Network Resilience of Star Graphs: A Comparative Analysis

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Abstract

Star graphs have been proposed as network topologies for the interconnection of multicomputer systems. They have been shown to compare very favorably with binary n-cubes networks (hypercubes) in terms of degree, diameter, fault-tolerance and applicability in VLSI design.

In this paper we study some fault-tolerance properties of star graphs with a specific focus on their network resilience. Network resilience is a measure of the robustness of a network that is based on the probability of occurrence of a network partition as a result of multiple node failures. We derive an approximate analytical expression for the probability of a network disconnection in star graphs that is verified using a Monte-Carlo simulation.

The results are compared and contrasted with those obtained for hypercubes and other networks[4]. These results show that, unlike other networks, the resilience of star graphs stays constant as the network size is increased.

1 Introduction

The recent development in VLSI technology have spurred increased research in the various aspects of large scale or massively parallel multicomputer systems. A multicomputer system consists of a large number of identical independent processing elements which communicate among each over an interconnection network. The application programs are modeled as collection of concurrently executable and communicating tasks. As the number of processing elements or nodes increases, so does the failure rate, and consequently the system reliability as well as availability become important issues in the design of such systems.

Excellent analyses of different issues involved in designing reliable parallel systems can be found in [7, 8]. In traditional reliable or fault-tolerant architectures the objective of failure-free operation is achieved mainly by hardware replication or redundancy. But in case of a large scale parallel computing system, this redundancy is provided inherently in the design of the interconnection topology and the system is allowed to degrade gracefully under conditions of failure down to the lowest acceptable performance level. Hence the design of the interconnection network becomes the most important issue in the design of large-scale systems.

The underlying topology of an interconnection network is modeled as a symmetric graph where the nodes represent the processing elements and the edges (arcs) represent the bi-directional communication channels. Design features for an efficient interconnection topology include properties like low degree, regularity, small diameter, high connectivity, efficient routing algorithm, high fault tolerance, low fault diameter etc. Since more and more processors must work concurrently in a large-scale system, the criteria of high fault tolerance and strong resilience [1, 2, 3] have become increasingly important. A computer system is said to be k-fault tolerant if it can allow up to k failures with continuing operations; k is called the fault tolerance of the system. Network fault tolerance has been defined as the maximum number of elements that can fail without inducing a possible disconnection in the network [8]. For example, in a regular graph with degree m, the network fault tolerance is $m - 1$.

Whenever a node fails, the fault tolerant routing algorithm bypasses the failed node. But when successive failures lead to a state of network disconnection whereby one or more healthy nodes are cut out from the rest of the system, distributed recovery is not possible because the state of the computation in the isolated nodes is unreachable. This situation is a failure state since distributed fault detection, recovery and restart procedures depend on graph connectedness. If we define coverage factor as the probability of a successful recovery from failure, then this coverage factor will depend on the disconnection probability of the graph. In this paper we attempt to analyze this dependence for star graphs.

In section 2 we present the necessary background information about star graphs and introduce certain graph theoretic definitions. Section 3 deals with the theoretical and experimental analyses of disconnection probability for star graphs while section 4 presents the analysis of resilience of star graphs and effects of disconnection probability on star graph resilience. Section 5 concludes the paper.

2 Star Graphs and Hypercubes

2.1 Background

In this section we briefly discuss the background information about star graphs and compare their network properties to those of binary n-cubes. Graph theoretic terms not defined here can be found in [9]. An extensive discussion of the properties of star graphs can be found in [1, 2].

A star graph S_n, of order n, is defined to be a symmetric graph $G = (V, E)$ where V is the set of $n!$ vertices each representing a distinct permutation of n elements and E is the set of symmetric edges such that two permutations (nodes) are connected by an edge iff one can be reached from the other by interchanging its first symbol with any other symbol. For example in S_3, the node representing permutation ABC will have edges to two other permutations (nodes) BAC and CBA. Figure 1 shows S_3 and S_4. The diameter of S_n is given by $\lfloor 3(n-1)/2 \rfloor$ and an efficient routing algorithm exists for such star graphs to compute the minimal path [1, 2]. Let $A(G : v)$ or simply $A(v)$ (when G is understood from the context) denote the set of vertices adjacent to vertex v in graph G. For any subset $X \subset V$, $A(X)$ is defined as $\cup_{v \in X} A(v) - X$. Then $|A(S_n : v)| = n - 1 = d(v)$, $\forall v \in S_n$, i.e., S_n is a $(n-1)$-regular graph, where $d(v)$ denotes the degree of vertex v. The vertex connectivity of a graph G is defined to be the least $|X|$ for a subset $X \subset V$ such that $G - X$ is disconnected. It has been shown in [1] that the vertex connectivity of star graph S_n is $n-1$, i.e., S_n is optimally or $(n-2)$-fault tolerant in the sense that whenever an arbitrary set of $(n-2)$ or fewer vertices are removed the remaining graph is still connected.

Table 1 summarizes the network properties of binary n-cubes and star graphs.

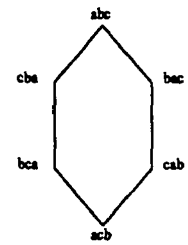

Figure 1(a) : The 3-Star

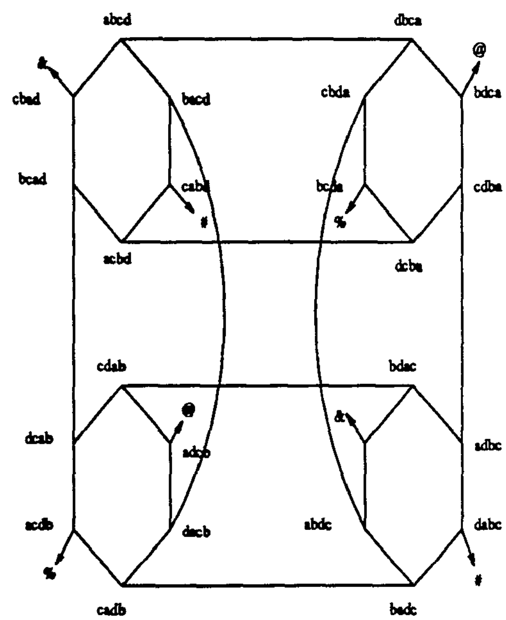

Figure 1(b) : The 4-Star

Figure 1: Star Graphs of order 3 and 4

Property	*Star*	*Binary Cube*
degree	n	n
number of nodes	$(n+1)!$	2^n
diameter	$\lfloor 3n/2 \rfloor$	n

Table 1: Properties of binary n-cubes and star graphs with the same degree

2.2 Some Network Properties

Let $S_n = G(V, E)$ represent a star graph of order n as defined earlier and $C_n = G(V, E)$ a binary n-cube of dimension n. An m-cluster is any *connected* subset

X_m of M nodes in G. R_m is the number of neighbor nodes to an m-cluster X_m, i.e., $R_m = |A(X_m)|$. Also let $N(m)$ be the number of m-clusters in G.

Lemma 1 *For an arbitrary edge (u, v) in S_n, we have $\{A(u) - v\} \cap \{A(v) - u\} = \emptyset$.*

Proof : Let the first symbol in u and v be X and Y respectively. For all vertices in $A(v) - u$, the symbol Y is in the same position as in v, say j. Now Y is in the first position of u and vertices in $A(u)$ are generated by interchanging Y with any other symbol in u. To bring Y to the j-th position will lead to vertex v. Thus the vertices in $A(u) - v$ cannot have Y in the j-th position. Hence the result. □

Corollary 1 *For any arbitrary $u, v \in S_n$, $|A(v) \cap A(u)| \leq 1$.*

Lemma 1, and its Corollary, essentially state that there are no cycles of size 3 in S_n. These results can also be derived for C_n.

Lemma 2 *Consider two edges (u, v) and (v, w) in S_n. Then $\{A(u) - v\} \cap \{A(w) - v\} = \emptyset$.*

Proof : Similar to that of lemma 1. □

Corollary 2 *Any edge $e = (u, v)$ in S_n is a 2-cluster and hence $|\{A(u) - v\} \cup \{A(v) - u\}| = 2n - 4$.*

Lemma 2, and its Corollary, state that there are no cycles of size 4 in S_n. These results, however, do not hold for C_n; C_2 being a trivial case of a cycle of size 4.

3 Disconnection Probability of Star Graphs

In this section we present an analytical evaluation of the disconnection probability of a star graph that is verified using a Monte-Carlo simulation approach. The model is a homogeneous, nonreconfigurable, large-scale system based on star graphs. This model is similar to the one used in [4, 5] to analyze hypercubes and cube-connected cycles.

3.1 Theoretical Analysis

Definition 1 *A system is in a disconnected state if and only if there exists a cluster of size m that is disconnected from the system and $m \geq 1$.*

Definition 2 *$P(i)$ = Probability that the system is disconnected exactly after i-th failure.*

Definition 3 *$Q(i)$ = Probability that a disconnected graph results with $N - i$ nodes at the i-th node removal provided that no disconnection occurred until the i-th node removal.*

Definition 4 *$Q_m(i)$ = Probability that a disconnected m-cluster results in a graph with $N - i$ nodes by removing a single node from a connected graph with $N - i + 1$ nodes.*

It readily follows from these two definitions [4] that

$$P(i) = Q(i) \prod_{j=1}^{i-1} (1 - Q(j)) \tag{1}$$

and

$$Q(i) = \sum_{m \geq 1} Q_m(i) \tag{2}$$

It is now evident that to compute the disconnection probability of a star graph for a given number of node removal, we have to enumerate all possible m-clusters of the graph. This is combinatorially an almost intractable problem. We try to develop an insight into the problem first by computing the number of m-clusters for smaller values of m.

Lemma 3 *For a star graph S_n,*

$$P(i) = \begin{cases} 0, & i < n - 1 \\ Q_1(i), & i = n - 1 \end{cases} \tag{3}$$

Proof : S_n is a regular graph of degree $n - 1$ and hence there cannot be any disconnection as long as the number of faults is less than $n - 1$. And also when $i = n - 1$ disconnection of only a single node is possible (i.e., a 1-cluster) since the graph is $n - 1$ regular.; this can happen when all these failed nodes are the only adjacent nodes of the disconnected node. □

Lemma 4 *For a given S_n, $Q_1(n - 1) = N / \binom{N}{n-1}$*

Proof : In S_n, there are $N = n!$ nodes which can be disconnected and there are $\binom{N}{n-1}$ ways to select a subset of $n - 1$ nodes. □

Lemma 5 *For a given S_n,*

$$Q_2(i) = \begin{cases} 0, & i < 2n - 4 \\ \frac{n!(n-1)/2}{\binom{n!}{2n-4}}, & i = 2n - 4 \end{cases} \tag{4}$$

Proof : In order to disconnect a 2-cluster i.e., an edge, all of the neighbors must fail and by lemma 2 and its corollary, any edge in S_n has $2n - 4$ neighbors. We can choose an edge in $n!(n - 1)/2$ ways in S_n since there are only that many edges and $\binom{n!}{2n-4}$ represents the number of ways one can choose a subset of (2n-4) vertices out of $n!$ ones. □

Lemma 6 *Consider two connecting edges (u,v) and (u,w) in S_n. Then $|A(X)| = 3n-7$ where $X = \{u,v,w\}$.*

Proof : Lemmas 1 and 2 indicate that $A(u)-\{v,w\}$, $A(v)-u$, and $A(w)-u$ are pairwise mutually disjoint. Since each vertex has degree $n-1$, the result readily follows. □

Lemma 7 *For a given S_n,*

$$Q_3(i) = \begin{cases} 0, & i < 3n-7 \\ \frac{\binom{n-1}{2}n!}{\binom{n!}{3n-7}}, & i = 3n-7 \end{cases} \tag{5}$$

Proof : To disconnect a 3-cluster (i.e., two adjoining edges), all the neighbors of this 3-cluster must be failed. By lemma 6 the number of such neighbors is $3n-7$. For a given node we can choose two incident edges in $\binom{n-1}{2}$ ways and there are $n!$ nodes; hence $n!\binom{n-1}{2}$ gives the number of possible 3-clusters. We can choose a subset of $(3n-7)$ vertices out of $n!$ in $\binom{n!}{3n-7}$ ways. □

Computing Q_m for $m > 3$ becomes increasingly involved. Instead, for simplicity we consider only those cases where m is a factorial of some integer and the cluster is an order-k star graph, $m = k!$. This will indeed provide an indication of the variation of $Q_m(i)$ as a function of m. This approach can be justified by the fact that of all the possible configurations of an m node cluster, a star graph S_k, $k! = m$, has the lowest number of neighbors and hence the highest probability of disconnection.

Lemma 8 *The number R_m of neighbors of a m-cluster, $m = k!$, in a star graph S_n, is given by*

$$R_{m=k!} = (n-k)k$$

Proof : There are $k!$ nodes in a subgraph S_k. Each of these nodes has $n-1$ neighbors, $k-1$ of which belong to the subgraph itself and $n-k$ are "external neighbors". □

Lemma 9 *The number of distinct substar graphs S_k of order k in a given S_n, when $k \leq n-2$, is given by*

$$N(k!) = \binom{n}{k}\frac{(n-1)!}{(k-1)!} \tag{6}$$

Proof : Each node is represented by a permutation of n symbols. Also the nodes in a subgraph S_k must have $(n-k)$ symbols in the same positions. We can choose k symbols out of n in $\binom{n}{k}$ ways and then place the remaining $(n-k)$ symbols in different possible positions to get different subgraphs. For example the first of the $(n-k)$ symbols can be placed in k different positions in a string of k symbols (we cannot place the new symbol at the beginning since if the leading symbol is fixed in position no edge can be generated). Hence the $(n-k)$ symbols can be placed in $\frac{(n-1)!}{(k-1)!}$ ways and hence the result. □

Theorem 1 *The disconnection probability of a star subgraph of size k (consisting of $m = k!$ nodes) in a star graph S_n of $n!$ nodes is given by*

$$Q_m(i) = \begin{cases} 0, & i < R_m \\ \frac{N(m)}{\binom{n!}{R_m}}, & i = R_m \end{cases} \tag{7}$$

Proof : The disconnection can occur when the number of failures is less than the number of neighbors of the subset to be disconnected. The probability of a disconnection when the number of failures i is less than the number of neighbors R_m is zero. For larger values of i, the probability of a disconnection of a subset of size m is proportional to the number of possible subsets which can be so disconnected. The disconnection of each of these subsets can occur when a specific R_m out of a total of $n!$ nodes failed. Thus, the total probability of disconnection is the ratio of two values. □

At this point we want to note that it was conjectured in [4] that in any regular graph $Q_1(i) \gg Q_m(i)$, for $m > 1$ provided that for any m-cluster the graph satisfies the relation $1 < m < N/2 \Rightarrow R_m > n$ where N is number of nodes in the graph and n is the degree of each node. Authors in [4] also gave an intuitive justification for their conjecture. While it seems extremely hard to prove the conjecture rigorously, our following examples and subsequent experimental results on star graphs lend further strong credibility to this conjecture.

Example : Consider a star graph S_6 with $n = 6$ and the number of nodes $N = 6! = 720$.

$$\begin{aligned} Q_1(5) &= 720/\tbinom{720}{5} &&= 45.27 \times 10^{-11} \\ Q_2(8) &= \frac{720\times 5}{2\times\binom{720}{8}} &&= 10.449 \times 10^{-16} \\ Q_3(11) &= 720 \times 10/\tbinom{720}{11} &&= 5 \times 10^{-20} \end{aligned} \tag{8}$$

This example shows that when the connection of a 2-node cluster is possible at $i = 2n-4$, the probability of a prior single node disconnection event is about half a million times larger and similarly when a disconnection of a 3-node cluster is possible at $i = 3n-7$, the probability of a prior 2-node disconnection is twenty thousand times larger. This example, although it does not prove the said conjecture of [4], is a further demonstration of the rationale behind the conjecture as was shown in [4] with examples from hypercubes, cube connected cycles etc.

Now we propose to give an approximate analytical expression for $P(i)$ based on the above mentioned results. We cannot give an exact expression for $P(i)$ but try to give an indication of its magnitude (and later verify it experimentally) by using the approximation $Q(i) \approx Q_1(i)$ which is based on the conjecture $Q_1(i) \gg Q_m(i)$ for all $m > 1$. Hence

$$P(i) = Q_1(i) \prod_{j=1}^{i-1} (1 - Q_1(j)) \tag{9}$$

To evaluate $P(i)$ for our star graphs, we need an expression for $Q_1(i)$ for $i > n - 1$.

Theorem 2 *The conditional probability of disconnecting a single node after i failures, where $n - 1 < i < 2n - 3$, is given by*

$$Q_1(i) = \frac{(n-1)N\binom{N-n}{i-n+1}}{(N-i+1)\binom{N}{i-1}} \tag{10}$$

Proof : By definition, no disconnection occurred at the $(i - 1)$st failure and hence the probability that one occurs at the i-th failure is the probability that some node had all but one of its neighbors failed and that neighbor was the i-th failure. $\binom{N}{i-1}$ represents the possible combinations of $i - 1$ failures among $N = n!$ nodes. $\binom{n-1}{n-2} = n - 1$ is the possible choices of $n - 2$ failed neighbors among $n - 1$ neighbors while $\binom{N-n}{i-n+1}$ is the combination of the remaining $i - n + 1$ failure in the rest of the system, where N is the number of nodes that can be isolated. Lastly $1/(N - i + 1)$ is the probability that the last remaining neighbor fails. □

Equation 10 corresponds to the single node disconnection probability when more than $n - 1$ nodes have failed. For $i \geq 2n - 3$, it is possible to have two or more single node disconnection. However, the probability of multiple single node disconnection is of the same order as that of a cluster disconnection when $m > 1$. Therefore, using the approximation $Q(i) \approx Q_1(i)$, we can extend the range of i in 10 to $i > n - 1$. We obtain

$$Q_1(i > n - 1) = \frac{(n-1)(N-n)!(i-1)!(N-i)}{(i-n+1)!(N-1)!} \tag{11}$$

From equation 11, we can write that

$$\frac{Q_1(i+1)}{Q_1(i)} = \frac{i}{i-n+2} \frac{N-i-1}{N-i} \approx \frac{i}{i-n+2} \tag{12}$$

This proves that the relation $Q_1(i+1) > Q_1(i)$ holds for $i > n-1$. Thus the approximation analysis for $P(i)$ of a star graph is complete.

3.2 Monte-Carlo Simulation

The objective of our simulation experiment was to measure the values of $P(i)$ for star graphs of different sizes and to compare those with similar results for hypercubes [4]. A program has been developed which simulates the failure of nodes and checks eventual disconnection in the graph. Each iteration stage of the simulation consisted of the following :

- Randomly choose any one of the remaining $(N-i)$ vertices and remove the vertex from the graph along with all the incident edges.
- Record the number and size of the connected components of the remaining graph.
- If more than one components are found, record the iteration number and size of the component and exit, else repeat.

In each case the number of samples were higher than 2000.

3.2.1 Frequency of Disconnection

Table 1 shows the frequency of occurrence of disconnections of different sized clusters for different sized star graphs. Table 1 also includes similar results for binary hypercubes and cube-connected cycles for comparison purposes; those are taken from [1]. Let $F_{st}(K)$ denote the probability that the disconnected cluster is of size K provided disconnection occurred in the star graph. Similarly $F_{cc}(K)$ and $F_{bc}(K)$ are defined. These values are shown in table 1 for $K = 1, 2, 3, 4$ as obtained by our simulation experiment. We make the following observations :

- For all values of N (number of nodes in the star graph), $F_{st}(1)$, frequency of single node disconnection is larger than 50 percent and $F_{st}(1)$ always increases with increasing N. Similar observations can be made about binary cubes and cube connected cycles [1].
- Dominance of the single node disconnection increases with increasing node degree n.

The above results do not give any indication of the value of $P(i)$ itself, only of its composition. In the following section we give the analytical (obtained by using the expressions derived in the earlier section) as well as simulation (obtained by our Monte-Carlo simulation) results for $P(i)$ of star graphs of different sizes.

3.2.2 Results for $P(i)$

Figures 2, 3, and 4 show the analytical and simulation plots of $P(i)$ versus the percentage of failed nodes (i/N percent) for a star graph of order 4, 5 and 6 respectively. We make the following observations :

- The curves are narrower for higher order star graphs and the discrepancy between analytical and simulation results is never beyond 20 percent.
- If P_{max} represents the maximum value of $P(i)$ and i_{peak} represents the corresponding value of i, there is a very close correlation in the value of i_{peak} between the simulation results and the analytical model. Also the value of i_{peak} tends to be lesser for higher order star graphs and i_{peak} for star graphs is always less than 50 percent. It may be recalled from [1] that i_{peak} for binary cubes was near 50 percent.
- The variation in the value of P_{max} is due to the approximation in the analytical model as well as to the statistical error in simulation experiment.
- P_{max} steadily falls with increasing size of the star graph.
- We can conclude that N is the dominant factor in determining the value of P_{max}.

4 Network Resilience

We have seen that a network disconnection can impede the recovery mechanism in a gracefully degradable system. Hence the probability of no disconnection is a multiplicative coefficient of the coverage factor, the probability of successful recovery. In other words, the coverage factor at i-th failure is $(1 - P(i))$ times the coverage factor in a fully connected network graph. The range of values of P_{max} as obtained from the figures is very high compared to acceptable values of coverage factor. In order to allow enough failures without reaching high values of $P(i)$, we use the concept of network resilience which was used in [4] to study hypercubes.

Network Resilience $NR(p)$ of a distributed system is defined as the maximum number of node failures that can be sustained while the network remains connected with a probability $(1 - p)$. It is formally defined as

$$\sum_{i=1}^{N} RP(i) \leq p \tag{13}$$

$(1-p)$ is therefore the certainty factor of no disconnection after $NR(p)$ failures. The measure of *relative network resilience* $RNR(p)$ is defined as $NR(p)/N$. Table 2 shows the values of $NR(p)$ and $RNR(p)$ for the star graph, hypercubes and cube connected cycle cases and for $p = 0.01$. The values for the hypercubes and cube connected cycles are taken from [1] for comparison purposes. These values represent the maximum number of nodes that can fail with less than 1 percent chance of network disconnection. The plots of $RNR(0.01)$ are shown in Figure 5. We make the following observations :

- Relative network resilience decreases for cube connected cycles while it increases for star graphs and hypercubes.
- The network resilience in all cases increases with increase in number of nodes in the network, i.e., larger systems allow a larger number of degradation states irrespective of the topology.
- When the degree of nodes remain constant the relative network resilience decreases with increasing N.
- A sublogarithmic increase in node degree, such as in a star graph results in a slight increase of the RNR and a logarithmic increase in node degree such as in hypercube results in more increase in relative network resilience.

5 Conclusion

In this paper we have studied the robustness of star graphs using the probability of disconnection as a probabilistic measure of network fault tolerance. An approximate analytical expression for the probability of a network partition was derived. The expression was further verified using a Monte-Carlo simulation of node failures.

The measure of network resilience was used as a criterion for the comparative evaluation of network fault-tolerance in star graphs, binary n-cube, the cube-connected-cycles and the mesh topologies. The results show that star graphs maintain a constant network resilience as the size of the network is increased. The hypercube, on the other hand, has an increasing network resilience. This, however, comes with the additional cost of a much larger node degree for large networks.

Our study lends further proof to two major points: disconnection probability can be used as a very meaningful criterion to measure network resilience in real life applications and the star graphs seem to be a very attractive alternative to hypercubes in VLSI design.

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K	*Star Graph*		
	24	120	720
1	63.6	82.6	94.3
2	15.6	12.4	5.1
3	6.3	2.7	0.43
4	3.1	0.9	0.07
K	*C-C Cycles*		
	24	160	896
1	25.2	71.3	84.0
2	18.6	15.3	12.2
3	11.0	5.0	2.5
4	7.0	3.0	0.9
K	*Binary Cube*		
	16	128	1024
1	67.75	89.1	97.3
2	11.25	6.65	2.4
3	8.05	1.85	0.3
4	5.95	0.65	0.0

Table 2: Frequencies of Disconnection of K-Clusters (in %)

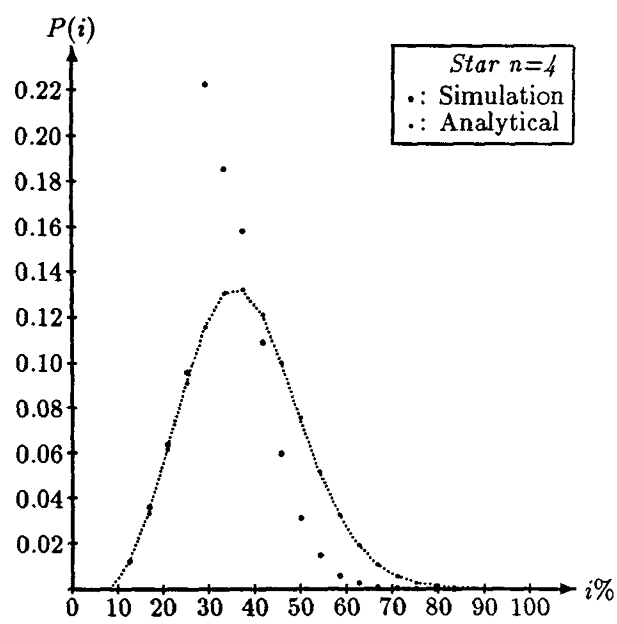

Figure 2: Probability of Disconnection for S_4, $N = 24$

Graph	N	$NR(p)$	$RNR(p)$
Star	24	2	8.33
	120	13	10.83
	720	80	11.11
	5040	577	11.45
	40320	4667	11.57
Binary Cube	32	8	25.0
	64	17	26.56
	128	36	28.12
	256	77	30.08
	512	161	31.45
	1024	337	32.91
	2048	700	34.18
	4096	1445	35.28
	8192	2971	36.27
	16384	6083	37.13
	32768	12418	37.90
Cube-Connected-Cycles	24	2	8.33
	48	3	6.25
	96	5	5.21
	192	8	4.17
	384	12	3.12
	768	19	2.47
	1536	29	1.89
	3072	46	1.5
	6144	73	1.19
	12288	116	0.94
	24576	183	0.74
	49152	290	0.59

Table 3: Resilience and Relative Resilience for $p = 0.01$

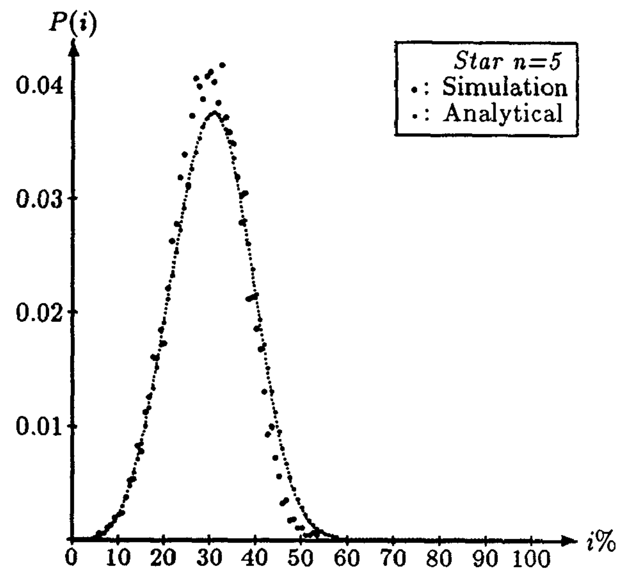

Figure 3: Probability of Disconnection for S_5, $N = 120$

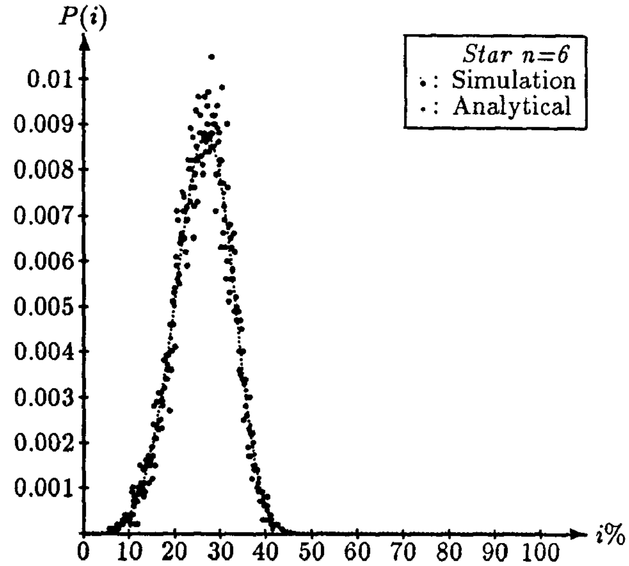

Figure 4: Probability of Disconnection for S_6, $N = 720$

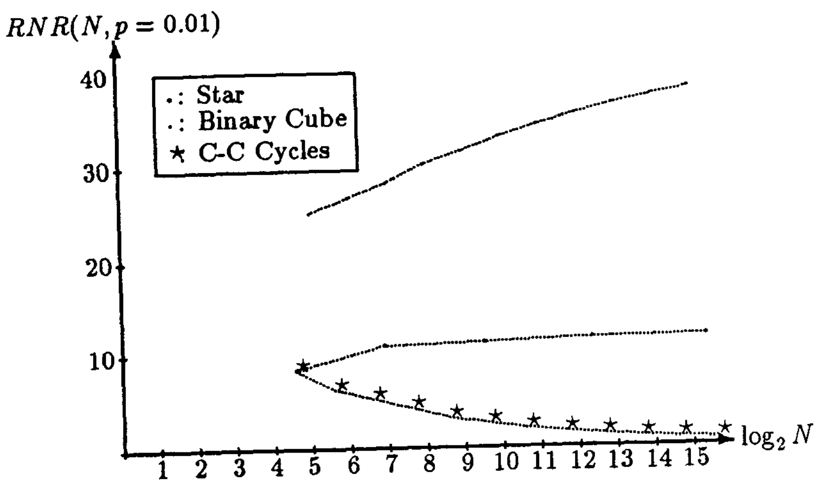

Figure 5: Relative resilience for $p = 0.01$