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COMMONSENSE REASONING BASED ON FUZZY LOGIC

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ABSTRACT

Although most human reasoning is approximate rather than precise in nature, traditional logical systems focus almost exclusively on those modes of reasoning which lend themselves to precise formalization. In recent years, however, in our attempt to design systems which are capable of performing tasks requiring a high level of cognitive skill, it has become increasingly clear that in order to attain this goal we need logical systems which can deal with knowledge that is imprecise, incomplete or not totally reliable.

Prominent among the systems which have been suggested for this purpose are those based on default reasoning (Reiter, 1983), circumscription (McCarthy, 1980), nonmonotonic reasoning (McDermott and Doyle, 1980, 1982), and probabilistic logic (Nilsson, 1984). These and related systems are basically extensions of first-order predicate calculus and probability theory, and are rooted in bivalent logic.

In a departure from reliance on bivalent logical systems, we have developed an approach to commonsense reasoning based on fuzzy logic (Zadeh, 1983, 1984). In this approach, a central role is played by the concept of *dispositionality* and the closely related concept of *usuality*. Furthermore, an extensive use is made of syllogistic reasoning (Zadeh, 1985), in which the premises are propositions containing fuzzy quantifiers such as *most*, *many*, *usually*, etc.

The point of departure in the fuzzy-logic-based approach to commonsense reasoning is the assumption that commonsense knowledge consists for the most part of dispositions, that is, propositions which are preponderantly, but not necessarily always, true. For example:

Birds can fly.
Slimness is attractive.
Glue is sticky if it is not dry.

Typically, a disposition contains one or more implicit fuzzy quantifiers. For example, *birds can fly* may be interpreted as *most birds can fly* or, equivalently, as *usually (if X is a bird then X can fly)*. A proposition of the general form *usually (p)* or *usually (if p then q)*, where p and q are propositions, is said to be *usuality-qualified*. In this sense, commonsense knowledge may be viewed as a collection of usuality-qualified propositions in which the fuzzy quantifier *usually* is typically implicit rather than explicit.

Our approach to inference from commonsense knowledge may be viewed as an application of fuzzy logic, under the assumption that a disposition may be expressed in the canonical form QA's are B's, where Q is a fuzzy quantifier, e.g., *most, almost all, usually*, etc., and A and B are fuzzy predicates such as *small, tall, slim, young*, etc. Fuzzy logic provides a basis for inference from dispositions of this type through the use of *fuzzy syllogistic reasoning* (Zadeh, 1985). As the name implies, fuzzy syllogistic reasoning is an extension of classical syllogistic reasoning to fuzzy predicates and fuzzy quantifiers. In its generic form, a fuzzy syllogism may be expressed as the inference schema

$$\begin{array}{l} Q_1A\text{'s are }B\text{'s} \\ \underline{Q_2C\text{'s are }D\text{'s}} \\ Q_3E\text{'s are }F\text{'s} \end{array}$$

in which A, B, C, D, E and F are interrelated fuzzy predicates and Q_1, Q_2 and Q_3 are fuzzy quantifiers.

The interrelations between A,B,C,D,E and F provide a basis for a classification of fuzzy syllogisms. The more important of these syllogisms are the following ($\wedge \triangleq$ *conjunction*, $\vee \triangleq$ *disjunction*):

(a) *Intersection/product syllogism:*

$$C = A \wedge B,\ E = A,\ F = C \wedge D$$

(b) *Chaining syllogism:*

$$C = B,\ E = A,\ F = D$$

(c) *Consequent conjunction syllogism:*

$$A = C = E,\ F = B \wedge D$$

(d) *Consequent disjunction syllogism:*

$$A = C = E,\ F = B \vee D$$

(e) *Antecedent conjunction syllogism:*

$$B=D=F,\ E=A \wedge C$$

(f) *Antecedent disjunction syllogism:*

$$B=D=F,\ E=A \vee C$$

In the context of expert systems, these and related syllogisms provide a set of inference rules for combining evidence through conjunction, disjunction and chaining (Zadeh, 1983).

One of the basic problems in fuzzy syllogistic reasoning is the following: Given A, B, C, D, E and F, find the maximally specific (i.e., most restrictive) fuzzy quantifier Q_3 such that the proposition Q_3 *E's are F's* is entailed by the premises. In the case of (a), (b) and (c), this leads to the following syllogisms:

Intersection/Product Syllogism.

$$Q_1\,A\text{'s are }B\text{'s} \tag{1}$$

$$\underline{Q_2\,(A \text{ and } B)\text{'s are } C\text{'s}}$$

$$(Q_1 \otimes Q_2)A\text{'s are }(B \text{ and } C)\text{'s}$$

where $\otimes$ denotes the product in fuzzy arithmetic (Kaufmann and Gupta, 1985). It should be noted that (4.1) may be viewed as an analog of the basic probabilistic identity

$$p(B,C/A) = p(B/A)p(C/A,B)$$

A concrete example of the intersection/product syllogism is the following:

most students are young (2)

most young students are single

most^2 *students are young* and *single*

where *most*^{2} denotes the product of the fuzzy quantifier *most* with itself.

Chaining Syllogism.

$$Q_1A\text{'s are }B\text{'s}$$

$$\underline{Q_2B\text{'s are }C\text{'s}}$$

$$(Q_1 \otimes Q_2)A\text{'s are }C\text{'s}$$

This syllogism may be viewed as a special case of the intersection product syllogism. It results when $B \subset A$ and Q_1 and Q_2 are monotone increasing, that is, $\geq Q_1 = Q_1$, and $\geq Q_2 = Q_2$,where $\geq Q_1$ should be read as *at least* Q_1, and likewise for Q_2. A simple example of the chaining syllogism is the following:

most students are undergraduates

most undergraduates are single

most^2 *students are single*

Note that *undergraduates* $\subset$ *students* and that in the conclusion F = *single*, rather than *young and single*, as in (2).

Consequent Conjunction Syllogism.

The consequent conjunction syllogism is a example of a basic syllogism which is not a derivative of the intersection/product syllogism. Its statement may be expressed as follows:

$$Q_1A\text{'s are }B\text{'s} \tag{3}$$

$$\underline{Q_2A\text{'s are }C\text{'s}}$$

$$Q\ A\text{'s are }(B \text{ and } C)\text{'s},$$

where Q is a fuzzy quantifier which is defined by the inequalities

$$0 \circledvee (Q_1 \oplus Q_2 \ominus 1) \leq Q \leq Q_1 \circledwedge Q_2 \tag{4}$$

in which $\circledvee$, $\circledwedge$, $\oplus$, and $\ominus$ are the operations of $\vee$ (max), $\wedge$ (min), + and - in fuzzy arithmetic.

An illustration of (3) is provided by the example

most students are young

most students are single

Q *students are single* and *young*

where

$$2most \ominus 1 \leq Q \leq most.$$

This expression for Q follows from (4) by noting that

$$most \circledwedge most = most$$

and

$$0 \circledvee (2most \ominus 1) = 2most \ominus 1$$

The three basic syllogisms stated above are merely examples of a collection of fuzzy syllogisms which may be developed and employed for purposes of inference from commonsense knowledge. In addition to its application to commonsense reasoning, fuzzy syllogistic reasoning may serve to provide a basis of rules for combining uncertain evidence in expert systems (Zadeh, 1983).

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