# Jointly Optimal Routing and Caching for Arbitrary Network Topologies

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#### Abstract

We study a problem of fundamental importance to ICNs, namely, minimizing routing costs by jointly optimizing caching and routing decisions over an arbitrary network topology. We consider both source routing and hop-by-hop routing settings. The respective offline problems are NP-hard. Nevertheless, we show that there exist polynomial time approximation algorithms producing solutions within a constant approximation from the optimal. We also produce distributed, adaptive algorithms with the same approximation guarantees. We simulate our adaptive algorithms over a broad array of different topologies. Our algorithms reduce routing costs by several orders of magnitude compared to prior art, including algorithms optimizing caching under fixed routing.

### 1 Introduction

Optimally placing resources in a network and routing requests toward them is a problem as old as the Internet itself. It is of paramount importance in information centric networks (ICNs) [31, 57], but also naturally arises in a variety of networking applications such as web-cache design [14, 36, 58], wireless/femtocell networks [42, 45, 51], and peer-to-peer networks [16, 39], to name a few. Motivated by this problem, we study a caching network, i.e., a network of nodes augmented with additional storage capabilities. In such a network, some nodes act as designated content servers, permanently storing content and serving as "caches of last resort". Other nodes generate requests for content that are forwarded towards these designated servers. If, however, an intermediate node in the path towards a server stores the requested content, the request is satisfied early: i.e., the request ceases to be forwarded, and a content copy is sent over the reverse path towards the request's source.

This abstract setting naturally captures ICNs. Designated servers correspond to traditional web servers permanently storing content, while nodes generating requests correspond to customer-facing gateways. Intermediate, cache-enabled nodes correspond to storage-augmented routers in the Internet's backbone: such routers forward requests but, departing from traditional network-layer protocols, immediately serve requests for content they store. An extensive body of research, both theoretical [9, 14, 24, 25, 30, 40, 47, 48] and experimental [14, 31, 36, 39, 49, 58], has focused on modeling and analyzing networks of caches in which routing is fixed, and requests follow predetermined paths. For example, shortest paths to the nearest designated server are often used. Given routes to be followed, and the demand for items, the above works aim to model and analyze (theoretically or empirically) the behavior of different caching algorithms deployed over intermediate nodes.

It is not a priori clear whether fixed routing and, more specifically, routing towards the nearest server is the appropriate design choice for such networks. This is of special interest in the context of ICNs, where delegating routing decisions to another protocol amounts to an "incremental" deployment. For example, in such a deployment, requests can be forwarded towards the closest designated web servers over paths determined according to,

e.g., existing routing protocols such as OSPF or BGP [35]. Subsequent caching decisions by intermediate routers affect only where—within a given path—requests are satisfied. An alternative is to *jointly* optimize both routing and caching decisions simultaneously. Doing so however poses a significant challenge, precisely because this joint optimization is inherently combinatorial. Indeed, jointly optimizing routing and caching decisions with the objective of, e.g., minimizing routing costs, is an NP-hard problem, and constructing a distributed approximation algorithm is far from trivial [11, 23, 30, 51].

This state of affairs gives rise to the following questions. First, is it possible to design distributed, adaptive, and tractable algorithms jointly optimizing both routing and caching decisions over arbitrary cache network topologies, with provable performance guarantees? Identifying such algorithms is important precisely due to the combinatorial nature of the problem at hand. Second, presuming such algorithms exist, do they yield significant performance improvements over fixed routing protocols? Answering this question in the affirmative may justify the potential increase in protocol complexity due joint optimization. It can also inform future ICN design, indicating whether full optimization is preferable, or whether an incremental approach in which routing and caching are separate suffices.

Our goal is to provide rigorous, comprehensive answers to these two questions. We make the following contributions:

- We show, by constructing a counterexample, that fixed routing (and, in particular, routing towards the nearest server) can be *arbitrarily suboptimal* compared to jointly optimizing caching and routing decisions. Intuitively, joint optimization affects routing costs drastically because *exploiting path diversity increases caching opportunities*.
- We propose a formal mathematical framework for joint routing and caching optimization. We consider both source routing and hop-by-hop routing strategies, the two predominant classes of routing protocols over the Internet [35].
- We study the offline version of the joint routing and caching optimization problem, which is NP-hard, and construct a polynomial-time 1-1/e approximation algorithm.
- We provide a distributed, adaptive algorithm that converges to joint routing and caching strategies that are, globally, within a 1 1/e approximation ratio from the optimal.
- We evaluate our distributed algorithm over 9 synthetic and 3 real-life network topologies, and show that it significantly outperforms the state of the art: it reduces routing costs by a factor between 10 and 1000, for a broad array of competitors, including both fixed and dynamic routing protocols.

The remainder of this paper is organized as follows. We review related work in Section 2, and present our mathematical model of a caching network in Section 3. Our main results are presented in Section 4. A numerical evaluation of our algorithms over several topologies is presented in Section 5, and we conclude in Section 6.

## 2 Related Work

There are several adaptive, distributed approaches determining how to populate caches under fixed routing. A simple, elegant, and ubiquitous algorithm is path replication [16], sometimes also referred to as "leave-copy-everywhere" (LCE) [36]: once a request for an item reaches a cache, every downstream node receiving the response caches the item. Several variants of this principle exist. In "leave-copy-down" (LCD), a copy is placed only in the node immediately preceding the cache storing the requested item [36, 37], while "move-copy-down" (MCD) also removes the present upstream copy. Probabilistic variants have also been proposed [46]. To evict items, traditional eviction policies like Least Recently Used (LRU), Least Frequently Used (LFU), First In First Out (FIFO), and Random Replacement (RR) are typically used. Several works [36, 46, 49, 50, 54] have experimentally studied the performance of these protocols and variants over a broad array of topologies. Despite the advantages of simplicity and elegance inherent in path replication, when targeting an optimization objective such as, e.g., minimizing total routing costs, path replication combined with all of the above eviction and replication policies is

known to be arbitrarily suboptimal [30].

There is a vast literature on the performance of eviction policies like LRU, FIFO, LFU, etc., on a single cache, and the topic is classic [2, 18, 22, 27, 33]. Nevertheless, the study of networks of caches still poses significant challenges. A central problem is that, even if arriving traffic in an LRU cache is, e.g., Poisson, the outgoing traffic is hard to describe analytically. Rosensweig et al. [48] study conditions under which path replication with LRU, FIFO, and variants, lead to an ergodic chain. A significant breakthrough in this area has been the so-called Che approximation [14, 25], which postulates that the hit rate of an LRU cache can be well approximated under the assumption that items stay in the cache for a constant time. This approximation is quite accurate in practice [25], and its success motivated extensive research in so-called time-to-live (TTL) caches. In TTL caches, items stay in a cache for predetermined times, and are evicted when TTL timers expire. A series of recent works have focused on identifying how to set TTLs to (a) approximate the behavior of known eviction policies, (b) describe hit-rates in closed-form formulas [9, 14, 20, 24, 40]. Despite these advances, none of the above works address issues of routing cost minimization over multiple hops, which is our goal.

In their seminal paper [16] introducing path replication, Cohen and Shenker also introduced the abstract problem of finding a content placement that minimizes routing costs. The authors show that path replication combined with a constant rate of evictions leads to an allocation that is optimal, in equilibrium, when nodes are visited through uniform sampling. Unfortunately, this optimality breaks down when uniform sampling is replaced by routing over arbitrary topologies [30]. Several papers have studied complexity and optimization issues of cost minimization as an offline caching problem under restricted topologies [4, 5, 7, 11, 23, 51]. With the exception of [51], these works model the network as a bipartite graph: nodes generating requests connect directly to caches, and demands are satisfied a single hop, and do not readily generalize to arbitrary topologies. In general, the *pipage rounding* technique of Ageev and Sviridenko [3] (see also [12, 53]) yields again a constant approximation algorithm in the bipartite setting, while approximation algorithms are also known for several variants of this problem [5, 7, 11, 23]. Excluding [11], all these works focus only on centralized solutions of the offline caching problem; none considers jointly optimizing caching and routing decisions.

In earlier work [30], we consider a setting in which routes are fixed, and only caching decisions are optimized in an adaptive, distributed fashion. We extend [30] to incorporate routing decisions, both through source and hop-by-hop routing. We show that a variant of pipage rounding [3] can be used to construct a poly-time approximation algorithm, that also lends itself to a distributed, adaptive implementation. Crucially, our evaluations in Section 5 show that jointly optimizing caching and routing significantly improves performance compared to fixed routing, reducing the routing costs by as much as three orders of magnitude compared to [30].

Several recent works study caching and routing jointly, in more restrictive settings than the ones we consider here. The benefit of routing towards nearest replicas, rather than towards nearest designated servers, has been observed empirically [13, 15, 21]. Deghan et al. [19], Abedini and Shakkotai [1], and Xie et al. [56] all study joint routing and content placement schemes in a bipartite, single-hop setting. In all three cases, minimizing the single-hop routing cost reduces to solving a linear program; Naveen et al. [42] extend this to other, non-linear (but still convex) objectives of the hit rate, still under single-hop, bipartite routing constraints. None of these approaches generalize to a multi-hop setting, which leads to non-convex formulations (see Section 3.6); addressing this lack of convexity is one of our technical contributions. Closer to our work, a multi-hop, multi-path setting is formally analyzed by Carofiglio et al. [13] under the assumption that requests by different users follow non-overlapping paths. The authors show that under appropriate conditions on request arrival rates, this assumption leads to a convex optimization problem. Our approach addresses the lack of convexity in its full generality, for arbitrary topologies, request arrival rates, and overlapping paths.

The problem we study is also related to more general placement problems, including the allocation of virtual machines (VMs) to hosts in cloud computing [8, 29, 38, 52]—see also [32], that jointly optimizes placement and routing in this context. This is a harder problem: heterogeneity of host resources and VM requirements leads to multiple

knapsack-like constraints (one for each resource) per host. Our storage constraints are simpler; as a result, in contrast to [8, 29, 32, 38, 52], we can provide poly-time, distributed algorithms with provable approximation guarantees.

### 3 Model

We begin by presenting our formal model, extending [30] to account for both caching and routing decisions. Our analysis applies to two routing variants: (a) source routing and (b) hop-by-hop routing. In both cases, we study two types of strategies: deterministic and randomized. For example, in source routing, requests for an item originating from the same source may be forwarded over several possible paths, given as input. In deterministic source routing, only one is selected and used for all subsequent requests with this origin. In contrast, a randomized strategy samples a new path to follow independently with each new request. We also use similar deterministic and randomized analogues both for caching strategies as well as for hop-by-hop routing strategies.

Randomized strategies subsume deterministic ones, and are arguably more flexible and general. This begs the question: why study both? There are three reasons. First, optimizing deterministic strategies naturally relates to submodular maximization subject to matroid constraints, allowing us to leverage related combinatorial optimization techniques. Second, the online, distributed algorithms we propose to construct randomized strategies rely on the solution to the offline, deterministic problem. Finally, and most importantly: deterministic strategies turn out to be equivalent to randomized strategies! As we show in Thm. 3, the smallest routing cost attained by randomized strategies is exactly the same as the one attained by deterministic strategies.

### 3.1 Network Model and Content Requests

Consider a network represented as a directed, symmetric graph G(V, E). Content items (e.g., files, or file chunks) of equal size are to be distributed across network nodes. Each node is associated with a cache that can store a finite number of items. We denote by  $\mathcal C$  the set of possible content items, i.e., the *catalog*, and by  $c_v \in \mathbb N$  the cache capacity at node  $v \in V$ : exactly  $c_v$  content items can be stored in v. The network serves content requests routed over the graph G. A request (i,s) is determined by (a) the item  $i \in \mathcal C$  requested, and (b) the source  $s \in V$  of the request. We denote by  $\mathcal R \subseteq \mathcal C \times V$  the set of all requests. Requests of different types  $(i,s) \in \mathcal R$  arrive according to independent Poisson processes with arrival rates  $\lambda_{(i,s)} > 0$ ,  $(i,s) \in \mathcal R$ .

For each item  $i \in \mathcal{C}$  there is a fixed set of designated server nodes  $\mathcal{S}_i \subseteq V$ , that always store i. A node  $v \in \mathcal{S}_i$  permanently stores i in excess memory outside its cache. Thus, the placement of items to designated servers is fixed and outside the network's design. A request (i,s) is routed over a path in G towards a designated server. However, forwarding terminates upon reaching any intermediate cache that stores i. At that point, a response carrying i is sent over the reverse path, i.e., from the node where the cache hit occurred, back to source node s. Both caching and routing decisions are network design parameters, which we define formally below.

# 3.2 Caching Strategies

We study two types or caches:  $deterministic\ {\rm and}\ randomized.$ 

**Deterministic caches.** For each node  $v \in V$ , we define v's caching strategy as a vector  $x_v \in \{0,1\}^{|\mathcal{C}|}$ , where  $x_{vi} \in \{0,1\}$ , for  $i \in \mathcal{C}$ , is the binary variable indicating whether v stores content item i. As v can store no more than  $c_v$  items, we have that:

$$\sum_{i \in \mathcal{C}} x_{vi} \le c_v, \text{ for all } v \in V.$$
 (1)

We define the global caching strategy as the matrix  $X = [x_{vi}]_{v \in V, i \in \mathcal{C}} \in \{0, 1\}^{|V| \times |\mathcal{C}|}$ , whose rows comprise the caching strategies of each node.

<sup>&</sup>lt;sup>1</sup>A directed graph is symmetric when  $(i, j) \in E$  implies that  $(j, i) \in E$ .

Common Notation										
G(V, E)	Network graph, with nodes $V$ and edges $E$									
$\mathcal{C}$	Item catalog									
$c_v$	Cache capacity at node $v \in V$									
$w_{uv}$	Weight of edge $(u, v) \in E$									
$\mathcal{R}$	Set of requests $(i, s)$ , with $i \in \mathcal{C}$ and source $s \in V$									
$\mathcal{S}_i^{(i,s)}$	Arrival rate of requests $(i, s) \in \mathcal{R}$									
${\mathcal S}_i$	Set of designated servers of $i \in \mathcal{C}$									
$x_{vi}$	Variable indicating whether $v \in V$ stores $i \in \mathcal{C}$									
$\xi_{vi}$	Marginal probability that $v$ stores $i$									
X	Global caching strategy of $x_{vi}$ s, in $\{0,1\}^{ V \times \mathcal{C} }$									
Ξ	Expectation of caching strategy matrix $X$									
T	Duration of a timeslot in online setting									
$w_{uv}$	weight/cost of edge $(u, v)$									
$supp(\cdot)$	Support of a probability distribution									
$conv(\cdot)$	Convex hull of a set									
Source Routing $\mathcal{P}_{(i,c)} \qquad \text{Set of paths request } (i,s) \in \mathcal{R} \text{ can follow}$										
$\mathcal{P}_{(i,s)}$										
$P_{\mathtt{SR}}$	Total number of paths A simple path of $G$									
$k_p(v)$	The position of node $v \in p$ in path $p$ .									
	Variable indicating whether $(i, s) \in \mathcal{R}$ is forwarded over $p \in \mathcal{P}_{(i,s)}$									
$r_{(i,s),p}$	Marginal probability that s routes request for i over p									
$ ho_{(i,s),p}$										
r	Routing strategy of $r_{(i,s),p}$ s, in $\{0,1\}^{\sum (i,s) \in \mathcal{R}}  \mathcal{P}(i,s) $ .									
$\rho$	Expectation of routing strategy vector $r$									
$\mathcal{D}_{\mathtt{SR}}$	Feasible strategies $(r, X)$ of MAXCG-S									
RNS	Route to nearest server									
RNR	Route to nearest replica									
	Hop-by-Hop Routing									
$G^{(i)}$	DAG with sinks in $S_i$									
$E^{(i)}$	Edges in DAG $G^{(i)}$									
$G^{(i,s)}$	Subgraph of $G^{(i)}$ including only nodes reachable from s									
$\mathcal{P}^{u}_{(i,s)}$	Set of paths in $G^{(i,s)}$ from s to u.									
$P_{HH}$	Total number of paths									
$r_{uv}^{(i)}$ $\rho_{uv}^{(i)}$	Variable indicating whether $u$ forwards a request for $i$ to $v$									
$ ho_{uv}^{(i)}$	Marginal probability that $u$ forwards a request for $i$ to $v$									
r	Routing strategy of $r_{u,v}^i$ s, in $\{0,1\}^{\sum_{i\in\mathcal{C}} E^{(i)} }$ .									
$\rho$	Expectation of routing strategy vector $r$									
$\mathcal{D}_{\mathtt{HH}}$	Feasible strategies $(r, X)$ of MAXCG-HH									

Table 1: Notation Summary

**Randomized caches.** In the case of randomized caches, the caching strategies  $x_v, v \in V$ , are random variables. We denote by:

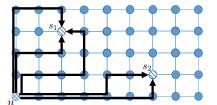
$$\xi_{vi} \equiv \mathbf{P}[x_{vi} = 1] = \mathbb{E}[x_{v,i}] \in [0,1], \text{ for } i \in C,$$
 (2)

the marginal probability that node v caches item i, and by  $\Xi = [\xi_{vi}]_{v \in V, i \in \mathcal{C}} = \mathbb{E}[X] \in [0,1]^{|V| \times |\mathcal{C}|}$ , the corresponding expectation of the global caching strategy.

# 3.3 Source Routing Strategies

Recall that requests are routed towards designated server nodes. In source routing, for every request  $(i,s) \in \mathcal{C} \times V$ , there exists a  $\mathcal{P}_{(i,s)}$  of paths that the request can follow towards a designated server in  $\mathcal{S}_i$ . A source node s can forward a request among any of these paths, but we assume each response follows the same path as its corresponding request.

Formally, a path p of length |p| = K is a sequence  $\{p_1, p_2, \ldots, p_K\}$  of nodes  $p_k \in V$  such that  $(p_k, p_{k+1}) \in E$ , for every  $k \in \{1, \ldots, |p| - 1\}$ . We make the following natural assumptions on the set of paths  $\mathcal{P}_{(i,s)}$ . For every  $p \in \mathcal{P}_{(i,s)}$ : (a) p starts at s, i.e.,  $p_1 = s$ ; (b) p is simple, i.e., it contains no loops; (c) the last node in p is a designated server for item i, i.e., if |p| = K,  $p_K \in S_i$ ; and (d) no other node in p is a designated server for i, i.e., if |p| = K,  $p_k \notin S_i$ , for  $k = 1, \ldots, K - 1$ . Given a path p and a  $v \in p$ , denote by  $k_p(v)$  is the position of v in p; i.e.,  $k_p(v)$  equals to  $k \in \{1, \ldots, |p|\}$  such that  $p_k = v$ . As in the case of caches, we consider both deterministic and randomized routing strategies.



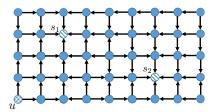


Figure 1: Source Routing vs. Hop-by-Hop routing. In source routing, shown left, source node u on the bottom left can choose among 5 possible paths to route a request to one of the designated servers storing i  $(s_1, s_2)$ . In hop-by-hop routing, each intermediate node in the network selects the next hop among one of its neighbors in a DAG, whose sinks are the designated servers.

**Deterministic Routing.** Given sets  $\mathcal{P}_{(i,s)}$ ,  $(i,s) \in \mathcal{R}$ , the routing strategy of a source  $s \in V$  w.r.t. request  $(i,s) \in \mathcal{R}$  is a vector  $r_{(i,s)} \in \{0,1\}^{|\mathcal{P}_{(i,s)}|}$ , where  $r_{(i,s),p} \in \{0,1\}$  is a binary variable indicating whether s selected path  $p \in \mathcal{P}_{(i,s)}$ . Routing strategies satisfy:

$$\sum_{p \in \mathcal{P}_{(i,s)}} r_{(i,s),p} = 1, \text{ for all } (i,s) \in \mathcal{R}.$$
(3)

indicating that exactly one path is selected. Let  $P_{SR} = \sum_{(i,s) \in \mathcal{R}} |\mathcal{P}_{(i,s)}|$  be the total number of paths. We refer to the vector  $r = [r_{(i,s),p}]_{(i,s) \in \mathcal{R}, p \in \mathcal{P}_{(i,s)}} \in \{0,1\}^P$ , as the global routing strategy.

Randomized Routing. In the case of randomized routing, variables  $r_{(i,s)}$ ,  $(i,s) \in \mathcal{R}$  are random. We randomize routing by allowing requests to be routed over a random path in  $\mathcal{P}_{(i,s)}$ , selected independently of all past requests (at s or elsewhere). We denote by

$$\rho_{(i,s),p} \equiv \mathbf{P}[r_{(i,s),p} = 1] = \mathbb{E}[r_{(i,s),p}], \text{ for } p \in \mathcal{P}_{(i,s)},$$
 (4)

the probability that path p is selected by s, and by  $\rho = [\rho_{(i,s),p}]_{(i,s)\in\mathcal{R},p\in\mathcal{P}_{(i,s)}} = \mathbb{E}[r] \in [0,1]^P$  the expectation of the global routing strategy r.

**Remark.** We make no a priori assumptions on  $P_{SR}$ , the total number of paths used during source routing; moreover, we allow paths to overlap. The complexity of our offline algorithm, and the rate of convergence of our distributed, adaptive algorithm depend on  $P_{SR}$ . In practice, if the number of possible paths is, e.g., exponential in |V|, it makes sense to restrict each  $\mathcal{P}_{(i,s)}$  to a small subset of possible paths, or to use hop-by-hop routing instead, which, as discussed below, restricts the maximum number of paths considered.

### 3.4 Hop-by-Hop Routing Strategies

Under hop-by-hop routing, each node along the path makes an individual decision on where to route a request message. When a request for item i arrives at an intermediate node  $v \in V$ , node v determines how to forward the request to one of its neighbors. The decision depends on i but not on the request's source. This limits the paths a request may follow, making hop-by-hop routing less expressive than source routing. On the other hand, reducing the space of routing strategies reduces complexity. In adaptive algorithms, it also speeds up convergence, as routing decisions w.r.t. i are "learned" across requests by different sources.

To ensure loop-freedom, we must assume that forwarding decisions are restricted to a subset of possible neighbors in G. For each  $i \in \mathcal{C}$ , we denote by  $G^{(i)}(V, E^{(i)})$  a graph that has the following properties: (a)  $G^{(i)}$  is a subgraph of G, i.e.,  $E^{(i)} \subseteq E$ ; (b)  $G^{(i)}$  is a directed acyclic graph (DAG); and (c) a node v in  $G^{(i)}$  is a sink if and only if it is a designated server for i, i.e.,  $v \in \mathcal{S}_i$ . Note that, given G and  $\mathcal{S}_i$ ,  $G^{(i)}$  can be constructed in polynomial time using, e.g., the Bellman-Ford algorithm [17]. Indeed, requiring that v

forwards requests for  $i \in \mathcal{C}$  only towards neighbors with a smaller distance to a designated server in  $\mathcal{S}_i$  results in such a DAG. A distance-vector protocol [35] can form this DAG in a distributed fashion. We assume that every node  $v \in V$  can forward a request for item i only to a neighbor in  $G^{(i)}$ . Then, the above properties of  $G^{(i)}$  ensure both loop freedom and successful termination.

**Deterministic Routing.** For any node  $s \in V$ , let  $G^{(i,s)}$  be the induced subgraph of  $G^{(i)}$  which results from removing any nodes in  $G^{(i)}$  not reachable from s. For any u in  $G^{(i,s)}$ , let  $\mathcal{P}^u_{(i,s)}$  be the set of all paths in  $G^{(i,s)}$  from s to u, and denote by  $P_{\mathtt{HH}} =$ 

 $\sum_{(i,s)\in\mathcal{C}}\sum_{u\in V}|\mathcal{P}^u_{(i,s)}|. \text{ We denote by } r^{(i)}_{uv}\in\{0,1\}, \text{ for } (u,v)\in E^{(i)}, \ i\in\mathcal{C}, \text{ the decision variable indicating whether } u \text{ forwards a request for } i \text{ to } v. \text{ The } global \ routing \ strategy \text{ is } r=[r^{(i)}_{uv}]_{i\in\mathcal{C},(u,v)\in E^{(i)}}\in\{0,1\}^{\sum_{i\in\mathcal{C}}|E^{(i)}|}, \text{ and satisfies}$ 

$$\sum_{v:(u,v)\in E(i)} r_{uv}^{(i)} = 1, \quad \text{for all } v \in V, i \in \mathcal{C}.$$
 (5)

Note that, in contrast to source routing strategies, that have length  $P_{SR}$ , hop-by-hop routing strategies have length at most  $|\mathcal{C}||E|$ .

Randomized Routing. As in the case of source routing, we also consider randomized hop-by-hop routing strategies, whereby each request is forwarded independently from previous routing decisions to one of the possible neighbors. We again denote by

$$\rho = [\rho_{uv}^{(i)}]_{i \in \mathcal{C}, (u,v) \in E^{(i)}} = [\mathbb{E}[r_{uv}^{(i)}]]_{i \in \mathcal{C}, (u,v) \in E^{(i)}} 
= [\mathbf{P}[r_{uv}^{(i)} = 1]]_{i \in \mathcal{C}, (u,v) \in E^{(i)}} \in [0,1]^{\sum_{i \in \mathcal{C}} |E^{(i)}|},$$
(6)

the vector of corresponding (marginal) probabilities of routing decisions at each node v.

### 3.5 Offline vs. Online Setting

To reason about the caching networks we have proposed, we consider two settings: the offline and online setting. In the offline setting, all problem inputs (demands, network topology, cache capacities, etc.) are known apriori to, e.g., a system designer. At time t=0, the system designer selects (a) a caching strategy X, and (b) a routing strategy r. Both can be either deterministic or randomized, but both are also static: they do not change as time progresses. In the case of caching, cache contents (selected deterministically or at random at t=0) remain static for all  $t\geq 0$ . In the case of routing decisions, the distribution over paths (in source routing) or neighbors (in hop-by-hop routing) remains static, but each request is routed independently of previous requests.

In the online setting, no a priori knowledge of the demand, i.e., the rates of requests  $\lambda_{(i,s)}, \ (i,s) \in \mathcal{R}$  is assumed. Both caching and routing strategies change through time via a distributed, adaptive algorithm. Time is slotted, and each slot has duration T>0. During a timeslot, both caching and and routing strategies remain fixed. Nodes have access only to local information: they are aware of their graph neighborhood and state information they maintain locally. They exchange messages, including both normal request and response traffic, as well as (possibly) control messages, and may adapt their state. At the conclusion of a time slot, each node changes its caching and routing strategies. Changes made by v depend only on its neighborhood, its current local state, as well as on messages that node v received in the previous timeslot. Both caching and routing strategies during a timeslot may be deterministic or randomized. Implementing a caching strategy at the conclusion of a timeslot involves changing cache contents, which incurs additional overhead; if T is large, however, this cost is negligible compared to the cost of transferring items during a timeslot.

# 3.6 Optimal Routing and Caching

We are now ready to formally pose the problem of jointly optimizing caching and routing. We pose here the *offline* problem, in which problem inputs are given, and static caching and routing strategies are determined (jointly) at time t=0. Nonetheless, we

will devise distributed, adaptive algorithms that do not a priori know the demand, but still converge to (probabilistic) strategies that are within a constant approximation of the (offline) optimal.

To capture costs (e.g., latency, money, etc.), we associate a weight  $w_{uv} \geq 0$  with each edge  $(u,v) \in E$ , representing the cost of transferring an item across this edge. We assume that costs are solely due to response messages that carry an item, while request forwarding costs are negligible. We do not assume that  $w_{uv} = w_{vu}$ . We describe the cost minimization objectives under source and hop-by-hop routing below.

**Source Routing.** The cost for serving a request  $(i, s) \in \mathcal{R}$  under source routing is:

$$C_{\text{SR}}^{(i,s)}(r,X) = \sum_{p \in \mathcal{P}_{(i,s)}} r_{(i,s),p} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \prod_{k'=1}^k (1 - x_{p_{k'}i}). \tag{7}$$

Intuitively, (7) states that  $C_{SR}^{(i,s)}$  includes the cost of an edge  $(p_{k+1}, p_k)$  in the path p if (a) p is selected by the routing strategy, and (b) no cache preceding this edge in p stores i. In the deterministic setting, we seek a global caching and routing strategy (r, X) minimizing the aggregate expected cost, defined as:

$$C_{SR}(r, X) = \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} C_{SR}^{(i,s)}(r, X),$$
 (8)

with  $C_{SR}^{(i,s)}$  given by (7). That is, we wish to solve:

MINCOST-SR

Minimize: 
$$C_{SR}(r, X)$$
 (9a)

subj. to: 
$$(r, X) \in \mathcal{D}_{SR}$$
 (9b)

where  $\mathcal{D}_{SR} \subset \mathbb{R}^{P_{SR}} \times \mathbb{R}^{|V| \times |\mathcal{C}|}$  is the set of (r, X) satisfying the routing, capacity, and integrality constraints, i.e.:

$$\sum_{i \in \mathcal{C}} x_{vi} = c_v, \qquad \text{for all } v \in V, \tag{10a}$$

$$\sum_{p \in \mathcal{P}_{(i,s)}} r_{(i,s),p} = 1, \qquad \text{for all } (i,s) \in \mathcal{R},$$
(10b)

$$x_{vi} \in \{0, 1\},$$
 for all  $v \in V, i \in \mathcal{C}$ , and (10c)

$$r_{(i,s),p} \in \{0,1\},$$
 for all  $p \in \mathcal{P}_{(i,s)}, (i,s) \in \mathcal{R}.$  (10d)

This problem is NP-hard, even in the case where routing is fixed: see Shanmugam et al. [51] for a reduction from the 2-Disjoint Set Cover Problem.

**Hop-By-Hop Routing.** Similarly to (7), under hop-by-hop routing, the cost of serving (i, s) can be written as:

$$C_{\text{HH}}^{(i,s)}(r,X) = \sum_{(u,v)\in G^{(i,s)}} w_{vu} \cdot r_{uv}^{(i)} (1 - x_{ui}) \cdot \sum_{p\in\mathcal{P}_{(i,s)}^{u}} \prod_{k'=1}^{|p|-1} r_{p_{k'}p_{k'+1}}^{(i)} (1 - x_{p_{k'}i}).$$

$$(11)$$

We wish to solve:

MinCost-hh

Minimize: 
$$C_{\text{HH}}(r, X)$$
 (12a)

subj. to: 
$$(r, X) \in \mathcal{D}_{HH}$$
 (12b)

where  $C_{\mathtt{HH}}(r,X) = \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} C_{\mathtt{HH}}^{(i,s)}(r,X)$  is the expected routing cost, and  $\mathcal{D}_{\mathtt{HH}}$  is the set of  $(r,X) \in \mathbb{R}^{\sum_{i \in \mathcal{C}} |E^{(i)}|} \times \mathbb{R}^{|V| \times |\mathcal{C}|}$  satisfying the constraints:

$$\sum_{i \in \mathcal{C}} x_{vi} = c_v, \qquad \text{for all } v \in V, \tag{13a}$$

$$\sum_{v:(u,v)\in E(i)} r_{uv}^{(i)} = 1 \qquad \text{for all } v\in V, i\in\mathcal{C},$$
(13b)

$$x_{vi} \in \{0, 1\},$$
 for all  $v \in V, i \in \mathcal{C}$ , and (13c)

$$r_{uv}^{(i)} \in \{0, 1\},$$
 for all  $(u, v) \in E^{(i)}, i \in \mathcal{C}.$  (13d)

**Randomization.** The above routing cost minimization problems can also be stated in the context of randomized caching and routing strategies. For example, in the case of source routing, assuming<sup>2</sup> (a) independent caching strategies across nodes selected at time t=0, with marginal probabilities given by  $\Xi$ , and (b) independent routing strategies at each source, with marginals given by  $\rho$  (also independent from caching strategies), all terms in  $C_{SR}$  contain products of independent random variables; this implies that:

$$\mathbb{E}[C_{SR}(r,X)] = C_{SR}[\mathbb{E}[r], \mathbb{E}[X]] = C_{SR}(\rho,\Xi),\tag{14}$$

where the expectation is taken over the randomness of both caching and routing strategies. The expected routing cost thus depends on the routing and caching strategies only through the expectations  $\rho$  and  $\Xi$ . As a result, under randomized routing and caching strategies, MINCOST-SR becomes (see Appendix A for the derivation):

Minimize: 
$$C_{SR}(\rho, \Xi)$$
 (15a)

subj. to: 
$$(\rho, \Xi) \in conv(\mathcal{D}_{SR})$$
 (15b)

where  $conv(\mathcal{D}_{SR})$  is the convex hull of  $\mathcal{D}_{SR}$ ; this is precisely the set defined by (10) with integrality constraints (10c), (10d) relaxed. The objective function  $C_{SR}$  is not convex and the relaxed problem (15) is therefore not a convex optimization problem. This is in stark contrast to single-hop settings, that often can naturally be expressed as linear programs [1, 19, 42].

A similar derivation can be done for hop-by-hop routing. Assuming again independent caches and independent routing strategies, it can be shown that optimizing over randomized hop-by-hop strategies is equivalent to

Minimize: 
$$C_{\rm HH}(\rho,\Xi)$$
 (16a)

subj. to: 
$$(\rho, \Xi) \in conv(\mathcal{D}_{HH}),$$
 (16b)

where  $conv(\mathcal{D}_{HH})$  the convex hull of  $\mathcal{D}_{HH}$ . This, again, is a non-convex optimization problem

### 3.7 Fixed Routing

When the global routing strategy r is fixed, (9) reduces to

Minimize: 
$$C_{SR}(r, X)$$
 (17a)

subj. to: 
$$X$$
 satisfies (10a) and (10c) (17b)

MINCOST-HH can be similarly restricted to caching only. We studied this restricted optimization in earlier work [30]. In particular, under given global routing strategy r, we cast (17) as a maximization problem as follows. Let

$$C_0^r = C_{SR}(r,0) = \sum_{(i,s)\in\mathcal{R}} \lambda_{(i,s)} \sum_{p\in\mathcal{P}_{(i,s)}} r_{(i,s),p} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k}$$
(18)

be the cost when all caches are empty (i.e., X is the zero matrix 0). Note that this is a constant that does not depend on X. Consider the following maximization problem:

Maximize: 
$$F_{SR}^r(X) = C_0^r - C_{SR}(r, X)$$
 (19a)

subj. to: 
$$X$$
 satisfies (10a) and (10c) (19b)

This problem is equivalent to (17), in that a feasible solution to (19) is optimal if and only if it also optimal for (17). The objective  $F_{\rm SR}^{\rm r}(X)$ , referred to as the *caching gain* in [30], is monotone, non-negative, and submodular, while the set of constraints on X

<sup>&</sup>lt;sup>2</sup>The independence of routing and caching strategies across nodes comes without loss of generality! The minimum expected cost attainable under independence is the same under cache contents and routing strategies sampled from an arbitrary joint distribution over  $\mathcal{D}_{SR}$ ; see Thm 3.

is a set of matroid constraints. As a result, for any r, there exist standard approaches for constructing a polynomial time approximation algorithm solving the corresponding maximization problem (19) within a 1-1/e factor from its optimal solution [30, 51]. In addition, we show [30] that an approximation algorithm based on a technique known as pipage rounding [3] can be converted into a distributed, adaptive version with the same approximation ratio.

# 3.8 Greedy Routing Strategies

In the case of source routing, we identify two "greedy" deterministic routing strategies, that are often used in practice, and play a role in our analysis. We say that a global routing strategy r is a route-to-nearest-server (RNS) strategy if all paths it selects are least-cost paths to designated servers, irrespectively of cache contents. Formally, for all  $(i,s) \in \mathcal{R}$ ,  $r_{(i,s),p^*}=1$  for some  $p^* \in \arg\min_{p \in \mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1},p_k}$ , while  $r_{(i,s),p}=0$  for all other  $p \in \mathcal{P}_{(i,s)}$  s.t.  $p \neq p^*$ . Similarly, given a caching strategy X, we say that a global routing strategy r is route-to-nearest-replica (RNR) strategy if, for all  $(i,s) \in \mathcal{R}$ ,  $r_{(i,s),p^*}=1$  for some  $p^* \in \arg\min_{p \in \mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1},p_k} \prod_{k'=1}^k (1-x_{p_{k'}i})$ , while  $r_{(i,s),p}=0$  for all other  $p \in \mathcal{P}_{(i,s)}$  s.t.  $p \neq p^*$ . In contrast to RNS strategies, RNR strategies depend on the caching strategy X. Note that RNS and RNR strategies can be defined similarly in the context of hop-by-hop routing.

# 3.9 Multi-Item Requests and Stationarity

We have assumed that all items in the catalog are of equal size. In practice, contents of unequal size can be partitioned into equally sized chunks, which would play the role of "items" in our formulation. Requesting, e.g., a file, amounts to requesting all of its chunks simultaneously. Our model can be easily extended to account for requests comprising multiple item/source tuples, i.e., of the form  $\{(i_1, s), (i_2, s), \ldots, (i_L, s)\}$ . Each such multi-item request contributes several summation terms to the routing cost  $C_{SR}$ , given by (8), one for each chunk in the requested set. Thus, the objective has the same form. Note that this formulation does not assume, or require, that chunks of the same file are stored in the same cache or that requests for them follow the same paths.

More broadly speaking, the time average routing cost is given by  $CS_{\mathbb{SR}}$  even if (a) the request arrival processes for requests of type  $(i,s) \in \mathcal{R}$  are not independent, and (b) are arbitrary stationary ergodic renewal processes with arrival rates  $\lambda_{(i,s)}$ . Our offline analysis and results therefore readily extend to general stationary ergodic arrival processes. Moreover our distributed, adaptive algorithm readily applies to non-independent Poisson arrivals (and in particular, to multi-item requests that would occur under file-chunking); it can also be extended to general stationary arrivals, provided that the subgradient estimators used in our projected gradient ascent algorithm are replaced with unbiased estimates matching the statistics of underlying renewal process.

# 4 Main Results

We present our main results in this section, extending the analysis in [30] to the joint optimization of both caching and routing decisions. We provide an analysis of both source and hop-by-hop routing; all proofs are provided in the appendix.

# 4.1 Routing to Nearest Server Is Suboptimal

A simple approach, followed by most works that optimize caching separately from routing, is to always route requests to the nearest designated server storing an item (i.e., use an RNS strategy). It is therefore interesting to ask how this simple heuristic performs compared to a solution that attempts to solve (9) by *jointly* optimizing caching and routing. It is easy to see that RNS and, more generally, routing that ignores caching strategies, can lead to arbitrarily suboptimal solutions:

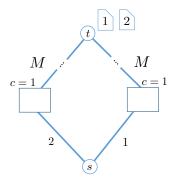


Figure 2: A simple example illustrating the benefits of path diversity. A source node s generates requests for items 1 and 2, permanently stored on designated server t. Intermediate nodes on the are two alternative paths towards t have capacity 1. Numbers above edges indicate costs. Under RNS, requests for both items are forwarded over the same path towards t, leading to an  $\Omega(M)$  routing cost irrespective of the caching strategy. In contrast the optimal solution uses different paths per item, leading to an O(1) cost.

**Theorem 1.** For any M>0, there exists a caching network for which the route-to-nearest-server strategy r' satisfies

$$\min_{X:(r',X)\in\mathcal{D}_{SR}} C_{SR}(r',X) / \min_{(r,X)\in\mathcal{D}_{SR}} C_{SR}(r,X) = \Theta(M).$$
 (20)

In other words, routing to the nearest server can be arbitrarily suboptimal, incurring a cost arbitrarily larger than the cost of the optimal jointly optimized routing and caching policy. The network that exhibits this behavior is shown in Fig. 2, and a proof of the theorem can be found in Appendix B.. In short, a source node s generates requests for items 1 and 2 that are permanently stored on designated server t. There are two alternative paths towards t, each passing through an intermediate node with cache capacity 1 (i.e., able to store only one item). Under shortest path routing, requests for both items are forwarded over the path of length M+1 towards t; fixing routes this way leads to a cost of M+1 for at least one of the items, irrespectively of which item is cached in the intermediate node. On the other hand, if routing and caching decisions are jointly optimized, requests for the two items can be forwarded to different paths, allowing both items to be cached, and reducing the cost for both requests to at most 2.

This example illustrates that joint optimization of caching and routing decisions benefits the system by increasing path diversity. In turn, increasing path diversity can increase caching opportunities, thereby leading to reductions in caching costs. This is consistent with our experimental results in Section 5.

# 4.2 Offline Source Routing.

**Expected Caching Gain.** Before presenting a distributed, adaptive joint routing and caching algorithm, we first turn our attention to the offline problem MINCOST. As in the solution by [30] described in Section 3.7, we cast this first as a maximization problem. Let  $C_0$  be the constant:

$$C_{SR}^{0} = \sum_{(i,s)\in\mathcal{R}} \lambda_{(i,s)} \sum_{p\in\mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_{k}}.$$
 (21)

Then, given a pair of strategies (r, X), we define the expected caching gain  $F_{SR}(r, X)$  as follows:

$$F_{SR}(r, X) = C_{SR}^0 - C_{SR}(r, X),$$
 (22)

where  $C_{\mathtt{SR}}$  is the aggregate routing cost given by (8). Note that  $C_{\mathtt{SR}}^0$  upper bounds the expected routing cost, so that  $F_{\mathtt{SR}}(r,X) \geq 0$ . We seek to solve the following problem, equivalent to MinCost:

Maximize: 
$$F_{SR}(r, X)$$
 (23a)

subj. to: 
$$(r, X) \in \mathcal{D}_{SR}$$
 (23b)

The selection of the constant  $C_{SR}^0$  is not arbitrary: this is precisely the value that allows us to approximate  $F_{SR}$  via the concave relaxation  $L_{SR}$  below-c.f. Eq. (26).

**Approximation Algorithm.** Its equivalence to MinCost implies that MaxCG-S is also NP-hard. Nevertheless, we show that there exists a polynomial time approximation algorithm for MaxCG-S:

**Theorem 2.** There exists an algorithm that terminates within a number of steps that is polynomial in |V|, |C|, and  $P_{SR}$ , and produces a strategy  $(r', X') \in \mathcal{D}_{SR}$  such that

$$F_{SR}(r', X') \ge (1 - 1/e) \max_{(r, X) \in \mathcal{D}_{SR}} F_{SR}(r, X).$$
 (24)

The proof can be found in Appendix C.1. In Sec. 5 we show that, in spite of attaining approximation guarantees w.r.t.  $F_{SR}$  rather than  $C_{SR}$ , the resulting approximation algorithm has excellent performance in practice in terms of minimizing routing costs. In particular, we can reduce routing costs by a factor as high as  $10^3$  compared to fixed routing policies, including [30]. We briefly describe the algorithm below, leaving details to the appendix. Consider the concave function  $L_{SR}: \operatorname{conv}(\mathcal{D}_{SR}) \to \mathbb{R}_+$ , defined as:

$$L_{SR}(\rho,\Xi) = \sum_{(i,s)\in\mathcal{R}} \lambda_{(i,s)} \sum_{p\in\mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \cdot \min\left\{1, 1 - \rho_{(i,s),p} + \sum_{k'=1}^{k} \xi_{p_{k'}i}\right\}.$$
(25)

Then,  $L_{SR}$  closely approximates  $F_{SR}$  (see Lemma 2):

$$(1 - 1/e) L_{SR}(\rho, \Xi) \le F_{SR}(\rho, \Xi) \le L_{SR}(\rho, \Xi), \tag{26}$$

for all  $(\rho, \Xi) \in \mathtt{conv}(\mathcal{D}_\mathtt{SR})$ . Our constant-approximation algorithm for MAXCG-S comprises two steps. *First*, obtain

$$(\rho^*, \Xi^*) \in \arg\max_{(\rho,\Xi) \in \operatorname{conv}(\mathcal{D}_{SR})} L_{SR}(\rho, \Xi). \tag{27}$$

As  $L_{SR}$  is a concave function and  $conv(\mathcal{D}_{SR})$  is convex, the above maximization is a convex optimization problem. In fact, it can be reduced to a linear program, so it can be solved in polynomial time [44]. Second, round the (possibly fractional) solution  $(\rho^*, \Xi^*) \in conv(\mathcal{D}_{SR})$  to an integral solution  $(r, X) \in \mathcal{D}_{SR}$  such that  $F_{SR}(r, X) \geq F_{SR}(\rho^*, \Xi^*)$ . This rounding is deterministic and also takes place in polynomial time.

The rounding technique used in our proof of Thm. 2 has the following immediate implication, whose proof is in Appendix C.3:

**Corollary 1.** There exists an optimal solution  $(r^*, X^*)$  to MaxCG-S (and hence, to MinCost-SR) in which  $r^*$  is an route-to-nearest-replica (RNR) strategy w.r.t.  $X^*$ .

Although, in light of Theorem 1, Corollary 1 suggests an advantage of RNR over RNS strategies, we note it does not give any intuition on how to construct an optimal RNR solution.

Equivalence of Deterministic and Randomized Strategies. We can also show the following result (see Appendix C.4) regarding randomized strategies. For  $\mu$  a probability distribution over  $\mathcal{D}_{SR}$ , let  $\mathbb{E}_{\mu}[C_{SR}(r,X)]$  be the expected routing cost under  $\mu$ . Then, the following equivalence theorem holds:

**Theorem 3.** The deterministic and randomized versions of MinCost-SR attain the same optimal routing cost, i.e.:

$$\min_{(r,X)\in\mathcal{D}_{SR}} C_{SR}(r,X) = \min_{(\rho,\Xi)\in conv(\mathcal{D}_{SR})} C_{SR}(\rho,\Xi)$$

$$= \min_{\mu: supp(\mu)=\mathcal{D}_{SR}} \mathbb{E}_{\mu}[C_{SR}(r,X)]$$
(28)

The first equality of the theorem implies that, surprisingly, there is no inherent advantage in randomization: although randomized strategies constitute a superset of deterministic strategies, the optimal attainable routing cost (or, equivalently, caching gain) is the same for both classes. The second equality implies that assuming independent caching and routing strategies is as powerful as sampling routing and caching strategies from an arbitrary joint distribution. Thm. 3 generalizes Thm. 5 of [30], which pertains to optimizing caching alone.

## 4.3 Online Source Routing

The algorithm in Thm. 2 is offline and centralized: it assumes full knowledge of the input, including demands and arrival rates, which are rarely a priori available in practice. To that end, we turn our attention to solving MaxCG-S in the online setting, in the absence of any a priori knowledge of the demand, and seek an algorithm that is both adaptive and distributed.

As described in 3.5, in the online setting, time is partitioned into slots of equal length T>0. Caching and routing strategies are randomized as described in Sec. 3: at the beginning of a timeslot, nodes place a random set of contents in their cache, independently of each other; upon arrival, a new request is routed over a random path, selected independently of (a) all past routes followed, and (b) of caching decisions. Our next theorem shows that, in steady state, the expected caching gain of the jointly constructed routing and caching strategies is within a constant approximation of the optimal solution to the offline problem MaxCG-S:

**Theorem 4.** There exists a distributed, adaptive algorithm under which the randomized strategies sampled during the k-th slot  $(r^{(k)}, X^{(k)}) \in \mathcal{D}_{SR}$  satisfy

$$\lim_{k \to \infty} \mathbb{E}[F_{SR}(r^{(k)}, X^{(k)})] \ge (1 - 1/e) \max_{(r, X) \in \mathcal{D}_{SR}} F_{SR}(r, X). \tag{29}$$

The proof can be found in Appendix D.6. Note that, despite the fact that the algorithm has no prior knowledge of the demands, the guarantee provided is w.r.t. an optimal solution of the offline problem (23). Moreover, in light of Thm. 3, our adaptive algorithm is  $1-\frac{1}{e}$ -competitive w.r.t. optimal offline randomized strategies as well. Our algorithm naturally generalizes [30]: when the path sets  $\mathcal{P}_{(i,s)}$  are singletons, and routing is fixed, our algorithm coincides with the cache-only optimization algorithm in [30]. Interestingly, the algorithm casts routing and caching in the same control plane: the same quantities are communicated through control messages to adapt both the caching and routing strategies. Algorithm Overview. We give a brief overview of the distributed, adaptive algorithm that attains the guarantees of Theorem 4 below. The algorithm is summarized in Algorithm 1. Recall that time is partitioned into slots of equal length T>0. Caching and routing strategies are randomized as described in Sec. 3: at the beginning of a timeslot, nodes place a random set of contents in their cache, independently of each other; upon arrival, a new request is routed over a random path, selected independently of (a) all past routes followed, and (b) of caching decisions.

More specifically, nodes in the network maintain the following state information. Each node  $v \in G$  maintains locally a vector  $\xi_v \in [0,1]^{|\mathcal{C}|}$ , determining its randomized caching strategy. Moreover, for each request  $(i,s) \in \mathcal{R}$ , source node s maintains a vector  $\rho_{(i,s)} \in [0,1]^{|\mathcal{P}(i,s)|}$ , determining its randomized routing strategy. Together, these variables represent the global state of the network, denoted by  $(\rho,\Xi) \in \mathsf{conv}(\mathcal{D}_\mathtt{SR})$ . When the timeslot ends, each node performs the following four tasks:

# Algorithm 1 Projected Gradient Ascent

```
1: Execute the following for each v \in V and each (i, s) \in \mathcal{R}:

2: Pick arbitrary state (\rho^{(0)}, \Xi^{(0)}) \in \operatorname{conv}(\mathcal{D}_{SR}).

3: for each timeslot k \geq 1 do

4: for each v \in V do
              Compute the sliding average \bar{\xi}_v^{(k)} through (45).
 5:
              Sample a feasible x_v^{(k)} from a distribution with marginals \bar{\xi}_v^{(k)}.
 6:
              Place items x_v^{(k)} in cache.
 7:
              Collect measurements and, at the end of the timeslot, compute estimate z_v of \partial_{\xi_v} L_{SR}(\rho^k, \Xi^{(k)})
 8:
              Adapt through (44). to new state \xi_v^{(k+1)} in the direction of the gradient with step-size \gamma_k,
 9:
10.
          end for
          for each (i, s) \in \mathcal{R} do
11:
              Compute the sliding average \bar{\rho}_{(i,s)}^{(k)} through (46).
12:
              Whenever a new request arrives, sample p \in \mathcal{P}_{(i,s)} from distribution \bar{\rho}_{(i,s)}^{(k)}.
Collect measurements and, at the end of the timeslot, compute estimate q_{(i,s)} of
13:
14:
              \partial_{\rho_{(i,s)}} L_{SR}(\rho^k, \Xi^{(k)})through (43).
              Adapt through (44). to new state \rho_{(i,s)}^{(k+1)} in the direction of the gradient with step-size \gamma_k,
15:
              projecting back to conv(\mathcal{D}_{SR}).
17: end for
```

- 1. Subgradient Estimation. Each node uses measurements collected during the duration of a timeslot to construct estimates of the gradient of  $L_{SR}$  w.r.t. its own local state variables. As  $L_{SR}$  is not everywhere differentiable, an estimate of a *subgradient* of  $L_{SR}$  is computed instead.
- 2. **State Adaptation.** Nodes adapt their local caching and routing state variables  $\xi_v$ ,  $v \in V$ , and  $\rho_{(i,s)}$ ,  $(i,s) \in \mathcal{R}$ , pushing them towards a direction that increases  $L_{SR}$ , as determined by the estimated subgradients.
- 3. State Smoothening. Nodes compute "smoothened" versions  $\bar{\xi}_v$ ,  $v \in V$ , and  $\bar{\rho}_{(i,s)}$ ,  $(i,s) \in \mathcal{R}$ , interpolated between present and past states. This is needed on account of the non-differentiability of  $L_{SR}$ .
- 4. Randomized Caching and Routing. After smoothening, each node v reshuffles the contents of its cache using the smoothened caching marginals  $\bar{\xi}_v$ , producing a random placement (i.e., caching strategy  $x_v$ ) to be used throughout the next slot. Moreover, each node  $s \in V$  routes requests  $(i,s) \in \mathcal{R}$  received during next timeslot over random paths (i.e., routing strategies  $r_{(i,s)}$ ) sampled in an i.i.d. fashion from the smoothened marginals  $\bar{\rho}_{(i,s)}$ .

Together, these steps ensure that, in steady state, the expected caching gain of the jointly constructed routing and caching strategies is within a constant approximation of the optimal solution to the offline problem MaxCG-S. We formally describe the constituent subgradient estimation, state adaptation, smoothening, and random sampling steps in detail in Appendices D.1 to D.4, respectively.

**Convergence Guarantees.** The proof of the convergence of the algorithm relies on the following key lemma:

**Lemma 1.** Let  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)}) \in \mathcal{D}_2$  be the smoothened state variables at the k-th slot of Algorithm 1, and  $(\rho^*, \Xi^*) \in \arg\max_{(\rho,\Xi) \in \mathsf{conv}(\mathcal{D}_{SR})} L_{SR}(\rho,\Xi)$ . Then, for  $\gamma_k$  the step-size used in projected gradient ascent,

$$\varepsilon_k \equiv \mathbb{E}[L_{\text{SR}}(\rho^*, \Xi^*) - L_{\text{SR}}(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})] \leq \frac{D^2 + M^2 \sum_{\ell=\lfloor k/2 \rfloor}^k \gamma_\ell^2}{2 \sum_{\ell=\lfloor k/2 \rfloor}^k \gamma_\ell},$$

where  $D = \sqrt{2|V| \max_{v \in V} c_v + 2|\mathcal{R}|}$ , and

$$M = W|V|\Lambda \sqrt{(|V||\mathcal{C}|P^2 + |\mathcal{R}|P)(1 + \frac{1}{\Lambda T})}.$$

In particular,  $\varepsilon_k = O(1/\sqrt{k})$  for  $\gamma = 1/\sqrt{k}$ .

Lemma 1 establishes that Algorithm 1 converges arbitrarily close to an optimizer of  $L_{\rm SR}$ . As, by (26), this is a close approximation of  $F_{\rm SR}$ , the limit points of the algorithm are with the 1-1/e from the optimal. Crucially, Lemma 1 can be used to determine the rate of convergence of the algorithm, by determining the number of steps required for  $\varepsilon_k$  to reach a desired threshold  $\delta$ . Moreover, through quantity M, Lemma 1 establishes a tradeoff w.r.t. T: increasing T decreases the error in the estimated subgradient, thereby reducing the total number of steps till convergence, but also increases the time taken by each step.

### 4.4 Hop-by-Hop Routing

A similar analysis to the one we outlined above applies to hop-by-hop routing, both in the offline and online setting. We state again the main theorems here; proofs can again be found in Appendix E.

Offline Setting. As in the case of source routing, we define the constant:  $C^0_{\mathtt{HH}} = \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} \sum_{(u,v) \in G^{(i,s)}} w_{vu} | \mathcal{P}^u_{(i,s)}|$ . Using this constant, we define the caching gain maximization problem to be:

Maximize: 
$$F_{HH}(r, X)$$
 (30a)

subj. to: 
$$(r, X) \in \mathcal{D}_{HH}$$
 (30b)

where  $F_{\rm HH}(r,X) = C_{\rm HH}^0 - \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} C_{\rm HH}^{(i,s)}(r,X)$  is the expected caching gain. This is again an NP-hard problem, equivalent to (12). We can again construct a constant approximation algorithm for MAXCG-HH:

**Theorem 5.** There exists an algorithm that terminates within a number of steps that is polynomial in |V|, |C|, and  $P_{HH}$ , and produces a strategy  $(r', X') \in \mathcal{D}_{HH}$  such that

$$F_{\text{HH}}(r', X') \ge (1 - 1/e) \max_{(r, X) \in \mathcal{D}_{\text{HH}}} F_{\text{HH}}(r, X).$$
 (31)

Online Setting. Finally, as in the case of source routing, we can provide a distributed, adaptive algorithm for hop-by-hop routing as well.

**Theorem 6.** There exists a distributed, adaptive algorithm under which the randomized strategies sampled during the k-th slot  $(r^{(k)}, X^{(k)}) \in \mathcal{D}_{\mathtt{HH}}$  satisfy

$$\lim_{k \to \infty} \mathbb{E}[F_{\text{HH}}(r^{(k)}, X^{(k)})] \ge (1 - 1/e) \max_{(r, X) \in \mathcal{D}_{\text{SR}}} F_{\text{HH}}(r, X). \tag{32}$$

We note again that the distributed, adaptive algorithm attains an expected caching gain within a constant approximation from the *offline* optimal.

#### 5 Evaluation

We simulate Algorithm 1 over a broad variety of both synthetic and real networks. We compare its performance to traditional caching policies, combined with both static and dynamic multi-path routing. **Experiment Setup.** We consider the topologies in Table 2. For each graph G(V, E) in Table 2, we generate a catalog of size  $|\mathcal{C}|$ , and assign to each node  $v \in V$  a cache of capacity  $c_v$ . For every item  $i \in \mathcal{C}$ , we designate a node selected u.a.r. from V as a designated server for this item; the item is stored outside the designate server's cache. We assign a weight to each edge in E selected u.a.r. from the interval [1,100]. We also select a random set of Q nodes as the possible request sources, and generate a set of requests  $\mathcal{R} \subseteq \mathcal{C} \times V$  by sampling exactly  $|\mathcal{R}|$  from the set  $\mathcal{C} \times Q$ , uniformly at random. For each such request  $(i,s) \in \mathcal{R}$ , we select the request rate  $\lambda_{(i,s)}$  according to a Zipf distribution with parameter 1.2; these are normalized so that average request rate over all |Q| sources is 1 request per time unit. For each request  $(i,s) \in \mathcal{R}$ , we

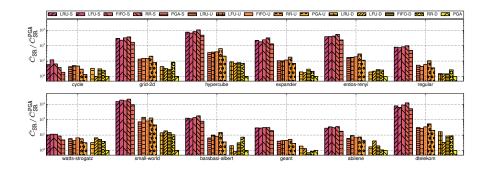


Figure 3: Ratio of expected routing cost  $\bar{C}_{SR}$  to routing cost  $\bar{C}_{SR}^{PGA}$  under our PGA policy, for different topologies and strategies. For each topology, each of the three groups of bars corresponds to a routing strategy, namely, RNS/shortest path routing (-S), uniform routing (-U), and dynamic routing (-D). The algorithm presented in [30] is PGA-S, while our algorithm (PGA), with ratio 1.0, is shown last for reference purposes; values of of  $\bar{C}_{SR}^{PGA}$  are given in Table 2.

Graph		E	C	$ \mathcal{R} $	Q	$c_v$	$ \mathcal{P}_{(i,s)} $	$ar{C}_{\mathtt{SR}}^{\mathtt{PGA}}$
cycle	30	60	10	100	10	2	2	20.17
grid-2d	100	360	300	1K	20	3	30	0.228
hypercube	128	896	300	1K	20	3	30	0.028
expander	100	716	300	1K	20	3	30	0.112
erdos-renyi	100	1042	300	1K	20	3	30	0.047
regular	100	300	300	1K	20	3	30	0.762
watts-strogatz	100	400	300	1K	20	3	2	35.08
small-world	100	491	300	1K	20	3	30	0.029
barabasi-albert	100	768	300	1K	20	3	30	0.187
geant	22	66	10	100	10	2	10	1.28
abilene	9	26	10	90	9	2	10	0.911
dtelekom	68	546	300	1K	20	3	30	0.025

Table 2: Graph Topologies and Experiment Parameters.

generate  $|\mathcal{P}_{(i,s)}|$  paths from the source  $s \in V$  to the designated server of item  $i \in \mathcal{C}$ . In all cases, this path set includes the shortest path to the designated server. We consider only paths with stretch at most 4.0; that is, the maximum cost of a path in  $P_{(i,s)}$  is at most 4 times the cost of the shortest path to the designated source. The values of  $|\mathcal{C}|$ ,  $|\mathcal{R}|$  |Q|,  $c_v$ , and  $\mathcal{P}_{(i,s)}$  for each experiment are given in Table 2. Online Caching and Routing Algorithms. We compare the performance of our joint caching and routing projected gradient ascent algorithm (PGA) to several competitors. In terms of caching, we consider four traditional eviction policies for comparison: Least-Recently-Used (LRU), Least-Frequently-Used (LFU), First-In-First-Out (FIFO), and Random Replacement (RR). We combine these policies with path-replication [16, 31]: once a request for an item reaches a cache that stores the item, every cache in the reverse path on the way to the query source stores the item, evicting stale items using one of the above eviction policies. We combine the above caching policies with three different routing policies. In route-to-nearest-server (-S), only the shortest path to the nearest designated server is used to route the message. In uniform routing  $(-\mathbf{U})$ , the source s routes each request (i, s) on a path selected uniformly at random among all paths in  $\mathcal{P}_{(i,s)}$ . We combine each of these (static) routing strategies with each of the above caching strategies use. For instance, LRU-U indicates LRU evictions combined with uniform routing. Note that PGA-S, i.e., our algorithm restricted to RNS routing, is exactly the single-path routing algorithm proposed in [30]. To move beyond static routing policies for LRU, LFU, FIFO, and RR, we also combine the above traditional caching strategies with an adaptive routing strategy, akin to our algorithm, with estimates of the expected routing cost at each path used to adapt routing strategies. During a slot, each source node s maintains an average of the routing cost incurred when routing a

Graph	LRU-S	LFU-S	FIFO-S	RR-S	PGA-S	LRU-U	LFU-U	FIFO-U	RR-U	PGA-U	LRU	LFU	FIF0	RR	PGA
cycle	0.47	0.47	4.38	0.47	865.29	0.47	0.47	3.62	0.47	436.14	6.62	24.08	12.91	0.47	148.20
grid-2d	0.08	3.77	0.08	0.53	657.84	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
hypercube	0.21	2.27	1.62	19.48	924.75	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
expander	0.38	1.31	0.38	70.02	794.27	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
erdos-renyi	3.08	0.25	0.93	5.40	870.84	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
regular	1.50	7.53	1.64	13.66	1183.97	0.05	0.05	0.05	0.05	8.52	0.05	0.05	2.54	1.40	11.49
watts-strogatz	11.88	8.15	19.47	19.47	158.39	7.80	2.88	9.01	8.22	54.90	19.22	7.01	14.00	17.79	37.05
small-world	0.30	1.02	0.30	20.07	955.48	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
barabasi-albert	1.28	1.28	1.28	2.12	1126.24	1.28	1.28	1.28	1.28	6.86	1.28	1.28	1.28	1.28	7.58
geant	0.09	0.09	20.75	4.13	1312.96	1.85	0.09	0.09	0.09	12.71	0.09	13.34	0.09	7.88	14.41
abilene	3.44	3.44	3.44	3.44	802.66	3.44	3.44	3.44	3.44	23.08	5.75	6.13	6.04	7.09	14.36
dtelekom	0.30	0.30	0.30	0.42	927.24	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30

Table 3: Convergence times, in simulation time units.

request over each path. At the end of the slot, the source decreases the probability  $\rho_{(i,s),p}$  that it will follow the path p by an amount proportional to the average, and projects the new strategy to the simplex. For fixed caching strategies, this dynamic routing scheme converges to a route-to-nearest-replica (RNS) routing, which we expect by Cor. 1 to have good performance. We denote this routing scheme with the extension -D. Note that all algorithms we simulate are online.

Experiments and Measurements. Each experiment consists of a simulation of the caching and routing policy, over a specific topology, for a total of 5000 time units. To leverage PASTA, we collect measurements during the duration of the execution at exponentially distributed intervals with mean 1.0 time unit. At each measurement epoch, we extract the current cache contents in the network and construct  $X \in \{0,1\}^{|V| \times |\mathcal{C}|}$ . Similarly, we extract the current routing strategies  $\rho_{(i,s)}$  for all requests  $(i,s) \in \mathcal{R}$ , and construct the global routing strategy  $\rho \in [0,1]^{P_{SR}}$ . Then, we evaluate the expected routing cost  $C_{SR}(\rho,X)$ . We report the average  $\bar{C}_{SR}$  of these values across measurements collected after a warmup phase, during 1000 and 5000 time units of the simulation; that is, if  $t_i$  are the measurement times, then  $\bar{C}_{SR} = \frac{1}{t_{tot} - t_v} \sum_{t_i : \in [t_v, t_{tot}]} C_{SR}(\rho(t_i), X(t_i))$ . Performance w.r.t Routing Costs. The relative performance of the different strategies

Performance w.r.t Routing Costs. The relative performance of the different strategies to our algorithm is shown in Figure 3. With the exception of cycle and watts-strogatz, where paths are scarce, we see several common trends across topologies. First, simply moving from RNS routing to uniform, multi-path routing, reduces the routing cost by a factor of 10. Even without optimizing routing or caching, simply increasing path options increases the available caching capacity. For all caching policies, optimizing routing through the dynamic routing policy (denoted by -D), reduces routing costs by another factor of 10. Finally, jointly optimizing routing and caching leads to a reduction by an additional factor between 2 and 10 times. In several cases, PGA outperforms RNS routing combined with either traditional policies or [30] by 3 orders of magnitude.

Convergence. In Table 3, we show the convergence time for all algorithms we simulated. We define the convergence time to be the time at which the time-average caching gain reaches 95% of the expected caching gain attained at steady state. LRU converges faster than PGA, though it converges to a sub-optimal stationary distribution. Interestingly, both -U and adaptive routing reduce convergence times for PGA, in some cases (like grid-2d and dtelekom) to the order of magnitude of LRU: this is because path diversification reduces contention: it assigns contents to non-overlapping caches, which are populated quickly with distinct contents.

### 6 Conclusions

We have constructed joint caching and routing schemes with optimality guarantees for arbitrary network topologies. Identifying schemes that lead to improved approximation guarantees, especially on the routing cost directly rather than on the caching gain, is an important open question. Equally important is to incorporate queuing and congestion. In particular, accounting for queueing delays and identifying delay-minimizing strategies is open even under fixed routing. Such an analysis can also potentially be used to understand how different caching and routing schemes affect both delay optimality and throughput optimality.

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# A Cost Minimization Under Randomized Strategies

We prove here that, under the randomized routing and caching strategies with the independence assumptions described in Section 3, problem (9) reduces to:

$$\min_{(\rho,\Xi)\in\operatorname{conv}(\mathcal{D}_{SR})} C_{SR}(\rho,\Xi) \tag{33}$$

where  $conv(\mathcal{D}_{SR})$  is the convex hull of  $\mathcal{D}_{SR}$ . The set  $conv(\mathcal{D}_{SR})$  consists of pairs  $(\rho, \Xi)$  where

$$\begin{split} \rho &= [\rho_{(i,s),p}]_{(i,s) \in \mathcal{R}, p \in \mathcal{P}_{(i,s)}} \in [0,1]^P, \quad \text{and} \\ \Xi &= [\xi_{vi}]_{v \in V, i \in \mathcal{C}} \in [0,1]^{|V| \times |\mathcal{C}|}, \end{split}$$

satisfy (10a) and (10b).

Consider a randomized routing strategy r and a randomized caching strategy X, such that  $(r,X) \in \mathcal{D}_{\mathtt{SR}}$ . Let  $\rho = \mathbb{E}[r]$  and  $\Xi = \mathbb{E}[X]$ . Then  $(r,X) \in \mathcal{D}_{\mathtt{SR}}$  readily implies that  $(\rho,\Xi) \in \mathtt{conv}(\mathcal{D}_{\mathtt{SR}})$ . Indeed, as (r,X) satisfy (10c) and (10d), there expectiations must have coordinates within [0,1]. On the other hand, the constraints (10a) and (10b) are linear, so if they are satisfied by (r,X) they must be satisfied by their expectations. Hence, any randomized strategy pair  $(r,X) \in \mathcal{D}_{\mathtt{SR}}$  necessarily has expectation  $(\rho,\Xi)$  that is in the feasibility domain of problem (33); moreover, by (14), its expected routing cost would be exactly given by  $C_{\mathtt{SR}}(\rho,\Xi)$ .

To complete the proof, we need to show that any feasible solution  $(\rho, \Xi) \in \mathsf{conv}(\mathcal{D}_{\mathtt{SR}})$  of (33), we can constuct a MinCost feasible pair of randomized strategies  $(r, X) \in \mathcal{D}_{\mathtt{SR}}$  whose expectations are precisely  $(\rho, \Xi)$ ; then, by (14), it must be that  $\mathbb{E}[C_{\mathtt{SR}}(r, X)] = C_{\mathtt{SR}}(\rho, X)$ . Note that this construction is trivial for routing strategies: given  $(\rho, \Xi) \in \mathsf{conv}(\mathcal{D}_{\mathtt{SR}})$ , we can construct a randomized strategy r by setting  $r_{(i,s)}$  for each  $(i,s) \in \mathcal{R}$  to be an independent categorical variable over  $\mathcal{P}_{(i,s)}$  with  $\mathbf{P}[r_{(i,s),p} = 1] = \rho_{(i,s),p}$ , for  $p \in \mathcal{P}_{(i,s)}$ . It is less obvious how to do so for caching strategies; nevertheless, the technique by [10,30] presented in Appendix D.4 below (see also Figure 5) achieves precisely the desired property: given a feasible  $\Xi$ , it produces a feasible randomized placement X, independent across nodes that (a) satisfies capacity constraints, and (b) has marginals given by  $\Xi$ .  $\square$ 

### Algorithm 2 Offline Algorithm

- 1: Find  $(\rho^*, \Xi^*) \in \arg \max_{(\rho,\Xi) \in \text{conv}(\mathcal{D}_{SR})} L_{SR}(\rho,\Xi)$ 2: Fix  $\rho^*$ , and round  $\Xi^*$  to obtain integral, feasible X' s.t.  $F_{SR}(\rho^*, X') \geq F_{SR}(\rho^*, \Xi^*)$
- 3: Fix X', and round  $\rho^*$  to obtain integral, feasible r' s.t. .  $F_{\mathtt{SR}}(r',X')$   $\geq$  $F_{SR}(\rho^*, X')$
- 4: return (r', X')

### Proof of Theorem 1

Consider simple diamond network shown in Figure 2. A source node s generates requests for items 1 and 2 (i.e.,  $\mathcal{R} = \{(1,s),(2,s)\}$ ), that are permanently stored on designated server t, requesting each with equal rate  $\lambda_{(1,s)} = \lambda_{(2,s)} = 1 \text{sec}^{-1}$ . The path sets  $\mathcal{P}_{(i,s)}$ , i = 1, 2, are identical, and consist of the two alternative paths towards t, each passing through an intermediate node with cache capacity 1 (i.e., able to store only one item). The two paths have routing costs M+1 and M+2, respectively. Under the route-to-nearest server strategy r', requests for both items are forwarded over the path of length M+1towards t; fixing routes this way leads to a cost M+1 for at least one of the items. This happens irrespectively of which item is cached in the intermediate node; as a result the expected routing cost is  $\Theta(M)$ . On the other hand, if routing and caching decisions are jointly optimized, requests for the two items can be forwarded to different paths, allowing both items to be cached in the nearby caches, and reducing the cost for both requests to at most 2.

#### $\mathbf{C}$ Offline Source Routing

# Proof of Theorem 2

Following [3], the technique for producing an approximation algorithm to solve MAXCG-S is to: (a) relax the combinatorial joint routing and caching problem to a convex optimization problem, (b) solve this convex relaxation, and (c) round the (possibly fractional) solution to obtain an integral solution to the original problem.

To that end, consider the concave function  $L_{SR} : conv(\mathcal{D}_{SR}) \to \mathbb{R}_+$ , defined as:

$$L_{SR}(\rho,\Xi) = \sum_{(i,s)\in\mathcal{R}} \lambda_{(i,s)} \sum_{p\in\mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \cdot \min\left\{1, 1 - \rho_{(i,s),p} + \sum_{k'=1}^{k} \xi_{p_{k'}i}\right\}.$$
(34)

Then,  $L_{SR}$  closely approximates  $F_{SR}$ :

**Lemma 2.**  $(1-1/e) L_{SR}(\rho,\Xi) \leq F_{SR}(\rho,\Xi) \leq L_{SR}(\rho,\Xi)$  for all  $(\rho,\Xi) \in conv(\mathcal{D}_{SR})$ .

Proof. This follows from the Goemans-Williamson inequality [28, 30]: for any sequence of  $y_i \in [0,1], i \in \{1,\ldots,n\},\$ 

$$(1-1/e)\min\{1,\sum_{i=1}^{n}y_i\} \le 1-\prod_{i=1}^{n}(1-y_i) \le \min\{1,\sum_{i=1}^{n}y_i\}.$$

The lemma therefore follows by applying the inequality to every term in the summation making up  $F_{SR}$ , to all variables  $\xi_{vi}$ ,  $v \in V$ ,  $i \in C$ , and  $1 - \rho_{(i,s),p}$ ,  $(i,s) \in \mathcal{R}$ ,  $p \in \mathcal{P}_{(i,s)}$ .

Constructing a constant-approximation algorithm for MAXCG-S amounts to the following steps. First, obtain

$$(\rho^*, \Xi^*) \in \underset{(\rho,\Xi) \in \text{conv}(\mathcal{D}_{SR})}{\arg \max} L_{SR}(\rho,\Xi). \tag{35}$$

As  $L_{SR}$  is a concave function and  $conv(\mathcal{D}_{SR})$  is convex, the above maximization is a convex optimization problem. In fact, it can be reduced to a linear program, so it can be solved in polynomial time [44]: for completeness we outline this reduction in Appendix C.2. Second, round the (possibly fractional) solution  $(\rho^*, \Xi^*) \in \text{conv}(\mathcal{D}_{SR})$  to an integral solution  $(r, X) \in \mathcal{D}_{SR}$  such that  $F_{SR}(r, X) \geq F_{SR}(\rho^*, \Xi^*)$ . This rounding is deterministic and takes place in polynomial time.

The above steps are summarized in Algorithm 2. The following two lemmas hold. First, a feasible fractional solution can be converted—in polynomial time—to a feasible solution in which only  $\rho$  is fractional, while increasing  $F_{SR}$ :

**Lemma 3** ([51]). Given any  $(\rho, \Xi) \in \text{conv}(\mathcal{D}_{SR})$ , an integral X such that  $(\rho, X) \in \text{conv}(\mathcal{D}_{SR})$  and  $F_{SR}(\rho, X) \geq F_{SR}(\rho, \Xi)$  can be constructed in  $O(|V|^2|\mathcal{C}|P)$  time.

*Proof.* This is proved in [51] for fixed routing strategies; for completeness, we repeat the proof here. Given a fractional solution  $(\rho,\Xi) \in \mathcal{D}_{SR}$ , there must exist a  $v \in V$  that contains two fractional values  $\xi_{vi}, \xi_{vi'}$ ; this is because, as  $c_v \in \mathbb{N}$  for all  $v \in V$ , the capacity constraints in  $\mathcal{D}_{SR}$  imply that fractional elements of a row in matrix  $\Xi$  must come in pairs. Observe that, restricted to these two variables, function  $F_{SR}$  is an affine function of  $\xi_{vi}, \xi_{vi'}$ . As such, it is maximized at the extrema of the polytope in  $\mathbb{R}^2$  implied by the capacity and [0,1] constraints involving variables  $\xi_{vi}, \xi_{vi'}$ . As a result, there is a way to transfer equal mass from one of the two variables to the other so that (a) one of them becomes integral (either 0 or 1), (b) the resulting  $\Xi'$  remains feasible, and (c)  $F_{SR}$ does not decrease.<sup>3</sup> Performing this transfer of mass reduces the number of fractional variables in  $\Xi$  by one, while maintaining feasibility and, crucially, either increasing  $F_{SR}$ or keeping it constant. This rounding can be repeated so long as  $\Xi$  remains fractional: this eliminates all fractional variables in at most  $O(|V|\mathcal{C})$  steps. Each step requires at most four evaluations of  $F_{SR}$ , which can be done in O(|V|P) time. Note that the pair of fractional variables selected each time is arbitrary: the order of elimination (i.e., the order with which pairs of fractional variables are rounded) leads to a different rounding, but all such roundings are (a) feasible and, (b) either increase  $F_{\mathtt{SR}}$  or keep it constant.

The routing strategy  $\rho$  can also be rounded in polynomial time, while keeping the caching strategy X fixed:

**Lemma 4.** Given any  $(\rho, \Xi) \in \text{conv}(\mathcal{D}_{SR})$ , an integral r s.t.  $(r, \Xi) \in \text{conv}(\mathcal{D}_{SR})$  and  $F_{SR}(r, \Xi) \geq F_{SR}(\rho, \Xi)$  can be constructed in O(|V|P) time. Moreover, if  $\Xi$  is integral, then the resulting r is a route-to-nearest-replica (RNR) strategy.

Proof. Given  $(\rho, \Xi) \in \text{conv}(\mathcal{D}_{SR})$ , notice that, for fixed  $\Xi$ ,  $F_{SR}$  is an affine function of the routing strategy  $\rho$ . All coefficients involving variables  $\rho_{(i,s),p}$ ,  $p \in \mathcal{P}_{(i,s)}$ , are nonnegative, and the set of constraints on  $\rho$  is separable across requests  $(i,s) \in \mathcal{R}$ . Hence, given  $\Xi$ , maximizing  $F_{SR}$  w.r.t.  $\rho$  can be done by selecting the path  $p^* \in \mathcal{P}_{(i,s)}$  with the highest coefficient of  $F_{SR}$ , for every  $(i,s) \in \mathcal{P}$ ; this is precisely the lowest cost path, i.e.,  $p^*_{(i,s)} \in \mathcal{P}_{(i,s)}$  is such that

$$p_{(i,s)}^* = \underset{p \in \mathcal{P}_{(i,s)}}{\min} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \prod_{k'=1}^k (1 - \xi_{p_{k'}i}).$$
 (36)

Hence, given  $\Xi$ , setting  $\rho_{(i,s),p^*}=1$ , and  $\rho_{(i,s),p}=0$  for all remaining paths  $p\in\mathcal{P}_{(i.s.)}$  s.t.  $p\neq p^*$  can only increase  $F_{\mathtt{SR}}$ . Each  $p^*$  can be computed in  $O(|\mathcal{P}_{(i,s)}||V|)$  time and there is most  $O(\mathcal{R})$  such paths. This results in an integral, feasible strategy r, and the resulting  $F_{\mathtt{SR}}$  either increases or stays constant, i.e.,  $(r,\Xi)\in\mathsf{conv}(\mathcal{D}_{\mathtt{SR}})$  and  $F_{\mathtt{SR}}(r,\Xi)\geq F_{\mathtt{SR}}(\rho,\Xi)$ . Finally, if  $\Xi=X$  for some integral X, then the selection of each strategy  $p^*$  through (36) yields precisely a route-to-nearest-replica routing for (i,s). Note that, in contrast to rounding the caching strategy in the proof of Lemma 3, the order with which routing strategies are rounded does affect the final integral strategy.

Putting everything together, having  $(\rho^*, \Xi^*)$  a solution to (35), we can construct an integral solution (r', X') by:

<sup>&</sup>lt;sup>3</sup>This property is called  $\epsilon$ -convexity by Ageev and Sviridenko [3].

- 1. fixing the routing strategy to  $\rho^*$ , and rounding  $\Xi^*$  to get an integral X' as in Lemma 3, and then
- 2. fixing the caching strategy to X', and rounding  $\rho^*$  to get an integral r' as in Lemma 4.

To conclude the proof of Theorem 2, note that the complexity statement is a consequence of Lemmas 3 and 4, and the fact that (35) reduces to a linear program. By construction, the output of the algorithm (r', X') is such that:

$$F_{SR}(r', X') \geq F_{SR}(\rho^*, \Xi^*).$$

Let  $(r^*, X^*) \in \arg\max_{(r,X) \in \mathcal{D}_{SR}} F_{SR}(r, X)$  be an optimal solution to MaxCG-S. Then, by Lemma 2 and the optimality of  $(\rho^*, X^*)$  in  $\mathsf{conv}(\mathcal{D}_{SR})$ :

$$F_{\text{SR}}(r^*, X^*) \leq L_{\text{SR}}(r^*, X^*) \leq L_{\text{SR}}(\rho^*, \Xi^*) \leq \frac{e}{e-1} F_{\text{SR}}(\rho^*, \Xi^*).$$

Together, these imply that the constructed (r', X') is such that

$$F_{SR}(r', X') \ge (1 - 1/e)F_{SR}(r^*, X^*),$$

and the theorem follows.

# C.2 Reducing (35) to A Linear Program

We show here how to convert problem (35) into a linear program. This can be done by introducing appropriate auxiliary variables. In particular, we introduce an auxiliary variable

$$t_{(i,s),p,k}, (i,s) \in \mathcal{R}, p \in \mathcal{P}_{(i,s)}, k = 1, \dots, |p| - 1$$

for every term in the sum defining function  $L_{SR}$ . Then, the problem (35) is equivalent to:

$$\begin{split} \text{Maximize: } & \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} \sum_{p \in \mathcal{P}_{(i,s)}} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \cdot t_{(i,s),p,k} \\ \text{subj. to: } & t_{(i,s),p,k} \leq 1 - \rho_{(i,s),p} + \sum_{k'=1}^k \xi_{p_{k'}i}, \quad \text{and} \\ & t_{(i,s),p,k} \leq 1 \quad \text{for all } (i,s) \in \mathcal{R}, p \in \mathcal{P}_{(i,s)}, \\ & k = 1, \dots, |p| - 1 \\ & \sum_{p \in \mathcal{P}_{(i,s)}} \rho_{(i,s),p} = 1, \quad \text{for all } (i,s) \in \mathcal{R}, \\ & \sum_{i \in \mathcal{C}} \xi_{vi} \leq c_v, \quad \text{for all } v \in V, \\ & \xi_{vi} \in [0,1], \quad \text{for all } v \in V, i \in \mathcal{C}, \quad \text{and} \\ & \rho_{(i,s),p} \in [0,1], \quad \text{for all } p \in \mathcal{P}_{(i,s)}, (i,s) \in \mathcal{R}. \end{split}$$

There are a total of O(P|V|) auxiliary variables, so this is linear program with  $O(P|V|) + O(|V||\mathcal{C}|)$  variables and  $O(P|V|) + O(|\mathcal{R}|) + O(|V|) + O(|V||\mathcal{C}|) + O(P)$  constraints, so it can be solved in time that is in polynomial P, |V|, and  $|\mathcal{C}|$  [44].

#### C.3 Proof of Corollary 1

Let  $(r^*, X^*)$  be an optimal solution to MaxCG-S in which  $r^*$  is not a RNR strategy. Then, by Lemma 4, we can construct an r' that is an RNR strategy w.r.t. X such that (a)  $F_{SR}(r', X^*) \geq F_{SR}(r^*, X^*)$  and (b)  $(r', X^*) \in \mathcal{D}_{SR}$ . As  $(r^*, X^*)$  is optimal, so is  $(r', X^*)$ .

## C.4 Proof of Theorem 3

Clearly,

$$\min_{(r,X) \in \mathcal{D}_{\mathtt{SR}}} C_{\mathtt{SR}}(r,X) \geq \min_{(\rho,\Xi) \in \mathtt{conv}(\mathcal{D}_{\mathtt{SR}})} C_{\mathtt{SR}}(\rho,\Xi)$$

as  $\mathcal{D}_{\mathtt{SR}} \subset \mathtt{conv}(\mathcal{D}_{\mathtt{SR}}).$  Let

$$(\rho^*,\Xi^*) \in \mathop{\arg\min}_{(\rho,\Xi) \in \mathsf{conv}(\mathcal{D}_\mathtt{SR})} C_\mathtt{SR}(\rho,\Xi) = \mathop{\arg\max}_{(\rho,\Xi) \in \mathsf{conv}(\mathcal{D}_\mathtt{SR})} F_\mathtt{SR}(\rho,\Xi).$$

Then, Lemmas 3 and 4 imply that we can construct an integral  $(r'', X'') \in \mathcal{D}_{SR}$  s.t.

$$F_{SR}(r'', X'') > F_{SR}(\rho^*, \Xi^*),$$
 (37)

which implies that

$$\begin{split} \min_{(r,X)\in\mathcal{D}_{\mathrm{SR}}} C_{\mathrm{SR}}(r,X) &\leq C_{\mathrm{SR}}(r'',X'') \\ &\stackrel{(37)}{\leq} C_{\mathrm{SR}}(\rho^*,\Xi^*) \\ &= \min_{(\rho,\Xi)\in\mathrm{conv}(\mathcal{D}_{\mathrm{SR}})} C_{\mathrm{SR}}(\rho,\Xi), \end{split}$$

and the first equality follows.

To show the second equality, note that for

$$\mu^* \in \underset{\mu: \operatorname{supp}(\mu) = \mathcal{D}_{\operatorname{SR}}}{\operatorname{arg \, min}} \mathbb{E}_{\mu}[C_{\operatorname{SR}}(r, X)],$$

and

$$(r^*, X^*) = \underset{(r, X) \in \mathcal{D}_{SR}}{\arg \min} C_{SR}(r, X),$$

we have that

$$\begin{split} \mathbb{E}_{\mu^*}[C_{\mathtt{SR}}(r,X)] &= \min_{\mu: \mathtt{supp}(\mu) = \mathcal{D}_{\mathtt{SR}}} \mathbb{E}_{\mu}[C_{\mathtt{SR}}(r,X)] \\ &\leq \min_{(r,X) \in \mathcal{D}_{\mathtt{SR}}} C_{\mathtt{SR}}(r,X) = C_{\mathtt{SR}}(r^*,X^*), \end{split}$$

as deterministic strategies are a subset of randomized strategies. On the other hand,

$$\begin{split} \mathbb{E}_{\mu^*}[C_{\text{SR}}(r,X)] &= \sum_{(r,X) \in \mathcal{D}_{\text{SR}}} \mu\left((r,X)\right) C_{\text{SR}}(r,X) \\ &\geq C_{\text{SR}}(r^*,X^*) \sum_{(r,X) \in \mathcal{D}_{\text{SR}}} \mu\left((r,X)\right) = C_{\text{SR}}(r^*,X^*) \end{split}$$

and the second equality also follows.

# D ONLINE SOURCE ROUTING

Before proving Theorem 4, we formally describe in Sections D.1 to D.4 the constituent subgradient estimation, state adaptation, smoothening, and random sampling steps in detail. A modification of Algorithm 1, leading to reduced control traffic, at an increase of the corresponding variance of the subgradient estimate, can be found in Appendix D.7.

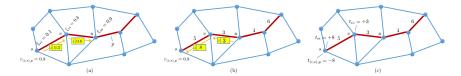


Figure 4: Example of control message trajectory over a single path. When node s generates a request  $(i,s) \in \mathcal{R}$ , it also generates a control message for every path  $p \in \mathcal{P}_{(i,s)}$ . Path p is indicated by red thick edges. In subfigure (a), the control message counter is initialized to  $(1-r_{(i,s),p})+\xi_{si}=1-0.8+0.1=0.3$  by s. It is is forwarded upstream on p to node v, that adds its own caching state variable w.r.t. item i, namely  $\xi_{ui}=0.3$ , to the counter. As the sum is below 1.0, the message is forwarded upstream, until it reaches node u with  $\xi_{ui}=0.9$ . As the total sum is now 1.5>1.0, the propagation over p terminates, and a response is sent downstream by u. The response is shown in subfigure (b), accumulating the weights of edges it traverses. Nodes in its path, namely v and s, sniff this information, as shown in subfigure (c), and collect measurements  $t_{vi}, t_{si}$  to be added to the averages estimating  $\partial_{\xi_{ui}} L_{\rm SR}$  and  $\partial_{\xi_{si}} L_{\rm SR}$ , respectively. The source s also collects measurement  $t_{(i,s),p}=-t_{si}$ , to be used in the average estimating  $\partial_{\rho_{(i,s),p}} L$ .

# D.1 Subgradient Estimation

We now describe how to estimate the subgradients of  $L_{\text{SR}}$  through measurements collected during a timeslot. These estimates are computed in a distributed fashion at each node, using only information available from control messages traversing the node. Let  $(\rho^{(k)}, \Xi^{(k)}) \in \text{conv}(\mathcal{D}_{\text{SR}})$  be the pair of global states at the k-th measurement period. At the conclusion of a timeslot, each  $v \in V$  produces a random vector  $z_v = z_v(\rho^{(k)}, \Xi^{(k)}) \in \mathbb{R}^{|\mathcal{C}|}$  that is an unbiased estimator of a subgradient of  $L_{\text{SR}}$  w.r.t. to  $\xi_v$ . Similarly, for every  $(i,s) \in \mathcal{R}$ , source node s produces a random vector  $q_{(i,s)} = q_{(i,s)}(\rho^{(k)}, \Xi^{(k)}) \in \mathbb{R}^{|\mathcal{P}(i,s)|}$  that is an unbiased estimator of a subgradient of  $L_{\text{SR}}$  with respect to (w.r.t.)  $\rho_{(i,s)}$ . Formally,

$$\mathbb{E}[z_v(\rho^{(k)}, \Xi^{(k)})] \in \partial_{\mathcal{E}_v} L_{SR}(\rho^{(k)}, \Xi^{(k)}), \tag{38}$$

$$\mathbb{E}[q_{(i,s)}(\rho^{(k)}, \Xi^{(k)})] \in \partial_{\rho_{(i,s)}} L_{SR}(\rho^{(k)}, \Xi^{(k)}), \tag{39}$$

where  $\partial_{\xi_v} L_{SR}(\rho, \Xi)$ ,  $\partial_{\rho_{(i,s)}} L_{SR}$  are the sets of subgradients of  $L_{SR}$  w.r.t.  $\xi_v$  and  $\rho_{(i,s)}$ , respectively. To produce these estimates, nodes measure the upstream cost incurred at paths passing through it using control messages, exchanged among nodes as follows:

- 1. Every time a node s generates a new request (i,s), it also generates additional control messages, one per path  $p \in \mathcal{P}_{(i,s)}$ . The message corresponding to path p is to be propagated over p, and contains a counter initialized to  $1 \rho_{(i,s),p} + \xi_{si}$ .
- 2. When following path p, the message is forwarded until a node  $u \in p$  s.t.  $1 \rho_{(i,s),p} + \sum_{\ell=1}^{k_p(u)} \xi_{p_\ell i} > 1$  is found, or the end of the path is reached. To keep track of this, every  $v \in p$  traversed adds its state variable  $\xi_{vi}$  to the message counter.
- 3. Upon reaching either such a node u or the end of the path, the control message is sent down in the reverse direction. Initializing its counter to zero, every time it traverses an edge in this reverse direction, it adds the weight of this edge into a weight counter.
- 4. Every node on the reverse path "sniffs" the weight counter of the control message, learning the sum of weights of all edges further upstream towards u; that is, recalling that  $k_p(v)$  is the position of visited node  $v \in p$ , v learns the quantity:

$$t_{vi} = \sum_{k'=k_v(p)}^{|p|-1} w_{p_{k'+1}p_{k'}} \mathbb{1}_{1-\rho_{(i,s),p} + \sum_{\ell=1}^{k'} \xi_{p_\ell i} \le 1}.$$
 (40)

5. In addition, the source s of the request, upon receiving the message sent over the

reverse path, sniffs the quantity

$$t_{(i,s),p} = -\sum_{k'=1}^{|p|-1} w_{p_{k'+1}p_{k'}} \mathbb{1}_{1-\rho_{(i,s),p} + \sum_{\ell=1}^{k'} \xi_{p_{\ell}} i \le 1}. \tag{41}$$

This is the (negative of) the sum of weights accumulated by the control message returning to the source s.

An example illustrating the above five steps can be found in Figure 4. Let  $\mathcal{T}_{vi}$  be the set of quantities collected in this way at node v regarding item  $i \in \mathcal{C}$  during a measurement period of duration T. At the end of the timeslot, each node  $v \in V$  produces  $z_v$  as follows:

$$z_{vi} = \sum_{t \in \mathcal{T}_{vi}} t/T, \quad i \in \mathcal{C}.$$
 (42)

Similarly, let  $\mathcal{T}_{(i,s),p}$  be the set of quantities collected in this way at source node s regarding path  $p \in \mathcal{P}_{(i,s)}$  during a measurement period of duration T. At the end of the measurement period, s produces the estimate  $q_{(i,s)}$ :

$$q_{(i,s),p} = \sum_{t \in \mathcal{T}_{(i,s),p}} t/T, \quad i \in \mathcal{C}.$$

$$(43)$$

We show that the resulting  $z_v$ ,  $q_{(i,s)}$  satisfy (38) and (39), respectively, in Lemma 5. In the above construction, control messages are sent over all paths in  $\mathcal{P}_{(i,s)}$ . It is important to note however that when sent over paths p such that  $\rho_{(i,s),p} \approx 0$  control messages do not travel far: the termination condition (the sum exceeding 1) is satisfied early on. Messages sent over unlikely paths are thus pruned early, and "deep" propagation only happens in very likely paths. Nevertheless, to reduce control traffic, in Section D.7 we modify the algorithm to propagate only a single control message over a single path.

## D.2 State Adaptation

Having estimates  $Z = [z_v]_{v \in V}, q = [q_{(i,s)}]_{(i,s) \in \mathcal{R}}$ , the global state is adapted as follows: at the conclusion of the k-th period, the new state  $(\rho^{(k+1)}, \Xi^{(k+1)})$  is computed as:

$$\mathcal{P}_{\text{conv}(\mathcal{D}_{\text{SR}})}(\rho^{(k)} + \gamma_k q(\rho^{(k)}, \Xi^{(k)}), \Xi^{(k)} + \gamma_k Z(\rho^{(k)}, \Xi^{(k)})), \tag{44}$$

where  $\gamma_k = 1/\sqrt{k}$  is a gain factor and  $\mathcal{P}_{\text{conv}(\mathcal{D}_{SR})}$  is the orthogonal projection onto the convex set  $\text{conv}(\mathcal{D}_{SR})$ . Note that this additive adaptation and corresponding projection is separable across nodes and can be performed in a distributed fashion: each node  $v \in V$  adapts its own relaxed caching strategy, each source s adapts its routing strategy, and all nodes project these strategies to their respective local constraints implied by (10b),(10a), and the [0,1] constraints. Note that these involve projections onto the rescaled simplex, for which well-known linear algorithms exist [41].

# D.3 State Smoothening.

Upon performing the state adaptation (44), each node  $v \in V$  and each source s, for  $(i, s) \in \mathcal{R}$ , compute the following "sliding averages" of current and past states:

$$\bar{\xi}_v^{(k)} = \sum_{\ell=\lfloor \frac{k}{2} \rfloor}^k \gamma_\ell \xi_v^{(\ell)} / \left[ \sum_{\ell=\lfloor \frac{k}{2} \rfloor}^k \gamma_\ell \right]. \tag{45}$$

$$\bar{\rho}_s^{(k)} = \sum_{\ell=\lfloor \frac{k}{2} \rfloor}^k \gamma_\ell \rho_v^{(\ell)} / \left[ \sum_{\ell=\lfloor \frac{k}{2} \rfloor}^k \gamma_\ell \right]. \tag{46}$$

This is necessary because of the non-differentiability of  $L_{SR}$  [43]. Note that  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)}) \in \text{conv}(\mathcal{D}_{SR})$ , as a convex combination of elements of  $\text{conv}(\mathcal{D}_{SR})$ .

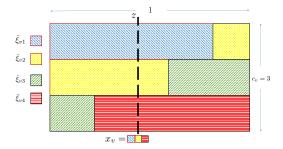


Figure 5: Construction of a feasible randomized caching strategy  $x_v$  that satisfies marginals  $\mathbf{P}[x_{vi}=1]=\bar{\xi}_{vi}$ , where  $\sum_{i\in\mathcal{C}}\bar{\xi}_{vi}=c_v$ . In this example,  $c_v=3$ , and  $\mathcal{C}=\{1,2,3,4\}$ . Given  $\bar{\xi}_v$ , 4 rectangles of height 1 each are constructed, such that the *i*-th rectangle has length  $\bar{\xi}_{vi}\in[0,1]$ , and the total length is  $c_v$ . After placing the 4 rectangles in a  $3\times 1$  box, cutting the box at z selected u.a.r. from [0,1], and constructing a triplet of items from the rectangles it intersects, leads to an integral caching strategy with the desired marginals.

# D.4 Randomized Caching and Routing.

The resulting  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})$  determine the randomized routing and caching strategies at each node during a timeslot. First, given  $\bar{\rho}^{(k)}$ , each time a request (i,s) is generated, path  $p \in \mathcal{P}_{(i,s)}$  is used to route the request with probability  $\bar{\rho}_{(i,s),p}$ , independently of past routing and caching decisions. Second, given  $\bar{\xi}_v^{(k)}$ , each node  $v \in V$  reshuffles its contents, placing items in its cache independently of all other nodes: that is, node v selects a random strategy  $x_v^{(k)} \in \{0,1\}^{|\mathcal{C}|}$  sampled independently of any other node in V. The random strategy  $x_v^{(k)}$  satisfies the following two properties:

- 1. It is a *feasible* strategy, i.e., satisfies the capacity and integrality constraints (10a) and (10c).
- 2. It is consistent with the marginals  $\bar{\xi}_v^{(k)}$ , i.e., for all  $i \in \mathcal{C}$ ,  $\mathbb{E}[x_{vi}^{(k)} \mid \bar{\xi}_v^{(k)}] = \bar{\xi}_{vi}^{(k)}$ . We note that there can be many random caching strategies whose distributions satisfy the

We note that there can be many random caching strategies whose distributions satisfy the above two properties. An efficient algorithm generating such a distribution is provided in [10] and, independently, in [30]. Given  $\bar{\xi}_v^{(k)}$ , a distribution over (deterministic) caching strategies can be computed in  $O(c_v|\mathcal{C}|\log|\mathcal{C}|)$  time, and has  $O(|\mathcal{C}|)$  support; for the sake of completeness, we briefly outline this below. We follow the high-level description of [10] here; a detailed, formal description of the algorithm, a proof of its correctness, and a computational complexity analysis, can be found in [30].

The input to the algorithm are the marginal probabilities  $\bar{\xi}_{vi} \in [0,1], i \in \mathcal{C}$  s.t.  $\sum_{i \in \mathcal{C}} \bar{\xi}_{vi} = c_v$ , where  $c_v \in \mathbb{N}$  is the capacity of cache v. To construct a randomized caching strategy with the desired marginal distribution, consider a rectangle box of area  $c_v \times 1$ , as illustrated in Figure 5. For each  $i \in \mathcal{C}$ , place a rectangle of length  $\bar{\xi}_{vi}$  and height 1 inside the box, starting from the top left corner. If a rectangle does not fit in a row, cut it, and place the remainder in the row immediately below, starting again from the left. As  $\sum_{i \in \mathcal{C}} \bar{\xi}_{vi} = c_v$ , this space-filling method tessellates the  $c_v \times 1$  box. The randomized placement then is constructed as follows: select a value in  $z \in [0,1]$  uniformly at random, and "cut" the box at position z. The value will intersect exactly  $c_v$  distinct rectangles: as  $\bar{\xi}_{vi} \leq 1$ , no rectangle "overlaps" with itself. The algorithm then produces as output the caching strategy  $x_v \in \{0,1\}^{|\mathcal{C}|}$  where:

$$x_{vi} = \begin{cases} 1, & \text{if the line intersects rectangle } i, \\ 0, & \text{o.w.} \end{cases}$$

As the line intersects  $c_v$  distinct rectangles,  $\sum_{i \in \mathcal{C}} x_{vi} = c_v$ , so the caching strategy is

indeed feasible. On the other hand, by construction, the probability that  $x_{vi} = 1$  is exactly equal to the length of the *i*-th rectangle, so the marginal probability that *i* is placed in the cache is indeed:

$$\mathbf{P}[x_{vi} = 1] = \bar{\xi}_{vi}.$$

and the randomized cache strategy  $x_v$  has the desired marginals.

### D.5 Proof of Lemma 1

We first show that (42) and (43) are unbiased estimators of the subgradient:

**Lemma 5.** The vectors  $z_v$ ,  $v \in V$ , and  $q_{(i,s)}$ ,  $(i,s) \in \mathcal{R}$  constructed through coordinates (42) and (43), satisfy properties (38) and (39), respectively. Moreover  $\mathbb{E}[||z_v||_2^2] < C_1$ , and  $\mathbb{E}[||q_{(i,s)}||_2^2] < C_2$ , where

$$C_1 = W^2 \bar{P}^2 |V|^2 |\mathcal{C}| (\Lambda^2 + \frac{\Lambda}{T}), \ C_2 = W^2 |V|^2 P(\Lambda^2 + \frac{\Lambda}{T}),$$

and

$$W = \max_{(i,j)\in E} w_{ij}, \ \bar{P} = \max_{(i,s)\in\mathcal{R}} |\mathcal{P}_{(i,s)}|, \ \text{and} \ \Lambda = \sum_{(i,s)\in\mathcal{R}} \lambda_{(i,s)}.$$

*Proof.* A vector  $\zeta \in \mathbb{R}^{|\mathcal{C}|}$  belongs to  $\partial_{\xi_v} L_{SR}(\rho, \Xi)$  if and only if  $\zeta_i \in [\underline{\partial_{\xi_{vi}} L_{SR}(\rho, \Xi)}, \overline{\partial_{\xi_{vi}} L_{SR}(\rho, \Xi)}]$ , where:

$$\begin{split} \overline{\partial_{\xi_{vi}} L}_{\text{SR}}(\rho, \Xi) &= \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} \sum_{p \in \mathcal{P}_{(i,s)}} \mathbb{1}_{v \in p} \cdot \\ & \sum_{k' = k_p(v)}^{|p| - 1} w_{p_{k'+1} p_{k'}} \mathbb{1}_{1 - \rho_{(i,s)} + \sum_{\ell = 1}^{k'} \xi_{p_{\ell}} i \le 1}, \end{split}$$

$$\begin{split} \frac{\partial_{\xi_{vi}} L_{\text{SR}}(\rho,\Xi) &= \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} \sum_{p \in \mathcal{P}_{(i,s)}} \mathbb{1}_{v \in p} \cdot \\ & \sum_{k' = k_p(v)}^{|p| - 1} w_{p_{k'+1} p_{k'}} \mathbb{1}_{1 - \rho_{(i,s)} + \sum_{\ell=1}^{k'} \xi_{p_{\ell}} i < 1}. \end{split}$$

If  $L_{\text{SR}}$  is differentiable at  $(\rho, \Xi)$  w.r.t  $\xi_{vi}$ , the two limits coincide and are equal to  $\frac{\partial L_{\text{SR}}}{\partial v_i}$ . It immediately follows from the fact that requests are Poisson that  $\mathbb{E}[z_{vi}(\rho,\Xi)] = \overline{\partial_{\xi_{vi}} L_{\text{SR}}}(\rho,\Xi)$ , so indeed  $\mathbb{E}[z_v(Y)] \in \partial_{\xi_v} L_{\text{SR}}(\rho,\Xi)$ . To prove the bound on the second moment, note that, for  $T_{vi}$  the number of requests generated for i that pass through v during the slot,

$$\mathbb{E}[z_{vi}^2] = \frac{1}{T^2} \mathbb{E}[(\sum_{t \in \mathcal{T}_{vi}} t)^2] \leq \frac{W^2 \bar{P}^2 |V|^2}{T^2} \mathbb{E}\Big[T_{vi}^2\Big],$$

as  $t \leq W\bar{P}|V|$ . On the other hand,  $T_{vi}$  is Poisson distributed with expectation

$$\sum_{(i,s)\in\mathcal{R}}\mathbb{1}_{\exists p\in\mathcal{P}_{(i,s)}\text{ s.t. }v\in p}\lambda_{(i,s)}T,$$

and the upper bound follows. The statement for  $q_{(i,s)}$  follows similarly.

We now establish the convergence of the smoothened marginals to a global maximizer of L. Under (44), (45) and (46), from Theorem 14.1.1, page 215 of Nemirofski [43],

$$\varepsilon_k \le \frac{D^2 + M^2 \sum_{\ell = \lfloor k/2 \rfloor}^k \gamma_\ell^2}{2 \sum_{\ell = \lfloor k/2 \rfloor}^k \gamma_\ell},$$

where  $\gamma_k = \frac{1}{\sqrt{k}}$ ,

$$D \equiv \max_{x,y \in \text{conv}(\mathcal{D}_{\text{SR}})} ||x - y||_2 = \sqrt{|V| \max_{v} 2c_v + 2|\mathcal{R}|},$$

and

$$M \equiv \sup_{(\rho,\Xi)} \sqrt{\mathbb{E}[\|Z(\rho,\Xi)\|_{2}^{2}] + \mathbb{E}[\|q(\rho,\Xi)\|_{2}^{2}]}.$$

From Lem. 5,  $M \leq \sqrt{|V|C_1 + |\mathcal{R}|C_2}$ , and the lemma follows.

## D.6 Proof of Theorem 4

By construction, conditioned on  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})$ , the  $|V| + |\mathcal{R}|$  variables  $x_v, v \in V$ , and  $r_{(i,s)}$ , (i,s), are independent. Hence, conditioned on  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})$ , all monomial terms of  $F_{SR}$  involve independent random variables. Hence,

$$\mathbb{E}[F_{\text{SR}}(r^{(k)}, X^{(k)}) \mid \bar{\rho}^{(k)}, \bar{\Xi}^{(k)}] = F_{\text{SR}}(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)}),$$

and, in turn,

$$\lim_{k \to \infty} \mathbb{E}[F_{\mathtt{SR}}(r^{(k)}, X^{(k)})] = \lim_{k \to \infty} \mathbb{E}[F_{\mathtt{SR}}(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})].$$

Lemma 1 implies that, for  $\nu^{(k)}$  the distribution of  $(\bar{\rho}^{(k)}, \bar{\Xi}^{(k)})$ , and  $\Omega$  the set of  $(\rho^*, \Xi^*) \in \text{conv}(\mathcal{D}_{SR})$  that are maximizers of  $L_{SR}$ ,

$$\lim_{k\to\infty}\nu^{(k)}(\operatorname{conv}(\mathcal{D}_{\operatorname{SR}})\setminus\Omega)=0.$$

By Lemma 2,

$$F_{\text{SR}}(\rho^*, \Xi^*) \ge (1 - 1/e) \max_{(r,X) \in \mathcal{D}_{\text{SR}}} F_{\text{SR}}(r,X),$$

for any  $(\rho^*, \Xi^*) \in \Omega$ . The theorem follows from the above observations, and the fact that  $F_{SR}$  is bounded in  $conv(\mathcal{D}_{SR}) \setminus \Omega$ .

## D.7 Reducing Control Traffic.

We now discuss how control messages generated by the protocol can be reduced by modifying the algorithm to propagate only a single control message over a single path with each request. The path is selected uniformly at random over paths in the support of  $\rho_{(i,s)}$ . That is, when a request  $(i,s) \in \mathcal{R}$  arrives at s, a single control message is propagated over p selected uniformly at random from  $\sup(\rho_{(i,s)}) = \{p \in \mathcal{P}_{(i,s)} : \rho_{(i,s),p} > 0\}$ . This reduces the number of control messages generated by s by at least a  $c = |\sup(\rho_{(i,s)})|$  factor. To ensure that (38) and (39) hold, it suffices to rescale measured upstream costs by c. To do this, the (single) control message contains an additional field storing c. When extracting weight counters from downwards packets, nodes on the path compute  $t'_{vi} = c \cdot t_{vi}$ , and  $t'_{(i,s),p} = c \cdot t_{(i,s)p}$ , where  $t_{vi}$ ,  $t_{(i,s),p}$  are as in (40) and (41), respectively. This randomization reduces control traffic, but increases the variance of subgradient estimates, also by a factor of c. This, in turn, slows down the algorithm convergence; this tradeoff can be quantified through, e.g., the constants in Lemma 1.

# E Hop-By-Hop Routing

The proofs for the hop-by-hop setting are similar, *mutatis-mutandis*, as the proofs of the source routing setting. As such, in our exposition below, we focus on the main technical differences between the algorithms for the two settings.

## E.1 Proof of Thm 5 (Sketch)

A constant approximation algorithm for solving MAXCG-HH can be constructed following the same steps as for source routing. Consider the function  $L_{\text{HH}} : \text{conv}(\mathcal{D}_{\text{HH}}) \to \mathbb{R}_+$ , defined as:

$$\begin{split} L_{\text{HH}}(\rho,\Xi) &= \sum_{(i,s) \in \mathcal{R}} \lambda_{(i,s)} \sum_{(u,v) \in G^{(i,s)}} \sum_{p \in \mathcal{P}^u_{(i,s)}} w_{vu} \cdot \\ & \min\{1, 1 - \rho^{(i)}_{uv} + \xi_{ui} + \sum_{k'=1}^{|p|-1} (1 - \rho^{(i)}_{p_{k'}p_{k'+1}} + \xi_{p_{k'}i})\} \end{split}$$

As in Lemma 2, we can show that this concave function approximates  $F_{\mathtt{HH}}$ , in that for all  $(\rho, \Xi) \in \mathtt{conv}(\mathcal{D}_{\mathtt{HH}})$ :

$$(1 - 1/e)L_{\mathrm{HH}}(\rho, \Xi) \le F_{\mathrm{HH}}(\rho, \Xi) \le L_{\mathrm{HH}}(\rho, \Xi). \tag{47}$$

To construct a constant approximation solution, first, a fractional solution

$$(\rho^*, \Xi^*) = \underset{(\rho,\Xi) \in \text{conv}(\mathcal{D}_{\text{HH}})}{\arg \max} L_{\text{HH}}(\rho,\Xi), \tag{48}$$

can be obtained. This again involves a convex optimization, which again can be reduced (as in Section C.2) to a linear program. Subsequently, the solution can be rounded to obtain an integral solution  $(r, X) \in \mathcal{D}_{SR}$  such that  $F_{SR}(r, X) \geq F_{SR}(\rho^*, \Xi^*)$ .

Rounding follows the same steps as for source routing, with an exception for rounding the routing strategy  $\rho^*$ . To round  $\rho^*$ , one first rounds each node's strategy individually, i.e., for every  $v \in V$  and every  $i \in \mathcal{C}$ , we would pick the neighbor that maximizes the objective. This again follows from the fact that, given a caching strategy  $\Xi$ , and given the routing strategies of all other nodes, the objective is an affine function of  $\{r_{uv}^{(i)}\}_{v:(uv)\in E^{(i)}}$ , for all  $u\in V$ , with positive coefficients. Hence, keeping everything else fixed, if each node chooses a cost minimizing decision, this will round its strategy, and all nodes in V can do this sequentially. Note that, contrary to source routing, the order with which this rounding happens affects the final integral solution. Moreover, the DAG property ensures that all requests eventually reach a designated server, irrespectively of the routing strategies resulting from the rounding decisions.

# E.2 Proof of Theorem 6 (Sketch)

A distributed algorithm can be constructed by performing projected gradient ascent over  $L_{\rm HH}$ . Beyond the same caching state variables  $\xi_v$  stored at each node  $v \in V$ , each node  $v \in V$  maintains routing state variables  $\rho_u^{(i)} = [\rho_{uv}^{(i)}]_{v:(u,v)\in E^{(i)}} \in [0,1]^{|E^{(i)}|}$ , for each  $i \in \mathcal{C}$ , containing the marginal probabilities  $\rho_{uv}^{(i)}$  that u routes request message for item i towards  $v \in E^{(i)}$ . Time is slotted, and nodes perform subgradient estimation, state adaptation, state smoothening, and randomized sampling of caching and routing strategies. As the last three steps are nearly identical to source routing, we focus below on how to estimate subgradients, which is the key difference between the two algorithms.

Whenever a request  $(i,s) \in \mathcal{R}$  is generated, a control message is propagated in all neighbors of s in  $E^{(i)}$ . These messages contain counters initialized to  $1-\rho_{sv}^{(i)}+\xi_{si}$ . Each node  $v \in V$  receiving such a message generates one copy for each of its neighbors in  $E^{(i)}$ . For each neighbor u,v adds  $1-\rho_{vu}^{(i)}+\xi_{vi}$  to the counter, and forwards the message to u if the counter is below 1.0. Formally, a control message originating at s and reaching a node v after having followed path  $p \in G^{(i,s)}$  is forwarded to u if the following condition is satisfied:

$$1 - \rho_{vu}^{(i)} + \xi_{ui} + \sum_{\ell=1}^{k_p(v)-1} \left(1 - \rho_{p_{\ell}p_{\ell+1}}^{(i)} + \xi_{p_{\ell}i}\right) \le 1$$
 (49)

If this condition is met, v forwards a copy of the control message to u; the above process is repeated at each of each neighbors. If the condition fails  $for\ all\$ neighbors, a response message is generated by v and propagated over the reverse path, accumulating the weights of edges it passes through. Moreover, descending control messages are merged as follows. Each node v waits for all responses from neighbors to which it has sent control messages; upon the last arrival, it adds their counters, and sends the "merged" message containing the accumulated counter reversely over path v.

As before, messages on the return path are again "sniffed" by nodes they pass through, extracting the upstream costs. Their averages are used as estimators of the subgradients w.r.t. both the local routing and caching states, in a manner similar to how this was performed in source routing. As each edge is traversed at most twice, the maximum number of control messages is  $O(|E^{(i)}|)$ . As in the case of source routing, however, messages on low-probability paths are pruned early on due to condition (49). Moreover, as in Section D.7, randomization can again reduce the number of messages: only a single message need be propagated to a neighbor selected uniformly at random; in this case, the message needs to also contain a field keeping track of the product of the size of neighborhoods of nodes it has passed through, and updated by each node by multiplying the entry by the size of its own neighborhood. As in source routing, this is used as an additional scaling factor for quantities  $t_{vu}^{(i)}$ ,  $t_{vi}$ .

# F Detailed Description of Graphs in Table 2

Graph cycle is a simple cyclic graph; grid-2d is a two-dimensional square grid; hypercube is a 7-dimensional hypercube. Graph expander is a Margulies-Gabber-Galil expander [26]. The next 5 graphs are random, i.e., were sampled from a probability distribution. Graph erdos-renyi is an Erdős-Rényi graph with parameter p=0.1, and regular is a 3-regular graph sampled uniformly at random (u.a.r.). The watts-strogatz graph is generated according to the Watts-Strogatz model of a small-world network [55], i.e., a cycle and 4 randomly selected edges, while small-world is the graph by Kleinberg [34], i.e., a grid with additional long range edges. The preferential attachment model of Barabási and Albert [6], which yields powerlaw degrees, is used for barabasi-albert. The last 3 graphs are the GEANT, Abilene, and Deutche Telekom backbone networks [49].