Decoupled uplink/downlink user association in HetNets: A matching with contracts approach

Decoupled Uplink/Downlink User Association in HetNets: A Matching with Contracts Approach

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ABSTRACT

In light of the prevalent trend towards dense HetNets, the conventional coupled user association, where mobile device uses the same base station (BS) for both uplink and downlink traffic, is being questioned and the alternative and more general downlink/uplink decoupling paradigm is emerging. We focus on designing an effective user association mechanism for HetNets with downlink/uplink decoupling, which has started to receive more attention. We use a combination of matching theory and stochastic geometry. We model the problem as a matching with contracts game by drawing an analogy with the hospital-doctor matching problem. In our model, we use stochastic geometry to derive a closed-form expression for matching utility function. Our model captures different objectives between users in the uplink/downlink directions and also from the perspective of BSs. Based on this game model, we present a matching algorithm for decoupled uplink/downlink user association that results in a stable allocation. Simulation results demonstrate that our approach provides close-to-optimal performance, and significant gains over alternative approaches for user association in the decoupled context as well as the traditional coupled user association; these gains are a result of the holistic nature of our approach that accounts for the additional cost associated with decoupling and inter-dependence between uplink and downlink associations. Our work is also the first in the wireless communications domain to employ matching with contracts approach.

Keywords

HetNets; Downlink and uplink decoupling (DUDe); cell association; stochastic geometry

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CCS Concepts

•Networks → Mobile networks; •Theory of computation → Network games;

1. INTRODUCTION

Mobile data traffic has been dramatically growing in the past several years and this growth trend is expected to continue into the foreseeable future. It is widely agreed that shifting to a multi-tier heterogeneous cellular network (HetNet) architecture, with dense deployments of low-cost small cells overlaid within the coverage area of a macro cell, is a cost-effective way to cope with the growing traffic demand. Unlike the traditional homogenous architecture with macro cells all using similar transmit power, dense HetNets feature base stations (BSs) with widely different transmit powers and deployment topologies. This has implications on several aspects including interference management and resource allocation but of particular relevance to this paper is the issue of *user association* that concerns which BS a mobile device (UE) associates in the uplink (UL) and downlink (DL) directions.

The conventional approach to user association is to have a UE associate with the same BS in both directions and this is sometimes referred to as coupled user association, typically based on maximizing downlink SINR. In light of the shift towards dense HetNets, the limitations of coupled user association are coming to the fore. There is now an emerging body of work that argues in favor of departing from the convention and instead adopting the more general *downlink/uplink decoupling model for user association* [3, 9, 18] in which a UE could be associated with different BSs in the uplink and downlink directions. In [4], authors discuss the need for uplink/downlink decoupling in the context of device-centric architectures for 5G. Boccardi et al. [3] quantitatively show gains of such decoupling in dense HetNets in terms of several aspects, including: increased uplink SNR and data rate, different and better load balancing in the uplink and downlink, and allowing more device-to-device (D2D) transmissions that will share uplink bands as per 3GPP Rel. 12.

In this paper, we focus on the user association problem in HetNets with uplink/downlink decoupling. The user association problem in HetNets is considered more challenging due to the conflicting objectives of different entities in the

system and applying classical user association schemes result in load imbalances and inefficient operation. Moving to the decoupled uplink/downlink setting, the problem of user association gets even more challenging because in addition to reconciling competing objectives from different angles (uplink, downlink, macro BSs, small-cell BSs), both uplink and downlink association should be *jointly* tackled and cost associated with decoupling needs to be taken into account. While (coupled) user association in HetNets has received a fair amount of attention recently and there are several simulation and analytical studies investigating the decoupling benefits (e.g., [3, 9, 18]), very little work exists on the design of user association mechanisms suitable for the decoupled context [10, 16, 17] and these works fails to capture the inter-dependent nature of uplink/downlink associations and the impact of decoupling related cost, latter dependent on the bandwidth of the backhaul link connecting the two BSs involved in a decoupled user association.

To address the decoupled uplink/downlink user association problem in HetNets, we propose a novel mechanism based on matching theory and stochastic geometry. Matching theory is particularly proved to accommodate heterogeneity of system entities and their objectives to obtain stable and optimal algorithms that can be implemented in distributed (self-organizing) manner. Specifically we formulate the decoupled uplink/downlink user association problem as a *matching with contracts* game. We draw an analogy between the user association problem in the decoupled context with the doctor-hospital matching problem that exemplifies a matching with contracts game. In the doctor-hospital matching problem [11], there are a set of hospitals which seek to hire doctors by handing them contracts that respect ranked preferences of hospitals and doctors. To map this problem to our setting, BSs play the role of hospitals and UEs are the doctors, and there can be two types of contracts between the two sets: UL and DL association.

In our model, users have different objectives determining their BS preferences in the uplink and downlink directions: based on *long-term* throughput in the UL and DL directions; long-term throughput is used instead of instantaneous throughput to limit the need to frequently redo associations (by computing a new matching) in response to time-varying channel conditions. We use stochastic geometry to model this long-term throughput[1] as the average ergodic rate of a typical user and its associated BS. Users, then, use this model to rank their BS preferences in both uplink and downlink directions. From the perspective of BSs, the preferences capture the need to offload the traffic from the macro-cell BS to small-cell BSs. We present a matching algorithm that considers these diverse objectives and the decoupling related cost; this algorithm results in a stable allocation and we also empirically demonstrate its convergence. We show the effectiveness of our proposed mechanism based on matching with contracts approach via simulations in comparison with traditional coupled user association approach, conventional (one-to-many) matching based decoupled user association mechanism proposed in [17] and another recently proposed uplink oriented user association mechanism for decoupled context from [10]. The gains from our mechanism stem from its holistic nature, accounting for the additional cost incurred by decoupling while choosing the specific BSs to associate in the uplink and downlink directions. Our solution is also shown to result in performance that is a close to centralized and computationally expensive optimal solution representing the approach taken in [16]. It is also important to note that this is the first work that proposes using matching with contracts game in the wireless communication context.

[1]Henceforth, the terms throughput and rate are used interchangeably.

1.1 Our Contribution

The contributions of the paper can be summarized as follows:

- Using tools from matching theory, we formulate the decoupled uplink/downlink user association problem as a *matching with contracts* game.
- We use stochastic geometry to derive a closed-form expression for matching utility function in both uplink and downlink directions. In addition, we formulate the decoupled user association problem as an optimization problem to benchmark our solution.
- We model the decoupling cost as a function of backhaul links between base stations to tackle the inter-dependent nature of uplink/downlink association in decoupling scenario. This highlights the advantages of our approach which jointly optimize both uplink and downlink association in order to maximize the overall mean rate for the UE.
- Via simulation results, we show the superiority of our solution over the existing works in literature and the feasibility of proposed solution in both fast and slow fading environment.

1.2 Related Work

Fair amount of research attention has been drawn towards the resource allocation/cell association problem in HetNets. Most of these works focus on coupled scenario either in downlink direction [1, 2, 8, 13] using different techniques such as game theory [1, 13], markov decision processes [8] and stochastic geometry [2], or in uplink direction and joint downlink and uplink association [5, 15]. [15] tackles the uplink user association problem using matching theory, where authors have used college admission framework (so called, one-to-many matching) to model the mobile users as students and the BSs as colleges. However, this work does not consider the decoupled uplink/downlink user association, the focus of our work. In [17], authors extend the work in [15] to decoupled user association by applying matching algorithm similar to that in [15] separately for uplink and downlink associations, but it fails to consider the inter-dependency between uplink and downlink associations, and it also does not account the decoupling related cost. In [10],

authors tackled the decoupled user association based on load and backhaul capacity, however, this work only focuses on the uplink association and does not consider the downlink perspective and BSs' objectives. Another recent work [16] proposed a centralized approach to solve the decoupled user association problem with expensive solution in terms of communication overhead where each BS needs to continually send some statistics of system parameters to a centralized controller which decides the optimal association. In contrast to these related works, our proposed solution captures the inter-dependence between uplink and downlink associations and the impact of decoupling related cost. Moreover we propose a fully distributed solution by formulating this user association problem in decoupled context as a matching with contracts game and using stochastic geometry for modeling utility functions.

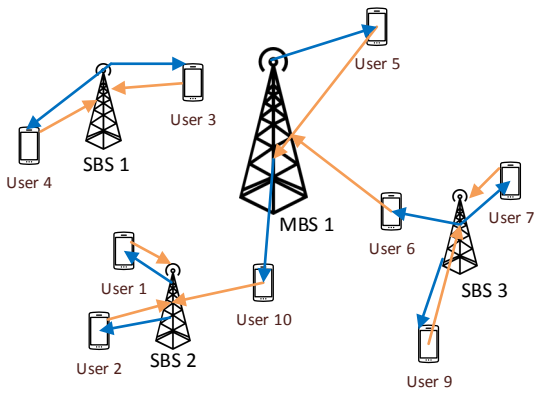

Figure 1: Illustration of decoupled uplink/downlink user association in a HetNet.

2. SYSTEM MODEL

2.1 Network Model

We consider a multi-tier HetNet including set of BSs $B = S \cup m$ where we denote by small-cell BSs (SBSs) $S = \{1,\ldots,|S|\}$, one macro-cell BS (MBS) m, and set of users $N = \{1,\ldots,|N|\}$ which are deployed and seek to transmit in both uplink and downlink directions. We consider that SBSs, and users are arranged in space following homogeneous Poisson point process (PPP) Φ of intensity λ_S, λ_N in the Euclidean plane, respectively. SBSs are overlaid on the MBS area to increase coverage and improve the performance of users. Each SBS j has a maximum *quota* q_j which is the maximum number of users that it can serve. We also assume that there is no intra-cell interference between users within a same cell as they can be assigned non-interfering set of resource blocks; however, the cell-edge users could suffer from inter-cell interference. We assume Rayleigh fading channel model. Transmit power is denoted by P_i where i can be either users, MBS or SBSs, i.e. $i \in \{m,S,N\}$. In this case, the received power at a typical user i in DL (or BS i in UL) at distance $d_{i,j}$ from BS j in DL (or user j in UL) is $P_j g_{i,j} d_{i,j}^{-\alpha}$, where $g_{i,j}$ is a random variable that follows an exponential distribution, and α is path loss exponent.

2.2 User Association: Decoupled Scenario

From DL perspective, in *traditional coupled user association*, the common criterion to drive user-BS association is the max downlink SINR [6], where each user by default shares the same BS both in UL and in the DL, which makes the association scheme unfair and inefficient for UL traffic in a dense HetNet architecture with BSs widely varying in terms of their transmit powers. Therefore, in order to achieve better network operation, it is potentially beneficial to decouple the UL and DL associations.

An illustrative scenario is shown in Figure 1 where the network consists of one MBS, 3 SBSs, and 10 users. The main purpose of SBSs is to offload traffic from MBS and would provide better performance to users. As a result, the main objective for each user is to associate with the BS enhancing its performance. The concept of traffic offloading is very clear from Figure 1. Instead of having 10 users associated with the MBS, only two users are connected to the MBS and the remaining are connected to the other 3 SBSs. Figure 1 also shows that every user attempts to maximize its own utility by connecting to the best BS for the UL and DL directions. For example, user 10 prefers to connect to the MBS in DL and SBS 2 in UL. In uplink association, the users use the fractional path loss compensation power control mechanism for UL power control which depends on the path loss model. Consequently, the users prefer to associate with BS with respect to the path-loss, which will allow the users to reduce their transmission power and the interference on the BS in turn. In other words, the users prefer to associate with the nearest BS in the UL direction in order to reduce its transmission power as well as the interference level at the BS. Thus, this makes the boundaries of BS to be different in UL and DL.

Based on the above network model, we seek to address the following question: *What is the best user-BS association in UL and DL directions?* This is non-trivial because UL and DL associations are inter-dependent and decoupling related cost also needs to be accounted.

2.3 Decoupling Cost

From the foregoing discussion and recent literature on DL/UL decoupling (DUDe), decoupled user association is seen to offer advantages over the coupled association in term of UL user throughput and UL/DL traffic load balancing. Dual connectivity is considered as an easier to realize practical route for deploying decoupled association where the user can connect to one BS in DL (e.g., max-SINR) and another BS in UL (e.g., based on path-loss). In such a decoupled user association deployment, there are two type of BSs, called, master-BS (M-BS) and secondary-BS (S-BS). Based on 3GPP specification, there are two type of architecture that agreed by 3GPP to support dual connectivity, named, *1A* and *3C alternatives*. 1A considers no bearer split. In this architecture, the core network deals with UL/DL BSs as two

disjoint cells where no coordination is needed. Due to this core network separation, S-BS mobility is visible to core network which significantly increases the signalling overhead. In addition, utilization of radio resources for the same bearer across two BSs is not possible. Furthermore, security impact due to the fact that ciphering will be required for both UL/DL BSs. The alternative 3C architecture that assumes bearer split (at the BS chosen for user association in the DL direction) does not share these limitations. In 3C architecture, only M-BS is visible to the core network which makes the mobility at S-BSs to be hidden from core network which limits the signalling overhead problem in 1A architecture. However, a coordination and flow control is needed at M-BS to forward traffic to S-BS.

In case of bearer split (i.e. 3C architecture), the key question is how much data should be forwarded between S-BS and M-BS. The nature of link between two BSs is the main factor to consider. The limitation of backhaul link capacity (between S-BS and M-BS) compared to access link would cause buffer overflow at S-BS, high packet loss and performance degradation. On the other hand, if the backhaul link capacity is over- provisioned and the limitation is on access link side, then S-BS would not forward enough data to the M-BS, and the S-BS buffer may often run out of data, thus limiting the user performance. Therefore, the decoupling scenario has a cost function mainly in UL direction due to bearer split architecture.

Considering the dual connectivity (3C) architecture and given the fact that uplink association in a HetNet context with DUDe would likely be via a nearby small cell, S-BS would correspond to the UL direction. Bandwidth of the backhaul link (X2 interface) between M-BS and S-BS could therefore be a limiting factor and affect the UL throughput in a decoupled user association so we account for this backhaul communication effect as a decoupling cost.

The decoupling cost is modelled as follows: let us assume that BSs reserve a queue for each user associated with it. We denote by τ_{slot} as the duration of a time-slot. Let $\Pi^R_{i,j}(t)$ be the number of received bits from user i to BS j in time-slot t:

$$\Pi^R_{i,j}(t) = \tau_{\text{slot}} w_{i,j} \log_2\left(1 + \frac{P_i g_{i,j}(t) d_{i,j}^{-\alpha}}{I + w_{i,j} N_0}\right),$$

where $w_{i,j}$ is the access link bandwidth between user i and BS j, and N_0 is noise power spectral density. Similarly, $\Pi^O_{j,k}(t)$ is the number of output bits from BS j to BS k at time-slot t:

$$\Pi^O_{j,k}(t) = \tau_{\text{slot}} w_{j,k} \log_2\left(1 + \frac{P_j g_{j,k}(t) d_{j,k}^{-\alpha}}{w_{j,k} N_0}\right),$$

where $w_{j,k}$ is the backhaul link bandwidth between BSs j and k which we assume as a wireless link. Ergodic mean of received and output bits can be calculated as:

$$\mathbb{E}[\Pi^R_{i,j}] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \Pi^R_{i,j}(t),\ \mathbb{E}[\Pi^O_{j,k}] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \Pi^O_{j,k}(t)$$

and we define the ratio of mean of output bits from BS j to k and mean of all received bits at BS j as:

$$r_{j,k} = \frac{\mathbb{E}[\Pi^O_{j,k}]}{\sum_{i\in N} \mathbb{E}[\Pi^R_{i,j}]}. \quad (1)$$

Then, the decoupling cost is given by:

$$c_{\text{DC}}(j,k) = \begin{cases} r(j,k), & \text{if } r(j,k) \le 1 \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

This cost can essentially be viewed as capturing the mismatch between access bandwidth of cell used for UL association and bandwidth between BSs involved in the decoupled user association.

3. REFERENCE PROBLEM: DOCTOR-HOSPITAL MATCHING

The user-BS matching describes the matching of users to BSs by allowing the decoupling of UL and DL associations. We model this problem as a matching with contracts. Basically, we assume that the contract between a particular user and BS is to choose UL or DL or neither.

The many-to-many matching with contracts [14] has been introduced to tackle problems such as United Kingdom Medical Intern match [14], the market used to allocate blood from blood banks to hospitals [12], worker-firm matching problem [7]. These works model the interaction between two set of agents in which one of them has limited quota and with conflict preferences in term of contracts, it is of interest to study how the assignment can occur between both sets of agents while satisfying, as much as possible, all preferences. We believe that this model is suitable for user association in a multi-tier HetNet architecture with DUDe as it naturally captures the dependency between UL/DL associations in our scenario, accommodates diverse objectives of system entities and allows stable associations.

We draw an analogy between our user association problem and the doctor-hospital matching problem [11]. As an illustrative example scenario for matching with contracts, consider a setting with a set of 3 doctors, i.e. $\{d_1,d_2,d_3\}$ and a set of 2 hospitals, i.e. $\{h_1,h_2\}$. Contracts can specify one or two of the following terms: a doctor works in the morning (MO) shift only; works in the afternoon (AF) shift only; works in both the morning and the afternoon, a full-time (FT) shift. Suppose that the set of contracts the hospitals can offer to doctors is given by $\{\text{MO}_{11},\text{MO}_{12},\text{MO}_{21},\text{AF}_{21},\text{AF}_{32},\text{MO}_{32},\text{FT}_{22},\text{FT}_{31}\}$ where MO_{ij}, AF_{ij} and FT_{ij} denote different contracts involving doctor d_i and hospital h_j. This list of contracts reflect constraints for the matching. For example, the hospital 1 is only willing to hire doctor 3 on full-time basis and not for morning or afternoon shifts. Now consider following example that shows the preferences of doctors and hospitals as follows:

$$\text{Preferences of } d_1 : \{h_1^{\text{MO}},h_2^{\text{AF}}\} \succ_{d_1} \{h_1^{\text{MO}}\} \succ_{d_1} \{h_2^{\text{MO}}\} \succ_{d_1} \emptyset$$

$$\text{Preferences of } d_2 : \{h_1^{\text{AF}}\} \succ_{d_2} \{h_1^{\text{MO}}\} \succ_{d_2} \emptyset$$

$$\text{Preferences of } d_3 : \{h_2^{\text{AF}}\} \succ_{d_3} \{h_2^{\text{MO}}\} \succ_{d_3} \emptyset$$

Preferences of $h_1 : \{d_1^{\mathrm{MO}}, d_2^{\mathrm{MO}}\} \succ_{h_1} \{d_1^{\mathrm{MO}}, d_2^{\mathrm{AF}}\} \succ_{h_1} \{d_2^{a}\} \succ_{h_1} \emptyset$

Preferences of $h_2 : \{d_3^{\mathrm{MO}}\} \succ_{h_2} \{d_3^{\mathrm{AF}}\} \succ_{h_2} \emptyset$

From the above, we can observe that doctor 1 prefers the contract combining hospital 1 in the morning and hospital 2 in the afternoon over all other contracts. Similarly, hospital 2 prefers to hire doctor 3 for morning shift d_3^{MO} over all other possibilities. The null contract $\emptyset$ for a doctor means that the doctor remains unemployed in the doctor-hospital matching problem, while null contract for a hospital means no doctors are hired at that hospital.

4. THE GAME MODEL AND ASSOCIATION RULES

Based on the discussion in the previous section, the user-BS matching game can be defined by four components $\langle N, B, X, \succ \rangle$ where

- X is the set of *contracts* acting as possible connection between users and BSs in which each user can have either UL or DL connection or both with a BS, and
- *preference relations* $\{\succ_1, \ldots, \succ_{|N|}\}$ and $\{\succ_1, \ldots, \succ_{|B|}\}$ for users and BSs, respectively allowing them to build preferences over the available contracts. The preference relations are defined as a *complete*, *transitive*, and *reflexive* binary relations over the set of all contracts including the null contract $\emptyset$. The null contract in our setting implies that there is no association between the user and BS in question.

Note that *with this matching with contracts approach, the preference relations are over the available set of contracts* rather than building the preferences over one another as in conventional matching (e.g., one-to-many matching in [15, 17]).

4.1 Performance Objectives in Downlink and Uplink

Consider DL transmission where each user $i \in N$ chooses a BS $j \in B$ and this choice corresponds to a certain SINR. It is reasonable to assume that the performance objective for DL would be to maximize the DL rate which is a function of SINR. The SINR of a user i from its associated BS j can be expressed as:

$$\mathsf{SINR}_{i,j}^{\mathrm{DL}} = \frac{P_j g_{i,j} d_{i,j}^{-\alpha}}{\sum_{k \in B \setminus j} P_k g_{i,k} d_{i,k}^{-\alpha} + N_0} \tag{3}$$

where P_j (P_k) is equal to P_s, the transmit power of a SBS, if j (k) is a SBS; P_m, the transmit power of a MBS, otherwise.

LEMMA 4.1. *We consider the average ergodic rate of a typical user i and its associated BS j in DL direction as follows:*

Case 1: If BS j is a SBS:

$$\theta_{i,j}^{\mathrm{DL}} \triangleq \mathbb{E}\left[\ln\left(1+\mathsf{SINR}_{i,j}^{\mathrm{DL}}\right)\right]$$

$$= \int_{t>0} \frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}} \left(\frac{e^t N_0}{P_s d_{i,j}^{-\alpha}} + \frac{\lambda_S \pi d_{i,j}^2 e^t}{(e^t-1)^{1-\frac{2}{\alpha}}} \frac{4\pi/\alpha^2}{\sin(2\pi/\alpha)} \right) \cdot \exp\left(-\frac{(e^t-1)N_0}{P_s d_{i,j}^{-\alpha}} - \frac{\lambda_S \pi d_{i,j}^2}{(e^t-1)^{-\frac{2}{\alpha}}} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)} \right) t dt \tag{4}$$

Case 2: If BS j is the MBS:

$$\theta_{i,j}^{\mathrm{DL}} \triangleq \mathbb{E}\left[\ln\left(1+\mathsf{SINR}_{i,j}^{\mathrm{DL}}\right)\right]$$

$$= \int_{t>0} \left(\frac{e^t N_0}{P_m d_{i,j}^{-\alpha}} + \frac{\lambda_S \pi d_{i,j}^2 e^t}{(e^t-1)^{1-\frac{2}{\alpha}}} \left(\frac{P_s}{P_m}\right)^{2/\alpha} \frac{4\pi/\alpha^2}{\sin(2\pi/\alpha)} \right) \cdot \exp\left(-\frac{(e^t-1)N_0}{P_m d_{i,j}^{-\alpha}} - \frac{\lambda_S \pi d_{i,j}^2 e^t}{(e^t-1)^{-\frac{2}{\alpha}}} \left(\frac{P_s}{P_m}\right)^{2/\alpha} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)} \right) t dt \tag{5}$$

PROOF. See appendix. □

In UL, as in [10], we also consider the rate as the objective for the user and this also depends on SINR. The SINR for user i which is associated with BS j is given by (as in [17]):

$$\mathsf{SINR}_{i,j}^{\mathrm{UL}} = \frac{P g_{i,j} d_{i,j}^{-\alpha}}{\sum_{k \in N \setminus i} P g_{k,j} d_{k,j}^{-\alpha} + N_0} \tag{6}$$

We consider the average ergodic rate of typical user and its associated BS as follows:

$$\theta_{i,j}^{\mathrm{UL}} \triangleq \mathbb{E}\left[\ln\left(1+\mathsf{SINR}_{i,j}^{\mathrm{UL}}\right)\right]$$

$$= \int_{t>0} \left(\frac{e^t N_0}{P d_{i,j}^{-\alpha}} + \lambda_N \pi d_{i,j}^2 e^t (e^t-1)^{\frac{2}{\alpha}-1} \frac{4\pi/\alpha^2}{\sin(2\pi/\alpha)} \right) \cdot \exp\left(-\frac{(e^t-1)N_0}{P d_{i,j}^{-\alpha}} - \lambda_N \pi d_{i,j}^2 (e^t-1)^{\frac{2}{\alpha}} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)} \right) t dt \tag{7}$$

4.2 Ranking Criteria

4.2.1 The Users' Ranking Criterion

Assume that each user $i \in N$ selects a BS $j \in B$ so as to optimize its rate. For this purpose, we propose a utility function that captures the user's rate in the UL and DL directions. For a particular user $i \in N$, the utility function is defined as follows:

$$\theta_i(j,k) = \begin{cases} \theta_{i,k}^{\mathrm{DL}} + (1 - c_{\mathrm{DC}}(j,k))\theta_{i,j}^{\mathrm{UL}}, & \text{if } j \neq k \\ \theta_{i,k}^{\mathrm{DL}} + \theta_{i,j}^{\mathrm{UL}}, & \text{if } j = k \end{cases} \tag{8}$$

4.2.2 The Base Stations' Ranking Criterion

Basically, each SBS has two objectives: 1) traffic offloading from the MBS, to extend its coverage, and enhance the user's performance, which is achieved by accepting the users

from MBS; this objective is indirectly captured by the following objective; 2) to select users that can potentially experience good rate on that base station.

Therefore, in general, the benefit or utility that any BS $j \in B$ obtains by serving user $i \in N$ is given by $H_j(i) = f(\theta_{i,j},\theta_{i,m})$, where $\theta_{i,j}$ is UL or DL rate that user i can achieve if it is associated with BS j and $\theta_{i,m}$ is UL or DL rate that user i can achieve if it is associated with the MBS. We let $f(\cdot)$ to be a function increasing with respect to total rate and use the following function to define the ranking criteria of BSs in the DL and UL directions, respectively:

$$\text{for DL : } H_j^{\text{DL}}(i) = \frac{\theta_{i,j}^{\text{DL}}}{\theta_{i,m}^{\text{DL}}} \quad \text{and for UL:} \quad H_j^{\text{UL}}(i) = \frac{\theta_{i,j}^{\text{UL}}}{\theta_{i,m}^{\text{UL}}} \tag{9}$$

4.3 The Contracts

A set of contract specify a user, a BS and a connection between the user and the BS, i.e., UL or DL connection, $X \equiv N \times B \times T$, where $T = \{\text{UL},\text{DL}\}$ is considered as contract terms which are in our model UL and DL associations, respectively. The contracts can either consist of two contract elements such as the users' contracts or only one contract element such as base stations' contract (see the next subsections 4.3.1, 4.3.2). We now give some essential definitions.

DEFINITION 4.1 (**THE ALLOCATION**). A set of contract $Z \subseteq X$ is an allocation if it contains at most one contract element (UL) and one contract element (DL) for each user-BS pair. Note that the empty set is considered as an assignment. The objective of matching with contracts problem is to find the stable allocation.

DEFINITION 4.2 (**CHOSEN SET**). User i's chosen set is denoted by $C_i(X')$ where $X' \subseteq X$. Chosen set is either the null set, if no acceptable contracts are offered, or the set of most preferred contracts.

Similarly, the chosen set of a BS j *chosen set* $C_j(X^{'})$ is a subset of contracts based on the preferences of BS j. Now consider $C_N(X') = \cup_{i \in N} C_i(X')$ as a the set of contracts chosen across all users from the set of contracts X'. Hence, the remaining offers from the set of contracts is called the *rejected set* which is formalized as: $R_N(X') = X' - C_N(X')$. Similarly, the chosen and rejected sets of the BSs are formalized as: $C_B(X') = \cup_{j \in B} C_j(X')$ and $R_B(X') = X' - C_B(X')$.

DEFINITION 4.3 (**STABLE ALLOCATION**). The allocation $Y \subseteq X$ is *stable allocation* if and only if: (i) Y is individually rational, and (ii) there are no blocking contracts in Y.

An allocation is said to be *individually rational* if no user and BS deviates from the allocation.

A *blocking contract* x is a contract in which user i strictly prefers BS j with contract x to its current BS and contract, and/or the BS j strictly prefers user i with contract x over a currently allocated user and associated contract.

4.3.1 The Users' Contracts

Each user $i \in N$ can sign a contract which includes the identity of UL and DL BSs. We denote by $x = \{\text{UL}_j,\text{DL}_k\}$ a contract of user i. Think of a two BS example. Let possibilities for contract be the following: user i can sign UL contract with BS 1 and DL contract with BS 2. For example, user 1 prefers contract $\{\text{UL}_1,\text{DL}_2\}$ means that user prefers association with BS 1 in the UL direction and BS 2 in the DL direction based on the utility function in (8). For the two BS example, if we write the preferences as following: $\{\text{UL}_1,\text{DL}_2\} \succ_1 \{\text{UL}_1,\text{DL}_1\} \succ_1 \emptyset$. Then, this implies that user i prefers to transmit its UL traffic over BS 1 and receive the DL traffic from BS 2 in comparison with transmitting UL and receiving DL traffic both via BS 1. The contract $x \in X$ is acceptable for user i if $x \succ_i \emptyset$ else it would reject that contract. For any user $i \in N$, a preference relation over the set of contracts X is defined as follows: for any two contracts $x,y \in X, x \neq y$, the preference relation becomes

$$x \succeq_i y \iff \theta_i(x) \geq \theta_i(y) \tag{10}$$

where $\theta_i(x) = \theta_i(\text{UL}_j,\text{DL}_k) = \theta_i(j,k)$ as given in equation (8).

4.3.2 The Base Stations' Contracts

For each BS $j \in B$, we define two separate list of preference relations for UL and DL direction, over the set of contracts X. For example, a contract may be user i in UL direction, i.e. $x = \{\text{UL}_i\}$, or another contract may be user i' in UL direction, i.e. $x' = \{\text{UL}_{i'}\}$. For any two contracts $x,x' \in X, x \neq x'$:

$$\text{UL}: \quad x \succeq_j x' \iff H_j(x) \geq H_j(x') \tag{11}$$

Similarly for downlink, a contract may be user i in DL direction, i.e. $y = \{\text{DL}_i\}$, or another contract may be user i' in DL direction, i.e. $y' = \{\text{DL}_{i'}\}$. For any two contracts $y,y' \in X, y \neq y'$:

$$\text{DL}: \quad y \succeq_j y' \iff H_j(y) \geq H_j(y') \tag{12}$$

4.4 The Matching Algorithm

As per the solution to the user-BS matching problem posed above, we propose an algorithm, shown in Algorithm 1, which seeks to provide a stable and close-to-optimal allocation.

At the initial network state, there is no user associated with any BS. We also assume that any user can be associated with any BS in order to maximize its utility function. Therefore, we generate a set of all possible contracts that the BS can offer to the users. Besides, we only have two terms in the set of contracts, either to accept UL user's connection then the contract will be UL_j or to accept DL user's connection and the contract will be DL_j where these two kind of contracts are between the user and BS j. As a result, we will have a list $X \equiv N \times B \times T$, where $|T| = 2$. Finally, at the initial stage, we assume that users accept all available contracts. In the main phase, users start ranking their preferences over the available set of contracts according to the utility function defined in (8) and start by submitting their requests for assignment to the most preferred contracts with the corresponding BSs. At this step, the algorithm generates two set of contracts. The first set is the chosen set of contracts which contains the most preferred contracts between the users and the BSs from the users' perspective based on their utility function. The second set is the rejected set of

Algorithm 1 The proposed matching with contracts algorithm for DUDe user association.

Initialization:

(a) The network starts where no users are assigned to any BS.

(b) Let $X \equiv N \times B \times T$ denote all possible contracts.

(c) Initialize the chosen set of contracts for the users as $C_N(0) = X$ where X is all the available contracts at iteration = 0.

Main Phase:

(a) Each user builds its preference list over the available set of contracts based on the utility function as per (8).

(b) Each user chooses its most preferred set of contracts, and generates the rejected set as $R_N(\text{iteration}) = X - C_N[\text{iteration}]$.

(c) Each BS builds its preference list over the available set of contracts based on the utility function as per (9).

(d) After all users submit their requests, each BS $j \in B$ chooses user contracts as per its preference list (and limited by quota q_j for each SBS $j \in S$) while rejecting the rest of the user contracts.

(e) From the previous step, the chosen set $C_B[\text{iteration}]$ which is the complement of $R_N(\text{iteration})$ is generated, and the rejected set is $R_B(\text{iteration})$

Repeat (iteration = iteration + 1)**:**

(a) The rejected users re-apply to their next best choice, in which $C_N[\text{iteration}]$ is the complement of $R_B(\text{iteration} - 1)$.

(b) Each BS j picks the top ranked contracts considering its previous preference list, new user contracts and its quota (if SBS), and rejects the rest.

Until: $R_B(\text{iteration}) = R_B(\text{iteration} - 1)$. At this stage, no more allocation is possible and the algorithm converges.

contracts which is the complement of the chosen set, such that given $X' \subset X$, $R_N(X_N) = X' \setminus C_N(X_N)$. Note that at the initial state $X' = X$. Then, each SBS $j \in S$ receives the requests and place the top q_j requests between the user and the BS on the waiting list and rejects the rest. Note that the MBS does not have a quota limitation (i.e., as no physical constraints) so all users not associated with a SBS in the UL or DL direction are associated with the MBS. The BSs order their preferences based on different utility function given in (9). In our model, since the user's preferences are not singleton sets like in conventional matching, we consider that an accepted preference must include both UL and DL contracts. For example, for a set of contracts $\{\text{UL}_j, \text{DL}_k\}$ of user i, if only BS j accepts the UL transmission and BS k rejects the DL transmission, then this set of contracts is considered as rejected. At this step, the rejected set of contracts, named as $R_B(X_B)$, is generated from BSs' perspective. The algorithm repeats as the rejected users submit requests for assignment to their next preferred set of contracts in which the remaining set of contracts $X' = X \setminus R_B(X_B(\text{iteration} - 1))$. Again, each SBS $j \in S$ creates a new waiting list of the top ranked q_j users among the previous waiting list and the new users, and rejects the rest. This algorithm is repeated and converges once $R_B(X_B(\text{iteration})) = R_B(X_B(\text{iteration} - 1))$ which means that every user $i \in N$ is associated with some BSs based on both users' and BSs' preferences. The following lemma states that the above described matching algorithm results in a stable allocation.

LEMMA 4.2. *Let $(X_N, X_B) \subset X \times X$ is a solution to the system of equations, $X_N = X \setminus R_B(X_B)$ and $X_B = X \setminus R_N(X_N)$, then $X_N \cap X_B$ is a stable allocation and $X_N \cap X_B = C_N(X_N) = C_B(X_B)$. Conversely, for any stable collection of contracts X', there exists some pair (X_N, X_B) satisfying the above two equations such that $X' = X_N \cap X_B$.*

PROOF. As our matching algorithm is an adaptation of the doctor-offering algorithm in [11], we refer to [11] for detailed proof. □

5. OPTIMAL DECOUPLED USER ASSOCIATION

The matching based algorithm described in the last section results in feasible and stable associations, and can also be implemented in a distributed (self-organizing) manner. To benchmark our matching based solution in terms of nearness to optimality, here we formulate the optimal user association problem in the decoupled context as a mixed integer linear program. It also serves as a centralized computationally expensive but optimal alternative (representative of [16]) to other approaches considered in the evaluation, including our proposed solution. We consider maximizing total rate across DL and UL directions as the objective for the optimization problem. We define the following variables:

$$z_{i,j}^{\text{DL}} = \begin{cases} 1, & \text{user } i \text{ is associated with BS } j \text{ in DL} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

$$z_{i,j}^{\text{UL}} = \begin{cases} 1, & \text{user } i \text{ is associated with BS } j \text{ in UL} \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

The UL rate can be calculated by taking into account the decoupling in the following way:

$$z_{i,j}^{\text{DL}} z_{i,k}^{\text{UL}} (1 - c_{\text{DC}}(j,k)) \theta_{i,k}^{\text{UL}} \tag{15}$$

where note that $c_{\text{DC}}(j,j) = 0$, $\forall j \in B$. We need to define a new set of variables given by

$$y_{i,j,k} = z_{i,j}^{\text{DL}} z_{i,k}^{\text{UL}}, \quad \forall i \in N, \forall j,k \in B \tag{16}$$

Thus, the optimal total rate can be calculated as following:

$$\begin{aligned}
&\max_{z,y} \sum_{i \in N} \sum_{j \in B} \left\{ \theta_{i,j}^{\text{DL}} z_{i,j}^{\text{DL}} + \sum_{k \in B} (1 - c_{\text{DC}}(j,k)) \theta_{i,k}^{\text{UL}} y_{i,j,k} \right\} \text{ s. t.} \\
&\sum_{j \in B} z_{i,j}^{\text{DL}} = 1, \quad \forall i \in N \\
&\sum_{j \in B} z_{i,j}^{\text{UL}} = 1, \quad \forall i \in N \\
&\sum_{i \in N} z_{i,j}^{\text{DL}} \leq q_j, \quad \forall j \in S \\
&\sum_{i \in N} z_{i,j}^{\text{UL}} \leq q_j, \quad \forall j \in S \\
&z_{i,j}^{\text{DL}} + z_{i,k}^{\text{UL}} - y_{i,j,k} \leq 1, \quad \forall i \in N, \forall j,k \in B \\
&z_{i,j}^{\text{DL}} + z_{i,k}^{\text{UL}} - 2y_{i,j,k} \geq 0, \quad \forall i \in N, \forall j,k \in B
\end{aligned} \tag{17}$$

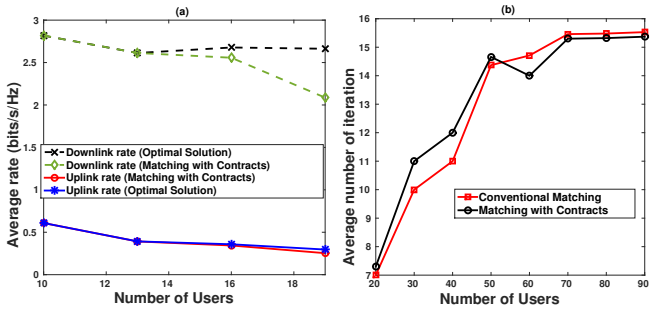

Figure 2: (a) Performance comparison with the optimal, in terms of average UL and DL rates with varying number of users; (b) Average number of iterations till convergence of matching with contracts algorithm with varying number of users.

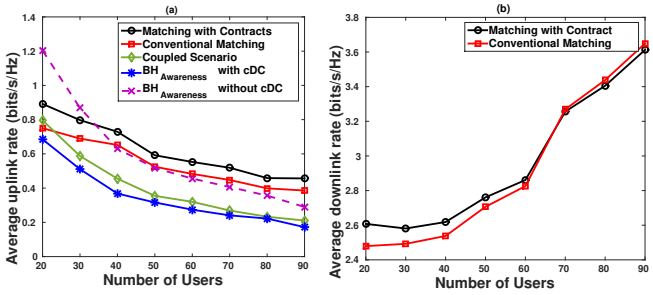

Figure 3: (a) Uplink and (b) downlink average rate results for different approaches with varying number of users.

First and second constraints ensure that a user must be associated to only one BS. Third and fourth constraints mean that at most q_j users can be associated with SBS $j \in S$. Last two constraints come from linearization of the product of two binary variables.

6. EVALUATION

We evaluate our matching with contracts approach in three stages. First, we assess the goodness of the user associations using our approach compared to the optimal solution obtained from solving the optimization problem presented in section 5 (that also represents [16] from the literature). Then, we compare our matching with contract approach with the decoupled user association based on conventional matching approach from [17], another recent work on decoupled user association [10] which we refer to as Decoupling with Backhaul awareness ($BH_{\text{Awareness}}$), and the coupled association case where each user uses the same BS in UL and DL based on classical max-SINR criterion applied in DL direction [6]. We use uplink and downlink average rates (i.e. bits/s/Hz) (corresponding to those presented earlier in section 4.1) to evaluate the performance of our algorithm. Lastly, we study the conditions where long-term matching has better performance result compared to instantaneous matching with different BS intensity (spatial density) levels and fading duration.

Concerning the first set of evaluations, Fig. 2(a) shows the results as a function of varying number of users, $|N| = 10$ to $|N| = 19$[2] and with number of SBSs $|S| = 3$, and with one MBS. We see that our matching with contracts approach performs close to optimal solution in terms of the average UL and DL rate. However, the performance of DL rate is slightly different from UL rate. This due to the fact that in matching approach, base stations play a role in association problem where it decide either to accept user association or not based on preferences list. Consider this example: user 1 prefers contract $\{UL_1, DL_2\}$ over contract $\{UL_1, DL_3\}$; on the other hand, in downlink direction BS 2 prefers contract $\{DL_3\}$ over $\{DL_1\}$, this means that BS 2 could accept user 3 association over user 1 which in turn affect the downlink performance as user 1 would associate with BS 3 which has lower DL performance compared to BS 2. Hence, this role of base stations is not reflected in optimal solution.

Fig. 2(b) shows the average number of iterations, the matching with contracts algorithm needs for convergence with increasing number of users compared to the conventional matching algorithm. We observe that the convergence time is almost the same in both cases which can be explained by the fact that both algorithms are based on Gale-Shapley deferred acceptance algorithm. In fact, we notice that as the number of users becomes large relative to the number of BSs, i.e., at $|N| \geq 50$ for $|B| = 10$, the average number of iterations becomes almost constant. This is due to the fact that SBSs have limited quota and so when number of users increases total number of users become greater than total available quota on all SBSs, the number of iteration will be almost constant as SBSs will accept the same preferred users and not require further iterations.

In the second set of evaluations, we compare our approach against alternative approaches mentioned above using simulations and with larger number of users and SBSs. For simulation parameters, we consider simulation parameters similar to those in [15], which model a macro-cell in a square area of $50m \times 50m$ with the MBS at the center, however, our approach is not limited only to these parameters. In this macro-cell area, we randomly deploy the SBSs and users. We set all users' transmit powers to $13dBm$, transmit power of SBSs to $20dBm$, MBS transmit power to $40dBm$ and the quota of the SBS is set to a typical value of $q_j = 4, \forall j \in S$. Each data point in the plots is an average of 1000 runs, over various possible locations of the SBSs and users. For approaches in [17] and [10], we run the simulation for uplink and downlink separately with the same parameters we used in matching with contracts approach and then we consider the cost of decoupling after getting the result; this is justified as neither of these approaches are designed to account for decoupling cost.

Fig. 3(a) shows the uplink rate gain in case of matching with contracts compared to conventional matching approach,

[2]results for higher number of users are not included because of very high computational times needed to compute the optimal solution.

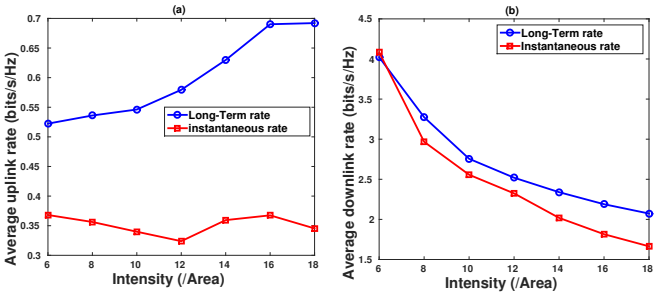

Figure 4: (a) uplink average rate, (b) downlink average rate, when base station intensity increases.

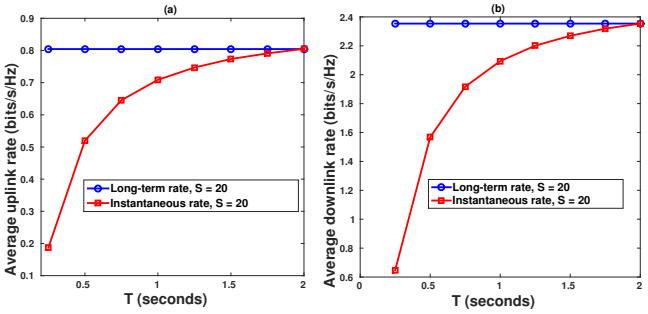

Figure 5: The effect of average fading duration on Long-term and Instantaneous matching (a) in uplink and (b) downlink directions.

$BH_{\text{Awareness}}$ approach and coupled association case. We observe that matching with contract approach has advantage over other approaches aided by its awareness of decoupling cost that helps avoid user associations with BSs with high decoupling cost (lower uplink rate). As a result, matching with contracts algorithm has the highest average uplink average rate. It improves the average uplink rate compared to the conventional matching algorithm in the case of number of users varies from $|N| = 20$ to $|N| = 90$, and number of base station equals 10 with intensity, $\lambda_S = 0.004$. When the number of users increases, the matching with contracts approach still has better performance compared to other approaches. On the other hand, the performance of downlink rate in case of matching with contracts is close to the conventional matching approach, as shown in Fig. 3(b). This is because both algorithms uses the same max-SINR which is used also in the other approaches (e.g., coupled association and decoupling with $BH_{\text{Awareness}}$). Also note that true performance of $BH_{\text{Awareness}}$ approach is after accounting for the decoupling cost which in fact makes it worse than the baseline coupled association case.

In the third set of evaluations, we studied the question of *which type of matching is more practical: Instantaneous matching or Long-term matching?* The difference between the two types of matching lies in deciding when the mobile user updates its user association (matching decision), whether instantaneously every few milliseconds or relatively infrequently (keeping the association for a longer term) say every few seconds/minutes. We study the conditions where the long-term matching could be better or worse than instantaneous matching. First, we studied the effect of BS density (intensity) on both cases. We find that when BS intensity increases the downlink average rate also decreases in both cases. This is because mobile users suffers more interference compared to lower density conditions, as in Fig. 4(b). On the other hand, both Long-term and Instantaneous matching perform differently in case of uplink average rate when the intensity increases. In Long-term matching the average uplink rate increases when the number of base station increases as mobile users start to find closer base station to associate with, however, the average uplink rate in Instantaneous matching is unaffected by base station intensity, as shown in Fig. 4(a). Beside, base station intensity, we study the effect of average fading duration T (Fig. 5). In both cases of uplink and downlink and when the number of base station equals $B = 20$, it is clear that using Long-term matching is better than Instantaneous matching as long as the average fading duration is below *2 seconds* (i.e., fast fading conditions). This is because of the cost of redoing association, in case of instantaneous matching, in response to time-varying channel conditions. In addition, in the case of slow fading conditions (> *2 seconds*), both long-term and instantaneous matching have almost similar performance. Thus, it is very clear that the *Long-term* matching that we adopt becomes more attractive compared to *Instantaneous* matching when the BS density increases and as the channel conditions vary rapidly.

7. CONCLUSION

In this paper, we have studied the problem of decoupled UL/DL user association in a HetNet context, given the diverse objectives, preferences and capabilities of the three types of entities involved: the users, the small-BSs, and macro BSs. We formulate this problem as a combination of stochastic geometry and a matching with contracts game by drawing an analogy with the doctors-to-hospitals matching problem. Our user-BS matching algorithm results in stable associations. This is also the first time matching with contracts was employed for addressing any wireless resource allocation problem. Through extensive simulation based evaluations, we show that the proposed approach outperforms existing matching based solutions for decoupled user association as well as traditional coupled user association. In addition, our solution also provides close to optimal performance.

8. APPENDIX

8.1 Proof of Lemma 4.1

Case 1: If BS j is a small BS:

$$\theta_{i,j}^{\text{DL}} \triangleq \mathbb{E}\left[\ln\left(1+\text{SINR}_{i,j}^{\text{DL}}\right)\right] = \mathbb{P}\left[\ln\left(1+\text{SINR}_{i,j}^{\text{DL}}\right) < t\right] =$$

$$1-\mathbb{P}\left[\frac{P_j g_{i,j} d_{i,j}^{-\alpha}}{\sum_{k\in S\setminus j} P_s g_{i,k} d_{i,k}^{-\alpha} + P_m g_{i,m} d_{i,m}^{-\alpha} + N_0} > e^t - 1\right]$$

$$\mathbb{P}\left[g_{i,j} - \frac{(e^t-1)P_m d_{i,m}^{-\alpha}}{P_s d_{i,j}^{-\alpha}} g_{i,m} > \frac{(e^t-1)(\sum_{k\in S\setminus j} P_s g_{i,k} d_{i,k}^{-\alpha})}{P_s d_{i,j}^{-\alpha}}\right]$$

Let $\chi = g_{i,j} - \frac{(e^t-1)P_m d_{i,m}^{-\alpha}}{P_s d_{i,j}^{-\alpha}} g_{i,m}$ where χ is a difference between two exponentially distributed random variables. Therefore, the PDF $f(\chi)$ and CDF $F(\chi)$ equals:

$$f(\chi) = \frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}} \begin{cases} e^{-\chi} & \text{if } \chi > 0 \\ e^{(e^t-1)P_m d_{i,m}^{-\alpha}} & \text{if } \chi < 0 \end{cases}$$

$$F(\chi) =$$

$$\frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}} \cdot \begin{cases} 1 + \frac{(e^t-1)P_m d_{i,m}^{-\alpha}}{P_s d_{i,j}^{-\alpha}} - e^{-\chi}, \text{ if } \chi > 0 \\ \frac{(e^t-1)P_m d_{i,m}^{-\alpha}}{P_s d_{i,j}^{-\alpha}} e^{(e^t-1)P_m d_{i,m}^{-\alpha}}, \text{ if } \chi < 0 \end{cases}$$

$$\mathbb{P}\left[g_{i,j} - \frac{(e^t-1)P_m d_{i,m}^{-\alpha}}{P_s d_{i,j}^{-\alpha}} g_{i,m} > \frac{(e^t-1)(\sum_{k\in S\setminus j} P_s g_{i,k} d_{i,k}^{-\alpha})}{P_s d_{i,j}^{-\alpha}}\right]$$
$$= \frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}} \exp\left(-\frac{(e^t-1)N_0}{P_s d_{i,j}^{-\alpha}}\right)$$
$$\cdot \exp\left(-\frac{(e^t-1)}{d_{i,j}^{-\alpha}} \sum_{k\in S\setminus j} g_{i,k} d_{i,k}^{-\alpha}\right)$$

Therefore, the PDF and CDF of the downlink direction can be found as:

$$F^{DL}(t) = 1 - \frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}} \exp\left(-\frac{(e^t-1)N_0}{P_s d_{i,j}^{-\alpha}}\right)$$
$$\cdot \exp\left(-\lambda_B \pi d_{i,j}^2 (e^t-1)^{2/\alpha} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right)$$
$$f^{DL}(t) = \frac{P_s d_{i,j}^{-\alpha}}{(e^t-1)P_m d_{i,m}^{-\alpha} + P_s d_{i,j}^{-\alpha}}$$
$$\cdot \left(\frac{e^t N_0}{P_s d_{i,j}^{-\alpha}} + \lambda_B \pi d_{i,j}^2 e^t (e^t-1)^{2/\alpha-1} \frac{4\pi/\alpha^2}{\sin(2\pi/\alpha)}\right)$$
$$\cdot \exp\left(-\frac{(e^t-1)N_0}{P_s d_{i,j}^{-\alpha}} - \lambda_B \pi d_{i,j}^2 (e^t-1)^{2/\alpha} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right)$$

Case 2: Similarly, If BS j is the macro BS, we have only on random variable $g_{i,m}$:

$$F^{DL}(t) = 1 - \exp\left(-\frac{(e^t-1)N_0}{P_m d_{i,j}^{-\alpha}}\right)$$
$$\cdot \exp\left(\lambda_B \pi d_{i,j}^2 (e^t-1)^{2/\alpha} \left(\frac{P_s}{P_m}\right)^{2/\alpha} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right)$$
$$f^{DL}(t) =$$
$$\left(\frac{e^t N_0}{P_m d_{i,j}^{-\alpha}} + \lambda_B \pi d_{i,j}^2 e^t (e^t-1)^{2/\alpha-1} \left(\frac{P_s}{P_m}\right)^{2/\alpha} \frac{4\pi/\alpha^2}{\sin(2\pi/\alpha)}\right)$$
$$\cdot \exp\left(-\frac{(e^t-1)N_0}{P_m d_{i,j}^{-\alpha}} - \lambda_B \pi d_{i,j}^2 (e^t-1)^{2/\alpha} \left(\frac{P_s}{P_m}\right)^{2/\alpha} \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right)$$

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