x)=1-\tau(x)$. By $\operatorname{ta}(X)$ we denote the set of all truth assignments $\tau: X \rightarrow\{0,1\}$. We usually say variable instead of atom in the context of formulas. Given a propositional formula $F$, the problem Sat asks whether $F$ is satisfiable. We can consider Sat as a parameterized problem by simply associating with every formula the parameter 0 .

## 3 Backdoors of Programs

In the following we give the main notions concerning backdoors for answer set programming, as introduced by Fichte and Szeider [2012]. Let $P$ be a program, $X$ a set of atoms, and $\tau \in \operatorname{ta}(X)$. The truth assignment reduct of $P$ under $\tau$ is the logic program $P_{\tau}$ obtained from $P$ by removing all rules $r$ for which at least one of the following holds: (i) $H(r) \cap \tau^{-1}(1) \neq \emptyset$, (ii) $H(r) \subseteq X$, (iii) $B^{+}(r) \cap \tau^{-1}(0) \neq \emptyset$, and (iv) $B^{-}(r) \cap \tau^{-1}(1) \neq \emptyset$, and then removing from the heads and bodies of the remaining rules all literals $v, \neg v$ with $v \in X$. In the following, let $\mathcal{C}$ be a class of programs. We call $\mathcal{C}$ to be rule induced if for each $P \in \mathcal{C}, P^{\prime} \subseteq P$ implies $P^{\prime} \in \mathcal{C}$. A set $X$ of atoms is a strong $\mathcal{C}$-backdoor of a program $P$ if $P_{\tau} \in \mathcal{C}$ for all truth assignments $\tau \in \operatorname{ta}(X)$. Given a strong $\mathcal{C}$-backdoor $X$ of a program $P$, the answer sets of $P$ are among the answer sets we obtain from the truth assignment reducts $P_{\tau}$ where $\tau \in X$, more formally $\operatorname{AS}(P) \subseteq\left\{M \cup \tau^{-1}(1) \mid \tau \in \operatorname{ta}(X \cap \operatorname{at}(P)), M \in \operatorname{AS}\left(P_{\tau}\right)\right\}$ where $\operatorname{AS}(P)$ denotes the set of all answer sets of $P$. For a program $P$ and a set $X$ of atoms we define $P-X$ as the program obtained from $P$ by deleting all atoms contained in $X$ and their negations from the heads and bodies of all the rules of $P$. A set $X$ of atoms is a deletion $\mathcal{C}$-backdoor of a program $P$ if $P-X \in \mathcal{C}$.
Example 2. Consider the program $P$ from Example 1. The set $X=\{b, c, h\}$ is a strong Normal-backdoor since the truth assignment reducts $P_{b=0, c=0, h=0}=P_{000}=\{i \leftarrow g ; a ; g \leftarrow \neg i\}, P_{001}=P_{010}=$ $P_{011}=P_{101}=\{a ; g \leftarrow \neg i\}, P_{100}=\{a ; i \leftarrow g ; g \leftarrow \neg i\}$, and $P_{110}=P_{111}=\{g \leftarrow \neg i\}$ are in the class Normal.

In the following we refer to $\mathcal{C}$ as the target class of the backdoor. For most target classes $\mathcal{C}$, deletion $\mathcal{C}$-backdoors are strong $\mathcal{C}$-backdoors. For $\mathcal{C}=$ Normal even the opposite direction is true.
Proposition 1 (Fichte and Szeider, 2012). If $\mathcal{C}$ is rule induced, then every deletion $\mathcal{C}$-backdoor is a strong $\mathcal{C}$-backdoor.

Lemma 1. Let $P$ be a program. A set $X$ is a strong Normal-backdoor of a program $P$ if and only if it is a deletion Normal-backdoor of $P$.

Proof. We observe that the class of all normal programs is rule-induced. Thus the if direction holds by Proposition 1. We proceed to show the only-if direction. Assume $X$ is a strong Normal-backdoor of $P$. Consider a rule $r^{\prime} \in P-X$ which is not tautological. Let $r \in P$ be a rule from which $r^{\prime}$ was obtained in forming $P-X$. We define $\tau \in \operatorname{ta}(X)$ by setting all atoms in $H(r) \cup B^{-}(r)$ to 0 , all atoms in $B^{+}(r)$ to 1 , and all remaining atoms in $X \backslash$ at $(r)$ arbitrarily to 0 or 1 . Since $r$ is not tautological, this definition of $\tau$ is sound. It remains to observe that $r^{\prime} \in P_{\tau}$. Since $X$ is a strong Normal-backdoor of $P$, the rule $r^{\prime}$ is normal. Hence, the lemma follows.

Each target class $\mathcal{C}$ gives rise to the following problems:

## $\mathcal{C}$-Backdoor-Asp-CHECK

Given: $\quad$ A program $P$, a strong $\mathcal{C}$-backdoor $X$ of $P$, a set $M \subseteq \operatorname{at}(P)$, and the size of the backdoor $k=|X|$.
Parameter: The integer $k$.
Question: $\quad I s M$ an answer set of $P$ ?

## $\mathcal{C}$-Backdoor-Brave-Reasoning

Given: $\quad$ A program $P$, a strong $\mathcal{C}$-backdoor $X$ of $P$, an atom $a^{*} \in$ $\operatorname{at}(P)$, and the size of the backdoor $k=|X|$.
Parameter: The integer $k$.
Question: Does $a^{*}$ belong to some answer set of $P$ ?

## $\mathcal{C}$-Backdoor-Skeptical-REASONing

Given: $\quad$ A program $P$, a strong $\mathcal{C}$-backdoor $X$ of $P$, an atom $a^{*} \in$ at $(P)$, and the size of the backdoor $k=|X|$.
Parameter: The integer $k$.
Question: Does $a^{*}$ belong to all answer sets of $P$ ?
Problems for deletion $\mathcal{C}$-backdoors can be defined similarly.

## 4 Using Backdoors

In this section, we show results regarding the use of backdoors with respect to the target class Normal.
Theorem 1. The problem Normal-Backdoor-Asp-Check is fixed-parameter tractable. More specifically, given a program $P$ of input size $n$, a strong Normal-backdoor $N$ of $P$ of size $k$, and a set $M \subseteq \operatorname{at}(P)$ of atoms, we can check in time $O\left(2^{k} n\right)$ whether $M$ is an answer set of $P$.

The most important part for establishing Theorem 1 is to check whether a model is a minimal model. In general, this is a co-NP-complete task, but in the context of Theorem 1 we can achieve fixed-parameter tractability based on the following construction and lemma.

Let $P$ be a given program, $X$ a strong Normal-backdoor of $P$ of size $k$, and let $M \subseteq \operatorname{at}(P)$. For a set $X_{1} \subseteq M \cap X$ we construct a program $P_{X_{1} \subseteq X}$ as follows: (i) remove all rules $r$ for which $H(r) \cap X_{1} \neq \emptyset$ and (ii) replace for all remaining rules $r$ the head $H(r)$ with $H(r) \backslash X$ and the positive body $B^{+}(r)$ with $B^{+}(r) \backslash X_{1}$.

Recall that by definition we exclude programs with tautological rules. Since $X$ is a strong Normal-backdoor of $P$, it is also a deletion Normal-backdoor of $P$ by Lemma 1. Hence $P-X$ is normal. Let $r$ be an arbitrarily chosen rule in $P$. Then there is a corresponding rule $r^{\prime} \in P-X$ and a corresponding rule $r^{\prime \prime} \in P_{X_{1} \subseteq X}$. Since we remove in both constructions exactly the same literals from the head of every rule, $H\left(r^{\prime}\right)=H\left(r^{\prime \prime}\right)$ holds. Consequently, $P_{X_{1} \subseteq X}$ is normal and $P_{X_{1} \subseteq X}^{M}$ is Horn (here $P_{X_{1} \subseteq X}^{M}$ denotes the GL-reduct of $P_{X_{1} \subseteq X}$ under $M$ ).

For any program $P^{\prime}$ let $\operatorname{Constr}\left(P^{\prime}\right)$ denote the set of constrains of $P^{\prime}$ and $\operatorname{Pos}\left(P^{\prime}\right)=P^{\prime} \backslash \operatorname{Constr}\left(P^{\prime}\right)$. If $P^{\prime}$ is Horn, $\operatorname{Pos}\left(P^{\prime}\right)$ has a least model $L$ and $P^{\prime}$ has a model if and only if $L$ is a model of Constr $\left(P^{\prime}\right)$ [Dowling and Gallier, 1984].

Let $X$ be a strong Normal-backdoor of $P$ and $X_{1} \subseteq X$. Given $M \subseteq \operatorname{at}(P)$, the algorithm $\operatorname{MinCheck}\left(X_{1}\right)$ below performs the following steps:

1. Return True if $X_{1}$ is not a subset of $M$.
2. Compute the Horn program $P_{X_{1} \subseteq X}^{M}$.
3. Compute the least model $L$ of $\operatorname{Pos}\left(P_{X_{1} \subseteq X}^{M}\right)$.
4. Return True if at least one of the following conditions holds:
(a) $L$ is not a model of $\operatorname{Constr}\left(P_{X_{1} \subseteq X}^{M}\right)$.
(b) $L$ is not a subset of $X$,
(c) $L \cup X_{1}$ is not a proper subset of $M$,
(d) $L \cup X_{1}$ is not a model of $P^{M}$.
5. Otherwise return False.

Lemma 2. Let $X$ be a strong Normal-backdoor. A model $M \subseteq \operatorname{at}(P)$ of $P^{M}$ is a minimal model of $P^{M}$ if and only if $\operatorname{MinCheck}\left(X_{1}\right)$ returns True for each set $X_{1} \subseteq X$.

Proof. $(\Rightarrow)$. Assume that $M$ is a minimal model of $P^{M}$, and suppose to the contrary that there is some $X_{1} \subseteq M \cap X$ for which the algorithm returns False. Consequently, none of the conditions in Step 4 of the algorithms holds. That means, the least model $L$ of $P_{X_{1} \subseteq X}^{M}$ satisfies Constr$\left(P_{X_{1} \subseteq X}^{M}\right)$ and is therefore a model of $P_{X_{1} \subseteq X}^{M}$. Moreover, since $L \cup X_{1} \nsubseteq M$ and $L \cup X_{1}$ is a model of $P^{M}, M$ cannot be a minimal model of $P^{M}$, a contradiction to our assumption. So we conclude that the algorithm succeeds and the only-if direction of the lemma is shown.
$(\Leftarrow)$. Assume that the algorithm returns True for each $X_{1} \subseteq M \cap X$. We show that $M$ is a minimal model of $P^{M}$. Suppose to the contrary that $P^{M}$ has a model $M^{\prime} \subsetneq M$.

We run the algorithm for $X_{1}:=M^{\prime} \cap X$. Let $L$ be the least model of $\operatorname{Pos}\left(P_{X_{1} \subseteq X}^{M}\right)$. By assumption, the algorithm returns True, hence some of the conditions of Step 4 of the algorithm must hold for $L$. We will show, however, that none of the conditions can hold, which will yield to a contradiction, and so establish the if direction of the lemma, and thus completes its proof.

First we show that $M^{\prime} \backslash X$ is a model of $P_{X_{1} \subseteq X}^{M}$. Consider a rule $r^{\prime} \in P_{X_{1} \subseteq X}^{M}$ and let $r \in P^{M}$ such that $r^{\prime}$ is obtained form $r$ by removing $X$ from $H(r)$ and by removing $X_{1}$ from $B^{+}(r)$. Since $M^{\prime}$ is a model of $P^{M}$, we have (i) $B^{+}(r) \backslash M^{\prime} \neq \emptyset$ or (ii) $H(r) \cap M^{\prime} \neq \emptyset$. Moreover, since $B^{+}\left(r^{\prime}\right)=B^{+}(r) \backslash X_{1}$ and $X_{1}=M^{\prime} \cap X$, (i) implies $\emptyset \neq B^{+}(r) \backslash M^{\prime}=B^{+}(r) \backslash X_{1} \backslash M^{\prime}=B^{+}\left(r^{\prime}\right) \backslash M^{\prime} \subseteq B^{+}\left(r^{\prime}\right) \backslash\left(M^{\prime} \backslash X\right)$, and since $H(r) \cap X_{1}=\emptyset$, (ii) implies $\emptyset \neq H(r) \cap M^{\prime}=H(r) \cap\left(M^{\prime} \backslash X_{1}\right)=H(r) \cap\left(M^{\prime} \backslash X\right)=$ $(H(r) \backslash X) \cap\left(M^{\prime} \backslash X\right)=H\left(r^{\prime}\right) \cap\left(M^{\prime} \backslash X\right)$. Hence $M^{\prime} \backslash X$ satisfies $r^{\prime}$. Since $r^{\prime} \in P_{X_{1} \subseteq X}^{M}$ was chosen arbitrarily, we conclude that $M^{\prime} \backslash X$ is a model of $P_{X_{1} \subseteq X}^{M}$.

Since $P_{X_{1} \subseteq X}^{M}$ has some model (namely $M^{\prime} \backslash X$ ), the least model $L$ of $\operatorname{Pos}\left(P_{X_{1} \subseteq X}^{M}\right)$ must be a model of $P_{X_{1} \subseteq X}^{M}$, thus Condition (a) cannot hold for $L$.

Next we show that the other conditions cannot hold either. Since $M^{\prime} \backslash X$ is a model of $P_{X_{1} \subseteq X}^{M}$, as shown above, we have $L \subseteq M^{\prime} \backslash X$. We obtain $L \subseteq M \backslash X$ since $M^{\prime} \backslash X \subseteq M \backslash X$. Further, we obtain $L \cup X_{1} \subsetneq M$ since $L \cup X_{1} \subseteq\left(M^{\prime} \backslash X\right) \cup X_{1}=\left(M^{\prime} \backslash X\right) \cup\left(M^{\prime} \cap X\right)=M^{\prime} \subsetneq M$. Hence we have excluded Conditions (b) and (c), and it remains to exclude Condition (d).

Consider a rule $r \in P^{M}$. If $X_{1} \cap H(r) \neq \emptyset$, then $L \cup X_{1}$ satisfies $r$; thus it remains to consider the case $X_{1} \cap H(r)=\emptyset$. In this case there is a rule $r^{\prime} \in P_{X_{1} \subseteq X}^{M}$ with $H\left(r^{\prime}\right)=H(r) \backslash X$ and $B^{+}\left(r^{\prime}\right)=B^{+}(r) \backslash X_{1}$. Since $L$ is a model of $P_{X_{1} \subseteq X}^{M}, L$ satisfies $r^{\prime}$. Hence (i) $B^{+}\left(r^{\prime}\right) \backslash L \neq \emptyset$ or (ii) $H\left(r^{\prime}\right) \cap L \neq \emptyset$. Since $B^{+}\left(r^{\prime}\right)=B^{+}(r) \backslash X_{1}$, (i) implies that $B^{+}(r) \backslash\left(L \cup X_{1}\right) \neq \emptyset$; and since $H\left(r^{\prime}\right) \subseteq H(r)$, (ii) implies that $H(r) \cap\left(L \cup X_{1}\right) \neq \emptyset$. Thus $L \cup X_{1}$ satisfies $r$. Since $r \in P^{M}$ was chosen arbitrarily, we conclude that $L \cup X_{1}$ is a model of $P^{M}$, which excludes also the last Condition (d).

We are now in a position to establish Theorem 1.
Proof of Theorem 1. First we check whether $M$ is a model of $P^{M}$. If $M$ is not a model of $P^{M}$ then it is not an answer set of $P$, and we can neglect it. Hence assume that $M$ is a model of $P^{M}$. Now we run the algorithm MinCheck. By Lemma 2 the algorithm decides whether $M$ is an answer set of $P$.

In order to complete the proof, it remains to bound the running time. The check whether $M$ is a model of $P^{M}$ can clearly be carried out in linear time. For each set $X_{1} \subseteq M \cap X$ the algorithm MinCheck runs in linear time. This follows directly from the fact that we can compute the least model of a Horn program in linear time [Dowling and Gallier, 1984]. As there are at most $2^{k}$ sets $X_{1}$ to consider, the total running time is $O\left(2^{k} n\right)$ where $n$ denotes the input size of $P$ and $k=|X|$. Thus, in particular, the decision is fixed-parameter tractable for parameter $k$.
Example 3. Consider the program $P$ from Example 1 and the backdoor $X=\{b, c, h\}$ from Example 2. Let $N=\{a, b, c, g\} \subseteq \operatorname{at}(P)$. Obviously $N$ is a model of $P$. We apply the algorithm MinCheck for each $X_{1}$ of $\{b, c\}$. For $X_{1}=\emptyset$ we obtain $P_{X_{1} \subseteq X}^{N}=\{a \leftarrow b ; \leftarrow a ; \leftarrow e ; a ; g \leftarrow \neg i\}$ and the least model $L=\{a, g\}$ of $\operatorname{Pos}\left(P_{\emptyset \subseteq X}^{N}\right)$. Since Condition $4 a$ holds $\left(L\right.$ is not a model of Constr $\left(P_{X_{1} \subseteq X}^{N}\right)$ ), the algorithm returns True. For $X_{2}=\{b\}$ we have $P_{X_{2} \subseteq X}^{N}=\{a ; \leftarrow a ; g\}$ and $L=\{g\}$ is the least model of $\operatorname{Pos}\left(P_{X_{2} \subseteq X}^{N}\right)$. Since Condition 4 a holds ( $L$ is not a model of Constr $\left(P_{X_{2} \subseteq X}^{N}\right)$ ), the algorithm returns True for $X_{2}$. For $X_{3}=\{c\}$ we obtain $P_{X_{3} \subseteq X}^{N}=\{a ; g\}$. The set $L=\{a, g\}$ is the least model of $\operatorname{Pos}\left(P_{X_{3} \subseteq X}^{N}\right)$. Since none of the Conditions $4 a-d$ hold, more precisely $L$ is a model of $\operatorname{Constr}\left(P_{X_{1} \subseteq X}^{N}\right)$, $L$ is a subset of $X, L \cup X_{1}$ is a proper subset of $N$, and $L \cup X_{1}$ is a model of $P^{N}$. Hence, the algorithm returns False. Thus MinCheck does not succeed, and $M$ is not a minimal model of $P^{M}$.

Example 4. Again, consider the program $P$ from Example 1 and the backdoor $X=\{b, c, h\}$ from Example 2. Let $M=\{b, c, g\} \subseteq \operatorname{at}(P)$. Since $M$ satisfies all rules in $P$, the set $M$ is a model of $P$. We apply the algorithm MinCheck for each subset of $\{b, c, h\}$. For $X_{1}=\emptyset$ we obtain $P_{X_{1} \subseteq X}^{M}=\{a \leftarrow b ; \leftarrow$ $a ; \leftarrow e ; a ; g\}$. The set $L=\{a, g\}$ is the least model of $\operatorname{Pos}\left(P_{X_{1} \subseteq X}^{M}\right)$. Since Condition $4 a$ holds, the algorithm returns True for $X_{1}$. For $X_{2}=\{b\}$ we have $P_{X_{2} \subseteq X}^{M}=\{a ; \leftarrow a ; g ; \leftarrow\}$ and the least model $L=\{a, g\}$ of $\operatorname{Pos}\left(P_{X_{2} \subseteq X}^{M}\right)$. Since Condition $4 a$ holds, MinCheck returns True for $X_{2}$. For $X_{3}=\{c\}$ we gain $P_{X_{3} \subseteq X}^{M}=\{a ; g\}$ and the least model $L=\{a, g\}$ of $\operatorname{Pos}\left(P_{X_{3} \subseteq X}^{M}\right)$. Since Condition $4 c$ holds, the algorithm returns True for $X_{3}$. For $X_{4}=\{b, c\}$ we obtain $P_{X_{4} \subseteq X}^{M}=\{g\}$. The set $L=\{g\}$ is the least model of $\operatorname{Pos}\left(P_{X_{4} \subseteq X}^{M}\right)$. Since Condition $4 c$ holds, the algorithm returns True for $X_{4}$. For all remaining subsets of $X$ the Algorithm MinCheck returns True according to Condition 1. Consequently, $M$ is a minimal model of $P^{M}$ and thus an answer set of $P$.

Next, we state and prove that there are fpt-reductions from Normal-Backdoor-Brave-Reasoning and Normal-Backdoor-Skeptical-Reasoning to Sat which is the main result of this paper.

Theorem 2. Given a disjunctive logic program $P$ of input size n, a strong Normal-backdoor $X$ of $P$ of size $k$, and an atom $a^{*} \in \operatorname{at}(P)$, we can produce in time $O\left(2^{k} n^{2}\right)$ propositional formulas $F_{\text {Brave }}\left(a^{*}\right)$ and $F_{\text {Skept }}\left(a^{*}\right)$ such that (i) $F_{\text {Brave }}\left(a^{*}\right)$ is satisfiable if and only if $a^{*}$ is in some answer set of $P$, and (ii) $F_{\text {Skept }}\left(a^{*}\right)$ is unsatisfiable if and only if $a^{*}$ is in all answer sets of $P$.

Proof. We would like to use a similar approach as in the proof of Theorem 1. However, we cannot consider all possible models $M$ one by one, as there could be too many of them. Instead, we will show that it is possible to implement $\operatorname{MinCheck}\left(X_{1}\right)$ for each set $X_{1} \subseteq X$ nondeterministically in such a way that we do not need to know $M$ in advance. Possible sets $M$ will be represented by the truth values of certain variables, and since the truth values do not need to be known in advance, this will allow us to consider all possible sets $M$ without enumerating them.

Next, we describe the construction of the formulas $F_{\text {Brave }}\left(a^{*}\right)$ and $F_{\text {Skept }}\left(a^{*}\right)$ in detail.
Among the variables of our formulas will be a set $V:=\{v[a] \mid a \in \operatorname{at}(P)\}$ containing a variable for each atom of $P$. The truth values of the variables in $V$ represent a subset $M \subseteq$ at $(P)$, such that $v[a]$ is true if and only if $a \in M$.

We define

$$
\begin{gathered}
F_{\text {Brave }}\left(a^{*}\right):=F^{\bmod } \wedge F^{\min } \wedge v\left[a^{*}\right] \text { and } \\
F_{\text {Skept }}\left(a^{*}\right):=F^{\bmod } \wedge F^{\min } \wedge \neg v\left[a^{*}\right],
\end{gathered}
$$

where $F^{\text {mod }}$ and $F^{\text {min }}$ are formulas, defined below, that check whether the truth values of the variables in $V$ represent a model $M$ of $P^{M}$, and whether $M$ is a minimal model of $P^{M}$, respectively.

The definition of $F^{\mathrm{mod}}$ is easy:

$$
F^{\bmod }:=\bigwedge_{r \in P}\left(\bigwedge_{b \in B^{-(r)}} \neg v[b] \rightarrow\left(\bigvee_{b \in B^{+}(r)} \neg v[b] \vee \bigvee_{b \in H(r)} v[b]\right)\right)
$$

The definition of $F^{\mathrm{min}}$ is more involved. First we define:

$$
F^{\min }:=\bigwedge_{1 \leq i \leq 2^{k}} F_{i}^{\min }
$$

where $F_{i}^{\mathrm{min}}$, defined below, encodes the Algorithm MinCheck $\left(X_{i}\right)$ for each set $X_{i}$ where $X_{1}, \ldots, X_{2^{k}}$ is an enumeration of all the subsets of $X$.

The formula $F_{i}^{\text {min }}$ will contain, in addition to the variables in $V, p$ distinct variables for each atom of $P, p:=\min \{|P|,|a t(P)|\}$. In particular, the set of variables of $F_{i}^{\text {min }}$ is the disjoint union of $V$ and $U_{i}$ where $U_{i}:=\left\{u_{i}^{j}[a] \mid a \in \operatorname{at}(P), 1 \leq j \leq p\right\}$. We write $U_{i}^{j}$ for the subset of $U_{i}$ containing all the variables $u_{i}^{j}[a]$. We assume that for $i \neq i^{\prime}$ the sets $U_{i}$ and $U_{i^{\prime}}$ are disjoint. For each $a \in \operatorname{at}(P)$ we also use the propositional constants $X(a)$ and $X_{1}(a)$ that are true if and only if $a \in X$ and $a \in X_{1}$, respectively.

The truth values of the variables in $U_{i}^{p}$ represent the unique minimal model of $\operatorname{Pos}\left(P_{X_{s} \subseteq X}^{M}\right)$.
We define the formula $F_{i}^{\text {min }}$ by means of the following auxiliary formulas.
The first auxiliary formula checks whether the truth values of the variables in $V$ represent a set $M$ that contains $X_{i}$ :

$$
F_{i}^{\subseteq}:=\bigwedge_{a \in X} X_{i}(a) \rightarrow v[a] .
$$

The next auxiliary formula encodes the computation of the least model ("lm") $L$ of $\operatorname{Pos}\left(P_{X_{i} \subseteq X}^{M}\right)$ where $M$ and $L$ are represented by the truth values of the variables in $V$ and $U_{i}^{p}$, respectively.

$$
\begin{gathered}
F_{i}^{\operatorname{lm}}:=\bigwedge_{a \in \operatorname{at}(P), 0 \leq i \leq p} F_{i}^{(a, i)}, \quad \text { where } \\
F_{i}^{(a, 0)}:=u_{i}^{0}[a] \leftrightarrow \text { false, }
\end{gathered}
$$

$$
\begin{aligned}
& F_{i}^{(a, j)}:=u_{i}^{j}[a] \leftrightarrow\left[u _ { i } ^ { j - 1 } [ a ] \vee \bigvee _ { r \in P _ { X _ { i } \subseteq X } , a \in H ( r ) } \left(\bigwedge_{b \in B^{+}(r)} u_{i}^{j-1}[b] \wedge\right.\right.\left.\left.\bigwedge_{b \in B^{-}(r)} \neg v[b]\right)\right] \\
&(\text { for } 1 \leq j \leq p-1) .
\end{aligned}
$$

The idea behind the construction of $F_{i}^{\mathrm{lm}}$ is to simulate the linear-time algorithm of Dowling and Gallier [1984]. Initially, all variables are set to false. This is represented by variables $u_{i}^{0}[a]$. Now we flip a variable from false to true if and only if there is a Horn rule where all the variables in the rule body are true. We iterate this process until a fixed-point is reached, then we have the least model. The flipping is represented in our formula by setting a variable $u_{i}^{j}[a]$ to true if and only if either $u_{i}^{j-1}[a]$ is true, or there is a rule $r \in \operatorname{Pos}\left(P_{X_{i} \subseteq X}^{M}\right)$ such that $H(r)=\{a\}$ and $u_{i}^{j}[b]$ is true for all $b \in B^{+}(r)$. The truth values of the variables $u_{i}^{p}$ now represent the least model of $\operatorname{Pos}\left(P_{X_{i} \subseteq X}^{M}\right)$.

The next four auxiliary formulas check whether the respective condition (a)-(d) of algorithm $\operatorname{MinCheck}\left(X_{i}\right)$ does not hold for $L$.
$F_{i}^{(a)}$ expresses that there is a rule in $\operatorname{Constr}\left(P_{X_{i} \subseteq X}^{M}\right)$ that is not satisfied by $L$ :

$$
F_{i}^{(\mathrm{a})}:=\bigvee_{r \in P_{X_{i}} \subseteq X}, H(r) \subseteq X\left(\bigwedge_{b \in B^{-}(r)} \neg v[b] \wedge \bigwedge_{b \in B^{+}(r)} u_{i}^{p}[b]\right)
$$

$F_{i}^{(\mathrm{b})}$ expresses that $L$ contains an atom that is not in $M \backslash X$ :

$$
F_{i}^{(\mathrm{b})}:=\bigvee_{a \in \operatorname{at}(P) \backslash X}\left(\neg v[a] \wedge u_{i}^{p}[a]\right)
$$

$F_{i}^{(c)}$ expresses that $L \cup X_{i}$ equals $M$ or $L \cup X_{i}$ contains an atom that is not in $M$ :

$$
F_{i}^{(\mathrm{c})}:=\left(\bigwedge_{a \in \operatorname{at}(P)} v[a] \leftrightarrow\left(u_{i}^{p}[a] \vee X_{i}(a)\right)\right) \vee\left(\bigvee_{a \in \operatorname{at}(P)}\left(u_{i}^{p}[a] \vee X_{i}(a)\right) \wedge \neg v[a]\right) .
$$

$F_{i}^{(\mathrm{d})}$ expresses that $P^{M}$ contains a rule that is not satisfied by $L \cup X_{i}$ :

$$
F_{i}^{(\mathrm{d})}:=\bigvee_{r \in P}\left[\bigwedge_{a \in B^{-}(r)} \neg v[a] \wedge \bigwedge_{a \in H(r)}\left(\neg u_{i}^{p}[a] \wedge \neg X_{i}(a)\right) \wedge \bigwedge_{b \in B^{+}(r)}\left(u_{i}^{p}[b] \vee X_{i}(b)\right)\right]
$$

Now we can put the auxiliary formulas together and obtain

$$
F_{i}^{\min }:=\neg F_{i}^{\subseteq} \vee\left(F_{i}^{\operatorname{lm}} \wedge\left(F_{i}^{(\mathrm{a})} \vee F_{i}^{(\mathrm{b})} \vee F_{i}^{(\mathrm{c})} \vee F_{i}^{(\mathrm{d})}\right)\right)
$$

It follows by Lemma 2 and by the construction of the auxiliary formulas that (i) $F_{\mathrm{Brave}}\left(a^{*}\right)$ is satisfiable if and only if $a^{*}$ is in some answer set of $P$, and (ii) $F_{\text {Skept }}\left(a^{*}\right)$ is unsatisfiable if and only if $a^{*}$ is in all answer sets of $P$.

Hence, it remains to observe that for each $i \leq 2^{k}$ the auxiliary formula $F_{i}^{\mathrm{lm}}$ can be constructed in quadratic time, whereas the auxiliary formulas $F_{i}^{\subseteq}$ and $F_{i}^{(\mathrm{a})} \vee F_{i}^{(\mathrm{b})} \vee F_{i}^{(\mathrm{c})} \vee F_{i}^{(\mathrm{d})}$ can be constructed in linear time. Since $|X|=k$ by assumption, we need to construct $O\left(2^{k}\right)$ auxiliary formulas in order to obtain $F_{\text {Skept }}\left(a^{*}\right)$ and $F_{\text {Brave }}\left(a^{*}\right)$. Hence, the running time as claimed in Theorem 2 follows and the theorem is established.

We would like to note that Theorem 2 remains true if we require that the formulas $F_{\text {Skept }}\left(a^{*}\right)$ and $F_{\text {Brave }}\left(a^{*}\right)$ are in Conjunctive Normal Form (CNF), as we can transform in linear time any propositional formula into a satisfiability-equivalent formula in CNF, e.g., using the well-known transformation due to Tseitin [1968], see also [Kleine Büning and Lettman, 1999]. This transformation produces for a given propositional formula $F^{\prime}$ in linear time a CNF formula $F$ such that both formulas are equivalent with respect to their satisfiability, and the length of $F$ is linear in the length of $F^{\prime}$.

Furthermore, the SAT encoding can be improved. For instance, one could share parts between the formulas $F_{i}^{\min }$ or replace the quadratic formula $F_{i}^{\mathrm{lm}}$ for the computation of least models with a smaller and more sophisticated SAT encoding [Janhunen, 2004] or a SAT(DL) encoding [Janhunen et al., 2009] for the Smт framework which combines propositional logic and linear constraints.

We would like to point out that our approach directly extends to more general problems, when we look for answer sets that satisfy a certain global property which can be expressed by a propositional formula $F^{\text {prop }}$ on the variables in $V$. We just check the satisfiability of $F^{\bmod } \wedge F^{\text {min }} \wedge F^{\text {prop }}$.

Example 5. Consider the program $P$ from Example 1 and the strong Normal-backdoor $X=\{b, c, h\}$ of $P$ from Example 2. We ask whether the atom $b$ is contained in at least one answer set. To decide the question, we check that $F_{\text {brave }}(b)$ is satisfiable and we answer the question positively. Since $M=\{b, c, g\}$ is model of $P^{M}$ we can satisfy $F^{\text {mod }}$ with a truth assignment $\tau$ that maps 1 to each variable $v[x]$ where $x \in\{b, c, g\}$ and 0 to each variable $v[x]$ where $x \in \operatorname{at}(P) \backslash\{b, c, g\}$. For $i=1$ let $X_{1}=\emptyset$. Then we have for the constants $X_{1}(x)=0$ where $x \in\{b, c, h\}$. Observe that $\tau$ already satisfies $F_{i}^{\subseteq}$ and that $F_{i}^{l m}$ encodes the computation of the least model $L$ of $\operatorname{Pos}\left(P_{X_{1} \subseteq X}^{M}\right)$ where $L$ is represented by the truth values of the variables in $U_{i}^{P}=\left\{u_{i}^{p}[x] \mid x \in \operatorname{at}(P)\right\}$. Thus $\tau$ also satisfies $F_{i}^{l m}$ if $\tau$ maps $u_{i}^{p}[a]$ to $1, u_{i}^{p}[g]$ to 1 , and $u_{i}^{p}[x]$ to 0 where $x \in \operatorname{at}(P) \backslash\{a, g\}$. As $\tau$ satisfies $F_{1}^{(a)}$, the truth assignment $\tau$ satisfies the formula $F_{1}^{\text {min }}$. It is not hard to see that $F_{i}^{\text {min }}$ is satisfiable for other values of $i$. Hence the formula $F_{\text {brave }}(b)$ is satisfiable and $b$ is contained in at least one answer set.

## Completeness for paraNP and co-paraNP

The parameterized complexity class paraNP contains all parameterized decision problems $L$ such that $(I, k) \in L$ can be decided nondeterministically in time $O\left(f(k)\|I\|^{c}\right)$, for some computable function $f$ and constant $c$ [Flum and Grohe, 2006]. By co-paraNP we denote the class of all parameterized decision problems whose complement (the same problem with yes and no answers swapped) is in paraNP.

If a non-parameterized problem is NP-complete, then adding a parameter that makes it paraNP-complete does not provide any gain, as this holds even true if the parameter is the constant 0 . Therefore a paraNP-completeness result for a problem that without parameterization is in NP, is usually considered as an utterly negative result. However, if the considered problem without parameter is outside NP, and we can show that with a suitable parameter the problem becomes paraNP-complete, this is in fact a positive result. Indeed, we get such a positive result as a corollary to Theorem 2.
Corollary 1. Normal-Backdoor-Brave-Reasoning is paraNP-complete, and Normal-Backdoor-Skeptical-Reasoning is co-paraNP-complete.
Proof. If a parameterized problem $L$ is NP-hard when we fix the parameter to a constant, then $L$ is paraNP-hard (Flum and Grohe, 2006, Th. 2.14). As Normal-Backdoor-Brave-Reasoning is

NP-hard for backdoor size 0, we conclude that Normal-BAckdoor-Brave-Reasoning is paraNPhard. A similar argument shows that Normal-Backdoor-Skeptical-Reasoning is co-paraNP-hard. SAT, considered as a parameterized problem with constant parameter 0 , is clearly paraNP-complete, this also follows from the mentioned result of Flum and Grohe [2006]; hence UnSat is co-paraNP-complete. As Theorem 2 provides fpt-reductions from Normal-Backdoor-Brave-Reasoning to Sat, and from Normal-Backdoor-Skeptical-Reasoning to UnSat, we conclude that Normal-Backdoor-Brave-Reasoning is in paraNP, and Normal-Backdoor-Skeptical-Reasoning is in co-paraNP.

## 5 Finding Backdoors

In this section, we study the problem of finding backdoors, formalized in terms of the following parameterized problem:

## Strong $\mathcal{C}$-Backdoor-Detection

Given: $\quad$ A (disjunctive) program $P$, and an integer $k$.
Parameter: The integer $k$.
Question: $\quad$ Find a strong $\mathcal{C}$-backdoor $X$ of $P$ of size at most $k$, or report that such $X$ does not exist.

We also consider the problem Deletion $\mathcal{C}$-Backdoor-Detection, defined similarly.
Let $P$ be a program. Let the head dependency graph $U_{P}^{H}$ be the undirected graph $U_{P}^{H}=(V, E)$ defined on the set $V=\operatorname{at}(P)$ of atoms of the given program $P$, where two atoms $x, y$ are joined by an edge $x y \in E$ if and only if $P$ contains a non-tautological rule $r$ with $x, y \in H(r)$. A vertex cover of a graph $G=(V, E)$ is a set $X \subseteq V$ such that for every edge $u v \in E$ we have $\{u, v\} \cap X \neq \emptyset$.

Lemma 3. Let $P$ be a program. A set $X \subseteq \operatorname{at}(P)$ is a deletion Normal-backdoor of $P$ if and only if $X$ is a vertex cover of $U_{P}^{H}$.

Proof. Let $X$ be a deletion Normal-backdoor of $P$. Consider an edge $u v$ of $U_{P}^{H}$, then there is a rule $r \in P$ with $u, v \in H(r)$ and $u \neq v$. Since $X$ is a deletion Normal-backdoor set of $P$, we have $\{u, v\} \cap X \neq \emptyset$. We conclude that $X$ is a vertex cover of $U_{P}^{H}$.

Conversely, assume that $X$ is a vertex cover of $U_{P}^{H}$. Consider a rule $r \in P-X$ for proof by contradiction. If $|H(r)| \geq 2$ then there are two variables $u, v \in H(r)$ and an edge $u v$ of $U_{P}^{H}$ such that $\{u, v\} \cap X=\emptyset$, contradicting the assumption that $X$ is a vertex cover. Hence the lemma prevails.

Theorem 3. The problems Strong Normal-Backdoor-Detection and Deletion Normal-Back-DOor-Detection are fixed-parameter tractable. In particular, given a program $P$ of input size $n$, and an integer $k$, we can find in time $O\left(1.2738^{k}+k n\right)$ a strong Normal-backdoor of $P$ with a size $\leq k$ or decide that no such backdoor exists.

Proof. In order to find a deletion Normal-backdoor of a given program $P$, we use Lemma 3 and find a vertex cover of size at most $k$ in the head dependency graph $U_{P}^{D}$. A vertex cover of size $k$, if it exists, can be found in time $O\left(1.2738^{k}+k n\right)$ [Chen et al., 2006]. Thus the theorem holds for deletion Normal-backdoors. Lemma 1 states that the strong Normal-backdoors of $P$ are exactly the deletion Normal-backdoors of $P$ (as we assume that $P$ does not contain any tautological rules). The theorem follows.

In Theorem 2 we assume that a strong Normal-backdoor of size at most $k$ is given when solving the problems Strong Normal-Backdoor-Brave-Reasoning and Skeptical-Reasoning. As a direct consequence of Theorem 3, this assumption can be dropped, and we obtain the following corollary.

Corollary 2. The results of Theorem 2 and Corollary 1 still hold if the backdoor is not given as part of the input.

## 6 Backdoors to Tightness

We associate with each program $P$ its positive dependency graph $D_{P}^{+}$. It has the atoms of $P$ as vertices and a directed edge $(x, y)$ between any two atoms $x, y \in \operatorname{at}(P)$ for which there is a rule $r \in P$ with $x \in H(r)$ and $y \in B^{+}(r)$. A program is called tight if $D_{P}^{+}$is acyclic [Lee and Lifschitz, 2003]. We denote the class of all tight programs by Tight.

It is well known that the main Asp reasoning problems are in NP and co-NP for tight programs; in fact, a reduction to SAT based on the concept of loop formulas has been proposed by Lin and Zhao [2004]. This was then generalized by Lee and Lifschitz [2003] with a reduction that takes as input a disjunctive normal program $P$ together with the set $S$ of all directed cycles in the positive dependency graph of $P$, and produces a CNF formula $F$ such that answer sets of $P$ correspond to the satisfying assignments of $F$. This provides an fpt-reduction from the problems Brave Reasoning and Skeptical Reasoning to Sat, when parameterized by the number of all cycles in the positive dependency graph of a given program $P$, assuming that these cycles are given as part of the input.

The number of cycles does not seem to be a very practical parameter, as this number can quickly become very large even for very simple programs. Lifschitz and Razborov [2006] have shown that already for normal programs an exponential blowup may occur, since the number of cycles in a normal program can be arbitrarily large. Hence, it would be interesting to generalize the result of Lee and Lifschitz [2003] to a more powerful parameter. In fact, the size $k$ of a deletion Tight-backdoor would be a candidate for such a parameter, as it is easy to see, it is at most as large as the number of cycles, but can be exponentially smaller. This is a direct consequence of the following two observations: (i) If a program $P$ has exactly $k$ cycles in $D_{P}^{+}$, we can construct a deletion Tight-backdoor $X$ of $P$ by taking one element from each cycle into $X$. (ii) If a program $P$ has a deletion Tight-backdoor of size 1, it can have arbitrarily many cycles that run through the atom in the backdoor.

In the following, we show that this parameter $k$ is of little use, as the reasoning problems already reach their full complexity for programs with a deletion Tight-backdoor of size 1.

Theorem 4. The problems Tight-Backdoor-Brave-Reasoning and Tight-Backdoor-Skeptical-REASOning are $\Sigma_{2}^{P}$-hard and $\Pi_{2}^{P}$-hard, respectively, even for programs that admit a strong Tight-backdoor of size 1, and the backdoor is provided with the input. The problems remain hard when we consider a deletion Tight-backdoor instead of a strong Tight-backdoor.

Proof. Consider the reduction from Eiter and Gottlob [Eiter and Gottlob, 1995] which reduces the $\Sigma_{2}^{P}$-hard problem $\exists \forall$-QBF Model Checking to the problem Consistency (which decides whether given a program $P$ has an answer set). A $\exists \forall$ quantified boolean formula (QBF) has the form $\exists x_{1} \cdots \exists x_{n} \forall y_{1} \cdots \forall y_{m} D_{1} \vee \ldots \vee D_{r}$ where each $D_{i}=l_{i, 1} \wedge l_{i, 2} \wedge l_{i, 3}$ and $l_{i, j}$ is either an atom $x_{1}, \ldots, x_{n}, y_{1}, \ldots, y_{m}$ or its negation. Their construction yields a program $P:=\left\{x_{i} \vee v_{i} ; y_{i} \vee z_{j} ; y_{j} \leftarrow\right.$ $\left.w ; z_{j} \leftarrow w ; w \leftarrow y_{j}, z_{j} ; w \leftarrow g\left(l_{k, 1}\right), g\left(l_{k, 2}\right), g\left(l_{k, 3}\right) ; w \leftarrow \neg w\right\}$ for each $i \in\{1, \ldots, n\}, j \in\{1, \ldots, m\}$, $k \in\{1, \ldots, r\}$, and $g$ maps as follows $g\left(\neg x_{i}\right)=v_{i}, g\left(\neg y_{j}\right)=z_{j}$, and otherwise $g(l)=l$. Since $P_{w=0}=\left\{x_{i} \vee v_{i} \leftarrow ; y_{j} \vee z_{j}\right\}$ and $P_{w=1}=\left\{x_{i} \vee v_{i} ; y_{j} \vee z_{j} ; y_{j} ; z_{j} ;\right\}$ are both in Tight, the set $X=\{w\}$ is a strong Tight-backdoor of $P$ of size 1. Thus the restriction does not yield tractability. The intractability of Skeptical Reasoning follows directly by the reduction of Eiter and Gottlob [Eiter and Gottlob, 1995] from the problem Consistency. Hardness of the other problems can be observed easily. Since $P-\{w\}:=\left\{x_{i} \vee v_{i} ; y_{i} \vee z_{j} ; y_{j} ; z_{j} ; \leftarrow y_{j}, z_{j} ; \leftarrow g\left(l_{k, 1}\right), g\left(l_{k, 2}\right), g\left(l_{k, 3}\right) ;\right\}$ for each $i \in\{1, \ldots, n\}$, $j \in\{1, \ldots, m\}, k \in\{1, \ldots, r\}$ is tight, we obtain a deletion Tight-backdoor of size 1. In consequence we established the theorem.

## 7 Experiments

Although our main results are theoretical, we have performed first experiments to determine the size of smallest strong Normal-backdoors for answer set programs representing structured and random sets of instances. Our experimental results summarized in Table 1 indicate, as expected, that structured instances have smaller backdoors than random instances. As instances from ConformantPlanning

| instance set | atoms | backdoor (\%) | stdev |
| :--- | ---: | ---: | ---: |
| ConformantPlanning | 1378.21 | 0.69 | 0.39 |
| MinimalDiagnosis | 97302.5 | 14.19 | 3.19 |
| MUS | 49402.3 | 1.90 | 0.35 |
| StrategicCompanies | 2002.0 | 6.03 | 0.04 |
| Mutex | 6449.0 | 49.94 | 0.09 |
| RandomQBF | 160.1 | 49.69 | 0.00 |

Table 1: Size of smallest strong Normal-backdoor for benchmark sets, given as $\%$ of the total number of atoms by the mean over the instances. ConformantPlanning: secure planning under incomplete initial states [To et al., 2009] encodings provided by Gebser and Kaminski [2012]. MinimalDiagnosis: an application in systems biology [Gebser et al., 2008] instances provided by Calimeri et al. [2011]. MUS: problem whether a clause belongs to some minimal unsatisfiable subset [Janota and Marques-Silva, 2011] encoding provided by Gebser and Kaminski [2012]. StrategicCompanies: encoding the $\Sigma_{2}^{P}$-complete problem of producing and owning companies and strategic sets between the companies [Gebser et al., 2007b]. Mutex: equivalence test of partial implementations of circuits, provided by Maratea et al. [2008] based on QBF instances of Ayari and Basin [2000]. RandomQBF: translations of randomly generated 2-QBF instances using the method by Chen and Interian [2005] instances provided by Gebser [2007b].
have rather small backdoors our translation seems to be feasible for these instances. Furthermore, we have compared the size of a smallest strong Normal-backdoor with the size of a smallest strong Horn-backdoor [Fichte and Szeider, 2012] for selected sets. It turns out that for ConformantPlanning smallest strong Normal-backdoors are significantly smaller ( $0.7 \%$ vs. $8.8 \%$ of the total number of atoms).

## 8 Conclusion

We have shown that backdoors of small size capture structural properties of disjunctive AsP instances that yield to a reduction of problem complexity. In particular, small backdoors to normality admit an fpttranslation from Asp to SAT and thus reduce the complexity of the fundamental AsP problems from the second level of the Polynomial Hierarchy to the first level. Thus, the size of a smallest Normal-backdoor is a structural parameter that admits a fixed-parameter tractable complexity reduction without making the problem itself fixed-parameter tractable.

Our complexity barrier breaking reductions provide a new way of using fixed-parameter tractability and enlarges its applicability. In fact, our approach as exemplified above for AsP is very general and might be applicable to a wide range of other hard combinatorial problems that lie beyond NP or co-NP. We hope that our work stimulates further investigations into this direction such as the application to abduction very recently established by Pfandler et al. [2013].

Our first empirical results suggest that with an improved SAT encoding and preprocessing techniques to reduce the size of Normal-backdoors (for instance, shifting, Janhunen et al., 2007), our approach could be of practical use, at least for certain classes of instances, and hence might fit into a portfoliobased solver.

## References

[Ayari and Basin, 2000] Abdelwaheb Ayari and David Basin. Bounded model construction for monadic second-order logics. In E. Emerson and A. Sistla, editors, Computer Aided Verification, volume 1855 of Lecture Notes in Computer Science, pages 99-112. Springer Verlag, 2000.
[Ben-Eliyahu and Dechter, 1994] R. Ben-Eliyahu and R. Dechter. Propositional semantics for disjunctive logic programs. Ann. Math. Artif. Intell., 12(1):53-87, 1994.
[Bidoít and Froidevaux, 1991] Nicole Bidoít and Christine Froidevaux. Negation by default and unstratifiable logic programs. Theoret. Comput. Sci., 78(1):85-112, 1991.
[Biere et al., 2009] Armin Biere, Marijn Heule, Hans van Maaren, and Toby Walsh, editors. Handbook of Satisfiability, volume 185 of Frontiers in Artificial Intelligence and Applications. IOS Press, 2009.
[Brass and Dix, 1998] Stefan Brass and Jürgen Dix. Characterizations of the disjunctive well-founded semantics: Confluent calculi and iterated GCWA. Journal of Automated Reasoning, 20:143-165, 1998.
[Brewka et al., 2011] G. Brewka, T. Eiter, and M. Truszczyński. Answer set programming at a glance. Communications of the ACM, 54(12):92-103, 2011.
[Calimeri et al., 2011] Francesco Calimeri, Giovambattista Ianni, Francesco Ricca, Mario Alviano, Annamaria Bria, Gelsomina Catalano, Susanna Cozza, Wolfgang Faber, Onofrio Febbraro, Nicola Leone, Marco Manna, Alessandra Martello, Claudio Panetta, Simona Perri, Kristian Reale, Maria Santoro, Marco Sirianni, Giorgio Terracina, and Pierfrancesco Veltri. The third answer set programming competition: Preliminary report of the system competition track. In James Delgrande and Wolfgang Faber, editors, Logic Programming and Nonmonotonic Reasoning, volume 6645 of Lecture Notes in Computer Science, pages 388-403. Springer Verlag, 2011.
[Chen and Interian, 2005] Hubie Chen and Yannet Interian. A model for generating random quantified boolean formulas. In Leslie Pack Kaelbling and Alessandro Saffiotti, editors, Proceedings of the 19th International Joint Conference on Artificial Intelligence (IJCAI'05), volume 19, pages 66-71, Edinburgh, Scotland, August 2005. Morgan Kaufmann.
[Chen et al., 2006] J. Chen, I. Kanj, and G. Xia. Improved parameterized upper bounds for vertex cover. In Proceedings of the 31st International Symposium on Mathematical Foundations of Computer Science (MFCS'06), pages 238-249. Springer Verlag, 2006.
[Clark, 1978] Keith L. Clark. Negation as failure. Logic and Data Bases, 1:293-322, 1978.
[Dowling and Gallier, 1984] William F. Dowling and Jean H. Gallier. Linear-time algorithms for testing the satisfiability of propositional horn formulae. J. Logic Programming, 1(3):267-284, 1984.
[Downey and Fellows, 1999] Rod G. Downey and Michael R. Fellows. Parameterized Complexity. Monographs in Computer Science. Springer Verlag, New York, 1999.
[Drescher et al., 2008] Christian Drescher, Martin Gebser, Torsten Grote, Benjamin Kaufmann, Arne König, Max Ostrowski, and Torsten Schaub. Conflict-driven disjunctive answer set solving. In Gerhard Brewka and Jérôme Lang, editors, Proceedings of the 11th International Conference on Principles of Knowledge Representation and Reasoning (KR'08), pages 422-432. AAAI Press, 2008.
[Eiter and Gottlob, 1995] Thomas Eiter and Georg Gottlob. On the computational cost of disjunctive logic programming: Propositional case. Ann. Math. Artif. Intell., 15(3-4):289-323, 1995.
[Fages, 1994] Francois Fages. Consistency of Clark's completion and existence of stable models. Journal of Methods of Logic in Computer Science, 1(1):51-60, 1994.
[Fichte and Szeider, 2012] Johannes Klaus Fichte and Stefan Szeider. Backdoors to tractable answerset programming. Technical report, arXiv:1104.2788, 2012. Extended and updated version of a paper that appeared in Proceedings of the 22nd International Conference on Artificial Intelligence (IJCAI'11).
[Fichte, 2012] Johannes Fichte. The good, the bad, and the odd: Cycles in answer-set programs. In Daniel Lassiter and Marija Slavkovik, editors, New Directions in Logic, Language and Computation, volume 7415 of Lecture Notes in Computer Science, pages 78-90. Springer Verlag, 2012.
[Flum and Grohe, 2006] Jörg Flum and Martin Grohe. Parameterized Complexity Theory, volume XIV of Theoret. Comput. Sci. Springer Verlag, Berlin, 2006.
[Gaspers and Szeider, 2012] Serge Gaspers and Stefan Szeider. Backdoors to satisfaction. In Hans Bodlaender, Rod Downey, Fedor Fomin, and Dániel Marx, editors, The Multivariate Algorithmic Revolution and Beyond, volume 7370 of Lecture Notes in Computer Science, pages 287-317. Springer Verlag, 2012.
[Gebser and Kaminski, 2012] Martin Gebser and Roland Kaminski. Personal communication, 2012.
[Gebser et al., 2007a] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub. Conflict-driven answer set solving. In Manuela M. Veloso, editor, Proceedings of the 20th International Joint Conference on Artificial Intelligence (IJCAI'07), pages 386-392, Hyderabad, India, January 2007.
[Gebser et al., 2007b] Martin Gebser, Lengning Liu, Gayathri Namasivayam, André Neumann, Torsten Schaub, and Mirosław Truszczyński. The first answer set programming system competition. In Chitta Baral, Gerhard Brewka, and John Schlipf, editors, Proceedings of the 9th Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR'07), volume 4483 of Lecture Notes in Computer Science, pages 3-17. Springer Verlag, 2007.
[Gebser et al., 2008] Martin Gebser, Torsten Schaub, Sven Thiele, Björn Usadel, and Philippe Veber. Detecting inconsistencies in large biological networks with answer set programming. In Maria Garcia de la Banda and Enrico Pontelli, editors, Logic Programming, volume 5366 of Lecture Notes in Computer Science, pages 130-144. Springer Verlag, 2008.
[Gelfond and Lifschitz, 1988] Michael Gelfond and Vladimir Lifschitz. The stable model semantics for logic programming. In Robert A. Kowalski and Kenneth A. Bowen, editors, Proceedings of the 5th International Conference and Symposium (ICLP/SLP'88), volume 2, pages 1070-1080. MIT Press, 1988.
[Gelfond and Lifschitz, 1991] Michael Gelfond and Vladimir Lifschitz. Classical negation in logic programs and disjunctive databases. New Generation Comput., 9(3/4):365-386, 1991.
[Giunchiglia et al., 2006] E. Giunchiglia, Y. Lierler, and M. Maratea. Answer set programming based on propositional satisfiability. Journal of Automated Reasoning, 36(4):345-377, 2006.
[Gomes et al., 2008] Carla P. Gomes, Henry Kautz, Ashish Sabharwal, and Bart Selman. Chapter 2 satisfiability solvers. In Vladimir Lifschitz Frank van Harmelen and Bruce Porter, editors, Handbook of Knowledge Representation, volume 3 of Foundations of Artificial Intelligence, pages $89-134$. Elsevier Science Publishers, North-Holland, 2008.
[Gottlob and Szeider, 2008] G. Gottlob and S. Szeider. Fixed-parameter algorithms for artificial intelligence, constraint satisfaction and database problems. The Computer Journal, 51(3):303-325, 2008.
[Janhunen et al., 2006] T. Janhunen, I. Niemelä, D. Seipel, P. Simons, and J.H. You. Unfolding partiality and disjunctions in stable model semantics. ACM Trans. Comput. Log., 7(1):1-37, 2006.
[Janhunen et al., 2007] Tomi Janhunen, Emilia Oikarinen, Hans Tompits, and Stefan Woltran. Modularity aspects of disjunctive stable models. In Chitta Baral, Gerhard Brewka, and John S. Schlipf, editors, Proceedings of the 9th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR'07), volume 4483 of Lecture Notes in Computer Science, pages 175-187, Berlin, Heidelberg, 2007. Springer-Verlag.
[Janhunen et al., 2009] Tomi Janhunen, Ilkka Niemela, and Mark Sevalnev. Computing stable models via reductions to difference logic. In Esra Erdem, Fangzhen Lin, and Torsten Schaub, editors, Proceedings of the 10th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR '09), volume 5753 of Lecture Notes in Computer Science, pages 142-154. Springer Verlag, 2009.
[Janhunen, 2004] Tomi Janhunen. Representing normal programs with clauses. In Ramon López de Mántaras and Ramon Saitta, editors, Proceedings of the 16th Eureopean Conference on Artificial Intelligence (ECAI'04), volume 16, pages 358-362. IOS Press, 2004.
[Janhunen, 2006] Tomi Janhunen. Some (in)translatability results for normal logic programs and propositional theories. Journal of Applied Non-Classical Logics, 16(1-2):35-86, 2006.
[Janota and Marques-Silva, 2011] Mikoláš Janota and Joao Marques-Silva. A tool for circumscriptionbased mus membership testing. In James Delgrande and Wolfgang Faber, editors, Logic Programming and Nonmonotonic Reasoning, volume 6645 of Lecture Notes in Computer Science, pages 266-271. Springer Verlag, 2011.
[Kleine Büning and Lettman, 1999] Hans Kleine Büning and Theodor Lettman. Propositional logic: deduction and algorithms. Cambridge University Press, Cambridge, 1999.
[Lee and Lifschitz, 2003] Joohyung Lee and Vladimir Lifschitz. Loop formulas for disjunctive logic programs. In Catuscia Palamidessi, editor, Logic Programming, volume 2916 of Lecture Notes in Computer Science, pages 451-465. Springer Verlag, 2003.
[Lee, 2005] J. Lee. A model-theoretic counterpart of loop formulas. In Leslie Pack Kaelbling and Alessandro Saffiotti, editors, Proceedings of the 19th International Joint Conference on Artificial Intelligence (IJCAI’05), volume 19, pages 503-508. Professional Book Center, 2005.
[Lifschitz and Razborov, 2006] V. Lifschitz and A. Razborov. Why are there so many loop formulas? ACM Transactions on Computational Logic (TOCL), 7(2):261-268, 2006.
[Lin and Zhao, 2004] F. Lin and Y. Zhao. ASSAT: Computing answer sets of a logic program by SAT solvers. Artificial Intelligence, 157(1-2):115-137, 2004.
[Maratea et al., 2008] Marco Maratea, Francesco Ricca, Wolfgang Faber, and Nicola Leone. Look-back techniques and heuristics in dlv: Implementation, evaluation, and comparison to qbf solvers. Journal of Algorithms, 63(1-3):70-89, 2008.
[Marek and Truszczynski, 1991a] Wiktor Marek and M. Truszczynski. Computing intersection of autoepistemic expansions. In Proceedings of the 1st International Conference on Logic Programming and Nonmonotonic Reassoning (LPNMR'91), pages 37-50. MIT Press, 1991.
[Marek and Truszczyński, 1991b] Wiktor Marek and Mirosław Truszczyński. Autoepistemic logic. J. ACM, 38(3):588-619, 1991.
[Marek and Truszczynski, 1999] Victor W. Marek and Miroslaw Truszczynski. Stable models and an alternative logic programming paradigm. In Krzysztof R. Apt, Victor W. Marek, Miroslaw Truszczynski, and David S. Warren, editors, The Logic Programming Paradigm: a 25-Year Perspective, pages 375-398. Springer Verlag, September 1999.
[Niedermeier, 2006] Rolf Niedermeier. Invitation to Fixed-Parameter Algorithms. Oxford Lecture Series in Mathematics and its Applications. Oxford University Press, 2006.
[Niemelä, 1999] Ilkka Niemelä. Logic programs with stable model semantics as a constraint programming paradigm. Ann. Math. Artif. Intell., 25(3):241-273, 1999.
[Nishimura et al., 2004] Naomi Nishimura, Prabhakar Ragde, and Stefan Szeider. Detecting backdoor sets with respect to Horn and binary clauses. In Holger H. Hoos and David G. Mitchell, editors, Proceedings of the 17th International Conference on Theory and Applications of Satisfiability Testing (SAT'04), volume 3542 of Lecture Notes in Computer Science, pages 96-103, Vancouver, BC, Canada, May 2004. Springer Verlag.
[Pfandler et al., 2013] A. Pfandler, S. Rümmele, and S. Szeider. Backdoors to abduction. In Proceedings of the 23nd International Joint Conference on Artificial Intelligence (IJCAI'13). AAAI Press/IJCAI, 2013. To appear.
[To et al., 2009] S.T. To, E. Pontelli, and T.C. Son. A conformant planner with explicit disjunctive representation of belief states. In Alfonso Gerevini, Adele E. Howe, Amedeo Cesta, and Ioannis Refanidis, editors, Proceedings of the 19th International Conference on Automated Planning and Scheduling (ICAPS'09), pages 305-312, Thessaloniki, Greece, September 2009. AAAI Press.
[Tseitin, 1968] G. S. Tseitin. On the complexity of derivation in propositional calculus. Zap. Nauchn. Sem. Leningrad Otd. Mat. Inst. Akad. Nauk SSSR, 8:23-41, 1968. Russian. English translation in J. Siekmann and G. Wrightson (eds.) Automation of Reasoning. Classical Papers on Computer Science 1967-1970, Springer Verlag, 466-483, 1983.
[Van Emden and Kowalski, 1976] M. H. Van Emden and R. A. Kowalski. The semantics of predicate logic as a programming language. J. ACM, 23:733-742, October 1976.
[Williams et al., 2003] Ryan Williams, Carla Gomes, and Bart Selman. Backdoors to typical case complexity. In Georg Gottlob and Toby Walsh, editors, Proceedings of the 18th International Joint Conference on Artificial Intelligence (IJCAI'03), pages 1173-1178, Acapulco, Mexico, August 2003. Morgan Kaufmann.


[^0]:    *Research supported by the ERC, Grant COMPLEX REASON 239962.
    ${ }^{\dagger}$ This is the author's self-archived copy including detailed proofs. A preliminary version of the paper was presented on the workshop ASPOCP'12.

