Memetic Multilevel Hypergraph Partitioning

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Abstract. Hypergraph partitioning has a wide range of important applications such as VLSI design or scientific computing. With focus on solution quality, we develop the first multilevel memetic algorithm to tackle the problem. Key components of our contribution are new effective multilevel recombination and mutation operations that provide a large amount of diversity. We perform a wide range of experiments on a benchmark set containing instances from application areas such VLSI, SAT solving, social networks, and scientific computing. Compared to the state-of-the-art hypergraph partitioning tools hMetis, PaToH, and KaHyPar, our new algorithm computes the best result on almost all instances.

1 Introduction

Given an undirected hypergraph H=(V,E), the k-way hypergraph partitioning problem is to find disjoint subsets of its vertex set, V_1,\ldots,V_k , called blocks, such that the blocks have roughly equal size and an objective function involving the cut hyperedges is minimized, e.g., the sum of the weights of those hyperedges that connect multiple blocks. The hypergraph partitioning problem has many important applications in practice such as scientific computing or VLSI design [44]. In particular VLSI design is a field where small improvements can lead to significant savings [60]. Hence, our focus in this work is on solution quality.

It is well known that the hypergraph partitioning problem (HGP) is NP-hard [40] so that mostly heuristic algorithms are used in practice. A successful heuristic to partition large hypergraphs is the *multilevel* approach [16]. Here, the hypergraph is recursively *contracted* to obtain smaller hypergraphs which should reflect the same basic structure as the input. After applying an *initial partitioning* algorithm to the smallest hypergraph, contraction is undone and, at each level, a *local search* method is used to improve the partitioning induced by the coarser level. The intuition behind this approach is that a good partition at one level of the hierarchy will also be a good partition on the next finer level. Hence, depending on the definition of the neighborhood, local search algorithms are able explore local solution spaces very effectively in this setting. However, they are also prone to get stuck in local optima [36]. The multilevel paradigm helps to some extent, since local search has a more global view on the problem on the coarse levels and a very fine-grained view on the fine levels of the multilevel hierarchy. In addition, as with many other metaheuristics, multilevel HGP gives better results if several repeated runs are made with some measures taken to diversify the search.

Still even a large number of repeated executions can only scratch the surface of the huge space of possible partitionings. In order to explore the global solution space extensively we need more sophisticated metaheuristics. This is where memetic algorithms (MAs), i.e., genetic algorithms combined with local search [37], come into play. Memetic algorithms allow for effective exploration (global search) and exploitation (local search) of the solution space. The general idea behind genetic algorithms is to use mechanisms inspired by biological evolution such as selection, mutation, recombination and survival of the fittest. A genetic algorithm (GA) starts with a population of individuals (in our case partitions of the hypergraph) and evolves the population over several generational cycles (rounds). In each round, the GA uses a selection rule based on the fitness of the individuals of the population to select good individuals and combines them to obtain improved offspring [29]. When an offspring is generated an eviction rule is used to select a member of the population to be replaced by the new offspring. For an evolutionary algorithm it is of major importance to preserve diversity in the population [10], i.e., the individuals should not become too similar in order to avoid a premature convergence of the algorithm. This is usually achieved by using mutation operations and by using eviction rules that take similarity of individuals into account.

Several genetic and memetic hypergraph partitioning algorithms have already been proposed in the literature [6,7,8,15,38]. However *none of them* is considered to be truly competitive with state-of-the-art tools [19]. We believe that this is due to the fact that all of them employ *flat* (i.e., non-multilevel) partitioning algorithms to drive the exploitation of the local solution space.

Our *main contribution* in this paper therefore is a technique that integrates a memetic algorithm with a *multilevel* hypergraph partitioner. To this end, we present sophisticated recombination and mutation operators as well as a replacement rule that uses a problem specific similarity measure. In contrast to previous work [6,7,8,15,38], which only considered small and outdated [1,3] ACM/SIGDA benchmark instances [46] (dating back to the late 1980s), we perform extensive experiments on a large benchmark set containing hypergraphs from several application areas. Our experiments indicate that our algorithm is able to compute partitions of very high quality and scales well to large networks. It performs better than KaHyPar, which seems to be the current method of choice among the available hypergraph partitioning tools unless speed

is more important than quality [30], and the state-of-the-art HGP tools hMetis [35,36] and PaToH [17]. In a setting where competing algorithms get the same fairly large amount of time to compute a solution, our new algorithm computes the best result on 648 out of the 700 benchmark instances. This is in contrast to previous *non-multilevel* evolutionary algorithms for the problem, which are not considered to be competitive with state-of-the-art tools [19].

2 Preliminaries

Notation and Definitions. An undirected hypergraph $H = (V, E, c, \omega)$ is defined as a set of n vertices V and a set of m hyperedges/nets E with vertex weights $c: V \to \mathbb{R}_{>0}$ and net weights $\omega: E \to \mathbb{R}_{>0}$, where each net is a subset of the vertex set V (i.e., $e \subseteq V$). The vertices of a net are called *pins*. We extend c and ω to sets, i.e., $c(U) := \sum_{v \in U} c(v)$ and $\omega(F) := \sum_{e \in F} \omega(e)$. A vertex v is incident to a net e if $v \in e$. I(v) denotes the set of all incident nets of v. The set $\Gamma(v) := \{u \mid \exists e \in E : \{v, u\} \subseteq e\}$ denotes the neighbors of v. The size |e| of a net e is the number of its pins. A k-way partition of a hypergraph H is a partition of its vertex set into k blocks $\Pi = \{V_1, \dots, V_k\}$ such that $\bigcup_{i=1}^k V_i = V$, $V_i \neq \emptyset$ for $1 \leq i \leq k$ and $V_i \cap V_j = \emptyset$ for $i \neq j$. We use b[v] to refer to the block of vertex v. We call a k-way partition Π ε -balanced if each block $V_i \in \Pi$ satisfies the balance constraint: $c(V_i) \leq L_{\max} := (1+\varepsilon) \lceil \frac{c(V)}{k} \rceil$ for some parameter ε . Given a k-way partition Π , the number of pins of a net e in block V_i is defined as $\Phi(e,V_i) := |\{v \in V_i \mid v \in e\}|$. For each net $e, \Lambda(e) := \{V_i \mid \Phi(e,V_i) > 0\}$ denotes the connectivity set of e. The connectivity of a net e is the cardinality of its connectivity set: $\lambda(e) := |\Lambda(e)|$. A net is called cut net if $\lambda(e) > 1$. The k-way hypergraph partitioning problem is to find an ε -balanced k-way partition Π of a hypergraph H that minimizes an objective function over the cut nets for some ε . Several objective functions exist in the literature [5,40]. The most commonly used cost functions are the cut-net metric $\operatorname{cut}(\Pi) := \sum_{e \in E'} \omega(e)$ and the *connectivity* metric $(\lambda - 1)(\Pi) := \sum_{e \in E'} (\lambda(e) - 1) \omega(e)$, where E'is the set of all cut nets [24,26]. Optimizing both objective functions is known to be NP-hard [40]. In this paper, we use the connectivity-metric $(\lambda - 1)(\Pi)$. Contracting a pair of vertices (u, v) means merging v into u. The weight of u becomes c(u) := c(u) + c(v). We connect u to the former neighbors $\Gamma(v)$ of v by replacing v with u in all nets $e \in I(v) \setminus I(u)$ and remove v from all nets $e \in I(u) \cap I(v)$. Uncontracting a vertex u reverses the contraction.

2.1 Related Work

Overview. Driven by applications in VLSI design and scientific computing, HGP has evolved into a broad research area since the 1990s. We refer to [5,11,44,54] for an extensive overview. In the following, we focus on issues closely related to the contributions of our paper. Memetic algorithms (MAs) were introduced in [42] and formalized in [45] as an extension to the concept of genetic algorithms (GAs) [31]. While GAs effectively explore the *global* solution space, MAs additionally allow for exploitation of the *local* solution space by incorporating local search methods into the genetic framework. We refer to [43] for an introduction to memetic algorithms. While several genetic and memetic flat (i.e., non-multilevel) hypergraph partitioning algorithms have been proposed in the literature, *none of them* is considered to be truly competitive with state-of-the-art tools [19]. Well-known multilevel HGP software packages with certain distinguishing characteristics include PaToH [17] (originating from scientific computing), hMetis [35,36] (originating from VLSI design), KaHyPar [2,30,52] (general purpose, *n*-level), Mondriaan [57] (sparse matrix partitioning), MLPart [4] (circuit partitioning), Zoltan [25], Parkway [55], and SHP [34] (distributed), UMPa [56] (directed hypergraph model, multi-objective), and kPaToH (multiple constraints, fixed vertices) [9].

Evolutionary Hypergraph Partitioning. Saab and Rao [47] present an evolution-based approach for solving a k-way multi-objective, multi-constraint hypergraph partitioning problem. Since the algorithm only works with one individual, it does not use any recombination operators. Instead, the solution initially generated via bin packing is evolved using a randomized algorithm that moves vertices to different blocks if their gain is greater than some random value. Hulin [33] provides a GA that uses a coding scheme specifically tailored to circuit bipartitioning along with crossover and mutation operations that respect the coding. Bui and Moon [15] present a steady-state MA for ratio cut bipartitioning of hypergraphs, which uses a weak variation of the FM algorithm [28] as local search engine. To improve the performance of the crossover operation, a preprocessing step re-indexes the vertices by the visiting order of a weighted depth first search on the clique-representation [32] of the hypergraph. Areibi [6] present a memetic algorithm that combines a GA with a modified version of Sanchis' k-way FM algorithm [48]. Areibi and Yang [7] enhance the MA presented in [6] with a preprocessing step that clusters and contracts vertices to reduce the complexity of the hypergraphs. Furthermore, the initial population contains both random as well as good solutions generated using the GRASP heuristic [27]. Armstrong et al. [8] propose a k-way MA that performs crossover, mutation and local search on multiple individuals in parallel. The traditional FM algorithm [28] and Sanchis' kway FM version [48] are used for local search. Kim et al. [38] present a steady-state MA for hypergraph bipartitioning, which uses a modified FM algorithm that works with lock-gains [39]. Note that none of these algorithms makes use of the multilevel paradigm.

Evolutionary Graph Partitioning. We refer to the survey of Kim et al. [37] for a general overview and more material on genetic approaches for graph partitioning. Soper et al. [53] provide the first algorithm that combined an evolutionary search algorithm within a multilevel graph partitioner. Here, crossover and mutation operators compute edge biases based on the input individuals. A similar approach based on perturbations of edge weights has been used by Delling et al. [23]. Benlic et al. [13] provide a multilevel memetic algorithm for balanced graph partitioning. PROBE [18] is a metaheuristic which can be viewed as a genetic algorithm without selection. It outperforms other metaheuristics, but it is restricted to the case k=2 and $\varepsilon=0$. KaHIP [51] contains KaFFPaE [50], which has a general recombine operator framework based on a multilevel algorithm.

2.2 k-way Hypergraph Partitioning using KaHyPar

Since our memetic algorithm builds on top of the KaHyPar framework, we briefly review its core components. While traditional multilevel HGP algorithms contract matchings or clusterings and therefore work with a coarsening hierarchy of $\mathcal{O}(\log n)$ levels, KaHyPar instantiates the multilevel paradigm in the extreme n-level version, removing only a single vertex between two levels. Vertex pairs (u,v) to be contracted are determined using the heavy-edge rating function $r(u,v) := \sum_{e \in E'} \omega(e)/(|e|-1)$, where $E' := \{I(u) \cap I(v)\}$. The coarsening process stops as soon as the number of vertices drops below a certain threshold or no more contractions are possible. The framework currently contains two coarsening algorithms. The first algorithm [52] contracts vertices in decreasing rating score order using a priority queue to store and update the ratings. The second algorithm [2] immediately contracts each vertex with its highest-rated neighbor in random order. After coarsening, a portfolio of simple algorithms is used to create an initial partition of the coarsest hypergraph. During uncoarsening, strong localized local search heuristics based on the FM algorithm [28,48] are used to refine the solution by moving vertices to other blocks in the order of improvements in the optimization objective. The framework provides a recursive bisection algorithm to optimize the cutnet metric (KaHyPar-R [52]) as well as a direct k-way algorithm to optimize the $(\lambda-1)$ metric (KaHyPar-K [2]). Recently, Heuer and Schlag [30] integrated an improved coarsening scheme into KaHyPar-K that

incorporates global information about the structure of the hypergraph into the coarsening process. It uses community detection in a preprocessing step and prevents inter-community contractions during coarsening. This version is referred to as KaHyPar-CA. Unless mentioned otherwise, we use the default configurations provided by the authors¹.

3 Memetic Multilevel Hypergraph Partitioning

We now explain the components of our memetic multilevel hypergraph partitioning algorithm. Given a hypergraph H and a time limit t, the algorithm starts by creating an initial population of \mathcal{P} individuals, which in our case correspond to ε -balanced k-way partitions of H. The population size $|\mathcal{P}|$ is determined dynamically by first measuring the time t_I spend to create one individual. Then \mathcal{P} is chosen such that the time to create $|\mathcal{P}|$ individuals is a certain percentage δ of the total running time t: $|\mathcal{P}| := \max(3, \min(50, \delta \cdot (t/t_I)))$, where δ is a tuning parameter. The lower bound on the population size is chosen to ensure a certain minimum of diversity, while the upper bound is used to ensure convergence. In contrast to previous approaches [6,8,15,33,38] the population is not filled with randomly generated individuals, but high-quality solutions computed by KaHyPar-CA.

To judge the *fitness* of an individual we use the connectivity $(\lambda-1)(\Pi)$ of its partition Π . The initial population is evolved over several generational cycles using the *steady-state* paradigm [22], i.e., we generate only *one* offspring per generation. Our two-point and multi-point recombination operators described in Section 3.1 improve the average quality of the population by effectively combining different solutions to the HGP problem. In order to sufficiently explore the global search space and to prevent premature convergence, it is important to keep the population diverse [10]. This becomes even more relevant in our case, since with KaHyPar-CA we use powerful heuristics to exploit the local solution space. Previous work on evolutionary algorithms for HGP [7,8,15,33,38] used simple mutations that change the block of each vertex uniformly at random with a small probability. In contrast to these simple, problem agnostic operators, we propose mutation operators based on V-cycles [59] that exploit knowledge of the problem domain and create offspring solutions in the *vicinity* of the current population. Furthermore in Section 3.3 we propose a replacement strategy which considers fitness *and* similarity to determine the individual to be evicted from the population.

3.1 Recombination Operators

The evolutionary algorithms for HGP presented in Section 2.1 use simple multi-point crossover operators which split the parent partitions into several parts and then combine these parts to form new offspring (see Figure 1 (a)). Since these operators do not take the structure of the hypergraph into account, offspring solutions may have considerably worse fitness than their parents. By generalizing the recombine operator framework presented in [50] from graphs to hypergraphs, our two-point recombine operators described in this section assure that the fitness of the offspring is *at least as good as the best of both parents*. The edge frequency based multi-point recombination operator described afterwards gives up this property, but still generates good offspring.

Two-Point Recombine. The operator starts with selecting parents for recombination using binary tournament selection (without replacement) [14]. Two individuals I_1 and I_2 are chosen uniformly at random from \mathcal{P} and the individual with better fitness (i.e., lower $(\lambda - 1)$ objective) becomes the first parent P_1 . This process is then repeated to determine the second parent P_2 . A tournament size of two is chosen to keep the selection pressure low and to avoid premature convergence, since all our individuals already constitute high-quality solutions. Both individuals/partitions are then used as input of a modified multilevel partitioning scheme as

https://qithub.com/SebastianSchlag/kahypar/tree/master/config

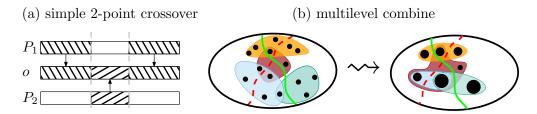


Fig. 1: (a) Traditional, problem agnostic crossover operation to combine parent partitions P_1 and P_2 to offspring o. (b) Recombination using modified multilevel coarsening to combine two partitions (dashed red line and solid green line). Each cut net e remains in the coarse hypergraph and maintains its connectivity $\lambda(e)$ regarding both partitions.

follows: During coarsening, two vertices u and v are only allowed to be contracted if both parents agree on the block assignment of both vertices, i.e., if $b_1[u] = b_1[v] \wedge b_2[u] = b_2[v]$. This is a generalization from multilevel evolutionary GP, i.e. [50], where edges running between two blocks are not eligible for contraction and therefore remain in the graph. In other words, our generalization allows two vertices of the same cut net to be contracted as long as the input individuals agree that they belong to the same block. For HGP, this restriction ensures that cut nets e remain in the coarsened hypergraph and maintain their connectivity $\lambda(e)$ regarding both partitions. This modification is important for our optimization objective, because it allows us to use the partition of the better parent as initial partition of the offspring. Since we can skip the initial partitioning phase and therefore do not need a sufficiently large number of vertices in the coarsest hypergraph to compute a good initial partition [36], we alter the stopping criterion of the coarsening phase such that it stops when no more contractions are possible. The high quality solution of the coarsest hypergraph contains two different classes of vertices: Those for which both parent partitions agree on a block assignment and those for which they don't (see Figure 1 (b) for an example). During the uncoarsening phase, local search algorithms can then use this initial partitioning to (i) exchange good parts of the solution on the coarse levels by moving few vertices and (ii) to find the best block assignment for those vertices, for which the parent partitions disagreed. Since KaHyPar's refinement algorithms guarantee nondecreasing solution quality, the fitness of offspring solutions generated using this kind of recombination is always at least as good as the better of both parents.

Edge-Frequency Multi-Recombine. The operator described previously is restricted to recombine p=2 partitions to improved offspring of nondecreasing quality. Sanders and Schulz [50] specifically restrict their operators to this case, arguing that in the course of the algorithm a series of two-point recombine operations to some extend emulates a multi-point recombine. We here present a reasonable multi-point recombine operation to partially evaluate this hypothesis in our experimental evaluation. Our recombine operator uses the concept of (hyper)edge frequency [60] to pass information about the cut nets of the t best individuals in the population on to new offspring. The frequency f(e) of a net e hereby refers to the number of times it appears in the cut in the t best solutions: $f(e) := |\{I \in t \mid \lambda(e) > 1\}|$. We use $t = \lceil \sqrt{|\mathcal{P}|} \rceil$, which is a common value in evolutionary algorithms [23]. Our multi-recombine operator then uses this information to create a new individual in the following way. The coarsening algorithm is modified to prefer to contract vertex pairs (u,v) which share a large number of small, low-frequency nets. This is achieved by replacing the standard heavy-edge rating function of KaHyPar with the rating function [60] shown in Eq. 1:

$$r(u,v) := \frac{1}{c(v) \cdot c(u)} \sum_{e \in \{I(v) \cap I(u)\}} \frac{\exp(-\gamma f(e))}{|e|}.$$
 (1)

This rating function disfavors the contraction of vertex pairs incident to cut nets with high frequency, because these nets are likely to appear in the cut of high quality solutions. The tuning parameter γ is used as a damping factor. After coarsening stops, we compute an initial partition of the coarsest hypergraph using KaHyPar's initial partitioning algorithms and refine it during the uncoarsening and local search phase.

3.2 Mutation Operations and Diversification

We define two mutation operators based on V-cycles. All operators are applied to a random individual I of the current population. The main idea of V-cycle based mutation operators is to reuse an already computed partition as input for the multilevel approach and to iterate coarsening and local search phases several times using different seeds for randomization. This approach has been applied successfully in evolutionary GP [50], therefore we also adopt it for HGP. Similar to the recombine operator described in Section 3.1, the quality of the solution is maintained by only contracting vertex pairs (u,v) belonging to the same block (b[u]=b[v]). By distinguishing two possibilities for initial partitioning, we define two different mutation operators: The first one uses the current partition of the individual as initial partition of the coarsest hypergraph and guarantees nondecreasing solution quality. The second one employs KaHyPar's portfolio of initial partitioning algorithms to compute a new solution for the coarsest hypergraph. During uncoarsening, local search algorithms improve the solution quality and thereby further mutate the individual. Since the second operator computes a new initial partition which might be different from the original partition of I, the fitness of offspring generated by this operator can be worse than the fitness of I.

3.3 Replacement Strategy

All recombination and mutation operators create one new offspring o. In order to keep the population diverse, we evict the individual $most\ similar$ to the offspring among all individuals whose fitness is equal to or worse than o. Previous work on bipartitioning [15,38] used the Hamming distance as a metric to measure the similarity between partitions. We propose a more sophisticated similarity measure that takes into account the connectivity $\lambda(e)$ of each cut net e. For each individual, we compute the multi-set $D:=\{(e,m(e)):e\in E\}$, where $m(e):=\lambda(e)-1$ is the multiplicity (i.e. number of occurrences) of e. Thus each cut net e is represented $\lambda(e)-1$ times in e. The difference of two individuals e and e is the computed as e and e is the symmetric difference.

4 Experimental Evaluation

System and Methodology. We implemented the memetic algorithm described in the previous section using the latest version of the KaHyPar framework. The code is written in C++ and compiled using g++-5.2 with flags -03 -mtune=native -march=native. We refer to the algorithm presented in this paper as EvoHGP. All experiments are performed on a cluster with 512 nodes, where each node has two Intel Xeon E5-2670 Octa-Core (Sandy Bridge) processors clocked at 2.6 GHz, 64 GB main memory, 20 MB L3- and 8x256 KB L2-Cache and runs RHEL 7.4.

We compare EvoHGP with two different configurations of KaHy-Par-CA [30], as well as to the *k*-way (hMetis-K) and the recursive bisection variant (hMetis-R) of hMetis 2.0 (p1) [35,36], and to PaToH 3.2 [17]. These HGP libraries were chosen because they provide the best solution quality [30,2]. The first configuration of KaHyPar-CA corresponds to the default configuration as described in [30]. Since it is known that global search strategies are more effective than plain restarts [49], we augment KaHyPar-CA with V-cycles (in a similar fashion as the first mutation operator) using a maximum number of 100 V-cycle iterations

per partitioner call. This *new* enhanced version of KaHyPar-CA constitutes the second configuration and is referred to as KaHyPar-CA-V. hMetis and PaToH are configured as described in [30]. Since PaToH ignores the random seed if configured to use the quality preset, we only report result of the default configuration (PaToH-D). For all five algorithms we perform repeated runs using different random seeds for each run.

To evaluate the impact of different algorithmic components of our algorithm in Section 4.1 each Evo-HGP configuration gets *two* hours time *per* test instance to compute a solution. For the final evaluation in Section 4.2 all algorithms get *eight* hours time per test instance. In both cases, we perform five repetitions with different seeds for each test instance and algorithm. Due to the large amount of computing time necessary to perform these experiments, we always partition 16 instances in parallel on a single node. We use the *arithmetic mean* when averaging over solutions of the same instance and the *geometric mean* when averaging over different instances in order to give every instance a comparable influence on the final result. In order to compare EvoHGP with the different algorithms, we present two kinds of plots: *Convergence plots* [50] show the evolution of solution quality over time normalized by instance size, while *performance plots* [52] are used to compare the best solutions of all algorithms on a per-instance basis.

Convergence Plots. We start by explaining how to compute the data for a single instance I, i.e., a k-way partition of a hypergraph H. Whenever an algorithm computes a partition that improves the solution quality, it reports a pair $(t, (\lambda - 1))$, where the timestamp t is the currently elapsed time. For r repetitions with different seeds s, these r sequences T_s^I of pairs are merged into one sequence T^I of triples $(t, s, (\lambda - 1))$, which is sorted by the timestamp t. Since we are interested in the *evolution* of the solution quality, we compute the sequence T_{\min}^{I} representing event-based average values. We start by computing the average connectivity \overline{c} and the average time \bar{t} using the first pair $(t, (\lambda - 1))$ of all r sequences T_s^I and insert (\bar{t}, \bar{c}) into T_{\min}^I . We then sweep through the remaining entries $(t, s, (\lambda - 1))$ of T^I . Each entry corresponds to a partition computed at timestamp t using seed s that improved the solution quality to $(\lambda - 1)$. For each entry we therefore replace the old connectivity value of seed s that took part in the computation of \bar{c} with the new value $(\lambda - 1)$, recompute \overline{c} and insert a new pair (t,\overline{c}) into T_{\min}^I . T_{\min}^I therefore represents the evolution of the average solution quality \overline{c} for instance I over time. In a final step, we create the *normalized* sequence N_{\min}^I , where each entry (t, \overline{c}) in T_{\min}^I is replaced by (t_n, \overline{c}) where $t_n := t/t_I$ and t_I is the average time that KaHyPar-CA needs to compute a k-way partition of H. Average values over multiple instances are then obtained as follows: All sequences N_{\min}^I of pairs (t_n, \overline{c}) are merged into a sequence N_{\min} of triples (t_n, \overline{c}, I) , which is then sorted by t_n . The final sequence $S_{\mathcal{G}}$ presenting event-based geometric averages values is then computed as follows: We start by computing the average normalized time $\overline{t_n}$ and the geometric mean connectivity \mathcal{G} over all instances I using the first value of all N_{\min}^I and insert $(\overline{t_n}, \mathcal{G})$ into $S_{\mathcal{G}}$. We then sweep through the remaining entries of N_{\min} . For each entry (t_n, \bar{c}, I) , we replace the old connectivity value of I that took part in the computation of \mathcal{G} with the new value \overline{c} , recompute \mathcal{G} and insert (t_n, \mathcal{G}) into $S_{\mathcal{G}}$. The sequence $S_{\mathcal{G}}$ therefore represents the evolution of the solution quality averaged over all instances and repetitions.

Performance Plots. These plots relate the smallest minimum connectivity of all algorithms to the corresponding connectivity produced by each algorithm on a per-instance basis. For each algorithm, these ratios are sorted in increasing order. The plots use a cube root scale for both axes to reduce right skewness [20] and show 1 - (best/algorithm) on the y-axis to highlight the instances were each partitioner performs badly. A point close to one indicates that the partition produced by the corresponding algorithm was considerably worse than the partition produced by the best algorithm. A value of zero therefore indicates that the corresponding algorithm produced the best solution. Points above one correspond to infeasible solutions that violated the balance constraint. Thus an algorithm is considered to outperform another algorithm if its corresponding ratio values are below those of the other algorithm. In order to include instances with a cut of zero into the results, we set the corresponding cut values to *one* for ratio computations.

Benchmark Instances. We evaluate our algorithm on a representative subset of 100 hypergraphs from the benchmark set of Heuer and Schlag [30]², which contains instances from four benchmark sets: the ISPD98 VLSI Circuit Benchmark Suite [3], the DAC 2012 Routability-Driven Placement Contest [58], the University of Florida Sparse Matrix Collection [21], and the international SAT Competition 2014 [12]. Sparse matrices are translated into hypergraphs using the row-net model [17], i.e., each row is treated as a net and each column as a vertex. SAT instances are converted to three different representations: For literal hypergraphs, each boolean *literal* is mapped to one vertex and each clause constitutes a net [44], while in the primal model each variable is represented by a vertex and each clause is represented by a net. In the dual model the opposite is the case [41]. The latter two models are more common in the SAT solving community than the literal model proposed in [44]. All hypergraphs have unit vertex and net weights. An overview of our benchmark sets is given in Tables 2 and 3 in Appendix A. To compare EvoHGP with the best competing algorithms, all 100 hypergraphs are partitioned into $k \in \{2, 4, 8, 16, 32, 64, 128\}$ blocks with $\varepsilon = 0.03$. For each hypergraph H and each value of k, a k-way partition H is considered to be *one* test instance, resulting in a total of 700 instances. In order to save running time, we choose a subset of 25 hypergraphs shown in Table 2, k=32, and $\varepsilon=0.03$ to evaluate the impact of different algorithmic components of our algorithm (recombine and mutation operations) before we run the algorithms on the large benchmark set.

4.1 Influence of Algorithmic Components

All configurations determine their population size \mathcal{P} dynamically such that $\delta=15\%$ of the total time is spent to create the initial population. According to the results of Wichlund and Aas [60],

the damping factor γ used for edge frequency calculations is set to $\gamma = 0.5$. We use a naming scheme to refer to different configurations of our algorithm. All configuration names start with EvoHGP followed by abbreviations for the added recombine and mutation operations (multiple abbreviations are used to add multiple operations). Abbreviation +C refers to EvoHGP using twopoint recombine operations, +ER refers to Evo-HGP using multi-recombine operations, and finally $+M_x$ adds mutation operations with a mutation chance of x. Whenever a mutation operation is performed, both operators have a 50 percent change of being chosen. Figure 2 compares different configurations of EvoHGP. Of all configurations, EvoHGP+ER, which relies only on multi-

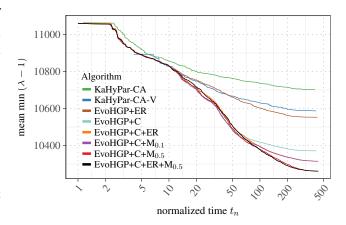


Fig. 2: Influence of EvoHGPs algorithmic components.

point recombine operations, performs worst, being only slighthly better than KaHyPar-CA-V. Comparing its performance with EvoHGP+C (which uses only two-point recombine operations), we can see that it is indeed beneficial to guarantee nondecreasing solution quality for combine operations. However combining both recombination operators results in a performance similar to EvoHGP+C+ $M_{0.1}$. This can be explained by the fact that multi-recombines also act as mutation operator in that they don't guarantee nondecreasing performance. Due to the fact that the strong multilevel local search engine KaHyPar-CA computes high quality solutions, we see that a significant amount of mutations is necessary to ensure diversity in the population. While EvoHGP+C+ $M_{0.1}$ (10% mutation chance performed best for evolutionary

² The benchmark set was downloaded from http://algo2.iti.kit.edu/schlag/sea2017/.

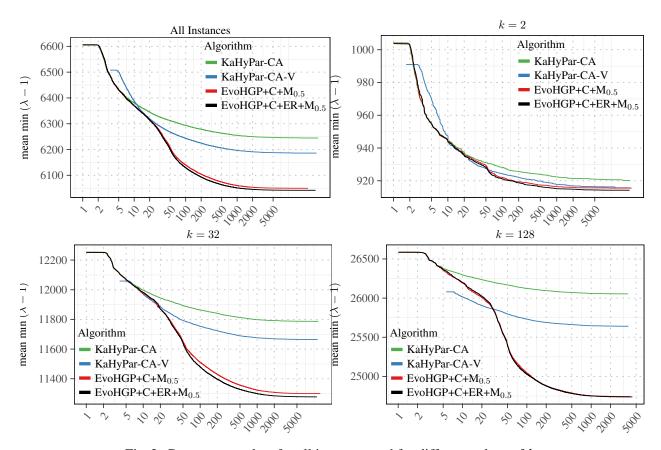


Fig. 3: Convergence plots for all instances and for different values of k.

graph partitioning in [50]) performs equally well as EvoHGP+C+ER, increasing the mutation rate to 50% (EvoHGP+C+M_{0.5}) improves the overall performance of the algorithm. Moreover, we see that using both recombination operators and mutations (EvoHGP+C+ER+M_{0.5}) also performs well. Since EvoHGP+C+M_{0.5} and EvoHGP+C+ER+M_{0.5} show the best convergence behavior, we restrict ourselves to these configurations for the remaining experiments performed in the paper.

4.2 Evaluation

We now switch to our large benchmark set to evaluate the performance of the different algorithms under consideration. Table 1 and Figure 3 compare the performance of our memetic algorithms with repeated executions of KaHyPar-CA and KaHyPar-CA-V. When looking at convergence plots, note that KaHyPar-CA-V starts later than all other algorithms and has an initially better solution quality. This is due to the fact it uses up to 100 V-cycles before reporting the first solution. The improvements of our memetic algorithms increase with increasing k. This is expected as the search space of possible partitionings increases with the number of blocks. Looking at Table 1, we see that both memetic algorithms on average outperform the best partitioner currently available (KaHyPar-CA), culminating in an improvement of 5.4% for k = 128. Furthermore both EvoHGP+C+M_{0.5} and EvoHGP+C+ER+M_{0.5} are able to improve upon the *new* V-cycling version KaHyPar-CA-V for all values of k and performs 3% better on average than KaHyPar-CA-V for $k \geq 32$. While the difference in solution quality between both memetic algorithms is small on average, a Wilcoxon matched pairs signed rank test [61] (using a 1% significance level) reveals that the improved solution quality of EvoHGP+C+ER+M_{0.5} is statistically significant (Z = -2.992857, p = 0.002763795). This is also confirmed by the performance plot shown in Figure 4 (left).

Table 1: Average improvement in connectivity metric over both KaHyPar-CA and the new version KaHyPar-CA-V.

k			KaHyPar-CA-V vs. EvoHGI +C+M _{0.5} +C+ER+M _{0.5}			
all	3.3%	3.4%	2.3%	2.4%		
2	0.9%	0.9%	0.3%	0.4%		
4	1.3%	1.4%	0.8%	1.0%		
8	2.7%	2.9%	1.9%	2.0%		
16	3.5%	3.6%	2.5%	2.6%		
32	4.3%	4.6%	3.2%	3.5%		
64	4.9%	5.0%	3.5%	3.6%		
128	5.4%	5.4%	3.7%	3.7%		

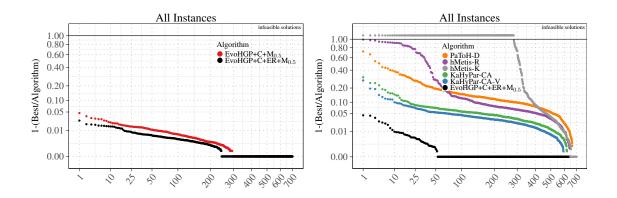


Fig. 4: Performance plots comparing the two final configurations of our evolutionary algorithm (left) and the best configuration of EvoHGP to the strongest non-evolutionary algorithms (right). EvoHGP+C+ER+ $M_{0.5}$ outperforms the non-evolutionary algorithms on almost all instances.

Looking at the performance plot that compares the strongest non-evolutionary algorithms with the strongest memetic configuration in Figure 4 (right), we see that EvoHGP+C+ER+M_{0.5} performs significantly better than *all* other algorithms. It produces the best partitions for 648 of the 700 instances. It is followed by KaHyPar-CA-V (107), KaHyPar-CA (79), hMetis-R (55), hMetis-K (46), and PaToH-D (35). Note that for some instances, multiple partitioners computed the same solution. Comparing the best solutions of EvoHGP+C+ER+M_{0.5} to each partitioner individually, it produced better partitions than PaToH-D, hMetis-K, hMetis-R, KaHyPar-CA, KaHyPar-CA-V in 661, 644, 636, 609, 585 cases, respectively.

This shows that even a large number of repeated executions helps only partially to explore the huge space of possible partitionings. By combining effective exploration (global search) with exploitation (in our case using powerful *n*-level HGP algorithms) our memetic algorithm can effectively help to break out of local minima and hence explore the global solution space more extensively.

5 Conclusion and Future Work

EvoHGP is the first *multilevel* memetic algorithm to tackle the balanced hypergraph partitioning problem. Key components of our contribution are new effective multilevel recombine and mutation operations that

incorporate information about the best solutions in the coarsening process and provide a large amount of diversity. Experiments comparing EvoHGP with a V-cycling version of KaHyPar-CA and the well known HGP tools hMetis and PaToH indicate that our evolutionary algorithm computes by far the best solutions on almost all instances. This confirms our conjecture that previous attempts to solve the HGP problem using memetic algorithms failed to be competitive with state-of-the-art tools because (i) only flat partitioning algorithms were used to drive the exploitation phase and (ii) evolutionary operators were problem agnostic and thus did not incorporate enough structural information into the algorithm. We therefore believe that EvoHGP is helpful in a wide area of application areas in which solution quality is of major importance. In the future, it would be interesting to apply EvoHGP in such application areas and to try other domain specific recombine operators that offer more specific knowledge of the application domain. In addition, it may be worth to investigate shared-memory parallelization as in [8] or a distributed memory parallelization based on islands as in [50]. Lastly, we plan to integrate our algorithm in the KaHyPar framework.

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A Benchmark Hypergraphs

Table 2: Basic properties of our small benchmark subset.

Hypergraph	n	m	p	Hypergraph	n	m	\overline{p}
ISPD98				SAT14Primal			
ibm06	32498	34826	128182	6s153	85646	245440	572692
ibm07	45926	48117	175639	aaai10-planning-ipc5	53919	308235	690466
ibm08	51309	50513	204890	$atco_enc2_opt1_05_21$	56533	526872	2097393
ibm09	53395	60902	222088	dated-10-11-u	141860	629461	1429872
ibm10	69429	75196	297567	hwmcc10-timeframe	163622	488120	1138944
SAT14Dual				SPM			
6s133	140968	48215	328924	laminar_duct3D	67173	67173	3833077
6s153	245440	85646	572692	mixtank_new	29957	29957	1995041
6s9	100384	34317	234228	mult_dcop_01	25187	25187	193276
dated-10-11-u	629461	141860	1429872	RFdevice	74104	74104	365580
dated-10-17-u	1070757	229544	2471122	vibrobox	12328	12328	342828
SAT14Literal							
6s133	96430	140968	328924				
6s153	171292	245440	572692				
aaai10-planning-ipc5	107838	308235	690466				
atco_enc2_opt1_05_21	112732	526872	2097393				
dated-10-11-u	283720	629461	1429872				

Table 3: Basic properties of our large benchmark subset.

Table 5. Basic properties of our rarge benchmark subset.								
Hypergraph	<u>n</u>	m	p	Hypergraph	<u>n</u>	<u>m</u>	<i>p</i>	
	DAC2012	511.605	1712706		AT14Prim		77.104	
superblue19	522 482			AProVE07-27	7 729	29 194	77 124	
superblue16	698 339		2 048 903	countbitssrl032	18 607	55 724	130 020	
superblue14	630 802				33 365	97 516 100 384	227 536	
superblue3	917 944 ISPD98	898 001	3 109 446	6s133	34 317		234 228	
ibm09	53 395	60 902	222 088		48 215	140 968	328 924	
ibm11	70 558	81 454		atco_enc1_opt2_10_16	85 646 9 643	245 440 152 744	572 692 641 139	
ibm10	69 429			aaai10-planning-ipc5	53 919			
ibm12	71 076	75 196 77 240		hwmcc10-timeframe	163 622	308 235 488 120	690 466 1 138 944	
ibm13	84 199	99 666		itox_vc1130	152 256	441 729	1 138 944	
ibm14	147 605	152772		dated-10-11-u	141 860	629 461	1 429 872	
ibm15	161 570	186 608		atco_enc1_opt2_05_4	14 636	386 163	1652800	
ibm16	183 484	190 048		manol-pipe-g10bid_i	266 405	792 175	1 848 407	
ibm18	210613	201 920		manol-pipe-c8nidw	269 048	792 173	1 866 355	
ibm17	185 495	189 581		atco_enc2_opt1_05_21	56 533	526 872	2 097 393	
	AT14Dual		800 030	dated-10-17-u		1 070 757	2471 122	
AProVE07-27	29 194	7 7 2 9	77 124	ACG-20-5p0		1390931	3 269 132	
countbitssrl032	55 724	18 607		ACG-20-5p0 ACG-20-5p1		1416850	3 3 3 3 5 3 1	
6s184	97 516	33 365	227 536	ACG-20-3p1	SPM	1410030		
6s9	100 384	34317		powersim	15 838	15 838	67 562	
6s133	140 968	48 215		as-caida	31 379	26 475	106 762	
6s153	245 440	85 646	572 692		24 842	24 842	159 981	
atco_enc1_opt2_10_16	152 744	9643		Ill_Stokes	20 896	20 896	191 368	
aaai10-planning-ipc5	308 235	53 9 1 9		mult_dcop_01	25 187	25 187	193 276	
hwmcc10-timeframe	488 120		1 138 944		108 175	33 798	232 647	
itox_vc1130	441 729		1 143 974		14 270	14 270	307 858	
dated-10-11-u	629 461		1 429 872		43 618	43 618	310 016	
atco_enc1_opt2_05_4	386 163			ckt11752_dc_1	49 702	49 702	333 029	
manol-pipe-g10bid_i	792 175		1 848 407		74 104	74 104	365 580	
manol-pipe-c8nidw	799 867			light_in_tissue	29 282	29 282	406 084	
atco_enc2_opt1_05_21	526 872			Pres_Poisson	14 822	14 822	715 804	
dated-10-17-u	1 070 757		2 471 122		60 000	60 000	760 154	
ACG-20-5p0	1390931			2D_54019_highK	54 019	54 019	996414	
ACG-20-5p1	1416850		3 333 531		40 216	40 216	1 042 160	
	AT14Litera			denormal	89 400	89 400	1 156 224	
AProVE07-27	15 458	29 194	77 124	2cubes_sphere	101 492	101 492	1647264	
countbitssrl032	37 213	55 724		av41092	41 092	41 092	1683 902	
6s184	66 730	97 516	227 536		256 000	256 000	1766400	
6s9	68 634	100 384	234 228		70 656	70 656	1828364	
6s133	96 430	140 968	328 924		15 449	15 449	1 930 655	
6s153	171 292	245 440		mixtank_new	29 957	29 957	1995 041	
atco_enc1_opt2_10_16	18 930	152 744		sme3Db	29 067	29 067	2 081 063	
aaai10-planning-ipc5	107 838	308 235		mc2depi	525 825	525 825	2 100 225	
hwmcc10-timeframe	327 243			poisson3Db	85 623	85 623	2 374 949	
itox_vc1130	294 326			rgg_n_2_18_s0	262 144	262 141	3 094 566	
dated-10-11-u	283 720		1 429 872		325 557	247 501	3 216 152	
atco_enc1_opt2_05_4	28 738		1652800		214 765	214 765	3 358 036	
manol-pipe-g10bid_i	532 810			laminar_duct3D	67 173	67 173	3 833 077	
manol-pipe-c8nidw	538 096		1 866 355		153 746	153 746	9 080 404	
atco_enc2_opt1_05_21	112 732			BenElechi1	245 874		13 150 496	
dated-10-17-u			2 471 122		504 855		17 588 875	
ACG-20-5p0		1390931		u1_5110111	201022	201022	11200013	
ACG-20-5p0 ACG-20-5p1		1416850						
	002372	1 110 050	000001					