Detection of Symmetry and Primary Axes in Support of Proactive Design for Assembly

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1. ABSTRACT

Assembly-oriented CAD has long been accepted as a necessary development from the current component-focused solid modelling systems. It is proposed that such an environment should incorporate assembly sequence generation and Design for Assembly (DFA) analyses to assist the designer, including some automatic inference to facilitate ease of use.

The key to enabling the various assembly analyses lies in interrogation of the CAD model and this poses some interesting challenges in the field of geometric reasoning. Statistics from case studies show that the identification of symmetry and primary axes is fundamental to many of the required geometric reasoning algorithms which have been identified. In particular, the determination of the major and minor axes of each component is necessary for the definition and evaluation of manufacturing complexity, feeding, gripping and insertion trajectories. The cross-sectional properties parallel and perpendicular to these primary axes can be used to validate the feasibility of the assembly sequence and for determination of other component attributes. Detection of both exact and partial symmetries associated with these axes can provide a useful means of evaluation of practical assembly issues such as component orientation.

This paper extends the recent review by Martin and Dutta of methods for symmetry detection. However, no pre-existing method suitable for this application is found and so a new technique is proposed which exploits the existence of loops within the CAD model. This entails the comparison of loop areas to discover exact symmetry, partial symmetry and repeated features. A preliminary implementation of this technique is described and in conclusion the benefits and problems associated with it are discussed.

Keywords: DFA, Symmetry, Assembly-Oriented CAD, Geometric Reasoning, Design Methodology

2. INTRODUCTION

It has long been recognised that up to 70% of a product's cost is determined in the design stage and that much of this cost is incurred during assembly. Hence, the aim of the project giving rise to this research is to provide an environment which encourages the designer to systematically consider assembly in the early stages of product design, thereby reducing the costs and time-to-market associated with high part-count and unnecessarily complex assembly procedures.

Design for Assembly (DFA) is a methodology which, although effective in reducing part-count and simplifying the assembly process, has traditionally been a reactive tool. Hence the work described in this paper forms the basis of a 'proactive' DFA methodology, developed as part of a first generation demonstrator which combines facilities for assembly sequence construction, validation and evaluation with an ACIS-based CAD environment.

3. ASSEMBLY-ORIENTED CAD

Computer-aided design software has been in evidence in industry for many years. Most current commercially available packages still tend to concentrate upon component oriented design, where individual parts are modelled and then assembled to create the final product. This fails to afford an assembly-oriented view of the product leading to problematic assembly, rework and ultimately redesign. Assembly-oriented CAD aims to overcome these issues by providing a top-down design software environment to enable the consideration of assembly design issues such as materials and processes, including the complex interactions between parts as well as the logistics of assembly sequences.

This approach is supported by the findings of Sodhi *et al* [13] who produced a comprehensive review of the literature and established basic requirements for assembly-oriented CAD and top-down design support to facilitate the creation of a useful and appropriate software environment. It was concluded that a top-down approach is fundamental for the appropriate support of the design process.

Integrating DFA [6] with a suitable solid modelling package has been proposed as one way to provide an assembly-oriented CAD system. Sturges *et al* [14] defined a framework for automating the DFA evaluation procedure within a CAD system. Kim *et al* [4] presented their system INSPIRE-2 which integrates assembly planning, DFA and redesign. However, these proposals implement DFA on completion of the design and little consideration is given to the potential benefits which could be gained from analysing products concurrently within the design process. Enabling earlier DFA investigations could identify possible assembly and manufacturing problems as the design progresses.

However, the DFA methodology in its current form is not suited to early application. The current analyses are very specific and require detailed part information, often not available until towards the end of the design process. Thus modifications must be made to the methodology to enable interactive assemblability analysis of a developing design. Since meaningful DFA results are dependent on the plausibility of the assembly sequence it is proposed that generation of the assembly sequence holds the key to enabling a proactive DFA methodology [1] and thus providing an assembly-oriented CAD environment.

4. GEOMETRIC REASONING SUPPORT

This approach generates some interesting problems associated with incomplete information, automatic inference and the timing of the different analyses. It has long been established that interrogation of the CAD model can provide much of the information required for DFA analyses [3]. In addition, knowledge engineering and the evaluation of case studies from industry have identified those heuristics which can assist in assembly sequence generation. The geometric reasoning elements for both these areas have been extracted and are shown in table 1.

Some of this information is input by the designer either implicitly or explicitly whilst the remainder must be extracted from the solid model. For instance the degrees of freedom of a component will be determined by the order of assembly operations including the defined type and timing of joining processes. However to analyse stability, the centre of gravity of the component relative to its position and orientation within the assembly must be evaluated. Most solid modellers are able to calculate this but many other required aspects of the geometry are difficult to infer. For example the determination of mating faces, which affect the degrees of freedom, requires a complex algorithm.

In particular, the definition and validation of insertion trajectories requires identification of mating faces and cross-sectional properties, particularly the maximum and minimum dimensions perpendicular to the axis of insertion. A large number of components from automotive industry case studies have been evaluated. Table 2 shows that in 45% of cases the component is inserted along its major axis defined, by the DFA methodology, according to its longest dimension (parallel to its surfaces). The remainder are inserted along a minor axis, perpendicular to the major axis, apart from 5% in which the axes cannot be defined according to the simple criteria above. 95% of components possess some degree of symmetry, of which 79% is rotational, and in 54% of cases the axis of symmetry and the major axis are coincident. Hence the importance of symmetry and major/minor axes in assembly can be seen.

The geometric reasoning requirements of DFA evaluations contain some common elements. A major part of a Manufacturing Analysis (MA) is the definition and evaluation of shape complexity which requires identification of major and minor axes and the global shape type such as rotational, prismatic or thin-walled. Analysis of component handling and feeding also requires knowledge of the major axis, axes of symmetry and features, internal and external, which corrupt the symmetry. Analysis of component gripping requires knowledge of the insertion trajectory and mating faces and is therefore also ultimately dependent on the evaluation of major axes.

Detection of both exact and partial symmetries associated with these axes can provide a useful means of evaluation of practical assembly issues such as component orientation. Indeed one of the major recommendations of the DFA methodology is that components should exhibit either exact symmetry or marked asymmetry to avoid confusion during the assembly process.

Hence the determination of mating faces, major and minor axes, axes of partial- and exact-symmetry and cross-sectional properties, have been identified as research priorities for support of DFA. They are all closely related and interdependent since if symmetry is found then the major and minor axes can be defined accordingly. If the major/minor axes are found then the search for symmetry can be confined to these axes. If the mating faces have been resolved then the definition of major/minor axes is simplified. If the major/minor axes are used to define insertion trajectories then cross-sectional properties can be found relative to these.

5. SYMMETRY DETECTION

The concept of symmetry is of major interest in many fields of research, from human psychology [Wehl] and medical imaging [2] to crystallography and atomic physics [11]. Hence methods for the detection of symmetry are many although their cross-disciplinary relevance is limited due to the domain-specific interpretation incorporated in each technique.

The detection of symmetry is an issue in many aspects of design. Besides its proven use in DFA analyses it is also useful for sectioning prototype or FEA models to reduce the size and hence the required resources and analysis times. In their work on symmetry detection for the correction of unintentional or inappropriate asymmetry, Martin and Dutta [8] provide a thorough review of the literature. These methods generally exploit pattern matching algorithms to detect exact symmetry or to find a measure of symmetry. Techniques which typify this approach are the symmetric difference method of Kulkarni *et al* [5] or the technique presented by Zabrodsky [16] which gauges the symmetry of a set of points relative to a 'symmetry transform', a symmetric set of points produced by averaging the original set. Perhaps the only method to date which tackles the detection of partial symmetry in a manner appropriate for this application is that of Parry-Barwick and Bowyer [10] which uses a multi-dimensional template matching algorithm. Unfortunately this is based on a CSG model as opposed to boundary representation and is very computationally expensive.

The relevance of symmetry has not been overlooked by the assembly modelling community. Lui and Popplestone [7] utilised symmetry group theory to express the relationships between two bodies in the field of automatic robotic assembly to solve the problem of finding all possible assembly configurations. Van Gool *et al* [15] used a more generalised concept of symmetry to solve a similar 'peg-in-hole' assembly problem.

Ong *et al* [9] devised methods for the detection of exact rotational symmetries perpendicular and parallel to the predefined insertion trajectory in a component-focused approach to assembly and DFA. These have been labelled as alpha (α) and beta (β) symmetry respectively. The method starts by locating the centre of gravity (CG) at the origin with the axis of insertion along the y-

Assembly Sequence & DFA Requirements		Current State of Assembly	Max/Min X Section	Swept X Section Props	Insertion trajectory	Degrees of Freedom	Mass	Volume	Mating Faces	Major/Minor Axis	C of G	Symmetry	Internal/External Features	Bounding Box
		Geometric Reasoning Algorithms												
Collisions	- component	✓		✓	✓				✓	✓				
	- gripping tool	✓		✓	✓									
	- joining tool			✓										
Stability	- current orientation	✓				✓			✓		✓			
	- turnover	✓				✓			✓		✓			
Compatability	- material/material								✓					
	- material/joining process								✓					
Manufacturing	- shape complexity		✓							✓		✓	✓	
	- cost analysis		✓					✓						✓
Handling/Feeding	- part characteristics		✓				✓	✓		✓		✓	✓	
	- part orientation									✓		✓	✓	
Fitting	- gripping				✓				✓	✓				
	- fitting process			✓	✓				✓					

Geometric Reasoning Requirements of an Assembly-Oriented CAD System

Table 1

Case Study	No. of components	Rotational Symmetry				Reflective Symmetry				Thin Wall Refl. Symm.				Other
		Exact	Partial	Symm = Major Axis	Major Axis Insertion	Exact	Partial	Symm = Major Axis	Major Axis Insertion	Exact	Partial	Symm = Major Axis	Major Axis Insertion	
Gear Change Assembly	22	15	2	11	7	1	1	1		3		3		
Oil Pump 1	33	20	5	14	14	1	1	2	1	2	2	3	1	2
Oil Pump 2	12	9	2	9	9	1		1	1					
Grating Arm Assembly 1	39	17	3	15	15	11		3	1	4		2	1	4
Grating Arm Assembly 2	8	3	2	5	5	1		1	1					2
Pintle Hook Mtg Assy 1	60	40	14	18	18	1				5		3	2	
Pintle Hook Mtg Assy 2	28	20	4	8	8	1				3		3	3	
Lens Assembly	18	14	2	9	9									2
Injector Assembly	25	16	7	14	14									2
Total	245	154	41	103	99	17	2	8	4	17	2	14	7	12

Case Studies from the Automotive Industry

Table 2

axis. Then separate subroutines are called for the detection of alpha and beta symmetry. In the Find_ALPHA subroutine the maximum and minimum y dimensions of the object are compared. If these are identical then it is possible that the object is symmetric and the algorithm progresses. The component is cut at the CG, perpendicular to the y axis, and then comparisons are made of the centroids and moments of inertia of each portion in the current orientation and rotated through 180 degrees. Finally the symmetric difference is found to clarify the findings. The Find_BETA subroutine starts by comparing moments of inertia I_x and I_z. If these are equal then the vertices at distinct levels along the y-axis are examined. For each group of vertices the object is rotated about the y axis through an angle corresponding to the number of vertices. A Boolean subtract operation between the rotated object and the original determines whether the object therefore contains beta symmetry and to what degree. If I_x and I_z are not equal it may be that although object is symmetric the symmetry is not lying conveniently across the x or z axes. In this case a comparison of vertex co-ordinates can determine equivalent points and the algorithm can continue as before.

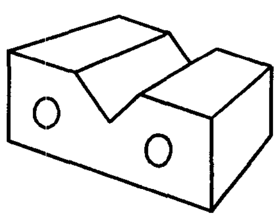
Ruixiao *et al* [12] also adopted the terminology of alpha and beta symmetry so solve the same problem. Once the axis of insertion has been defined by the user, rays are systematically passed from the centroid of the part to the outside. The numbers of times the ray is segmented by the object, the length of the segments and the classification (in or out of the object) are then used to make comparisons between portions of the objects boundary.

Unfortunately, none of these methods really satisfy the requirements described herein which are more wide ranging than pure orientation and require recognition of all elements of symmetry within the object. A novel method is proposed in this paper which aims to establish axes of symmetry, and those elements of the surface which corrupt that symmetry, in a form appropriate for use within the assembly-oriented CAD environment. The following definitions of symmetry have been adopted for the purposes of this application which coincide with definitions of shape complexity described in the DFA methodology [Lucas 93].

5.1 Definitions of Symmetry

- **Exact Reflective symmetry**

  This includes prismatic solid or thin-wall parts which exhibit reflective symmetry. A typical example is shown in Fig.1.

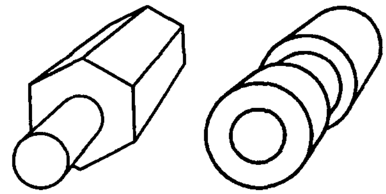


Example of Exact Reflective Symmetry

Figure 1

- **Exact Rotational Symmetry**

  This covers parts whose envelope is a solid of revolution and also those parts with *n*-fold rotational symmetry. It has been assumed throughout the course of this investigation that the threaded portion of fasteners is unlikely to be modelled and hence these have been assumed to have exact rotational symmetry. Typical examples of exact rotational symmetry are shown in Fig.2.



Examples of Exact Rotational Symmetry

Figure 2

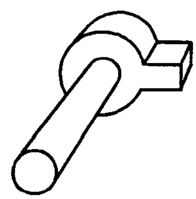
- **Partial Symmetry**

  This covers parts whereby portions of the boundary can be identified as symmetric. These will fall into three categories of part:

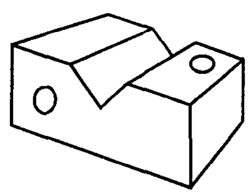
  (a) those where the part is a solid of revolution with additional features which corrupt the rotational symmetry (Fig.3a);

  (b) those where the part would have exact reflective symmetry but for the existence of additional features which corrupt that symmetry (Fig.3b);

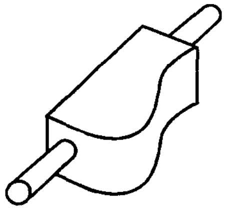
  (c) those parts whose basic overall shape is not symmetric but which contain repeated features at intervals around the boundary (Fig.3c).



(a)



(b)



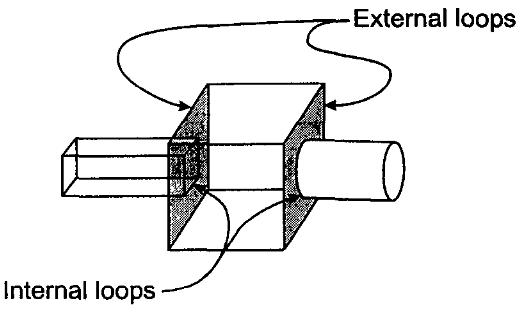
(c)

Examples of Partial Symmetry

Figure 3

5.2 Proposed Methodology

The proposed technique also involves the use of search algorithms. The general principle involves the calculation of the surface area bounded by each loop of a face, for each face belonging to the part. This enables the association of faces, whose only difference is the type of feature which they contain (Fig.4). In this way, partial symmetry and repeated features are as easy to identify as exact symmetry. The process can be broken down into 6 steps:



Shaded faces indicate partial symmetry based *on the area of surfaces bounded by external loops*

Figure 4

Step 1 - Determine the properties of each loop

Each loop of each face must be classified as either 'external', defining the overall size and type of the face, or 'internal', defining an intersecting feature.

Step 2 - Calculate loop area

The area of surface bounded by each loop is calculated for each loop of each face.

Step 3 - Rank loops

Loops are ranked according to their properties and bounded area, external loops ranking higher than internal loops.

Step 4 - Identify matching loops

Loops with matching type and area are identified and then further checks are required to validate their relationship such as face type and number of edges.

Step 5 - Construct a set of planes of symmetry

For each matching pair of loops a plane of symmetry can be constructed which bisects the centroids of the two loops.

Step 6 - Determine the primary axes

The major and minor axes can be defined by the intersection of the planes of symmetry ranked according to the number of pairs and area of the loops that they bisect.

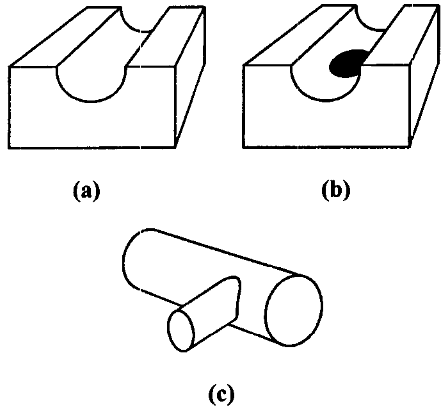
This algorithm is more complex that it would first appear. A loop is a continuous set of edges marking a boundary of a face and for planar faces there always exists an external loop, intersecting features resulting in internal loops. The same is true for all occurrences of non-intersecting B-spline surfaces. In many b-rep modellers it is a simple matter to identify the nature of each loop in this respect either through its explicit representation or use of a ready-made function. Unfortunately, the distinction is not clear for some commonly occurring face types of major importance in manufacturing and hence further interrogation is required in order to classify the loops defining each face.

For example, a cylinder can be considered as a special instance of a cone and hence the two can be treated similarly. The loops of these types of face cannot be distinguished as internal or external because the distinction is not relevant. For example, a complete cylindrical face possesses two circular loops bounding the ends. In such cases it is therefore necessary to differentiate between those 'end' loops and the 'internal' loops belonging to intersecting features in order to calculate the area of the cylindrical face without those intersecting features.

Hence it is necessary to first identify the type of face so that each face can be treated according to its particular properties. In many modellers this can be determined by examination of the surface equation. Fortunately, ACIS provides a simple function call to enable identification of particular face types; cylindrical, conical, spherical and toroidal in addition to planar and spline surfaces.

Once the type of face has been established, the loops belonging to each face can then be classified as external or internal. In order to cater for all types of face, the definition of an external loop is one such that upon its removal that face becomes unbounded. So for example when one end loop of a cylinder is removed it becomes a semi-infinite cylinder. Once the distinction has been made then it is possible to calculate the area of the surface defined only by the external loops. Subsequent calculations will determine the surface area bounded by each of the internal loops.

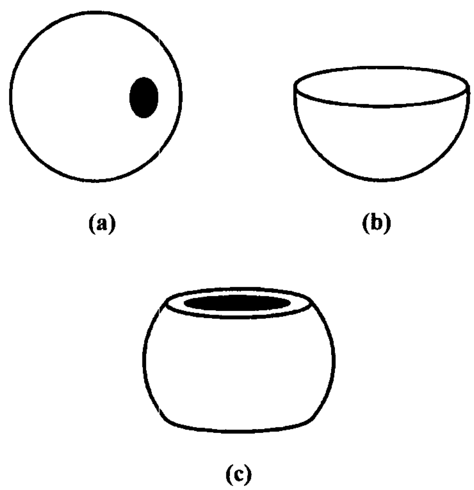
For planar faces it is a simple matter to compare the bounding box of each loop to identify the external loop. In the case of a cylindrical face several permutations can occur which necessitate a more sophisticated approach. If the face contains only one loop then it must be concluded that the face is just a portion of a cylinder or cone (Fig.5a). If the face contains two or more loops then one of two situations must exist; either the face is a portion of a cylinder with one or more intersecting features or the face is a complete cylinder with none or more intersecting features (Figs. 5b and 5c respectively). If it is found that the bounding box of one loop is identical to the bounding box of the face then it can be concluded that the former situation is true, in which case all other loops are 'internal'. Otherwise it is necessary to identify the two end loops. Various checks can be performed to differentiate between the different constructions.



Cylindrical faces with varying numbers of loops

Figure 5

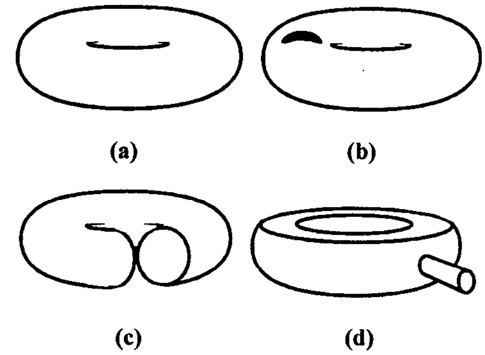
Similar reasoning can be applied in the case of a spherical face. If there are no loops associated with the face then this indicates a complete sphere with no intersecting features. If one or more loops exist then this indicates either a complete sphere with one or more intersecting features or a portion of a sphere with none or more intersecting features (Figs. 6a and 6b). In this situation the largest loop, as determined by its bounding box, can be considered as the 'external' loop and all others as 'internal'. If there are two equal loops then both should be considered as either external or internal loops (Fig.6c).



Spherical faces with varying numbers of loops

Figure 6

In the case of a toroidal face the situation is yet more complex. No loops indicates a complete toroidal face (Fig.7a). One loop indicates a complete toroidal face with one intersecting feature, i.e. an internal loop, or a portion of a torus with no intersecting features, i.e. an external loop, (Fig.7b). However, for greater numbers of loops it is very difficult to infer their characteristics (Figs. 7c and 7d).



Toroidal faces with varying numbers of loops

Figure 7

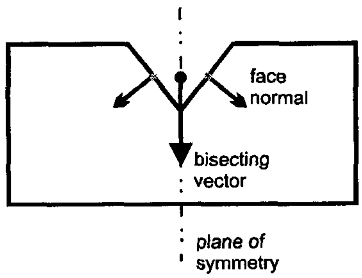
ACIS provides the functionality to discern external and internal loops for both planar and cylindrical faces but according to the definition described in this paper the function is not suitable for spherical or toroidal faces, all loops being identified as external. Hence, spherical faces can be dealt with as described above but as yet, toroidal faces are not completely dealt with.

This methodology is very much dependent on the way in which ACIS handles loops and in other modellers there may be subtle variations in the way that loops are handled. These variations, differences and problems may be exposed by pathological cases, yet to be fully enumerated. However, in the 'engineering world' these type of situations are unlikely to occur.

Once the nature of the loops in each face has been established, it is then possible to calculate the area of each portion of bounded surface. Of course it is possible to construct an algorithm to do this directly. However, using the ACIS functionality, a new face can be created with the appropriate loop/loops forming the external boundary of a portion of the same surface. It is then a matter of calculating the area of this new face using the appropriate ACIS function. Although much simpler, these extra steps will directly impinge upon the efficiency of the algorithm.

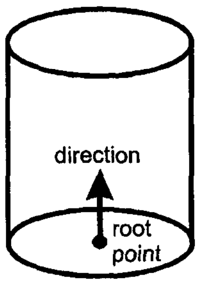
The accuracy of the area calculation has serious implications for the accuracy of results. Identical loops are identified by matching loop area and so, particularly for complex surfaces, a tolerance on the match is required. Other attributes, including face type and the number of edges in a loop, are stored for the purpose of validating matches.

As described in Step 5, each plane of symmetry will pass through a point bisecting the centroids of two matching loops. Its orientation will be defined by a vector bisecting the face normals at the centroids (Fig.8). Again, cylindrical and other types of face must be treated slightly differently. For instance, cones and cylinders are partly defined by a root point and a direction which can be used in a similar way to the centroid and face normal of a planar face (Fig.9).



Plane of symmetry defined by bisecting matching loops

Figure 8



Cylinder and Cone - Face Definition

Figure 9

This set of bisecting planes are ranked according to the area of the loops that they bisect and the number of pairs of loops that they bisect. The primary axes can then be defined by the intersection of these ranked planes. The major axis will be defined by the intersection of the plane associated with the maximum co-oriented loop area, with the plane associated with the next largest co-oriented loop area. Then the first minor axis can be defined as the intersection with the subsequent largest co-oriented loop area, perpendicular to the major axis and so on.

It was shown from the case studies in Table 2 that a large proportion of manufacturing parts have rotational symmetry but this would not necessarily be identified if the part is essentially cylindrical as there would be no 'matching' loop. Therefore the axes of cylindrical faces can be considered independently and ranked alongside the planes of symmetry according to the cylindrical face area. The major axis can then be defined according to the number and combined area of cylindrical faces relating to the same axis if this ranks highest.

This methodology is in its infancy and there are several areas where further research potential has been identified:

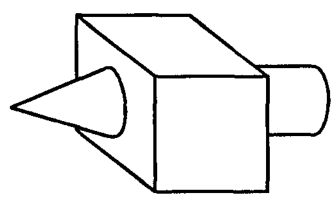
- This method involves the determination of major and minor axes associated with symmetry. The notion of the maximum co-oriented loop area is also effective for determining primary axes when no symmetry exists since by the DFA definition, the major axis is that lying parallel to the longest face.

- The method enables the identification of features as those elements of the topology associated with internal loops. The feeding analysis requires the identification of features apparent in silhouette which affect the orientation in transition. Hence the method requires further development in order to identify the nature of features in terms of protrusion and depression.

- The Lucas MA Shape Complexity Index categorises parts in one of three ways; largely a solid of revolution; largely a rectangular or cubic prism; or a flat or thin wall section component. The loop area methodology in its current form can be used to identify the general category of the part and to some degree the classification within that category. However, the classifications are based on empirical data and are subjective in nature. In adapting the MA shape complexity indices for the computer implementation a more objective and reliable system of classification can be adopted based on the number of axes associated with the component.

- For parts with *n*-fold rotational symmetry, several planes of reflective symmetry will be identified using this method. Although this will not affect the determination of the primary axes it is desirable in some instances to differentiate between the two types of symmetry. It is anticipated that by using the theory of symmetry groups [11] it will be possible to infer other forms of symmetry.

- In the majority of cases symmetrical faces will be identified by matching *external* loops. Symmetry of intersecting features, identified as *internal* loops, can be identified by the external loops of faces belonging to the features themselves. Hence, by considering only external loops the algorithm can be optimised for the majority of cases. There are exceptions and Figure 10 shows an example of such a pathological case. The bases of both the cone and the cylinder have the same diameter but this would not be detected if only external loops were considered. Only the symmetry of the cuboid would be recognised.



Pathological case if external loops only considered

Figure 10

- At the present time little consideration has been given to the robustness of the method. Further research is required in order to identify the effects of symmetry planes which are close together but not coplanar, either intentionally or due to accumulated errors.

6. CONCLUSIONS

This paper has demonstrated the need for geometric reasoning to support an assembly-oriented CAD environment and this has been supported by case studies gathered from industry. It proposes a solution to the problem of symmetry detection and determination of major and minor axes which is immune to the confusion that small changes to topology can generate.

At the time of writing this method has not been rigorously tested and it is anticipated that further refinement will be required both in those areas where problems have already been identified and others, as yet unforeseen. Despite the fact that loops are explicitly represented in the boundary representation of the solid model, this method has the disadvantages of many search algorithms which are inelegant and notoriously slow. Although the ACIS functionality has enabled a simple implementation of the technique, efficiency could be improved by a more direct approach to the calculation of loop areas which does not involve the creation of additional faces. Effectiveness could also be improved by the adoption of more efficient search algorithms.

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